Neural Nets

SELF-LEARNING FUZZY SPIKING NEURAL NETWORK AS A NONLINEAR PULSE-POSITION THRESHOLD DETECTION DYNAMIC SYSTEM BASED ON SECOND-ORDER CRITICALLY DAMPED RESPONSE UNITS

Yevgeniy Bodyanskiy, Artem Dolotov, Iryna Pliss

Abstract: *Architecture and learning algorithm of self-learning spiking neural network in fuzzy clustering task are outlined. Fuzzy receptive neurons for pulse-position transformation of input data are considered. It is proposed to treat a spiking neural network in terms of classical automatic control theory apparatus based on the Laplace transform. It is shown that synapse functioning can be easily modeled by a second order damped response unit. Spiking neuron soma is presented as a threshold detection unit. Thus, the proposed fuzzy spiking neural network is an analog-digital nonlinear pulse-position dynamic system. It is demonstrated how fuzzy probabilistic and possibilistic clustering approaches can be implemented on the base of the presented spiking neural network.*

Keywords: *computational intelligence, hybrid intelligent system, spiking neural network, fuzzy receptive neuron, fuzzy clustering, automatic control theory, analog-digital system, second order damped response system.*

ACM Classification Keywords: *I.2.6 [Artificial Intelligence]: Learning – Connectionism and neural nets; I.2.8 [Artificial Intelligence]: Problem Solving, Control Methods, and Search – Control theory; I.5.1 [Pattern Recognition]: Models – Fuzzy set, Neural nets; I.5.3 [Pattern Recognition]: Clustering – Algorithms.*

Conference: *The paper is selected from Seventh International Conference on Information Research and Applications – i.Tech 2009, Varna, Bulgaria, June-July 2009*

Introduction

Among a variety of computational intelligence means for data processing in the absence of a priori information [Haykin, 1999; Sato-Ilic, 2006], self-learning spiking neural networks (SLSNNs) are attracting growing attention both as biologically more realistic models than neural networks of the previous generations [Hopfield, 1995; Gerstner, 2002] and as considerably fast and computationally powerful processing systems [Natschlaeger, 1998; Maass, 1997]. For the last decade, SLSNNs have been successfully used in complex data processing problems solving, particularly in satellite image processing [Bohte, 2002]. Moreover, hybrid intelligent systems combining SLSNNs and fuzzy methodology approaches, known as self-learning fuzzy spiking neural networks (SLFSNNs), revealed a new area where spiking neural networks can be successfully applied, namely fuzzy clustering tasks [Bodyanskiy, 2008a-d].

Although spiking neural networks are becoming a popular computational intelligence tool for various technical problems solving, their architecture and functioning are treated in terms of neurophysiology rather than in terms of any technical sciences apparatus.

In this paper, a technically plausible description of a spiking neural network is introduced. It is proposed to define a spiking neural network in terms of well-known and widely used apparatus of classical automatic control theory based on the Laplace transform. It is shown that a spiking neural network is a pulse-position threshold detection system based on second-order damped response units. Such kind of description allows of, on the one hand, using it as an analog-digital system in technical problems solving. On the other hand, spiking neural network architecture and functioning formalizing simplifies the further spiking neural networks theoretical research.

Self-Learning Fuzzy Spiking Neural Network Architecture

A self-learning fuzzy spiking neural network is depicted on Figure 1. As illustrated, it is a heterogeneous three-layered feed-forward neural network with lateral connections in the second hidden layer.

