

## INTELLIGENCE ALGORITHMS FOR INCREASING NAVIGATION SYSTEMS ACCURACY

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**Abstract:** Application of neural network algorithm for increasing the accuracy of navigation systems are showing. Various navigation systems, where a couple of sensors are used in the same device in different positions and the disturbances act equally on both sensors, the trained neural network can be advantageous for increasing the accuracy of system. The neural algorithm had used for determination the interconnection between the sensors errors in two channels to avoid the unobservation of navigation system. Representation of thermal error of two-component navigation sensors by time model, which coefficients depend only on parameters of the device, its orientations relative to disturbance vector allows to predict thermal errors change, measuring the current temperature and having identified preliminary parameters of the model for the set position. These properties of thermal model are used for training the neural network and compensation the errors of navigation system in non-stationary thermal fields.

**Keywords:** neural network, navigation system, time model of the sensors errors, errors interconnection function, unobservation, model adequacy, verification, neural network algorithm, increasing the accuracy

**ACM Classification Keywords:** I.5.1 Models - Neural nets

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### Introduction

Neural networks (NN) can solve many problems that could not be approached previously in any practical way. NN have been trained to perform complex functions in various fields of application, where they already have been applied, including control and navigation systems. The basic trends of using this theory are connected with the solution of complicated practical tasks. At present time there are various types of NN, which are assigned to solve diverse tasks. These models differ in connection structure methods of weight determination or teaching principles [Heerman, 1992]. Control systems anyhow using artificial NN are one of possible alternatives to classical control mode. The opportunity of using NN for solving problems of control in many respects is based on that NN consists of two layers, where the first layer is sigmoid and the second layer is linear, can approximate any function of real numbers with the set degree of accuracy [Rauch, Winarske, 1988].

The purpose of paper is to show that in many navigation systems such as inertial navigation systems and strapdown navigation systems, where a couple of sensors are used in the same device in different positions and the disturbances act equally on both sensors, the trained NN can be advantageous for increasing accuracy of such navigation systems. As a particular case a NN to designate the interconnection function of dynamically tuned gyroscope (DTG), which is used in Kalman algorithm to avoid the unobservation of the system, is trained, after that the errors of corrected gyrocompass, that allow to increase the accuracy of course determination is estimated.

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### Errors time model

Math model of DTG can be presented as follows:

$$\omega_x^{dr} = \dot{\alpha}_1 = \dot{\alpha}_0 + \Delta\dot{\alpha}_T, \quad \omega_z^{dr} = \dot{\beta}_1 = \dot{\beta}_0 + \Delta\dot{\beta}_T$$

$$\begin{aligned}
\Delta\dot{\alpha}_T &= \delta\left(\frac{\Delta c}{H}\right)\beta_0 - \delta\left(\frac{1}{\tau}\right)\alpha_0 + \frac{1}{4}(\delta S_{13} + 4\delta S_{14})(\beta_0^3 + \alpha_0^2\beta_0) + \delta S_{15}(\alpha_0^3 + \\
&+ \alpha_0\beta_0^2) + \frac{\Delta c}{H}\delta\beta_0 - \frac{1}{\tau}\delta\alpha_0 + \frac{1}{4}(S_{13} + 4S_{14})(3\delta\beta_0 \cdot \beta_0^2 + 2\alpha_0\beta_0\delta\alpha_0 + \alpha_0^2\delta\beta_0) + \\
&+ S_{15}(3\alpha_0^2\delta\alpha_0 + 2\alpha_0\beta_0\delta\beta_0 + \beta_0^2\delta\beta_0) - \frac{H_1}{H^2}\omega_z\delta H + \frac{\omega_z}{H}\delta H_1 + \delta\left(\frac{M\beta}{H}\right), \\
\Delta\dot{\beta}_T &= -\delta\left(\frac{\Delta c}{H}\right)\alpha_0 - \delta\left(\frac{1}{\tau}\right)\beta_0 - \frac{1}{4}(\delta S_{13} + 4\delta S_{14})(\alpha_0^3 + \beta_0^2\alpha_0) + \delta S_{15}(\beta_0^3 + \\
&\beta_0\alpha_0^2) - \frac{\Delta c}{H}\delta\alpha_0 - \frac{1}{\tau}\delta\beta_0 - \frac{1}{4}(S_{13} + 4S_{14})(3\delta\alpha_0 \cdot \alpha_0^2 + 2\beta_0\alpha_0\delta\beta_0 + \beta_0^2\delta\alpha_0) + \\
&+ S_{15}(3\beta_0^2\delta\beta_0 + 2\alpha_0\beta_0\delta\beta_0 + \alpha_0^2\delta\beta_0) - \frac{H_1}{H^2}\omega_y\delta H + \frac{\omega_y}{H}\delta H_1 + \delta\left(\frac{M\alpha}{H}\right).
\end{aligned}$$

