APPLIED ASPECTS OF MATHEMATICAL MODELING AND OPTIMIZATION OF DYNAMICS OF CHARGED BEAMS

Fedir Garashchenko, Igor Kharchenko

Abstract: The complex of questions connected with the analysis, estimation and structural-parametrical optimization of dynamic system is considered in this article. Connection of such problems with tasks of control by beams of trajectories is emphasized. The special attention is concentrated on the review and analysis of spent scientific researches, the attention is stressed to their constructability and applied directedness. Efficiency of the developed algorithmic and software is demonstrated on the tasks of modeling and optimization of output beam characteristics in linear resonance accelerators.

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Introduction

The article covers applied aspects of use undifferentiated optimization algorithms in the tasks of optimal accelerating and focusing system's projecting. The review of methods of analyzing and estimation of calculation processes in dynamic systems as well as the estimates' optimization developed at the complex systems modelling department is in question. The special attention is paid to calculation experiments' specifics in the course of solving optimization problems of initial characteristics of charged particle beams for the waste range of radio engineering devices, namely: high frequencies generators, linear resonance accelerators, tri-electrode lens, electrostatic accelerators, etc. Authors observe unique algorithms and software corresponding to the modern possibilities of computer engineering development.

The scheme of linear accelerator and the dynamic equations for particles

Let us look at Fig.1, for example, to the scheme of linear resonance accelerator (LRA).

Here H – is the injector, DT – drift tube, AG – accelerator gap. Our work is connected with fulfillment of such methods of optimization, stability, sensitivity, which make it possible to define optimal parameters of LRA. And by means of this we may essentially increase efficient coefficient of such device.
The dynamic equations for particles in electromagnet field are in the form:

$$\frac{d}{dt}\left(m\vec{V}\right) = Ze\left\{\vec{E} + \left[\vec{V} \cdot \vec{B}\right]\right\} + \vec{F}_k, \qquad (1)$$

where, the vectors of electric $\vec{E}$ and magnetic $\vec{B}$ induction satisfy to Maxwell equations

$$rot\,\vec{E} = -\frac{d\vec{B}}{dt}, \quad div\,\vec{E} = \frac{\rho}{\varepsilon_0}, \quad rot\,\vec{B} = \frac{1}{c^2}\frac{d\overline{E}}{dt} + \mu_0\vec{j}, \quad div\,\vec{B} = 0. \qquad (2)$$

Here $\vec{V}$ – vector of participle velocity, c – velocity of light, ε_0, μ_0 – some electric and magnetic constants, $\vec{F}_k$ – Coulomb forces, Z – charge number, $m = \frac{m_0}{\sqrt{1-\frac{\upsilon^2}{c^2}}} = \frac{m_0}{\sqrt{1-\beta^2}} = \gamma\, m_0$ – dynamic mass, m_0 – rest mass.

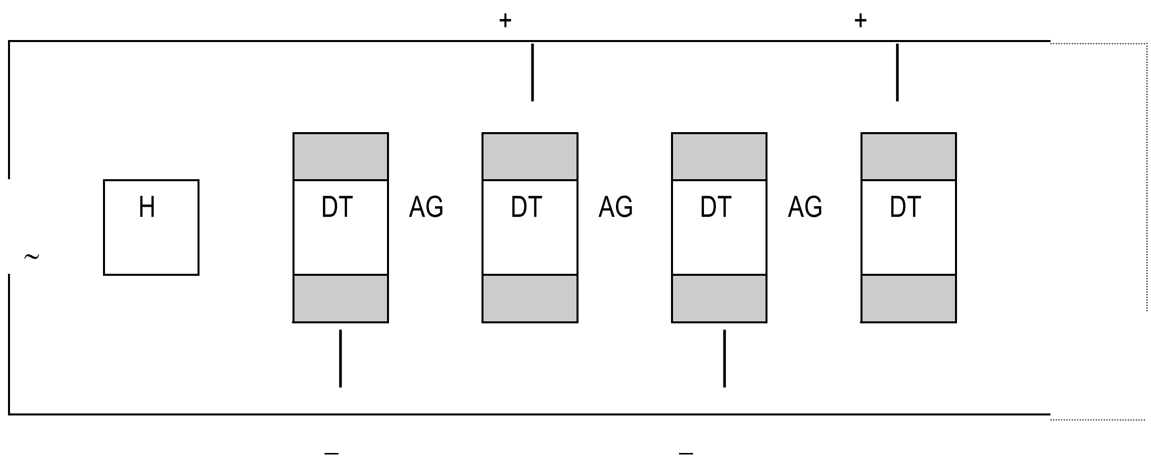

Fig.1.

