REDUCTION MEASUREMENTS FOR CALCULATION IN FUZZY EXPERIMENT SCHEME

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Abstract: The papers is dedicated to the questions of modeling and basing super-resolution measuring-calculating systems in the context of the conception "device + PC = new possibilities". By the authors of the article the new mathematical method of solution of the multi-criteria optimization problems was developed. The method is based on physic-mathematical formalism of reduction of fuzzy disfigured measurements. It is shown, that determinative part is played by mathematical properties of physical models of the object, which is measured, surroundings, measuring components of measuring-calculating systems and theirs cooperation as well as the developed mathematical method of processing and interpretation of measurements problem solution.

Keywords: measurements' interpretation, possibility-theoretical approach, fuzzy errors.

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Introduction

It is important to answer a question how much the result of design is exact when we deal with mathematical modeling of complex systems. A stochastic approach is most widespread. It is necessary that the observed value should be the result of meaning of independent test values for adequate application of stochastic principles to the complex system design. However, such description in probability terms is unnatural for the unique phenomena.

Methods of possibility theory allow to estimate an event truth with respect to other events and to take into account a subjective expert opinion. For example it is very important for prognostication of the social-economic phenomena, for medical diagnostic tasks, for mathematical modeling of human thinking process and other processes.

Measurement processing under fuzziness conditions based on multi-criteria optimization

The wide range of measuring processes would be modelled by the following equality

$$y = Gu + \nu, \tag{1}$$

where u is interpreted as a signal sent from the investigated object to an input of some device specified by G operator, ν characterizes environment impact on the device output and y is a measurements' result to be processed [2].

In wide enough mathematical terms it is possible to consider that $G : H_1 \to H_2$ – an linear operator, H_1 and H_2 – Hilbert space (H_2 – finite-dimensional), $u \in H_1$, $y \in H_2$. In regard to the characteristic of environment impact ν, till recently only such problem definitions have been considered where it was a random element of H_2 space, and it was expected that some statistical information was known about it (for example the first moment and covariance matrix). But in practice external impacts on measuring process not always would be adequately described by random variables. The lack of statistical information often forces to apply for expert estimations of environment impact which results in noise description by fuzzy values.

That very case will be examined in this paper, i.e. ν – a fuzzy element of H_2 space will be considered. Let we know a possibility distribution of the given value specified by an expert $\varphi^{\nu}(\cdot) \in L(H_2)$, i.e.

$\varphi^{\nu}(\cdot): H_2 \to L$, where L – scale on $[0;1]$ segment with natural order specified by inequality $\leq$ and two composition rules: addition in the sense of max and multiplication in the sense of min [1]. Let agree on $\varphi^{\nu}(\cdot)$ function which is monotone non-increasing relative to an argument norm. On this measurement model the measurements' result y itself becomes a fuzzy element of H_2 space and it's possible to express a joint distribution $\varphi^{y}(z,u) = \varphi^{y,\mu}(z,u) = \min\left(\varphi^{y|\mu}(z,u), \varphi^{\mu}(u)\right)$ by function $\varphi^{\nu}(\cdot)$. Since by the data u is an unknown arbitrary element of H_1 space, its distribution of possibilities will be $\varphi^{\mu}(u) = 1$. Conditional distribution of possibilities $\varphi^{y|\mu}(z,u) = \varphi^{\nu}(z - Gu)$, then $\varphi^{y}(z,u) = \varphi^{\nu}(z - Gu)$.

Let consider the problem of interpreting the measurements' results (1) for estimating the output of some given measuring device at signal u. We suppose that this device operation would be described by Hilbert-Shmidt operator $\Pi : H_1 \to H_3$, where H_3 – Hilbert space. One of known approaches to the problem of interpreting the measurements' results is to find such operator $B : H_2 \to H_3$ which would permit to interpret By as the exactest version of Πu in a certain sense. Forcing from the right on the equation (1) by B operator and subtracting Πu from both sides we obtain the equation