where  $\dot{\alpha}_0, \dot{\beta}_0$  - systematic errors and  $\Delta\dot{\alpha}_T, \Delta\dot{\beta}_T$  - thermal errors.

As long as the analysis of thermal errors components shows their mainly linear dependence on temperature, then subject to temperature variation in an unsteady thermal field can be written:

$$\Delta\dot{\alpha}_T = \sum_{k=1}^m L_k (1 - e^{-kt/\tau_k}); \quad \Delta\dot{\beta}_T = \sum_{k=1}^m R_k (1 - e^{-kt/\tau_k}); \quad (1)$$

$L_k, R_k$  - constant coefficients depend generally on ambient temperature, disturbance that acts on gyroscope and gyroscope parameters;  $\tau_k$  is determined only by gyroscope parameters.

The examination of the character of DTG thermal errors variation (1) enables the functional dependence of errors to be presented as power series

$$\dot{\alpha}_1 = \sum_{j=0}^n a_j t^j; \quad \dot{\beta}_1 = \sum_{j=0}^n b_j t^j, \quad (2)$$

which are regression equations.

Findings in (1) show that the errors variation in both measurement channels in time, in unsteady thermal fields, depend in the one way on the temperature change. The received dependences of DTG thermal errors show, that the change of its errors on both axes in time at non-stationary thermal fields is described in workmanlike manner qualitatively by similar dependences, and depend identically on temperature change.

Thus the unity of influence of factors (both determinate and casual), causing the change of disturbing moments of a gyroscope (thermal or magnetic fields, gravitation, etc.) on both measuring axes takes place.

In order to solve the task of algorithmically increasing the accuracy of gyrocompass by compensating DTG thermal errors, it is necessary to make sure that coefficients  $a_i, b_i$  (2) for the given device remain constant. With this purpose statistical equality of pairs coefficient  $b_i, b'_i, a_i, a'_i$ , received at various tests of DTG, is verified.

For an estimation of means of distribution  $M(a_i)$  and  $M(a'_i)$  - their best estimations of samples  $\bar{a}_i, \bar{a}'_i$  are utilized, and for an estimation of a dispersion  $\sigma^2$  - selective estimations:

$$\hat{S}_{a_i}^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (a_i - \bar{a}_i)^2, \quad \hat{S}_{a'_i}^2 = \frac{1}{n_2 - 1} \sum_{i=1}^{n_2} (a'_i - \bar{a}'_i)^2.$$

The best estimation for dispersion in this case is  $\hat{S}^2 = \frac{\hat{S}_{a_i}^2(n_1 - 1) + \hat{S}_{a'_i}^2(n_2 - 1)}{n_1 + n_2 - 2}$ .

If a hypothesis  $M(a_i) = M(a'_i)$  it is fair, then random variable  $(a_i - a'_i)$  submits to the normal law of distribution with average of distribution equal to zero and dispersion equal to  $\sigma^2 = (\frac{1}{n_1} + \frac{1}{n_2})$ .

