AN INFORMATION MODEL FOR PENSION FUND MANAGEMENT

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Abstract: In this paper the issues of Ukrainian new three-level pension system are discussed. First, the paper presents the mathematical model that allows calculating the optimal size of contributions to the non-state pension fund. Next, the non-state pension fund chooses an Asset Management Company. To do so it is proposed to use an approach based on Kohonen networks to classify asset management companies that work in Ukrainian market. Further, when the asset management company is chosen, it receives the pension contributions of the participants of the non-pension fund. Asset Management Company has to invest these contributions profitably. This paper proposes an approach for choosing the most profitable investment project using decision trees. The new pension system has been lawfully ratified only four years ago and is still developing, that is why this paper is very important.

Keywords: Information system, Pension System, Non-state pension fund, Assets, Liabilities, Decision Tree, Clustering, Kohonen network, Asset Management Company

ACM Classification Keywords: H.1 Models and principles

Conference: The paper is selected from XIVth International Conference "Knowledge-Dialogue-Solution" KDS 2008, Varna, Bulgaria, June-July 2008

Introduction

Four years ago Ukrainian former pension system was transferred into a new one. The new pension system is three-level like in many countries of Western Europe. One of the most important and underdeveloped parts of the new pension system is non-state pension funds. These pension funds are nowadays not very reliable for employees to save up pensions. This paper has several aims.

The first aim is to develop a new mathematical model that can be applied as a base for an automatic information system that would help the manager of a non-state pension fund make correct decisions, i.e. avoid default and calculate the optimal value of pension contributions. Some work in this direction has already been done by such famous researchers as Merton R. C [Merton R. C., 1974], Black, F., Scholes, M. [Black, F., and Scholes, M., 1973]. But their framework can only be applied to managing state pension funds.

Problems of non-state pension funds and life companies are not fully covered by classical Merton’s framework. These companies fund themselves by issuing highly structured form of debt: life insurance and pension policies. The path-dependent [P. Jorgensen, 2005] nature of the policies requires to apply a more elaborate pricing techniques to assess its value similar to a classical Merton’s framework.

The new model is based on the following ideas: the asset price of a non state pension fund is a random variable and is most often modelled using Wiener process, because this stochastic process most precisely reflects the random behaviour of financial markets (basic paper [Black, F., and Scholes, M., 1973]). The equation for modelling the behaviour of liabilities of a pension fund is different from what was developed before, because it reflects participation mechanism. In European countries nearly all non state pension funds nowadays apply participation mechanism, and it is natural to assume that soon Ukrainian non-state pension funds will do the same. The partial differential equation for determining the optimal value of pension contributions is derived basing on above assumptions about assets and liabilities behaviour.

The second aim of the paper is to solve the problem of automatic choice of a proper and experienced asset management company among the companies of Ukrainian market. This problem is nowadays crucial for many firms and enterprises that want to take part in creation of the non-state pension funds. But there is very little practical experience in this field because the new pension system is still under development. An asset management company plays a very important role: it invests pension contributions into the industry projects.

Many important financial results depend on the activity on the asset management company and first of all the safety of the pension contributions and the amount of pensions paid. Nowadays in Ukraine there are around two hundred asset management companies. Some of them came from European countries and brought wide experience, the other ones were created in Ukraine. So, which one to choose? Usually the asset management company is chosen empirically that is not very reliable. In this paper it is proposed to optimize and automate the choice of an asset management company by using Kohonen self-organizing maps.

The third goal of the paper is to discuss an approach that would help asset management company invest pension contributions most profitably. Having received pension contributions the non-state pension fund transfers the money to the assets management company. The assets management company invests pension contributions into the industry fields in order to make profit and pay pensions in the future. Decision Trees [4] and Bayesian methods are applied in this paper to develop a strategy that would help the asset management company invest pension contributions profitably, i.e. choose the optimal investment project.

Information System Creation

In order to create the information system for pension fund management it is necessary to solve the following tasks:

- evaluate the amount of pension contributions rationally;
- hedge the pension fund against default;
- choose the asset management company optimally;
- invest pension contributions profitably.