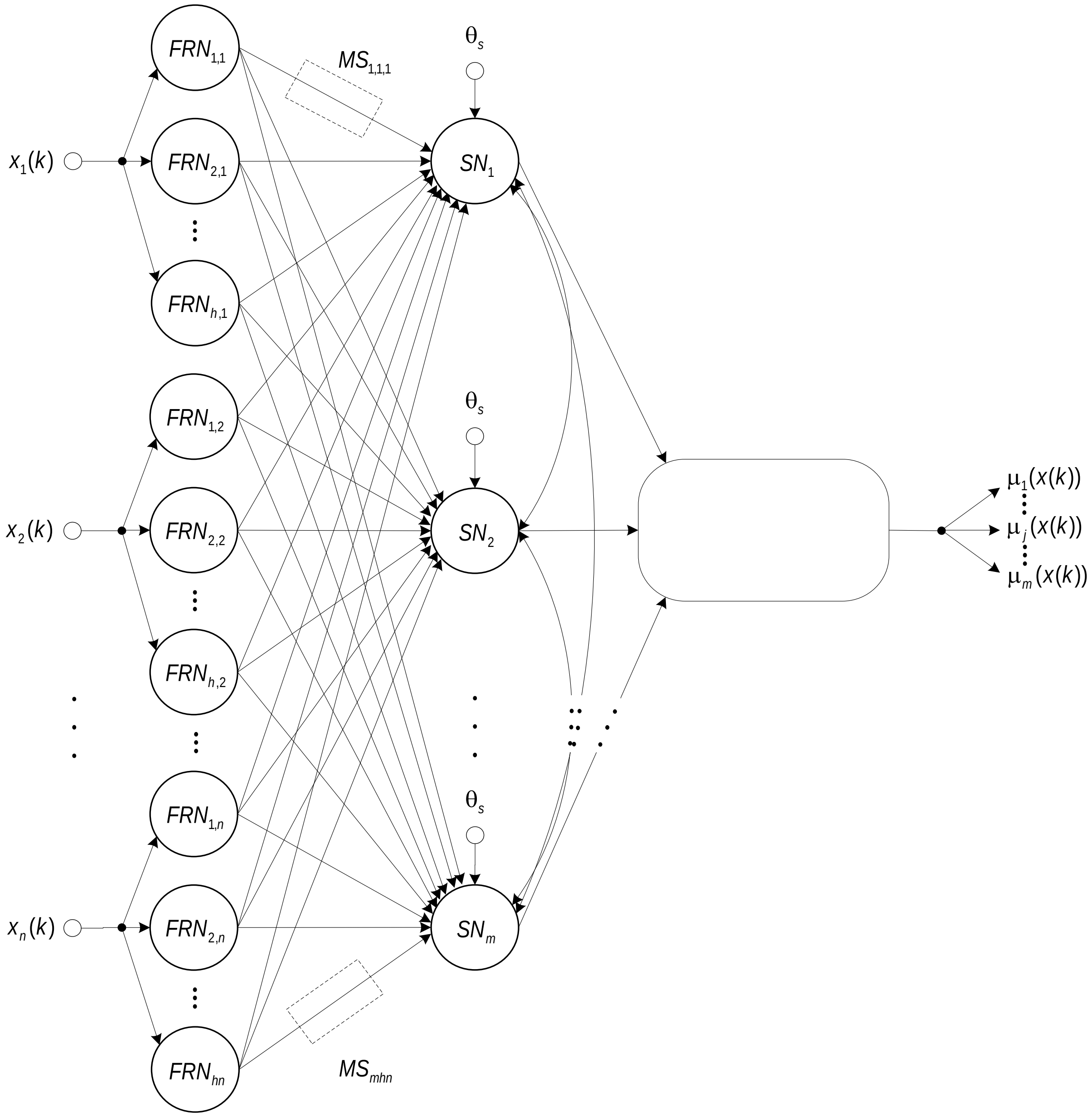

Figure 1. Self-learning fuzzy spiking neural network architecture

The first hidden layer performs pulse-position transformation of $n \times 1$-dimensional input patterns $x(k)$ (here, $k = 0,1,\ldots,N$ is a pattern number) to the input vector of spikes $\delta\left(t - t^{[0]}(x(k))\right)$ where each spike is defined by its firing time ($\delta(\bullet)$ is the Dirac delta function). The transformation is performed by population coding that implies that an input $x_i(k)$, $i = 1,2,...,n$, is processed at the same time by a pool of h fuzzy receptive neurons FRN_{li}, $l = 1,2,...,h$.