As sample estimate of dispersion  $D(a_i - a'_i)$  usually accept an estimation of  $S^2_{(a_i - a'_i)} = (\frac{1}{n_1} + \frac{1}{n_2})\widehat{S}^2$ .

If the random variable  $a_i - a'_i$  submits to the normal law of distribution, so statistics

$$t_c = \frac{(a_i - a'_i) - M(\bar{a}_i - \bar{a}'_i)}{\widehat{S}_{(a_i - a'_i)}} = \frac{(a_i - a'_i) - M(\bar{a}_i - \bar{a}'_i)}{\sqrt{(\frac{1}{n_1} + \frac{1}{n_2}) \frac{(n_1 - 1)S_{a_i}^2 + (n_2 - 1)S_{a'_i}^2}{n_1 + n_2 - 2}}}$$
 has  $t_c$ - Student distribution, and

$$k = n_1 + n_2 - 2.$$

Having chosen probability  $p = 1 - \alpha$ , according to the table  $t_c$  of distribution it is possible to determine critical value  $t_{cn1+n2-2;\alpha}$ , for which  $p(|t_c| > t_{cn1+n2-2;\alpha}) = \alpha$ .

If the calculated value  $|t_c| > t_{cn1+n2-2;\alpha}$  with the probability of  $p = 1 - \alpha$  then divergence of  $a_i, a'_i$  will be considered to be significant (not casual).

Experimental measurement of DTG drift change at the change of external temperature is shown on fig. 1.

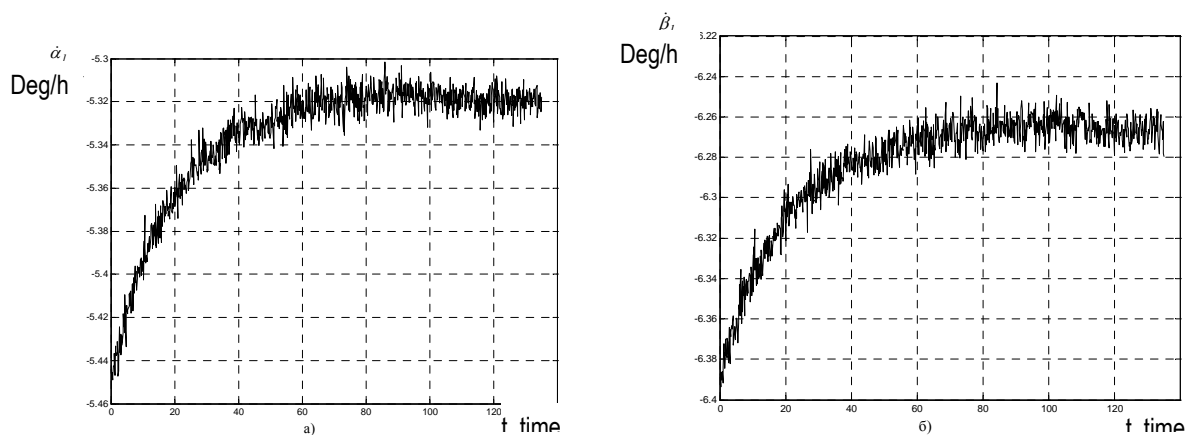


Fig. 1. Dependence of DTG drift at external temperature change on two channels.

The carried out statistical testing of coefficient values  $a_i, b_i$  according to the resulted technique has shown, that with the probability of 95 %, regression coefficients of the equations (2) at identical initial conditions are constant.

Function of the gyroscope drift, which approximated by polynomial (1), can be inadequate to observable values of the drift. Therefore it is necessary to check up its adequacy to the experiment data with the help of calculations deviation estimation of function values (2) from experimentally established ones, which are averaged by the number of experiences at factorial space points. For deviations estimation, Fisher's criterion is used.

In the table 1 the results of verification of statistical significance of regression coefficients estimation are adduced. It is seen, that the model (2) is adequate to the experiment at a significance value of  $q = 0.05$ , since

$$F_{calc} < F_{tab}.$$

The obtained and statistically estimated mathematical model of gyroscopes thermal drift has shown its adequacy to physical process that allows using it for solution tasks of algorithmically increasing the accuracy of gyrocompass.