In order to solve the first two tasks it is necessary to find the optimal value of pension contributions and to evaluate a put-to-default option (the instrument that allows to hedge pension fund against default). To derive a pricing equation we assume that the assets *W* of a non-state pension fund follow geometric Brownian motion (like in Black, Scholes, Merton's framework [Black, F., and Scholes, M., 1973]). Geometric Brownian motion is the most appropriate and widely used model that describes the behaviour of financial markets.

$$\frac{dW}{W} = \mu dt + \sigma dz \, , \tag{1}$$

where μ is the expected rate of return from the asset price, σ is the volatility of the asset price, t is time, z is Wiener process.

In order to better represent the path-dependent nature of policies issued by non-state pension funds, in this paper we assume that the liabilities *V* increases at a non-guaranteed rate $\upsilon = \upsilon(W/V)$ (unlike Merton's framework), i.e.,

$$\frac{dV}{V} = \upsilon(W/V)dt \, . \tag{2}$$

Most forms of liabilities (pension and insurance policies) that pension funds and insurance companies issue include participation mechanism: when the free asset ratio of the fund (*W/V*) is healthy (110% and above), the fund credits the liabilities with more generous reversionary bonus, than in case of free asset ratio just exceeding 100% mark. In the latter case policyholders are likely to get the contractually guaranteed rate only. When the solvency of the pension fund is compromised significantly, i.e., the free asset ratio is significantly below one the fund may either go into administration or undergo a restructuring plan, when some of its guarantees are taken back. That is why to model the path-dependent liability mechanism it is assumed that the bonus mechanism depends on the free asset ratio of the pension fund only and define the instantaneous bonus rate υ in terms of the function of the free asset ratio, i.e. $\upsilon = \upsilon(W/V)$.

Suppose that *p* is the fair value of pension (life) policy or put-to-default option. The variable *p* is function of *W*, *V* and *t*. Using Ito's lemma we obtain

$$dp = \left(\frac{\partial p}{\partial W} \mu W + \frac{\partial p}{\partial V} \upsilon\left(W/V\right) + \frac{\partial p}{\partial t} + \frac{1}{2}\sigma^2 W^2 \frac{\partial^2 p}{\partial W^2} \right) dt + \frac{\partial p}{\partial W} \sigma W dz \, .$$

We use the approach as in Black-Scholes framework for setting up a replicating portfolio consisting of -1 derivative security and $\partial p / \partial W$ units of the firm's assets. By such choice we will eliminate the Winner process and make the portfolio hedged and its return predictable. Thus we can pose the pricing equation for determining *p* (this corresponds to first two tasks stated above):

$$\frac{\partial p}{\partial t} + \upsilon\left(W/V\right)V\frac{\partial p}{\partial V} + rW\frac{\partial p}{\partial W} + \frac{1}{2}\sigma^2 W^2\frac{\partial^2 p}{\partial^2 W} - rp = 0, \quad (3)$$

where *r* is market rate of interest. This equation is derived basing on assumptions (1), (2). As in Black-Scholes framework the resulting partial differential equation (3) contains no stochastic components and is deterministic.

In our case pension fund credits liabilities with a guaranteed rate of interest r_g, if solvency W/V is below a certain level $1+\beta(\beta > 0)$, where *β* is a parameter, and distributes all the gains to the policyholders when solvency exceeds $1+\beta$. We obtain a reflecting boundary at point $1+\beta$ characterized by

$$\frac{\partial p}{\partial V} = 0 \,|_{W/V=1+\beta}. \quad (4)$$

Similarly, when solvency of the fund falls below a lower threshold $1-\gamma$ ($\gamma > 0$), where *γ* is a parameter, the pension fund (life company) may be forced to undergo a restructuring plan by reducing its obligations *V*. Hence, the lower threshold becomes the reflecting boundary as well

$$\frac{\partial p}{\partial V} = 0 \,|_{W/V=1-\gamma}. \quad (5)$$

In case of put-to-default option the key boundary condition is

$$p = \max(V - W, 0) \quad \text{when} \quad t=T. \quad (6)$$

In case of pension or insurance policy, it is

$$p = \min(W, V) \quad \text{when} \quad t=T. \quad (7)$$

This problem can be dealt with by means of similarity reduction.

