DYNAMIC SYSTEM QUALITY PROVIDING UNDER UNDETERMINED DISTURBANCES. MULTI-DIMENSIONAL CASE

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Abstract: *Multi-dimensional dynamic system under impact of the undetermined disturbing influences is reviewed. Control system that allows to influence over system reaction value in proportion to the disturbance is defined. An algorithm for system quality estimation and making decision about control aiming to provide required quality is proposed. Control algorithm for multi-dimensional system is developed.*

Keywords: *dynamic system quality; undetermined disturbances; condition estimation; resulting disturbance control; control algorithm; quality function.*

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Introduction

Dynamic system quality guaranteeing under the undetermined disturbances is one of the current problems of control theory. Existing methods presuppose either complete a priori information about disturbances, or their constraints are known [Lin, Su, 2000], [Poliak, Sherbakov, 2002], [Nikiforov, 2003], [Hou, Muller, 1992], while regulators with dynamic disturbance compensators might have high dimensions [Liubchyk, 2007].

Control Structure Forming

Dynamic system with n state variables can be described by a matrix equation

$$\dot{x} = -Ax - KU + EF , \tag{1}$$

where $x = \left[x_1 \dots x_n\right]^T$ - state variables; $x_{2k} = \dot{x}_{2k-1}$; disturbance $F = \left[F_1 \dots F_m\right]^T$, $E = \left\|e_{nm}\right\|$ - disturbance coefficient matrix $(n \times m)$, whose uneven lines are filled by zeros $\left(e_{nm} = 0,\ n = 2k,\ k = 1, \dots, 0.5n\right)$; controlling $U = Bx, B = \left[b_1 \dots b_n\right],\ K = \left[0\ k_2\ 0 \dots 0\ k_n\right]^T$; A – $(n \times n)$ system parameter matrix.

Let's put equation (1) in operator form

$$\left(sI + A\right)x = -KBx + EF , \tag{2}$$

s - Laplace operator, control matrix coefficients b_i are in general case the polynomials depending on s. Structure and order of those polynomials are defined by optimizing control functional.

From (2) we can get

$$x = \left(sI + A + KB\right)^{-1} EF = \left(sI + A + KB\right)^{aa} EF\Delta^{-1} , \tag{3}$$

$x = \left(sI + A + KB\right)^{aa} = \left\|a_{nn}^{aa}\right\|$ - algebraic complement matrix $(n \times n)$ for matrix $\left(sI + A + KB\right)$, Δ - characteristic polynomial of the system (3).

By analogy we can define

$$x = \left(sI + A\right)^{-1} \Delta F , \tag{4}$$

$$\Delta F = EF - KBx = (sI + A + KB)^{-1} \cdot (sI + A)EF = (sI + A + KB)^{aa} \cdot (sI + A)EF\Delta^{-1} . \tag{5}$$

If $(KB)_{ij} \gg (sI+A)_{ij},\ i,j=1,\dots,n$, then system (3) characteristic polynomial when $K_i = K_0$,

$$\Delta = K_0 \Delta_0 \tag{6}$$

where Δ_0 polynomial does not depend on K_i control coefficients.

If $(KB)_{ij} \sim (sI+A)_{ij},\ i,j=1,\dots,n$, let us form an additional control channel for the system (3) so that

$(sI+A+KB)x = -K_T(sI+A+KB)+EF,\ K_T = diag\left[K_{Ti}\right]$.

Then

$$x = (I+K_T)^{-1}(sI+A+KB)^{-1}EF,\ I = diag\left[1\right], \tag{7}$$

and while $K_{Ti} >> 1$

$$x = K_T^{-1}(sI+A+KB)^{-1}EF\ ,\ x_i = (1+K_{Ti})^{-1}\sum_{j=1}^{n} a_{ij}^{aa} f_j,\ f_j = \sum_{k=1}^{m} e_{jk}F_k, \tag{8}$$

$$\Delta F = EF - K_T(sI+A+KB)x = K_T^{-1}EF, \tag{9}$$

$$x = (sI+A+KB)^{-1}\Delta F\ . \tag{10}$$

In those cases K_0, K_{Ti} control coefficients alteration causes a proportional change in the state variables x value, (3), (8), and resulting disturbance ΔF (5), (9) affecting the dynamic system.

System Condition and Resulting Disturbance Estimation

Let us assume that system state variables x are measurable. System quality means having x variables in the certain range of alteration. Disturbances F (or ΔF) may take the system out of this range and cause errors.

To estimate disturbance effect over the state variables it is possible to use Duamel integral. For the system (4) we get

$$x = (sI+A)^{aa}\Delta F \Delta_1^{-1} = \left[\sum_{j=1}^{n} c_{1j}^{aa}\Delta f_j \dots \sum_{j=1}^{n} c_{nj}^{aa}\Delta f_j\right]^T \Delta_1^{-1},$$

$$x_i = \Delta_1^{-1}\sum_{j=1}^{n} c_{ij}^{aa}\Delta f_j,\ \Delta f_j = \sum_{k=1}^{m} e_{jk}\Delta F_K\ ,$$

Δ_1 - characteristic polynomial of the system (4).