Clusters detection takes place in the second hidden layer that consists of m spiking neurons SN_j, $j = 1,2,...,m$ (m is a number of clusters to be detected). They are connected with neurons of the previous layer by multiple synapses MS_{jli}. After learning phase, a spiking neuron SN_j emits outgoing spike $\delta\left(t - t_j^{[1]}(x(k))\right)$ for each input pattern $x(k)$, and the neuron firing time defines the distance of the input pattern to the neuron's center.

The third layer processes distances of the input patterns to spiking neurons' centers, performs fuzzy partitioning, and produces the membership levels $\mu_j(x(k))$, $j = 1,2,...,m$.

It is worth to note that two first hidden layers form conventional architecture of SLSNN [Bohte, 2002]. In case of such network using, the cluster that an input pattern belongs to is determined by the earliest fired spiking neuron.

Fuzzy Receptive Neurons

Architecture of fuzzy receptive neurons of the first hidden layer [Bodyanskiy, 2008c] is identical to the one of receptive neurons that were proposed to perform population coding in SLSNNs [Bohte, 2002]. The difference between them is an interpretation of their functioning and the method of activation functions setting.

As a rule, a receptive neuron activation function is bell-shaped (Gaussian usually), and activation functions of the neurons within a pool are shifted, overlapped, and of different widths. In a general case, firing time of a spike emitted by receptive neuron lies in a certain interval $[0, t_{max}^{[0]}]$ referred to as a coding interval and is defined by the expression

$$t_{li}^{[0]}(x_i(k)) = \left\lfloor t_{max}^{[0]}\left(1 - \psi\left(\left|x_i(k) - c_{li}^{[0]}\right|, \sigma_i\right)\right)\right\rfloor, \tag{1}$$

where $\lfloor \bullet \rfloor$ is the floor function, $\psi(\bullet,\bullet)$, $c_{li}^{[0]}$, and σ_i are the receptive neuron's activation function, center, and width respectively.

One can readily see that the layer of receptive neurons pools is identical to a fuzzification layer of neuro-fuzzy systems like Takagi-Sugeno-Kang networks, ANFIS, etc [Jang, 1997]. Considering activation function $\psi_{li}(x_i(k))$ as a membership function, the receptive neurons layer can be treated as the one that transforms input data set to a fuzzy set that is defined by values of activation-membership function $\psi_{li}(x_i(k))$ and is expressed over time domain in form of firing times $t_{li}^{[0]}(x_i(k))$. In fact, each pool of receptive neurons performs zero order Takagi-Sugeno fuzzy inference [Jang, 1997]

$$\text{IF } x_i(k) \text{ IS } X_{li} \text{ THEN OUTPUT IS } t_{li}^{[0]}, \tag{2}$$

where X_{li} is the fuzzy set with membership function $\psi_{li}(x_i(k))$. Thus, one can interpret a receptive neurons pool as a certain linguistic variable and each receptive neuron (more precisely, fuzzy receptive neuron) within the pool – as a linguistic term with membership function $\psi_{li}(x_i(k))$. This way, having any a priori knowledge of data structure, it is possible to adjust activation functions of the first layer neurons to fit them and thus, to get better clustering results.

Spiking Neuron as a Nonlinear Dynamic System

Spiking neuron as a nonlinear dynamic system is depicted on Figure 2. As illustrated, multiple synapses of spiking neuron SN_j transform the incoming pulse-position signal to a continuous-time form, and its soma transforms the incoming continuous-time signal back to pulse-position form.

In a scope of automatic control theory [Goodwin, 2001; Phillips, 2000; Dorf, 2001], multiple synapse MS_{jli} is a dynamic system that consists of different time delays, second-order damped response units, and adjustable gains that are connected in parallel. Each group of time delay, second-order damped response unit, and gain form a subsynapse of multiple synapse. As a response to incoming spike, the subsynapse produces delayed weighted postsynaptic potential $u_{jli}^{p}(t)$, and the multiple synapse produces total postsynaptic potential $u_{jli}(t)$ that arrives to spiking neuron soma.