In axial symmetric case from (1), (2) we may follow such equations:

$$\frac{d\gamma}{d\xi} = \alpha(\xi)\cos\varphi,$$
$$\frac{d\varphi}{d\xi} = \frac{2\pi\gamma}{\sqrt{\gamma^2 - 1}}, \qquad (3)$$
$$\frac{d^2 r}{d\xi^2} = -\frac{\gamma}{\gamma^2 - 1}\left(\frac{1}{2}\frac{d\alpha}{d\xi} r + \alpha\frac{dr}{d\xi}\right)\cos\varphi,\ \ \xi \in [0,T],$$

where $\xi = \dfrac{z}{\lambda}$, $\alpha(\xi) = ze|\lambda\, E_z(\xi)|m_0 c^2$, T – dimensionless length of LRA , λ – wave length .

Statement of problem

In real cases it is necessary to solve the following tasks:

Problem A. The initial dispersion is prescribed $\gamma(0),\ \varphi(0) \in M_0$, the constraint on the amplitude of accelerated field is given $\alpha(\xi) \in G_\alpha = \{\alpha(\xi) : 0 \le \alpha(\xi) \le C\}$.

It is necessary to solve the optimization problem for longitudinal motion (equations (1), (2) from system (3))

$$\min_{\alpha(\xi)\in G_\alpha}\ \max_{\gamma(0),\varphi(0)\in M_0} \left\{A_1(\gamma(T,\gamma(0),\varphi(0)) - \gamma_T)^2 + A_2(\varphi(T,\gamma(0),\varphi(0)) - \varphi_T)^2\right\}. \qquad (4)$$

Problem B. The limitation is given in such form:

$$\begin{cases} \max\limits_{\gamma(0),\varphi(0)} |\gamma(T) - \gamma_T| \le \varepsilon_1\ , \\ \max\limits_{\gamma(0),\varphi(0)} |\varphi(T) - \varphi_T| \le \varepsilon_2 \end{cases} \qquad (5)$$

and

$$x^2(\xi) + y^2(\xi) \le a^2(\xi) \qquad (6)$$

or

$$|r(\xi)| \le \varepsilon(\xi) \qquad (7)$$

in symmetric case, but the set of capture is in process of acceleration for particles under restriction (5)-(7) . It is necessary to choice the value of the field so that $m(\alpha(\xi))$ – it measure will be:

$$\max_{\alpha(\xi)\in G_\alpha} m(\alpha(\xi)). \tag{8}$$

The analysis of the problems connected with various accelerating and focusing system optimal designs leads to new mathematical statement in the area of stability and optimization. For example, to compute the particles capture area in accelerating process one needs a numeric algorithm for optimal practical stability area estimation. We mean area optimum as optimum in given structures (sphere, ellipsoid, generalized ellipsoid) as well as maximal on area volume. Thus, one of the important problems of area maximization of particle capture in accelerating mode is just that very practical stability problem in optimizing statement.

On the basis of the stated and proved general theorem the optimal estimations for the analysis of various dynamic systems practical stability are developed [Bublik, 1985], [Garashchenko, 2000]. The criterions have practical directness. They are easy algorithmic and modeled on computer. To calculate the practical stability area maximal on volume, the conception of stability in direction was introduced [Garashchenko, 1988].

The problems of some parametric system practical stability are also investigated. Latter helps us to develop new approaches to the solving of sensitivity problems, guaranteed sensitivity problems and tolerance computation.

The practical stability area optimal estimation is expressed as the maximum a certain function on independent variable, the number of constraints and partially defined initial conditions. That's why the estimate optimization is a non-smooth trajectory optimization problem. The necessary conditions of optimality and contractible algorithms for solving the minimization problem of maximum function on independent variable are adduced in paper.

The developed algorithms and programs were used when designing of heavy ions linear accelerators in Moscow Radio Technical Institute [Garashchenko, 1982, 1985], Institute Theoretical and Experimental Physics (ITEF), in the project of meson factory built in Troitsk city, when building the linear accelerators for medical goals on 3 MEV and 12 MEV in ITEF. We are taking part in the tandem accelerator computing in Nuclear Research Institute of National Academy of Science of Ukraine [Garashchenko, 1994]. The tandem will be supplement by a post-accelerator, according to the project we started to work out.