By analogy for the system (10)

$$x = (sI+A+KB)^{aa}\Delta F \Delta^{-1} = \left[\sum_{j=1}^{n} a_{1j}^{aa}\Delta f_j \dots \sum_{j=1}^{n} a_{nj}^{aa}\Delta f_j\right]^T \Delta\ ,$$

$$x_i = \Delta^{-1}\sum_{j=1}^{n} a_{ij}^{aa}\Delta f_j,\ \Delta f_j = \sum_{K=1}^{m} e_{jk}\Delta F_K\ .$$

Then

$$x_i = \int_0^t \sum_{j=1}^{n} w_{ij}(t-\tau)\Delta F_j(\tau)d\tau = \int_0^t \psi(t,\tau)d\tau, \tag{11}$$

$$\psi(t,\tau) = \sum_{j=1}^{n} w_{ij}(t-\tau)\Delta F_j(\tau) = \dot{x}(\tau) = x_{i+1}(\tau)\ , \tag{12}$$

where $w_{ij}(\tau)$ - system weight functions for i state variable from disturbance by j variable. Those are known functions for the systems (4) and (9).

In this way, to estimate system quality by its state variables x_i it is sufficient to know function (12) depending on the acting disturbances and system (4), (9) dynamic features. The state variables will be characterized by square limited by the function (12) at observation interval.

Making Decision about Starting Disturbance Control

Based on the system quality definition introduced, to provide quality it is necessary for the function (12) (quality function) to have value within predefined range, whose square S_Q does not exceed the limit value x_{ip} of the variable x_i in the interval where the function (12) has constant sign

$$S_Q \le x_{ip} . \tag{13}$$

To define the quality range it is necessary to estimate (or measure) x_i value in the $t \in \left(0, t_g\right)$ interval, where t_g - moment of time when $x_i\left(t_g\right) = \varepsilon x_{ip}$, $0 < \varepsilon < 1$,

and function (12) that in the same interval creates a range $S_1 = x_i(t_g)$ as part of the range (13). Second part S_2 of the range (13) should provide meeting the demand

$$S_1 + S_2 \le S_Q,\ S_2 \le (1-\varepsilon)S_Q . \tag{14}$$

When $\varepsilon = 0.5$ it is constructed as a reflection of the function (12) relative to $t = t_g$ line at the range $t > t_g$

$$\psi(\tau) = \psi(2t_g - \tau) . \tag{15}$$

$t = t_g$ is a moment of disturbance control start.

Disturbance Control and System Quality Providing Algorithm

Figures (3), (6), and (7) prove that value of external disturbance that impacts the system is altered by the operators K_0 or K_T. Meanwhile external disturbance value alteration is equal for the resulting disturbance as well as for quality function (12) and system state variables. Thus, disturbance control algorithm by the operators K_0 or K_T can be implemented by the drift of one of those functions from its permissible value. This permissible value should be forecasted considering system dynamic features. Let us use function (12) and let us develop a control algorithm relative to (14) and (15). Let us assume quality function (15) to be permissible

$$\psi(2t_g - \tau) = \psi_p(\tau),\ \tau = t_g,\ x_i > x_{ip} .$$

Let us define the drift of the existing x_i from the permissible state variable

$$y = x_i - x_{ip} = \int_0^t \left[\psi(t,\tau) - \psi_p(\tau)\right] d\tau,\ \left|x_i\right| > \left|x_{ip}\right| ,$$

and let us use direct Lyapunov method to define the control $K_T(t)$ (7) or $K_0(t) = 1 + K_*(t)$ (6).

Let us introduce Lyapunov function $V = y^2$ and provide

$$\frac{dV}{dt} = 2y\frac{dy}{dt} < 0 , \tag{16}$$

where $y(t)=\int_0^t\left[\psi_g(\tau)-\psi_p(\tau)\right]d\tau,\ \frac{dy}{dt}=\psi_g(\tau)-\psi_p(\tau),\ \ \psi_g=\frac{\psi}{1+K_{Ti}},\left(\psi_g=\frac{\psi}{1+K_*}\right).$

Condition (16) will be fulfilled if

$$K_{Ti}(t)>\frac{\psi(t)-\psi_p(t)}{\psi_p(t)},\ t>t_g,\ |x_i|>|x_{ip}|,\ K(t)>0. \qquad (17)$$

Condition (17) is fulfilled by the following rules of operator K_{Ti} (8) (or K_0 (6)) formation:

$$K_{Ti}(t)=\left[\frac{\psi(t)}{\psi_p(t)}-1\right]\cdot\left[1+sign\left(|x_i|-|x_{ip}|\right)\right]\cdot\left[1+sign\left(\psi-\psi_p\right)\right],$$

$$K_{Ti}(t)=\left[\frac{\psi(t)}{\psi_p(t)}-1\right]\cdot\left[1+sign\left(|x_i|-|x_{ip}|\right)\right]\cdot\left[1+sign\left(\psi-\psi_p\right)\right]\cdot\left[\frac{x_i}{x_{ip}}-\varepsilon\right]^2. \qquad (18)$$

Operator (18) becomes undefined when $\psi_p=0,\ y>0.$ To avoid uncertainty, it is sufficient to set in the algorithms (18)

$$K_{Ti}(t)=K(\psi_p\to 0_+, y>0)=const \qquad (19)$$

in the time interval starting from the moment where (19) value is accepted until the moment when $\psi(t)=0.$

Conclusion

Proposed algorithm of controlling the undetermined external disturbances is based upon the quality function estimation, that takes into account the disturbance itself and system dynamic features. Quality function allows to forecast the possible scale of its value range and to make a decision about control start. Control algorithm, developed on the basis of current and estimated quality function values, provides the necessary quality of the dynamic system.

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