Transfer function of a second-order damped response unit with unit gain factor is

$$G(s)=\frac{1}{(\tau_1 s+1)(\tau_2 s+1)}=\frac{1}{\tau_4^2 s^2+\tau_3 s+1}, \tag{3}$$

where s is the Laplace operator, $\tau_{1,2}=\frac{\tau_3}{2}\pm\sqrt{\frac{\tau_3^2}{4}-\tau_4^2}$, $\tau_1\ge\tau_2$, $\tau_3\ge 2\tau_4$, and its impulse response is

$$\tilde{\varepsilon}(t)=\frac{1}{\tau_1-\tau_2}\left(e^{-\frac{t}{\tau_1}}-e^{-\frac{t}{\tau_2}}\right). \tag{4}$$

Putting $\tau_1=\tau_2$ (that corresponds to a second-order critically damped response system) and applying l'Hôpital's rule, one can obtain

$$\tilde{\varepsilon}(t)=\frac{t}{\tau_1^2}e^{-\frac{t}{\tau_1}} \tag{5}$$

Comparing a spike-response function [Gerstner, 2002]

$$\varepsilon(t)=\frac{t}{\tau}e^{1-\frac{t}{\tau}}, \tag{6}$$

where τ is the membrane potential decay time constant, with (5) leads us to the following expression:

$$\varepsilon(t)=e\tau\tilde{\varepsilon}(t). \tag{7}$$

Thus, transfer function of the second-order critically damped response unit whose impulse response corresponds to a spike-response function is

$$G(s)=\frac{e\tau}{(\tau s+1)^2}. \tag{8}$$

Taking into account (8), transfer function of the p-th subsynapse of MS_{jli} takes form

$$U_{jli}^{p}(s)=\frac{\tau w_{jli}^{p}e^{1-d^{p}s}}{(\tau s+1)^2}, \tag{9}$$

where w_{jli}^{p} and d^{p} are synaptic weight and time delay of the subsynapse.

The Laplace transform of a spike $\delta\left(t-t_{li}^{[0]}(x_i(k))\right)$ is

$$L\left\{\delta\left(t-t_{li}^{[0]}(x_i(k))\right)\right\}=e^{-t_{li}^{[0]}(x_i(k))s}, \tag{10}$$

so taking into account transfer function of multiple synapse MS_{jli}

$$U_{jli}(s)=\sum_{p=1}^{q}U_{jli}^{p}(s)=\sum_{p=1}^{q}\frac{\tau w_{jli}^{p}e^{1-d^{p}s}}{(\tau s+1)^2}, \tag{11}$$

where q is a number of subsynapses within a multiple synapse, the Laplace transform of the multiple synapse output can be expressed in the following form:

$$u_{jli}(s)=e^{-t_{li}^{[0]}(x_i(k))s}U_{jli}(s)=e^{-t_{li}^{[0]}(x_i(k))s}\sum_{p=1}^{q}\frac{\tau w_{jli}^{p}e^{1-d^{p}s}}{(\tau s+1)^2}=\sum_{p=1}^{q}\frac{\tau w_{jli}^{p}e^{1-\left(t_{li}^{[0]}(x_i(k))+d^{p}\right)s}}{(\tau s+1)^2}. \tag{12}$$

