SIMULTANEOUS CONTROL OF CHAOTIC SYSTEMS USING RBF NETWORKS

Angel Castellanos, Rafael Gonzalo, Ana Martinez

Abstract: *Chaos control is a concept that recently acquiring more attention among the research community, concerning the fields of engineering, physics, chemistry, biology and mathematic. This paper presents a method to simultaneous control of deterministic chaos in several nonlinear dynamical systems. A radial basis function networks (RBFNs) has been used to control chaotic trajectories in the equilibrium points. Such neural network improves results, avoiding those problems that appear in other control methods, being also efficient dealing with a relatively small random dynamical noise.*

Keywords: *Neural Network, Radial basis function, Backpropagation, Chaotic Dynamic Systems, Control Feedback Methods.*

ACM Classification Keywords: *F.1.1 Models of Computation: Self-modifying machines (neural networks); F.1.2 Modes of Computation: Alternation and nondeterminism; G.1.7 Ordinary Differential Equations: Chaotic systems;*

Conference*: The paper is selected from Sixth International Conference on Information Research and Applications – i.Tech 2008, Varna, Bulgaria, June-July 2008*

Introduction

Nowadays, with the electronics communications growth, signals processing has become a technology of multiple facets. It has passed from implementation of tuned circuits to digital processors of signals. The base of the industry continues being the design and realization of filters to carry out noise elimination con carrier signals of information. Chaos is a special feature of parametric nonlinear dynamical systems. It is usually difficult to accurately predict its future behavior. The chaotic phenomena take place everywhere, so much in natural systems as in mechanisms built by the man. Previous works have been mainly focused in describing and characterizing the chaotic behavior in situations where there is not any intervention. A family of artificial neural networks has gotten good results on the prediction and control of the nonlinear plants [Haykin, S].

Chaotic systems are characterized by their sensitive dependence to small perturbations. An abundance of theorical and experimental research has been developed to capitalize on this fact and utilize it to control chaotic systems by applying very small, appropriately timed perturbations.

The control of chaotic signals is one of the most relevant research areas that have appeared in last years, taking the attention of computer scientists [Hübler A. W.]. Recently they have being proposed ideas and techniques to transform chaotic orbits into desired periodic orbits, using temporarily programmed controls [Chen G. & Dong X]. In 1990, Ott, Gebogi Yorke [Ott E., Grebogi C. & Yorke J. A.] developed methods to control two nonlinear system stabilizing one of the no stable periodic orbits embedded in the chaotic attractor.

This paper shows how mixed chaotic signals can be controlled using a radial basis function neural networks (RBFNs) as filter, in order to separate and to control at the same time.

The radial basis function networks provides a more effective control, it can be applied to the system at any point, even being too far from the desired state, avoiding long transient times. The control can be applied if there are only a few data of the systems, and it will remain stable much more time even with small random dynamical noise.

A Radial Basis Function Networks (RBFNs) Model.

Radial basis function emerged as a variant of artificial neural network in late 80's. Radial basis function (RBF) neural networks provide a powerful alternative to multilayer perceptron (MLP) neural networks to approximate or to classify a pattern set.

RBFs differ from MLPs in that the overall input-output map is constructed from local contributions of Gaussian axons, require fewer training samples and train faster than MLP. The most widely used method to estimate centers and widths consist on using an unsupervised technique called the k-nearest neighbor rule (see figure 1). The centers of the clusters give the centers of the RBFs and the distance between the clusters provides the width of the Gaussians. Computation of the centers, used in the kernels function of the RBF neural network, is being the main focus to study in order to achieve more efficient algorithms in the learning process of the pattern set.

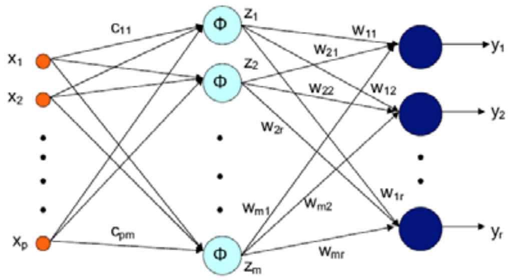

Figure 1.- Radial Basis Function Neural Network.

The choice of adequate centers implies a high performance, concerning the learning times, convergence and generalization. The activation function for RBFs network is given by $\phi_i = \phi\left(\frac{\|X(n) - C_i\|}{d_i}\right)$ $for\ i = 1,2...,m$

where $C_i = (c_{i1},...c_{ip})$ are the center of the function radial, d_i is standard deviation. The Gaussian function $\phi(r) = e^{\left(\frac{-x^2}{2}\right)}$ is the most useful in these cases [Moody, J. and Darken C].

Simultaneous control of chaotic systems.

A discrete dynamical function is going to be controlled, all the trajectories are focused towards the stable point $x_{n+1} = f(x_n)$ where $x_n \in \Re^2$, $f: \Re^2 \to \Re^2$.