Optimization problem

Many problems arise when such structures are designed. Let's mark the following:

1. The initial particle M_0 is set. One needs to select the parameter vector a according to the condition :

$$\min_{\alpha\in G_\alpha} \max_{x_0\in M_0} \Phi\left(x\left(T, x_0, \alpha\right)\right). \tag{9}$$

2. Let G_0 – is a particle capture area, with the measure $m(G_0) = m(\alpha)$. It is necessary to maximize the capture area :

$$\max_{\alpha\in G_\alpha} m(\alpha) . \tag{10}$$

The complexity of such problem solving consists in minimax statement and the function $m(\alpha)$ calculation as well and in general case fields are defined from the corresponding equations in partial derivations.

Algorithmic and software are formed by the principle of mathematical model complication:

- the choice of initial control and optimization longitudinal motion;
- the solving problems of maximum capture or cooperate optimization with account longitudinal and radial structures (homogeneous fields);
- the optimization with account non-homogeneous fields or experimental gates;
- the determination of boundary influence of coulomb forces on optimal regimes and the optimization;
- the determination of parameters tolerances.

Determination 1. The system of n - dimension:

$$\frac{dx}{dt} = f(x,t),\ t \in [t_0, T] \tag{11}$$

will be called $\{G_0, \Phi_t, t_0, T\}$- stability if $x(t, x_0) \in \Phi_t,\ t \in [t_0, T]$ $\forall x_0 \in G_0$.

Let us $G_0 = \{x : W(x) \le 1\}$, where $W(x)$ – positive determinate function.

Theorem 1. For $\{G_0, \Phi_t, t_0 T\}$-stability necessary and sufficiently existence of positive determinate function $V(x,t)$ which satisfies conditions:

$$V_t = \{x : V(x,t) \le 1\} \subset \Phi_t, t \in [t_0, T];$$

$$\left.\frac{dV(x,t)}{dt}\right|_{(11)} \le 0,\ x \in V_t, t \in [t_0, T];$$

$$G_0 \subset V_{t_0}.$$

The criteria basing on these theorems permit to estimate as optimal in some structures.

Let us write some model as equation of particle's motion:

$$\frac{dx}{dt} = f(x,t,\alpha),\ \frac{dy}{dt} = A(x,t,\alpha)y, \tag{12}$$

$$y(t) \in \Gamma_t = \{y : |l_s^*(t)y| \le 1,\ s = 1,2,\ldots N\}, \tag{13}$$

where x, y – vectors of longitudinal and radial components $x(t_0) \in D_0$

Theorem 2. For $G_0 = \{y : y^* By \le c^2\}$ would be the area of capture of particles in process of acceleration by radial co-ordinates for some $x(t_0) \in D_0$, necessary and sufficiently that

$$c^2 \le \min_{x_0 \in D_0} \min_{t_0 \in [t_o, T]} \min_{s=1,2,\ldots,N} \left[l_s^t(t) Q(t,\alpha,x) l_s(t)\right]^{-1}, \tag{14}$$

$$\frac{dQ}{dt} = A(x.t,\alpha)Q + QA^*(x,t,\alpha),\ Q(t_0,\alpha,x_0) = B^{-1}. \tag{15}$$

For this case problem 2 may be written in following manner:

$$\min_{\alpha} \min_{x_0 \in D_0} \min_{t_0 \in [t_o, T]} \min_{s=1,2,\ldots,N} l_s^t(t) Q(t,\alpha,x) l_s(t). \tag{16}$$

For solving the trajectory parametric optimization task let us consider two types of problems:

$$\min_{\alpha} \min_{x_0 \in D_0} \Phi(x(T,\alpha,x_0)), \tag{17}$$

$$\min_{\alpha} \min_{x_0 \in D_0} \Phi_1(x(T,\alpha)). \tag{18}$$

The necessary optimal conditions which were written for tasks (17),(18): derivatives under some direction on optimal $\alpha^{(0)}$ – are positive. On the basis of these conditions the necessary conditions are formulated for more complex problem (16).

The iterative procedure on k - step is used:

$$\alpha^{k+1} = P_G\left(\alpha^{(k)} - \rho_k G(\alpha^{(k)})\right). \tag{19}$$

Here $G(\alpha^{(k)})$ – sub-gradient of functional by α.