$$By - \Pi u = (BG - \Pi)u + B\nu$$

From this equation follows that By differs from Πu by two components: some artifact $(BG - \Pi)u$ and $B\nu$ – noise, which is fuzzy value. Since there is no initial information about u signal, minimization condition on B norm of Hilbert-Shmidt operator discrepancy, i.e. $\|BG - \Pi\|_2 \to \min\limits_{B}$ would be one of criterions of Πu optimal estimation as By. Introducing the definition of necessity of fuzzy value estimation by analogy with the definition of estimation mistake necessity correctness [1] the last definition would be offered as another optimality criterion. So the necessity of fuzzy value estimation correctness is

$$C_n(d(z)) = \theta \sup_{z} \sup_{u} \min\left(\varphi^{y}(z,u), \theta\, l(\Pi u, d(z))\right), \tag{2}$$

where $d(z)$ – estimation strategy; $l(\Pi u, d(z))$ – mistake absence possibility accompanying a choice of $d(z)$ as a value of Πu for each value $u \in H_1$ (fuzzy relation of correctness); θ – involution determined in [1] – dual isomorphism from L to $\tilde{L}$ such as $\theta(0) = 1$, $\theta(1) = 0$ and for any $a, b \in [0;1]$ relations $a \leq b$, $\theta(a) \geq \theta(b)$ and $\theta(a) \mathrel{\tilde{\leq}} \theta(b)$ are equivalent (here $\tilde{L}$ – scale on $[0;1]$ dual to L with operation of addition in the sense of min, operation of multiplication in the sense of max, and order relationship $\tilde{\leq}$ – opposite to natural). Essentially an integral by necessity from $l(\Pi u, d(z))$ is written in (2). Taking into consideration that $d(z) = Bz$, let direct the integral (2) to maximum by estimation strategy. As a result we obtain the optimality criterion $C_n(d(z)) = \theta \sup\limits_{z} \sup\limits_{u} \min\left(\varphi^{y}(z,u), \theta\, l(\Pi u, Bz)\right) \to \max\limits_{B}$. By analogy with [1] for possibility and necessity of an estimation error it is possible to prove that the task $C_n(d(z)) \to \max\limits_{B}$ should be reduced to the tasks $C_n(d(z), z) = \theta \sup\limits_{u} \min\left(\varphi^{y}(z,u), \theta\, l(\Pi u, Bz)\right) \to \max\limits_{B}$ for every $z \in H_2$. As a result, using properties of involution θ it is possible to write down the task

$$\begin{cases} \|BG - \Pi\| \to \min\limits_{B} \\ \sup\limits_{u} \min\left(\varphi^{\nu}(z - Gu), \theta\, l(\Pi u, Bz)\right) \to \min\limits_{B} \end{cases}. \tag{3}$$

Found from (3) the operator B gives the estimation $\overset{\wedge}{\Pi u} = Bz$ which is optimum in terms of operator discrepancy minimization and estimation correctness necessity maximization. Let's take the possibilities' distribution $\varphi^{B\nu}(B(z-Gu))$ of a fuzzy value $B\nu$ as fuzzy relation of correctness $l(\Pi u, Bz)$. It's possible to express it through $\varphi^{\nu}(\cdot)$ as $\varphi^{B\nu}(B(z-Gu)) = \max_{w}\{\varphi^{\nu}(w) | Bw = B(z-Gu)\}$ or $\varphi^{B\nu}(B(z-Gu)) = \varphi^{\nu}\left(B^{-}B(z-Gu)\right)$, where B^{-} – pseudoinverse to B operator. Put the last expression to (3) and get

$$\begin{cases} \|BG - \Pi\| \to \min\limits_{B} \\ \sup\limits_{u} \min\left(\varphi^{\nu}(z-Gu), \theta\varphi^{\nu}\left(B^{-}B(z-Gu)\right)\right) \to \min\limits_{B} \end{cases}. \tag{4}$$