Several systems are employed: The Lozi, Ikeda and Tinkerbell.

Lozi system [Chen G. & Dong X.] is described by the following equations:

$$\begin{cases} x_{k+1} = l_1(x_k, y_k, p) = -p|x_k| + y_k + 1 \\ y_{K+1} = l_2(x_k, y_k, q) = qx_k \end{cases}$$ where *p* and *q* are two real parameters

The values of the parameters that are taken to study the Lozi system are $p = 1$, $q = 0.997$. The stable points of the systems are given by :

$$Q^+ = (q_1, q_2) = \left(\frac{1}{p-(q-1)}, \frac{q}{p-(q-1)}\right)$$

$$Q^- = (q_3, q_4) = \left(\frac{-1}{p-(q-1)}, \frac{-q}{p-(q-1)}\right)$$

The attractor of Lozi system is the one that figure 2 shows.

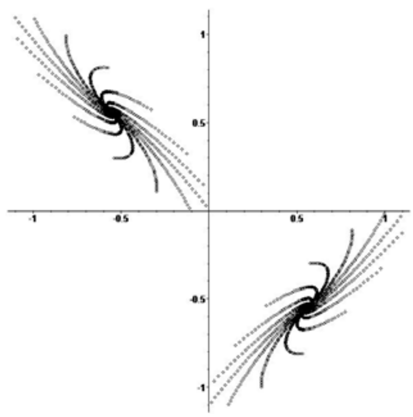

Figure 2

Ikeda system [Casdagli, M.] is described by the following equations:

$$\begin{cases} x_{k+1} = 1 + \mu(x_k \cos z - y \sin z) \\ y_{k+1} = \mu(x_k \sin z - y \cos z) \end{cases}$$, with

$$z = 0.4 - \frac{6}{1 + x^2 + y^2}, \mu = 0.7$$

The attractor of Ikeda function is the one that figure 3 shows.

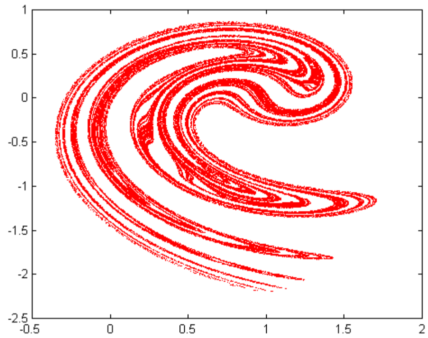

Figure 3 .

Applying the Newton methods with double precision, it can be found that the point $P = (0.60144697, 0.18998107)$ is the equilibrium point.

Thinkerbell system is described by:

$$\begin{cases} x_{k+1} = x_k^{\ 2} - y_k^{\ 2} + C_1 x_k + C_2 y_k \\ y_{k+1} = 2x_k y_k + C_3 x_k + C_4 y_k \\ C_1 = 0.9, C_2 = -0.6013 \\ C_3 = 2.0, C_4 = 0.4 \end{cases}$$

The attractor of Thinkerbel function is the one that figure 4 shows.

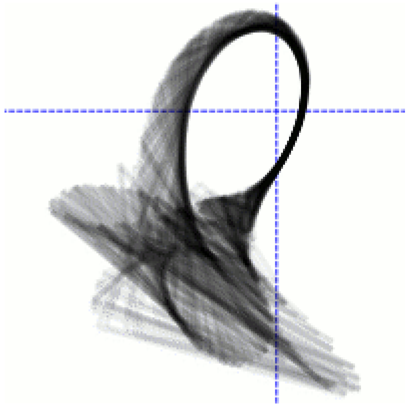

Figure 4.

The equilibrium point is $P = (0,0)$.

A radial basis function networks (RBFNs) has been used to control chaotic trajectories in the equilibrium points. The neural network employed as the main controller is a radial basis function networks.

The radial basis function network employed as the main controller consisting of three layers of neurons (input layer, hidden layer and output layer). The input layer has two neurons, one for each of the variables of the function ($f : \Re^2 \rightarrow \Re^2$) of the systems. In the hidden layer the configuration in the learning phase were seven neurons. And in the output layer again two neurons, one for each coordinate of equilibrium point of chaotic functions. We added noise too in the input patterns, in each case. We use a basis radial function with the competitive rule concienceful, the metric Euclidean, the function transfer tanhaxon and the learning rule momentum.

Learning Procedure

1. Input patterns. The input patterns are obtained of chaotic functions (Lozi, Ikeda and Thinkerbell) taking initial points $L_0 = (0.3, -1)$, $I_0 = (-0.9, 0.8)$ and $T_0 = (-0.3, 0.4)$. The time series of Lozi, Ikeda and Thinkerbell are calculated obtaining the collection of training patterns. The patterns set are obtained from mixed the tree time series previous. Also included a set of patterns with added noise.
2. Output patterns. The output patterns are the equilibrium points where the function must be controlled.
3. Hidden neurons. Several simulations have been performed in order to know how the number of hidden neurons affects the mean square error, in find the corresponding stable point. The best results obtained are with seven hidden neurons.