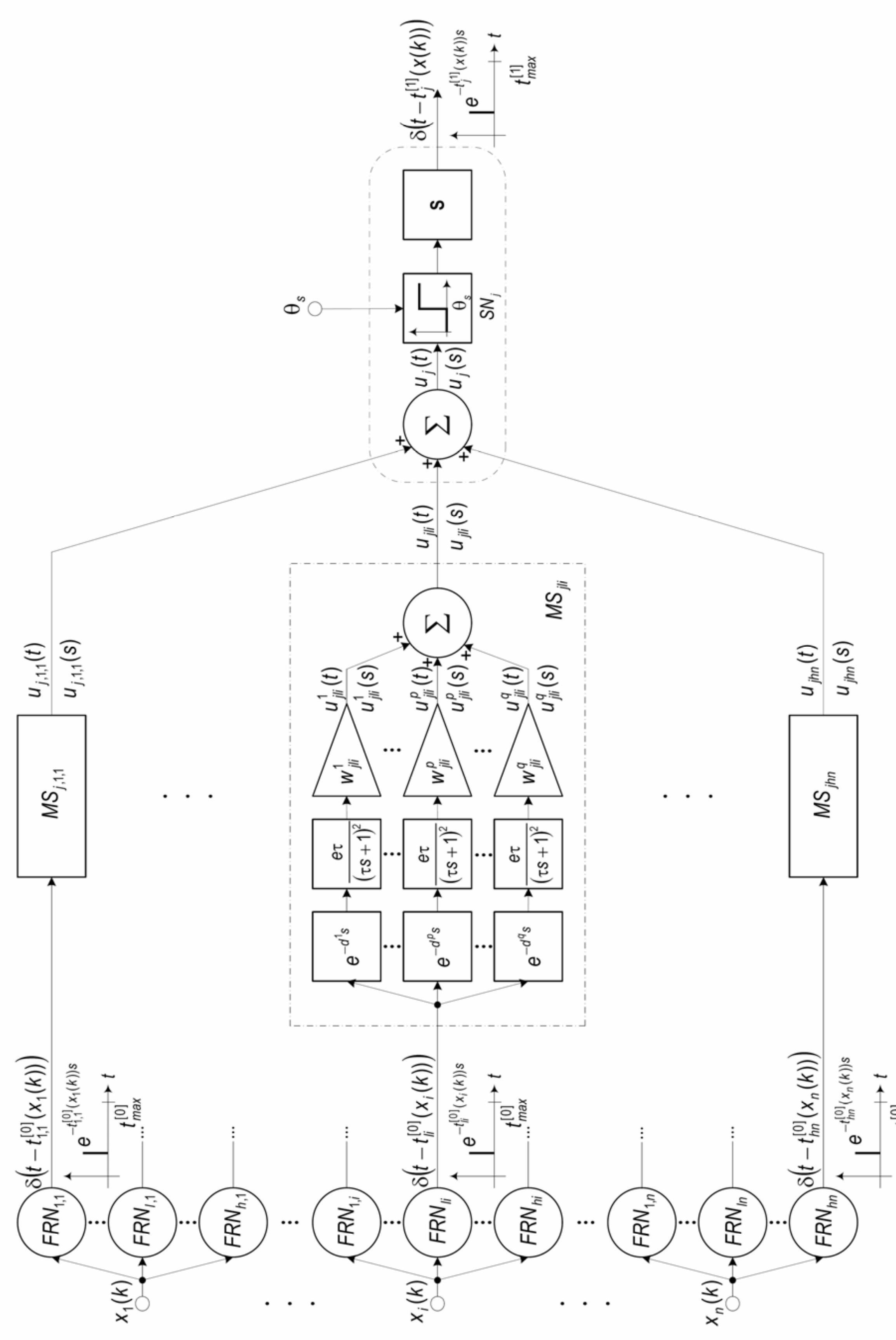

Figure 2. Spiking neuron as a nonlinear dynamic system

Here it is worth to note that since it is impossible to use δ-function in practice [Phillips, 2000], it is convenient to model it with impulse of a triangular form as shown on Figure 3. Such impulse is similar to a biological spike and satisfies the condition

$$\lim_{\Delta \to 0} \rho(t, \Delta) = \delta(t) . \tag{13}$$

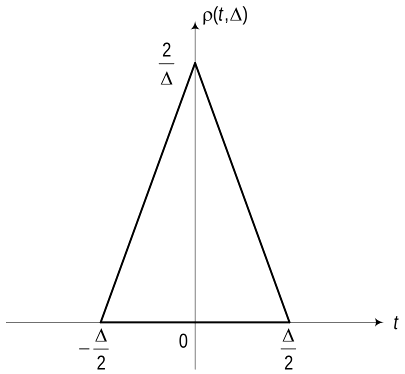

Figure 3. Triangular impulse

The Laplace transform of spiking neuron SN_j membrane potential can be expressed as follows:

$$u_j(s) = \sum_{i=1}^{n} \sum_{l=1}^{h} \sum_{p=1}^{q} \frac{\tau w_{jli}^{p} e^{1-\left(t_{li}^{[0]}(x_i(k))+d^p\right)s}}{(\tau s + 1)^2} . \tag{14}$$