Verification of DTG drifts has confirmed their repeatability. Let's determine interconnection function between drifts  $\dot{\alpha}_1, \dot{\beta}_1$  of a gyroscope in its different channels.

Table 1. Verification of model adequacy

Statistical characteristic	Gyroscopes orientation			
	$\vec{H}$ Vertical		$\vec{H}$ Horizontal	
	I channel	II channel	I channel	II channel
$S^2 (\text{deg/h})^2$	1,975 $10^{-2}$	1,144 $10^{-2}$	5,43 $10^{-2}$	5,83 $10^{-2}$
$S_{calc.}^2 (\text{deg/h})^2$	4,1969 $10^{-2}$	1,9837 $10^{-2}$	6,7338 $10^{-2}$	7,1884 $10^{-2}$
$F_{calc}$	2,125	1,734	1,566	1,233
Tabulated value $F$ - criterion at significance value $q=0.05$ , $F = 2.52$ .				

### Neural network algorithm

The possibility of using neural network algorithm to determine the interconnection between the DTG's two drifts in both its channels is approved by training the neural network and defining weight coefficients and biases, the algorithm's input and output are  $\dot{\alpha}_1 = \omega_z^{dr}$  and  $\dot{\beta}_1 = \omega_x^{dr}$  accordingly

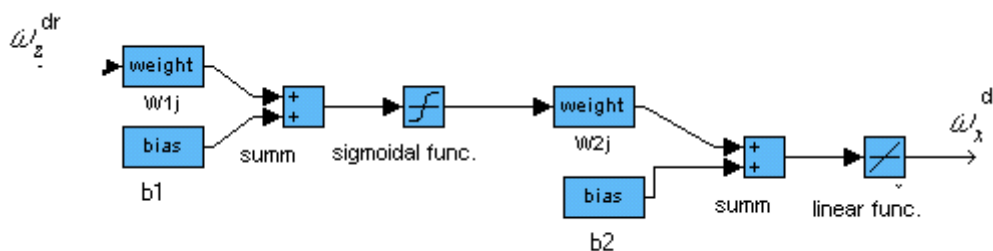


Fig. 2. Model of neural network of a straight propagation for DTG drift approximation

$$\omega_x^{dr}(t) = \sum_{j=1}^N W_{2j} \cdot (\text{sigm}(W_{1j} \cdot \omega_z^{dr}(t) + b_{1j})) + b_2, \quad (3)$$

where  $W_{1j}, W_{2j}$  – weight coefficients;  $b_1, b_2$  – biases, N- number of neurones in net's hidden layer.

The teaching algorithm of neuronet are the next.

1-st step. Weights of the net are given small initial values.

2-d step. The next teaching pair  $(X, Y)$  are selected from the teaching ensemble; vector  $X$  is delivered to net's input.

3-d step. The output of the net is calculated.

4-d step. The difference between the required (target,  $Y$ ) and the real (calculated) net's output is calculated.

5-th step. Weights are adjusted so, to minimize the accuracy (in the beginning the weights of the output layer, then with the use of differentiation complicated functions rule and the above mentioned derivative sigmoidal function, then the weights of previous layer and son on)

6-s step. Steps from the 2-nd to the 5-th are repeated for each pair of the teaching ensemble until the error in all ensembles does not reach the acceptable value.

Steps 2 and 3 similar to that carried out in the taught yet net.

The experimental investigations showed that the absolute error of estimating one drift by using the interconnection function (3) and the known other drift is less than 2%.