The solution is as follows:

$$u(x,\tau) = e^{B\tau}\int_{-\infty}^{x^+} u_0(z)e^{A(x-z)}G(x-x^+, z-x^+, \tau)dz \quad (8)$$

$$p(W,V,t;T) = Vu(\ln(W/V), \sigma^2(T-t)/2), \quad (9)$$

where *A* and *B* are parameters, $u_0(x) = u(x,0) = \max(1-e^x, 0)$ for put-option and $u_0(x) = u(x,0) = 1$ for a policy, $x^+ = ln(1+\beta)$, τ is a new time parameter defined by $(T-t)\frac{\sigma^2}{2} = \tau$, new variable *x* is defined by $x = ln(W/V)$.

The solution to this equation with the policy boundary condition allows the manager to calculate the optimal value of pension contributions the employee has to pay under a certain pension scheme.

The solution the equation (3) was modeled under different conditions and analyzed.

Using the framework above it is possible to determine the optimal size of pension contributions. Next the pension contributions are passed to an asset management company. The only problem that arises is to cluster the set of all asset management companies into classes “reliable” and “unreliable” i.e. those ones that are able to invest pension contributions profitably and those that are unable. At the same time the “reliable” companies can be also classified using other additional features.

Any asset management company has following features or characteristics: the time of existence, the amount of managed funds, the total value of assets of managed funds, work experience with European enterprises, the average return of the managed funds during last month and so on. That is why very often it is very difficult to choose an asset management company. In our country such decisions are made basing on intuition. In order to make decisions using formal criterions it is necessary to construct an information model that allows to cluster asset management companies basing on their features.

There exists a large amount of clustering methods. The systems of artificial intelligence are becoming more and more popular. Very useful are also expert methods and models. Methods of fuzzy logics are widely used. Regression methods, linear and logistic regression in particular, can also be applied for solving the clustering problems. And also decision trees are widely used for this kind of tasks. But all these methods have certain disadvantages. For example, methods of expert estimations highly depend on the opinion of a certain expert; in the above stated task intersecting clusters are not considered, that is why methods of fuzzy logics are not applicable; regression methods require some knowledge about the character of mathematical model in advance, but this is not possible in this case; methods of decision trees require lots of additional information, such as probabilities of certain objects belonging to certain clusters.

That is why to solve the problem of clustering of asset management companies it is better to use neural networks.

Neural networks are widely used in those parts of financial and investment business where it is necessary to process lots of information quickly in order to get estimates and forecasts. The advantages of neural nets are as follows: the ability to model and forecast non-linear processes; ability to work with noisy data; quick learning and adaptation ability.

In order to cluster asset management companies it is proposed to use Kohonen nets. The most common application of Kohonen nets is the solution of clusterization problem i.e. unsupervised classification.

For the given problem a network with seven input units and twelve output units was chosen. Figure 1 shows the structure of the net.

The experiment published in paper [Путятина А.Е., 2007] showed that it is possible to separate Ukrainian asset management companies into six clusters and these clusters are robust.

When the asset management company is chosen and the pension contributions are passed to it, it is necessary to invest contributions profitably.

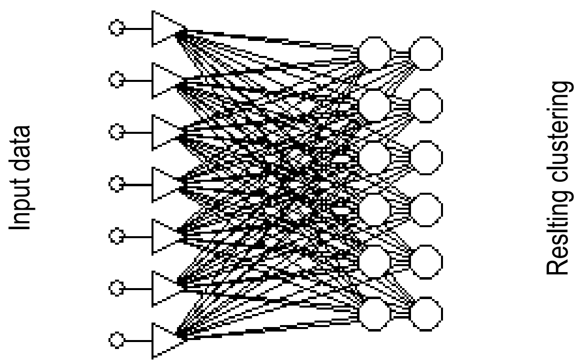

Figure 1. Kohonen network

The assets management company invests pension contributions into the industry fields in order to make profit and pay pensions in the future. Decision Trees [В.П. Романов, 2003] and Bayesian methods are applied in this paper to develop a strategy that would help the asset management company invest pension contributions profitably, i.e. choose the optimal investment project.

