IMAGE PARTITION TRANSFORMS FOR FAITHFUL SEGMENTATION SEARCH

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Abstract: The explosion of image content is closely connected with segmentations efficiency. High-level region-based interpretations are associated with some a priori information, measurable region properties, heuristics, and plausibility of computational inference. Conventional similarity analysis consists of following steps: images are segmented into disjoint regions, features are extracted from each region and the set of all features is used for high-level processing. Quite often simultaneous processing of several partitions is desired in order to produce reliable true conclusion. We propose operations with segmented images and a metric for nested partitions.

Keywords: image, spatial reasoning, partitions.

ACM Classification Keywords: I.4.6 Segmentation: region growing, partitioning

Introduction

Efficiency of image structuring and understanding strongly depends on a segmentation as a process of separating an image into several disjoint (or weakly intersecting) regions whose characteristics such as intensity, color, texture, shape, etc. are similar [see e.g. 1, 2]. Segmentation is a key step in early vision and it has been widely investigated in the field of image processing. Nevertheless image content formal descriptions using only low-level features extracted from each region are not necessarily the case for true conclusions. We may get totally correct segmentation, but most often we obtain under-segmentation, over-segmentation, missed regions, and noise regions. It should be emphasized that a fair segmentation can be provided if and only if we know exactly what we are looking for in an image.

To obtain a reliable image interpretation we have to transform a row image into an image data structure, then into an image knowledge structure and finally into a user-specific knowledge structure. Spatial reasoning plays a most important part in decision making. In this respect partition transforms (e.g. set theoretic) are serviceable in order to find regions that are heavily correlated with significant objects in the scene and it is essential to have tools in order to compare segmented image accurately.

The three classes of distance function (point to point, point to set, and set to set) are usually discussed as measures of proximity or dissimilarity in image processing [3, 4]. It is desirable to define an image metric that can be efficiently embedded in segmentations methods. A partition metric is consequently a candidate because it represents images as a finite subsets assemblage that takes into account mutual dependences of equivalence class corresponding to separate regions of interest. Metrics on nested partitions take on special significance since they give possibilities to define hierarchical content descriptions. We propose based on spatial relations operations with segmented images and a metric on nested partitions useful for such applications as object tracking and pattern matching.

Operations with segmented images

Let $\mathrm{B}(x)$ be an image and $x \in \mathrm{D} = \mathbb{Z}_n^+ \times \mathbb{Z}_m^+$ (here D is a viewing field). It should be noted that any faithful segmentation (a crisp clustering) generates a partition of the viewing field, i.e. $\mathrm{X} = \{[x]_1, \ldots, [x]_\alpha, \ldots, [x]_s\}$

where $[x]_\alpha \neq \varnothing$, $X = \bigcup_{\alpha=1}^{s} [x]_\alpha$, $\forall \alpha \neq \beta \Rightarrow [x]_\alpha \cap [x]_\beta = \varnothing$ (hereafter α, β, γ denote all allowable indices). Suppose that a region labeling $F : B(x) \to \mathbb{Z}_r^+$ corresponds to obtained segmentation then arbitrary two points $x', x'' \in D$ belong to the same equivalence class $x', x'' \in [x]_\alpha$ if the binary relation $\langle B(x'), B(x'') \rangle \in \tau \Leftrightarrow$ $\Leftrightarrow F(B(x')) = F(B(x'')) = \alpha$ is fulfilled.