### Increasing the accuracy of gyrocompass

Confining the analyze of those gyrocompass errors that caused by gyroscope drifts only, then the precession equations of gyrocompass motion when the ship is moving at a constant speed, heading and without heaving are [Zbrutsky, Nesterenko, Prokhorchuk, Lukjanenko, 1997]:

$$\begin{aligned}\dot{\alpha} - \omega_{\eta}\beta &= k_x\delta + \omega_z^{dr}; \\ \dot{\beta} + \omega_{\eta}\alpha &= -k_z\delta + \omega_x^{dr}; \\ T_n\dot{\delta} + \delta &= \beta,\end{aligned}\tag{4}$$

where  $\alpha, \beta$  – deviation angles of the gyrocompass principal axis from the meridian and horizon areas accordingly ( $\alpha$  – gyrocompass error);  $\delta$  – output signal of accelerometer amplifier;  $T_n$  – constant time of accelerometer amplifier;  $\omega_{\eta}$  – angular velocity northern projection of geographic accompanying trihedron turn;

$k_x, k_z$  – torque's pendular and damping slopes;  $\omega_z^{dr}, \omega_x^{dr}$  – gyroscopes drifts angular velocities around vertical and horizontal axes.

The application of Kalman optimal filter method to a gyrocompass in its standard statement (4) shows, that the positive result of increasing the accuracy cannot be achieved because of the nonobservability system. But the use of the offered interconnection function between gyroscope drifts (3) and the application of Kalman optimal filter method allows to estimate the heading identification errors and algorithmically compensate them.

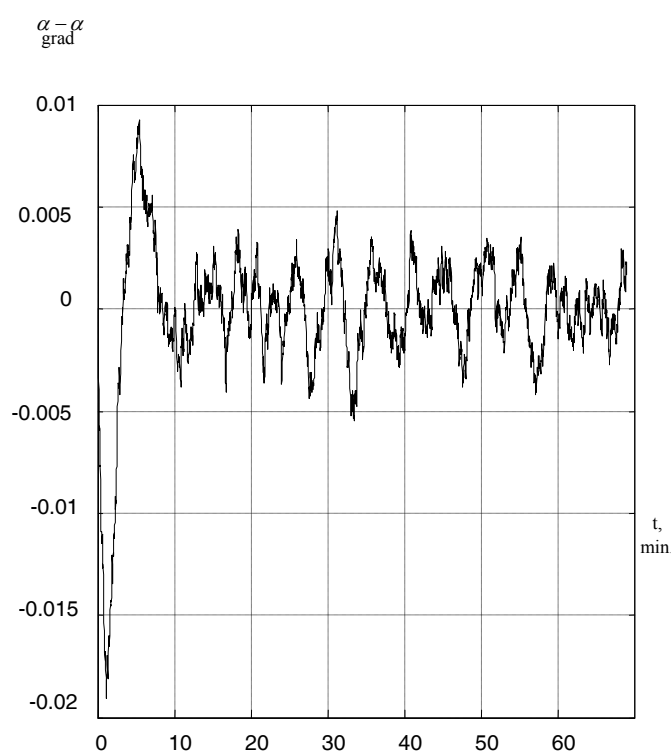


Fig. 3. Kalman optimal filter estimation of the gyrocompass heading error

Then (4) will be

$$\begin{aligned}
 \dot{\alpha} - \omega_{\eta}\beta &= k_x\delta + \omega_z^{dr}; \\
 \dot{\beta} + \omega_{\eta}\alpha &= -k_z\delta + f(\omega_z^{dr}); \\
 T_n\dot{\delta} + \delta &= \beta; \\
 \dot{\omega}_z^{dr} &= W_1.
 \end{aligned} \tag{5}$$

where  $f(\omega_z^{dr}) = w_x^{dr}$  - horizontal drift as function of vertical drift,  $W_1$  - white noise.

It is obvious from the fig. 3, that the estimated fault of the gyrocompass heading error, using the Kalman optimal filter and the proposed interconnection function between the two drifts of DTG, does not exceed 0.02 degrees and the standard deviation of this error is less than 0.003 degrees.

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## Conclusion

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Using the above discussed method we assume that it can be used for improving accuracy characteristics of many navigation systems where two one-component sensor are used in the same device in different positions such as accelerometers, strap down inertial systems etc.

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