Let's consider separately the first criterion of the task (4). It is easy to show, that it reaches the global minimum on value set

$$B = \Pi G^{-} + Y\left(I - GG^{-}\right). \tag{5}$$

Here $Y : H_2 \to H_3$ – any Hilbert-Shmidt operator. At the same time in the second criterion the value $\varphi^{\nu}(z-Gu)$ does not depend on B, so for any fixed u and z the value of the $\min\left(\varphi^{\nu}(z-Gu), \theta\varphi^{\nu}\left(B^{-}B(z-Gu)\right)\right)$ is as little, as little the norm of argument of $\theta\varphi^{\nu}(\cdot)$. Since the argument is given by $B^{-}B(z-Gu)$ where $B^{-}B$ – orthogonal projector, the value $\left\|B^{-}B(z-Gu)\right\|$ is as little, as wide the kernel of B. Choosing B in the form of (5) these requirements will be fulfilled if $Y\left(I - GG^{-}\right) = 0$. It follows from the fact that $B = \Pi G^{-}$ turn into zero any element from orthogonal addition to space of values of the operator G, and if the operator $Y\left(I - GG^{-}\right)$ is not zero at this element, kernel of $\Pi G^{-} + Y\left(I - GG^{-}\right)$ includes into the kernel of ΠG^{-}. So we conclude that $B = \Pi G^{-}$ is pareto-optimum decision of the task (4) because any changes of B leads to increasing either the first or the second criterion. If (5) doesn't include B then the first criterion grows up, or if (5) includes B but B doesn't equal ΠG^{-}, then the second criterion grows up.

Lets consider the way how to pick the operator B to reduce the second criterion of (4) better than it was at $B = \Pi G^{-}$. As it was above mentioned the wide the kernel of B is the little the value of the second criterion is. It is obvious that under widening of kernel of the operator B, not Πu but only it's some projection $P\Pi u$ should be restored completely, where $P : H_3 \to H_3$ – some orthogonal projector. It corresponds to the problem of measurement reduction to the device $\tilde{\Pi}$ which satisfies equations $(I-P)\tilde{\Pi}u = 0$ and $P\tilde{\Pi}u = P\Pi u$ for any signal u. As the signal u is a priori unknown lets add another condition to the task (4) and set up task

$$\begin{cases} \|BG - \tilde{\Pi}\|_2 \to \min\limits_{B} \\ \sup\limits_{u} \min\left(\varphi^{\nu}(z-Gu), \theta\varphi^{\nu}\left(B^{-}B(z-Gu)\right)\right) \to \min\limits_{B} \\ (I-P)\tilde{\Pi} \equiv 0 \end{cases}. \tag{6}$$

Much as the above-mentioned method it is simply to solve (6) and show that $B = P\tilde{\Pi}G^{-} = P\Pi G^{-}$ is pareto-optimum decision. By the way, the above received decision $B = \Pi G^{-}$ turns out from the last result by a choice

$P \equiv I$. The other extreme case, at which the second criterion of a task (4) is minimized as much as possible, turns out at $P \equiv 0$. Obviously that at such choice of an orthogonal projector the operator $B \equiv 0$, necessity of an estimation's correctness $C_n = 1$ and the norm of operator discrepancy is equal to $\|\Pi\|_2$. It is necessary to notice that under $B \equiv 0$ we get $R(G) \subseteq N(B)$ and as a result the signal passed through the device G turns into zero. The result doesn't contain any information about the signal Πu, so it's singular. Similarly the choice of an orthogonal projector P so that $\left\|\Pi G^-\right\|_2 \geq \left\|P\Pi G^-\right\|_2$ results in reduction of restored value dimension of Πu. So the choice of an orthogonal projector P is responsibility of HMD (human who makes decisions).

Thus, the decision of (4) is the set of operators $B = P\Pi G^-$. The norm of operator discrepancy is equal to $\left\|P\Pi G^- G - \Pi\right\|_2$ and the estimation's correctness necessity is equal to $C_n(B) = \theta \sup_u \min\left(\varphi^{\nu}(z - Gu), \theta\varphi^{\nu}\left(B^- B w_G(u)\right)\right)$ taking into account $Bz_{G_\perp} = 0$ ($z_{G_\perp} = \left(I - GG^-\right)z$). Here $w_G(u) = GG^- z - Gu$.