Let us introduce a characteristic function on equivalence classes

$$\lambda_{[x]_\alpha}(x) = \begin{cases} 0, \; x \in [x]_\alpha, \\ 1, \; x \in D \setminus [x]_\alpha. \end{cases} \tag{1}$$

It follows immediately that boundary conditions for spatial reasoning are

$$\lambda_D(x) = 0, \; \lambda_\varnothing(x) = 1.$$

In addition it is reasonable to indicate the expression providing certain duality in order to analyze image contents

$$\lambda_{D \setminus [x]_\alpha}(x) = 1 - \lambda_{[x]_\alpha}(x).$$

The direct check-up allows to introduce explicitly definable formulae of spatial interdependence between characteristic functions of two elements of arbitrary partitions X and Y

$$\lambda_{[x]_\alpha \cup [y]_\beta}(x) = \lambda_{[x]_\alpha}(x) \lambda_{[y]_\beta}(x), \tag{2}$$

$$\lambda_{[x]_\alpha \cap [y]_\beta}(x) = \lambda_{[x]_\alpha}(x) + \lambda_{[y]_\beta}(x) - \lambda_{[x]_\alpha}(x) \lambda_{[y]_\beta}(x), \tag{3}$$

$$\lambda_{[x]_\alpha \setminus [y]_\beta}(x) = 1 - \lambda_{[x]_\alpha}(x) + \lambda_{[y]_\beta}(x). \tag{4}$$

Appreciably intense interest consists in simultaneous transformations of equivalence class families since namely splitting and merging of partitions can get totally correct and complete segmentation of complex scenes. It easily seen that for any unions and intersections we get

$$\Xi = \bigcup_{\gamma \in \Gamma} [x]_\gamma \Rightarrow \lambda_\Xi(x) = \min_{\gamma \in \Gamma} \lambda_{[x]_\gamma}(x), \tag{5}$$

$$\Xi = \bigcap_{\gamma \in \Gamma} [x]_\gamma \Rightarrow \lambda_\Xi(x) = \max_{\gamma \in \Gamma} \lambda_{[x]_\gamma}(x). \tag{6}$$

The 169 types of spatial relations between two rectangles in 2-D space had been proposed in [2]. However, if we introduce a representation of each equivalence class as union of sets (rather points of boundaries and interior) it suffices to use combinations only of four relations in general case. Indeed, suppose that

$$\lambda_{[x]_\alpha}(x) = \partial \lambda_{[x]_\alpha} \cup \lambda^\circ_{[x]_\alpha}$$

where $\partial \lambda_{[x]_\alpha}$ denotes the boundary of the partition element describing by the characteristic function (1) and $\lambda^\circ_{[x]_\alpha}$ corresponds to interior points of this partition element. Let us introduce relations defining spatial relationships between any two objects $[x]_\alpha$ and $[x]_\beta$, $\alpha \neq \beta$

$$\begin{cases} \langle [x]_\alpha, [x]_\beta \rangle \in \tau_{11} \Leftrightarrow \partial \lambda_{[x]_\alpha} \cap \partial \lambda_{[x]_\beta} \neq \varnothing, \\ \langle [x]_\alpha, [x]_\beta \rangle \in \tau_{12} \Leftrightarrow \partial \lambda_{[x]_\alpha} \cap \lambda^\circ_{[x]_\beta} \neq \varnothing, \\ \langle [x]_\alpha, [x]_\beta \rangle \in \tau_{21} \Leftrightarrow \lambda^\circ_{[x]_\alpha} \cap \partial \lambda_{[x]_\beta} \neq \varnothing, \\ \langle [x]_\alpha, [x]_\beta \rangle \in \tau_{22} \Leftrightarrow \lambda^\circ_{[x]_\alpha} \cap \lambda^\circ_{[x]_\beta} \neq \varnothing. \end{cases} \tag{7}$$

Consequently, the (2×2) matrix (τ_{ij}) entirely determines all eight possible mutual locations of regions, viz:

i) $[x]_\alpha$ disjoins $[x]_\beta$, i.e. all parts of $[x]_\alpha$ are separated from all parts of $[x]_\beta$ iff $\langle [x]_\alpha, [x]_\beta \rangle \notin \tau_{ij}\ \forall i,\ j$;

ii) $[x]_\alpha$ contains $[x]_\beta$, i.e. all parts of $[x]_\beta$ are completely overlapping with any part of $[x]_\alpha$ iff τ_{21}, τ_{22}, are valid and the relations τ_{11}, τ_{12} are not true;