Let us assume that the manager of the asset management company has to make a decision concerning two projects. But in order to make a final decision the manager needs to make an additional expert examination (or several examinations).

Let t_1 and t_2 be two types of expert examination which the manager needs to make for the final decision. The case if manager decides not to make the examination let us define t_0.

Define the investment projects P_1 and P_2. The project P_1 requires X_1 to invest in, the expected return is R_1, as the result the asset management company makes the profit of $p_1 = R_1 – X_1$.

The project P_2 requires X_2 to invest in; the expected return equals R_2 and the expected profit is $p_2 = R_2 – X_2$.

The condition of the investment project is not known for sure, i.e. due to some economic situations, political factors and financial events the investment project may require some additional investments in the future. The expert examination makes the prediction about the future state of the project and, of cause, this prediction may be wrong or right with some probabilities. Let us define for simplicity the following: if the investment project requires additional investments then it is in a bad condition (denote $P_{1,2}^{bad}$); if it requires nothing then it is in a good condition (denote $P_{1,2}^{good}$).

The expert examination t_1 checks the state of the first investment project, and the expert examination t_2 checks the state of the second investment project. Define t_1^{good} the result of the first expert examination to consider the first investment project to be good, and t_1^{bad} the result of the first expert examination to consider the project bad. In the same way we can define t_2^{good} and t_2^{bad}.

If the project P_1 is in the bad condition, then the additional amount of money d_1 may be required to invest in the project. As the result the profit from P_1 totals $s_1 = R_1 – X_1 – d_1$.

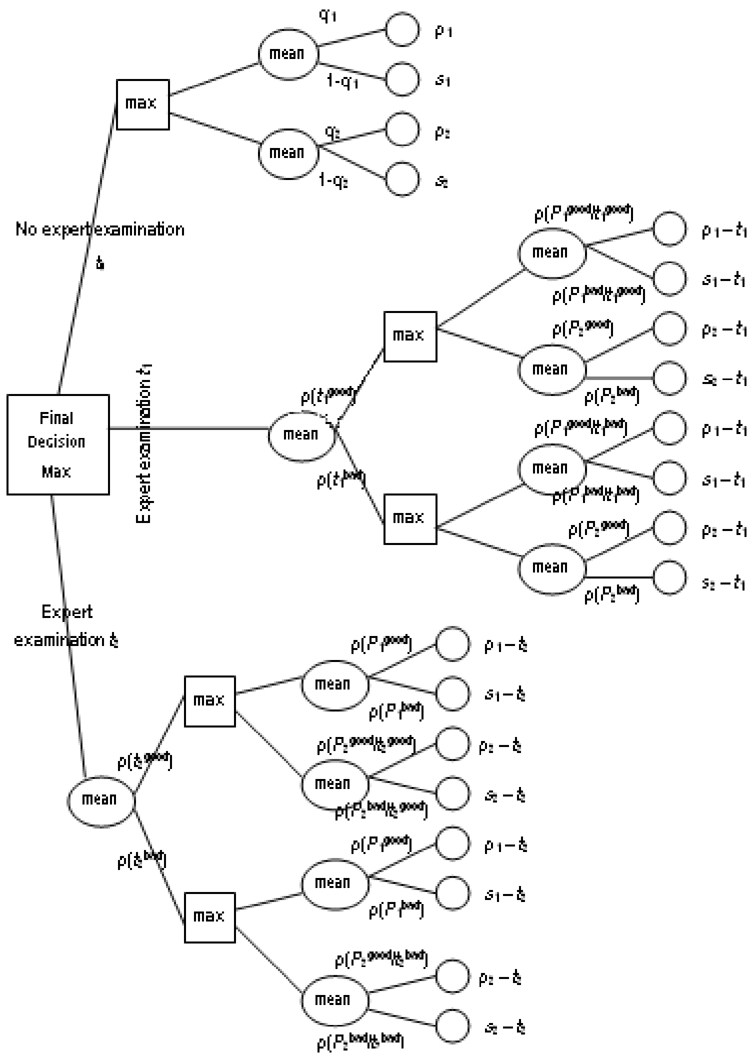

Figure 2. Decision Tree

If the object P_2 is in the bad condition, then additional investment equals d_2. Thus the assets management company makes the profit equal to $s_2 = R_2 - X_2 - d_2$.