Let us consider the parametric system for elaborated method for establish permits

$$\frac{dx}{dt} = f(x,t),\ t \in [t_0, T],\ x(t_0) \in G_0,\ \alpha \in G_\alpha . \qquad (20)$$

Determination 2. The non-disturbance motion is $x(t,0) = 0$ declared $\{G_0, G_\alpha, \Phi_t, t_0, T\}$- stability if $x(t,x,\alpha) \in \Phi_t,\ \ t \in [t_0, T],\ \forall x_0 \in G_0,\ \ \alpha \in G_\alpha$.

For (20) the theorems of practical stability by determination 2 are proved and criteria are elaborated.

It makes possible solving problems of counting permits on parameters, projecting systems with restrict ant guaranteed sensibility.

In conclusion let us describe numerical method of construction by volume estimates. The definition of stability by single direction is introduced as classical problems or as for problems of practical stability.

Let us formulate some results for the problem (12),(13).

Determination 3. The set of particles with initial condition:

$$y(t_0) = k_1 l, \quad 0 \le k_1 \le k$$

will be called the capture the particles in acceleration process in direction l by radial co-ordinates if

$$y(t, x(t_0)) \in \Phi_t,\ \ t \in [t_0, T],\ \ \forall y(t_0) = k_1 l, \quad 0 \le k_1 \le k\ ,\ \forall x(t_0) \in D_0 .$$

Theorem 3. The necessary and sufficiently conditions are given by correlations:

$$k_{(\alpha,l)} \le \vec{k}_{(\alpha,l)} = \min_{x_0 \in D_0} \min_{t_0 \in [t_o, T]} \min_{s=1,2,\dots,N} \left[l_s^t(t) Y(t, t_0, \alpha, x(t_0)) l_s(t) \right]^{-1} . \qquad (21)$$

Then the maximum of volume set of capture may be written as:

$$G_0^{\max} = \{ y : y = k_1 l,\ 0 \le k_1 \le k,\ \forall l,\ \|l\| = 1 \}. \qquad (22)$$

Using such algorithm it is easy to calculate $m(\alpha)$ for problem 2.

It is clear that such approaches may be carried and some tasks of sensitivity, calculation of maximum value set of permits.

These researches are fulfilled in general forms and may be used for solving other applied tasks [Garashchenko, 1993, 1998], for example: the systems with variable structure and explosive co-ordinates, digital-continuous systems and systems of equations in partial derivations.

The decision of concrete applied problems was based on the method of complication of mathematical model and was realized by such scheme:

1. The optimization of longitudinal motion (the structure of accelerator is chosen);
2. The correction of the parameters with regard to radial deviation.
3. The use of practical stability and stability by direction methods for beam estimation.
4. The problems of maximum capture of particles in accelerating process.
5. Taking account of dissimilarity field and experimental data.
6. Taking account of coulomb forces interactions.
7. Determination of parameters admittance of the system (size of drift tubes, errors of potential values in the tubes).

Notice, that we use parameters of previous stage on the next stage. They are the initial for next one on our calculations.

The applied task, which is solved on the basis of developed methodology.

1. The calculation of optimal parameters of the linear accelerators of heavy ions in different variants (for example 7-charged ions of uranium and for protons).
2. The projection of different types of linear accelerators with asymmetric phase variable focusing.
3. The minimization of effective cross-section phase beam volume in the case of longitudinal oscillation of the particles (for accelerator “MEGAN”).
4. The calculation of self-consistent distributions.
5. The determination of optimal parameters of electrons grouper.

Conclusion

Despite of certain difficulties connected with contact breach with leading Russian scientific centers, new results permitting to continue developing methods of technical systems’ optimization in stated structures were obtained by authors.

New studies in the domain of undifferentiated optimization and practical stability of trajectory beams were used to raise efficiency output and to lower energy consumption at stated intensity of particle beams at linear resonance and electrostatic accelerators. Modern algorithms and software created allows adapt it to the waste class of technical systems. These studies may be applied in tomography, scientific research and production, physics, science of materials, biology, medicine, etc.

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Authors’ Information

Fedir Garashchenko – *Professor, Taras Shevchenko National University of Kyiv, Faculty of Cybernetics, the Head of the Complex Systems Modelling Department, Kyiv, Ukraine, e-mail: garash@unicyb.kiev.ua*

Igor Kharchenko – *Associate professor, Taras Shevchenko National University of Kyiv, Faculty of Cybernetics, the Complex Systems Modelling Department, Kyiv, Ukraine, e-mail: ihar@unicyb.kiev.ua*