iii) similarly, $[x]_\alpha$ belongs $[x]_\beta$ iff $\langle [x]_\alpha, [x]_\beta \rangle \in \tau_{12}, \tau_{22}$ and $\langle [x]_\alpha, [x]_\beta \rangle \notin \tau_{11}, \tau_{21}$;

vi) $[x]_\alpha$ equals to $[x]_\beta$ iff $\langle [x]_\alpha, [x]_\beta \rangle \in \tau_{11}, \tau_{22}$ and $\langle [x]_\alpha, [x]_\beta \rangle \notin \tau_{12}, \tau_{21}$;

v) $[x]_\alpha$ is partly overlapping $[x]_\beta$ iff all relations τ_{ij} hold;

vi) $[x]_\alpha$ is externally bound to bound with $[x]_\beta$, i.e. there exist common points of boundaries and no part of $[x]_\alpha$ is overlapping with any part of $[x]_\beta$ iff $\langle [x]_\alpha, [x]_\beta \rangle \in \tau_{11}, \tau_{22}$ and $\langle [x]_\alpha, [x]_\beta \rangle \notin \tau_{12}, \tau_{21}$;

vii) $[x]_\alpha$ is internally bound to bound with $[x]_\beta$, i.e. there exist common points of boundaries and $[x]_\alpha$ belongs $[x]_\beta$ iff only the relation τ_{21} is not true;

viii) $[x]_\beta$ is internally bound to bound with $[x]_\alpha$, i.e. there exist common points of boundaries and $[x]_\alpha$ contains $[x]_\beta$ iff only the relation τ_{12} is not true.

All mentioned cases are illustrated by figure 1.

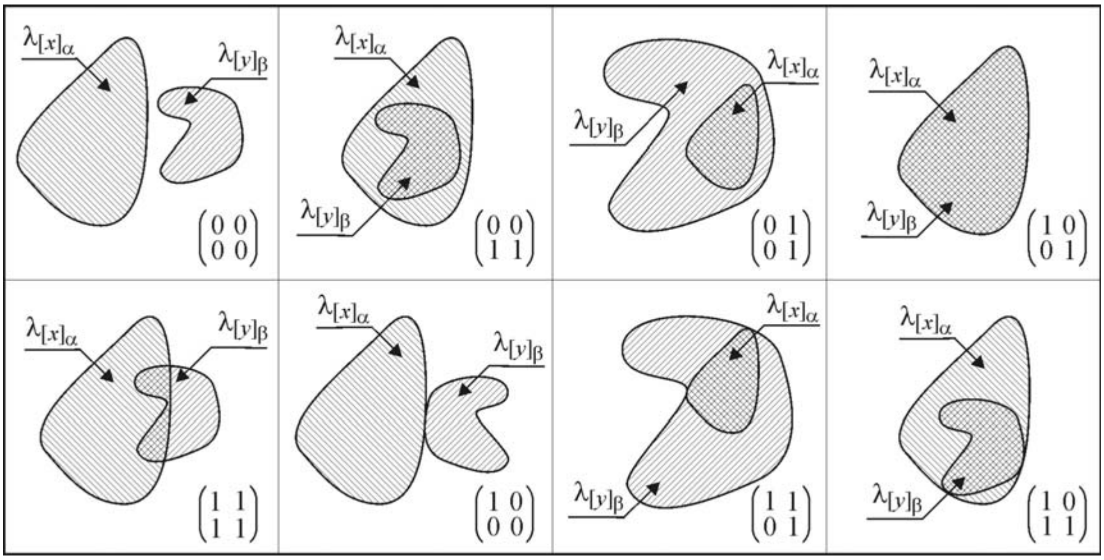

Figure 1. Possible mutual locations of equivalence classes

Now we can formalize intersection and conditional union operations with partitions. For simplicity of notations we write μ instead of a matrix (τ_{ij}) elements sum then introducing an indicator function

$$\varphi(\alpha,\beta) = \begin{cases} -1, s = 1; \\ 0, s = 0; \\ 1, s > 1. \end{cases}$$