After solving the task

$$\min\left(\varphi^{\nu}(z - Gu), \theta\varphi^{\nu}\left(B^- B w_G(u)\right)\right) \to \sup_u \tag{7}$$

and applying the involution θ to its decision we'll receive the value of estimation correctness necessity. As $z - Gu$ can be decomposed in the sum of orthogonal components $z - Gu = w_{GB^-}(u) + \left(w_{GB_\perp^-}(u) + z_{G_\perp B_\perp^-}\right)$, where $w_{GB^-}(u) = B^- B w_G(u)$, $w_{GB_\perp^-}(u) = \left(I - B^- B\right)w_G(u)$, $z_{G_\perp B_\perp^-} = \left(I - B^- B\right)z_{G_\perp} = z_{G_\perp}$, ($B^- B z_{G_\perp} = 0$ by the condition of the operator B construction) then it's possible to choose such u in (7) that $w_{GB_\perp^-}(u) = 0$ and $w_{GB^-}(u)$ remains without changes. Taking into consideration that on arrangement $\varphi^{\nu}(\cdot)$ does not monotonously increase by the argument norm, such u should not reduce $\varphi^{\nu}(z - Gu)$ and should not affect on $\varphi^{\nu}\left(w_{GB^-}(u)\right)$ in any way. So, is possible to express the task (7) as

$$\min\left(\varphi^{\nu}\left(w_{GB^-}(u) + z_{G_\perp B_\perp^-}\right), \theta\varphi^{\nu}\left(w_{GB^-}(u)\right)\right) \to \sup_{u \in \Omega}, \tag{8}$$

where $\Omega = \left\{ u \,\middle|\, w_{GB_\perp^-}(u) = 0 \right\}$.

Since the involution θ is monotonously decreasing function on $[0;1]$ by definition it becomes obvious from the type of the task (8) that its decision won't exceed some number $\alpha = \min_{x \in [0;1]}(x, \theta(x))$ which value depends on the involution definition. For example, if the involution is $\theta(x) = 1 - x$, $x \in [0;1]$ then $\alpha = 1/2$, or if the involution is in the form of hyperbola $\theta(x) = \dfrac{1 - x}{2x + 1}$, $x \in [0;1]$ then $\alpha = \dfrac{\sqrt{3} - 1}{2}$, etc. If we decide the task

$$\min\left(\varphi^{\nu}\left(w_{GB^-}(u)\right), \theta\varphi^{\nu}\left(w_{GB^-}(u)\right)\right) \to \sup_{u \in \Omega}$$

instead of the task (8) then its decision would be obviously described by the set of elements from H_1

$$U = \left\{ \tilde{u} \middle| \min_{u\in\Omega} \left| \varphi^{\,v}\left(w_{GB^-}(u)\right) - \alpha \right| \right\}, \quad (9)$$

since the minimum is got out between function $\varphi^{v}(\cdot)$ and dual to her at the same argument value. Thus for a special case of task (4) if $z_{G_\perp B_\perp^-} = 0$ we receive estimation correctness necessity's value

$$C_n = \theta \min\left(\varphi^{\,v}\left(w_{GB^-}(\tilde{u})\right), \theta\,\varphi^{\,v}\left(w_{GB^-}(\tilde{u})\right)\right),\ \tilde{u} \in U. \quad (10)$$

Let's consider possibility of occurrence of such special case. It is guaranteed if $R^{\perp}(G) \cap N(B) = \{0\}$. As mentioned above under constructing B the condition $R^{\perp}(G) \subseteq N(B)$ was obtained it should be concluded that the given special case should be inevitable only at $R^{\perp}(G) = N\left(G^{-}\right) = \{0\}$. The fact that the operator G has an opposite one follows from the last conclusion by the Banach theorem [3]. So if in the equality (1) the operator G is such as $\exists G^{-1}$ then the operator $B = P\Pi G^{-1}$ is the decision of the task (4) and the operator discrepancy norm is $\|BG - \Pi\|_2 = \|(P-I)\Pi\|_2$, and the value of estimation's correctness necessity can be calculated by the formula (10).