Let us assume that we also know the probabilities of both projects to be in good condition. For the first project this probability equals q_1, for the second project q_2.

Let us assume that we know the conditional probabilities for the expert examinations to detect the states of the investment projects correctly. These conditional probabilities can be written in matrix forms. For the first investment project the probability matrix takes the form

$$p(t_1 / P_1) = \begin{Vmatrix} p(t_1^{good} / P_1^{good}) & p(t_1^{bad} / P_1^{bad}) \\ p(t_1^{bad} / P_1^{good}) & p(t_1^{good} / P_1^{bad}) \end{Vmatrix}.$$

For the second investment project the probability matrix takes the form

$$p(t_2 / P_2) = \begin{Vmatrix} p(t_2^{good} / P_2^{good}) & p(t_2^{bad} / P_2^{bad}) \\ p(t_2^{bad} / P_2^{good}) & p(t_2^{good} / P_2^{bad}) \end{Vmatrix}.$$

In order to find the optimal strategy (choose the best investment project) it is necessary to calculate the probabilities of every tree branch. It is easy to calculate the probabilities of the t_0 branches (without the expert examination): they equal q_1 and q_2.

In order to calculate the probabilities of the random nodes along the branch t_1, we must find out the unconditional probabilities $p(t_1^{good})$ and $p(t_2^{good})$, and also the posterior probabilities $p(P_1^{good(bad)}/t_1^{good(bad)})$, $p(P_2^{good(bad)}/t_2^{good(bad)})$. These probabilities are not given explicitly that is why we use the Bayesian rule [4].

While the expert examination t_1 is applied only to the first investment project P_1, we can derive $p(P_2^{good(bad)}/t_1^{good(bad)}) = p(P_2^{good(bad)})$. In the same way we can calculate the probabilities of the random nodes along the branch t_2.

Next it is necessary to make the reverse analysis having chosen the maximal values:

- calculate the mean value for every random node and calculate the maximal profit of every decision node;
- track the branch with maximal profit down from the root in order to find the optimal strategy.

The structure of the decision tree for investment analysis is shown in the Figure 2.

The structure of this decision tree (the amount of branches) is the most widely used structure of decision trees that are used to compare, analyse, and choose market projects or objects in western economy.

Conclusion

In this paper a mathematical model for estimating the far value of pension contributions was reviewed. Then the pension contributions are passed to an asset management company which is chosen among the existing ones using Kohonen networks. Decision trees can be used in order to choose the optimal investment project and to invest pension contributions profitably. The pension contributions are then invested into the chosen project and the return is redistributed to pay pensions.

This entire framework together makes a complete information system for pension fund management.

The future research will consist of applying other methods and techniques and comparing the results.

Bibliography

[Black, F., and Scholes, M., 1973] “The Pricing of Options and Corporate Liabilities.” Journal of Political Economy 81 (1973):637.

[Merton R. C., 1974] On pricing of corporate debt: the risk structure of interest rates. Journal of Finance, 1974, 29: 449-470.

[P. Jorgensen, 2005], “Lognormal Approximation of Complex Path- dependent Pension Scheme Payoffs” pre-print of University of Aarhus Denmark, 2005, available electronically:http://skinance.com/Papers/2005/PLJ.pdf

[Путятина А.Е., 2007], “Применение сетей Кохонена для кластеризации компаний по управлению активами”, Радиоэлектроника и информатика, ХНУРЭ, Украина, accepted.

[В.П. Романов, 2003], Интеллектуальные информационные системы в экономике. – М.: «Экзамен», 2003. – 496 с.

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