we get for $X = \{[x]_\alpha\}$, $Y = \{[y]_\beta\}$

$$Z = X \otimes Y,\ Z = \{[z]_\gamma : \lambda_{[z]_\gamma}(x) = \lambda_{[x]_\alpha}(x) + \lambda_{[y]_\beta}(x) - \lambda_{[x]_\alpha}(x)\lambda_{[y]_\beta}(x)\} \tag{8}$$

and

$$Z = X \oplus Y\ ,\ Z = \{[z]_\gamma\}\ ,\ [z]_\gamma = \begin{cases} \{[x]_\alpha, [y]_\beta\}, \mathit{if}\ \varphi(\alpha,\beta) = 0; \\ [x]_\alpha \cup [y]_\beta, \mathit{if}\ \varphi(\alpha,\beta) = 1. \end{cases} \tag{9}$$

It is obvious evident that under $\varphi(\alpha,\beta) = -1$ a complementary analysis is required since merging of adjoining region is admissible action if features of $[z]_\gamma$ with the characteristic function $\lambda_{[z]_\gamma}(x) = \lambda_{[x]_\alpha}(x)\lambda_{[y]_\beta}(x)$ satisfy, for instance, requirements to the sought-for shape.

Thereby, expressions (2)–(4) determine operations with separate equivalence classes, relationships (5), (6) predetermine transformations of equivalence class families and (8), (9) on the base of relations (7) provide partition manipulations. The main goal of such segmented image reforming is a guaranteeing trade-off decision about regions of interest.

Results and outlook

Significant efforts are continuously being made in development of segmentation techniques. Cognitive-like approaches require obtaining of regions strongly correlated with meaningful objects in the scene. Mentioned operations create the necessary prerequisites for partitions transformations. However, efficiency of image structuring and understanding depends on the objectivity of partitions matching. Previously for finite sets we proved [5] that the functional

$$\rho(X,Y) = \sum_\alpha \sum_\beta \mathit{card}([x]_\alpha \Delta [y]_\beta)\mathit{card}([x]_\alpha \cap [y]_\beta) \tag{10}$$

(here the notation $X_i \Delta Y_j$ defines a symmetric difference) is a metric. Later for arbitrary measurable set with given measure $\mu(\circ)$, which can be interpreted as length, area, volume, mass distribution, probability distribution, and in special case cardinality, we had proved [6] that the functional

$$\rho(X,Y) = \sum_\alpha \sum_\beta \mu([x]_\alpha \Delta [y]_\beta)\mu([x]_\alpha \cap [y]_\beta) \tag{11}$$

is a metric also. Taking into consideration properties of nested partitions one can give concrete expression to metrics (10) and (11) for $X \subseteq Y$

$$\rho(X,Y) = \sum_\beta \mu([y]_\beta)^2 - \sum_\alpha \mu([x]_\alpha)^2 \tag{12}$$

Figure 2. Example of nested partitions

Substantially metric (12) intends for combination of visual features and metadata analysis to solve a semantic gap between low-level visual features and high-level human concept. Figure 2 illustrates nested partitions that are generated by algorithms based on adaptive thresholding, multithresholding and band-thresholding [7]. Simple geometrical shape parameters (the area and the perimeter of region, the diameters of circles with fixed area and perimeter, orthogonal projections of the figure on axes of abscissae and ordinates, the minimal and the maximal orthogonal projections of the figure on a line, the distance between opposite sides of the figure, the distance from an origin point in the figure to its boundary point for a given direction, the same average distance for all possible directions for a given point, the lengths of the long and short semi-axes of the ellipse with given area and perimeter, drainage-basin circularity, coefficient convexity ratio, etc.) were used for split and merging procedures along with relations (7) under operations (8) and (9).

The analysis of experimental results has shown that partition transforms and unbiased partitions matching substantially meant for the use at conceptual segmentation which not only builds partitions but can also explain why a set of regions confirms a desired pixel family.

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