Spiking neuron soma firing behavior is modeled by an element relay with dead zone θ_s that is defined by nonlinear function

$$f(u) = \frac{\operatorname{sign}(u - \theta_s) + 1}{2} , \tag{15}$$

and a derivative unit with transfer function

$$G_D(s) = s \tag{16}$$

being connected in series.

At the instance when soma membrane potential $u_j(t)$ reaches the firing threshold θ_s, the element relay triggers and emits the Heaviside step functions on its output. Differentiating the latter gives an outgoing spike $\delta\left(t - t_j^{[1]}(x(k))\right)$. Thus, spiking neuron soma functions as a threshold detection unit.

During learning phase, on each learning epoch, the temporal Hebbian rule updates weights of the spiking neuron-winner in the following way [Natschlaeger, 1998; Bohte, 2002]: the weights of those subsynapses which contributed to the neuron's firing are strengthened, whereas weights of subsynapses which did not contribute are weakened. Thus, weights are adjusted to move the center of the neuron-winner closer to input pattern. Lateral inhibitory connections in the second hidden layer are used only during the learning to implement 'winner-takes-all' mechanism. After learning phase is complete, the lateral connections are disabled.

Output Fuzzy Clustering Layer

The output layer, namely output fuzzy clustering layer, takes firing times of spikes $\delta\left(t - t^{[1]}(x(k))\right)$ arriving from the second layer, and either performs fuzzy partitioning of the input patterns $x(k)$ using probabilistic approach [Bodyanskiy, 2008a, b]

$$\mu_j(x(k)) = \frac{\left(t_j^{[1]}(x(k))\right)^{\frac{2}{1-\zeta}}}{\sum_{\iota=1}^{m}\left(t_\iota^{[1]}(x(k))\right)^{\frac{2}{1-\zeta}}}, \tag{17}$$

where ζ is the fuzzifier that determines boundary between clusters and controls the amount of fuzziness in the final partition, or possibilistic approach [Bodyanskiy, 2008d]

$$\mu_j(x(k)) = \left(1 + \left(\frac{\left(t_j^{[1]}(x(k))\right)^2}{\lambda_j}\right)^{\frac{1}{\zeta-1}}\right)^{-1}, \tag{18}$$

$$\lambda_j = \frac{\sum_{k=1}^{N}\mu_j^{\zeta}\left(t_j^{[1]}(x(k))\right)^2}{\sum_{k=1}^{N}\mu_j^{\zeta}(x(k))}, \tag{19}$$

It is readily seen that the output layer evaluates fuzzy membership similarly to well-known fuzzy c-means or possibilistic c-means algorithms [Bezdek, 2005] – depending on the used approach.

Output fuzzy clustering layer is disabled during learning phase and is used on classification phase only.

Conclusion

Spiking neural networks are more realistic models of real neuronal systems than artificial neural networks of the previous generations. Nevertheless, they can be described in a strict technically plausible way. Treating a spiking neural network in a scope of automatic control theory, it is easily seen that spiking neuron synapse is nothing other than a second-order damped response system, and the soma is a threshold detection system. Spiking neural network implemented on their basis is an analog-digital nonlinear dynamic system that conveys and processes information both in pulse-position and continuous-time forms. Such precise formal description of spiking neural network architecture and functioning presents a significant step toward evolving of artificial neural networks theory as a part of computational intelligence paradigm.

Acknowledgement

The paper is partially financed by the project ITHEA XXI of the Institute of Information Theories and Applications FOI ITHEA and the Consortium FOI Bulgaria (www.ithea.org, www.foibg.com).

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