4. Number of input patterns. The variation of error along the number of input patterns has been studied, among them files with 500, 1500 and 3000 for each system. The figure 5 shows error for 1500 patterns and figure 6 shows error for 3000 patterns of the input file.
5. To finish the learning phase of the network, another input pattern is obtained starting with other initial points for each chaotic function, finding the time series and training the neural network again.

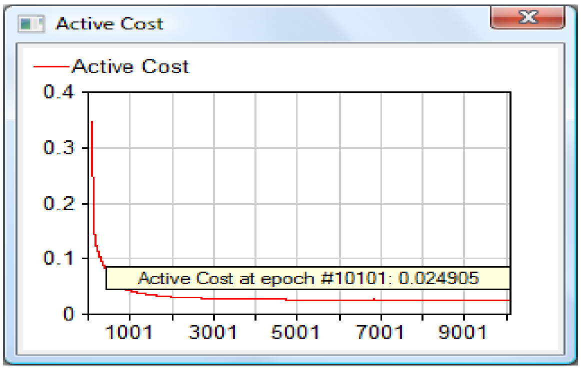

Figure 5

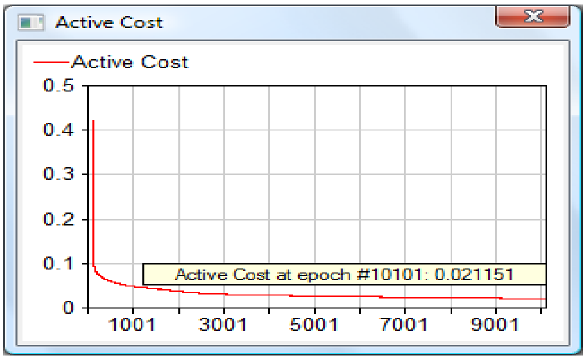

Figure 6

Achieving the control

Once the training phase is ended, it is necessary to check if the neural network is able to separate and control the function in the stable point in each case. Then choose several points for each system chaotic, are far enough from the stables points. These points are the basis for the pattern generation of the chaotic function. Each pattern set are made up of 1500 patterns for training RBFNs.

The network is also able to control the function when there exist only a few data and with some kind of noise. Next dynamical noise is added to the input, distributed on interval $[-0.01, 0.01]$. The results after training are similar to the obtained error without noise. We obtained the table 1 and table 2.

Active Performance

MSE	0.048677115489
NMSE	0.079412035298
r	0.959940877137
% Error	7.041473765294
AIC	-4287.819400732160
MDL	-4084.056346926107

Table 1

Active Performance

MSE	0.042866512744
NMSE	0.069932607445
r	0.966031616392
% Error	6.913486464162
AIC	-8084.993039748872
MDL	-6718.821699180137

Table 2

Conclusion

In this paper we have demonstrated the feasibility for using radial basis function neural network to implement schemes for simultaneous control of deterministic chaos in several nonlinear dynamical systems. The simulation experiments have illustrates the proposed method is effective for chaotic systems.

An important advantage of this control technique is that the obtained controllers are very stable, presenting a good behavior even with a small random dynamical noise or with a few data.

Bibliography

[Anderson, James A. 1995] Anderson, James. A. An Introduction to Neural Networks Cambridge. MA: MIT Press (1995).

[Chen G. & Dong X, (1992)] On feedback control of chaotic dynamical systems. Int. J. of Bifurcations and chaos, 2, 407-411 (1992).

[Chen G. & Dong X.; (1993)] From Chaos to Order. Int. J. of Bifurcations and Chaos, 3, 1363-1409 (1993).

[Haykin, S. 1994] Neural Networks. A Comprehensive Foundation. Macmillan, New York (1994).

[Hübler A. W. 1989] Adaptative control of chaotic systems. Helvetica Physica 62,343-346 (1989).

[Moody, J. and Darken C. (1989)]. Fast learning in networks of locally-tuned processing units. Neural Computation, 1:281-294 (1989).

[Ott E., Grebogi C. & Yorke J. A., (1990)]. Controling Chaos. Phys. Rev. Lett, 64, 1196-1199 (1990)

Authors' Information

Angel Castellanos– Departamento de Ciencias Basicas aplicadas a la Ingeniería Forestal. Escuela de Ingeniería Técnica Forestal. Universidad Politécnica de Madrid, Avda. de Ramiro de Maeztu s/n 28040 Madrid, Spain. angel.castellanos@upm.es

Ana Martinez– Natural computing group. Universidad Politécnica de Madrid, Spain. a.martinez@upm.es

Rafael Gonzalo– Natural computing group. Universidad Politécnica de Madrid, Spain. rgonzalo@fi.upm.es