Since we implicitly estimated u (let even using estimations' set (9)) we can use a priori distribution $\varphi^{\,v}(\cdot)$ to estimate the experiment model consistency. If $\exists u \in U : \varphi^{v}(z - Gu) > 0$ then it is possible to recognize the experiment model as consistent, and every $u \in U$ as consistent estimation of the input signal u. Otherwise it is necessary to recognize the experiment model as insolvent. Since under the arrangement $\varphi^{v}(\cdot)$ is monotonously non-increasing function by the argument norm then for definiteness lets choose estimation of input signal as

$$u = \arg\min_{\tilde{u}\in U} \left\| w_{GB^-}(\tilde{u}) \right\|_2. \quad (11)$$

It is obvious that if $\varphi^{v}(z - Gu) = 0$ then it is necessary to recognize the experiment model as insolvent, otherwise as solvent.

Lets get back to (8) and consider the case $z_{G_\perp B_\perp^-} \neq 0$. Its decision depends on a type of $\varphi^{v}(\cdot)$ in many respects. But it is possible to outline some limitations for required value. First, since in (8) the norm of the first argument of $\min$ (function $\varphi^{v}(\cdot)$) is greater by $\left\|z_{G_\perp B_\perp^-}\right\|_2$ then $\left\|w_{GB^-}(u)\right\|_2$ it is obvious that the value of estimation correctness' necessity is greater then $\theta \min\left(\varphi^{\,v}\left(w_{GB^-}(u)\right), \theta\,\varphi^{\,v}\left(w_{GB^-}(u)\right)\right)$ and less then $\theta \min\left(\varphi^{\,v}\left(w_{GB^-}(u) + z_{G_\perp B_\perp^-}\right), \theta\,\varphi^{\,v}\left(w_{GB^-}(u)\right)\right)$, where u – estimation (11). Secondly, if there is a nonempty set $\breve{U} = \left\{ \breve{u} \middle| \left(\left\|w_{GB^-}(\breve{u})\right\|_2 < \left\|w_{GB^-}(u)\right\|_2 \right) \& \left(\theta\varphi^{v}\left(w_{GB^-}(\breve{u})\right) \geq \varphi^{v}\left(w_{GB^-}(u) + z_{G_\perp B_\perp^-}\right) \right) \right\}$ then the optimum estimation of the input signal belongs to set $\overline{U} = \breve{U} \cup \{u\}$, otherwise (11) is the optimum estimation of the input signal and $C_n = \theta \min\left(\varphi^{\,v}\left(w_{GB^-}(\hat{u}) + z_{G_\perp B_\perp^-}\right), \theta\,\varphi^{\,v}\left(w_{GB^-}(\hat{u})\right)\right)$. Thirdly, using the above given definition of an experiment model's consistency it is possible to make its a prior estimation even before construction of the optimum operator B. Since $\left\|w_{GB^-}(u) + z_{G_\perp B_\perp^-}\right\|_2 = \left\|w_{GB^-}(u)\right\|_2 + \left\|z_{G_\perp B_\perp^-}\right\|_2$

and $\varphi^V(\cdot)$ is monotonously non-increasing function by the argument norm then it follows from $\varphi^V\left(z_{G_\perp B_\perp^-}\right)=0$ that $\forall u \in H_1\ \varphi^V(z-Gu)=\varphi^V\left(w_{GB^-}(u)+z_{G_\perp B_\perp^-}\right)=0$ and it is necessary to recognize the experiment model as inconsistent. At the same time it's easy to verify $\varphi^V\left(z_{G_\perp B_\perp^-}\right)=0$ because $R^\perp(G)\subseteq N(B)$ should be fulfilled under the construction of B and $\varphi^V\left(z_{G_\perp B_\perp^-}\right)=\varphi^V\left(\left(I-GG^-\right)z\right)$ follows from it. The further analysis of estimation correctness necessity value depends in many aspects on a type of function $\varphi^V(\cdot)$ and we shall not consider it in this work.

Conclusion

Analyzing the received results it is possible to say that they correlate to results received for the probabilistic experiment model [4], namely pareto-optimum estimations for the probabilistic experiment model [4] under increase of ignoring degree of statistical information will tend by the norm to the result obtained in this work. But it is necessary to note that the experiment models in probabilistic and possibility cases and the tasks of optimum estimation varies ideologically in spite of formal similarity of the models and vicinity of the received results.

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