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APPLICATION OF GENETIC ALGORITHMS TO VECTOR OPTIMIZATION OF THE AUTOMATIC CONTROL SYSTEMS

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Abstract: Methods for calculation of quality indexes for automatic control systems are presented. For the optimization of quality indexes defined only in a stability domain a vector objective function of varied parameters of the system is proposed. The stepwise principle of successive satisfaction of constraints for the passage into the definition domain of quality indexes is considered, as well as a rational mechanism of its realization in the form of the priority optimization of the vector objective function. For the optimization of the vector objective functions genetic algorithms as vector optimization methods are presented. Their application allows one to steer the optimization process from any initial point of the space of varied parameters into the stability domain of the system and to find the optimum of the quality indexes in this domain. The efficiency of the proposed application of vector genetic algorithms for the quality indexes optimization is confirmed by computational experiments.

Keywords: genetic algorithms, vector optimization methods, automatic control systems, quality indexes.

ACM Classification Keywords: G.1.6 Optimization - Nonlinear programming

Introduction

For the synthesis of automatic control systems (ACS) quality of their functioning can be presented by different criteria [Besekerskiy, Popov, 2004]. There are many difficulties even at the synthesis of the linear control systems, and the results of synthesis substantially depend on the applied criteria [Poljak, Scherbako, 2005]. Yet the task of synthesis of the nonlinear systems is even more difficult.

Quality criteria for both linear and nonlinear ACS can be reflected by the direct quality indexes (DQI) and improved integral quadratic estimations (IQE) [Severin, Nikulina, 2004], [Severin, 2004], [Severin, 2005]. The optimization tasks of these criteria have an identical feature — their objective functions domain is limited by stability conditions [Severin, 2008]. Therefore the standard optimization methods can not be effectively used for the optimization of ACS quality indexes.

For the synthesis of the linear control systems by means of DQI and improved IQE optimization the vector objective functions are offered and for their optimization the direct methods of unconstrained minimization are modified [Severin, 2005]. The methods optimization laboratory OPTLAB is developed in MATLAB system [Severin, 2009]. However, the offered vector optimization methods do not allow to find global extremes.

The decisions search of optimization tasks in very large and difficult domains is executed by genetic algorithms which are evolutionary computations variety and behave to heuristic search methods [Voronovskiy, Makhotilo, Petrashev, Sergeev, 1997], [Setlak, 2004]. Genetic algorithms are used for optimization tasks decision based on the principles and mechanisms, reminding biological evolution [Panchenko, 2007], [Weise, 2008].

The purpose of this paper consists of conception development of synthesis for linear and nonlinear ACS on the basis of DQI, improved IQE and modifications of genetic algorithms for vector optimization.

The methods of DQI and improved IQE calculation are considered for linear and nonlinear systems. Statements of systems optimization tasks are presented with using vector objective functions. Modification of genetic algorithms is offered for vector optimization of ACS quality indexes.

Quality Indexes Calculation of Linear Automatic Control Systems

Let linear model of ACS, depending on a vector of varied parameters $x \in R^p$, in state space looks like:

$$\frac{\partial X(x,t)}{\partial t} = A(x)X(x,t) + B(x)u(t), \qquad y(x,t) = C(x)X(x,t), \tag{1}$$

where X(x,t) is a state vector with an initial condition $X_0 = 0$; u(t) is entrance influence, y(x,t) is control output co-ordinate; A(x), B(x), C(x) are matrices of the control system parameters. For the stable watching system at standard input step signal u(t) = 1(t) the matrix of output C(x) is set so that the condition of output coordinate scaling was executed: $y(x,\infty)=1$. At the fixed parameters vector value x will build transient processes in model (1) on the quantum of time $[0, T_f]$. For this purpose at L integration steps of constant length $h = T_f / L$ with numbers $k = \overline{1, L}$ will enter denotations:

$$t_k = kh, \quad X_k(x) = X(x, t_k), \quad y_k(x) = C(x)X_k(x).$$
 (2)

We will designate matrix exponent and it's integral:

$$\varphi(x) = e^{A(x)h}, \quad \Phi(x) = \int_0^h e^{A(x)\tau} d\tau, \quad g(x) = \Phi(x)B(x).$$
 (3)

Then at an input signal u(t) = 1(t) transient process in ACS with model (1) it is possible to build on the recurrent formulas of matrix method [Severin, 2008]:

$$X_{k}(x) = \varphi(x)X_{k-1}(x) + g(x), \quad k = 1, L.$$
(4)

For a deviation $z(x,t) = y(x,t) - y(x,\infty)$ there are its values

$$z_k(x) = y_k(x) - y(x, \infty), \qquad k = 0, L,$$
 (5)

and their increments:

$$u_{lk}(x) = z_{k-2}(x) - z_{k-1}(x), \quad u_{rk}(x) = z_k(x) - z_{k-1}(x), \quad k = \overline{2, L}.$$
(6)

If the following condition, meaning that both successive increments are of the same sign, is met

$$u_{lk}(x)u_{rk}(x) > 0$$
, (7)

then with using of quadratic interpolation the extreme value is calculated $e_i(x)$:

$$d_{uk}(x) = [u_{lk}(x) - u_{rk}(x)]/2, \quad s_{uk}(x) = u_{lk}(x) + u_{rk}(x), \quad r_{uk}(x) = d_{uk}(x)/s_{uk}(x), \quad (8)$$

$$e_i(x) = z_{k-1}(x) - d_{uk}(x)r_{uk}(x)/2, \qquad (9)$$

where $i = 1, n_e(x), n_e(x)$ is extreme's number on segment $[0, T_f]$. By extreme's values of transient process the direct quality indexes are calculated: overshoot $\sigma(x)$, vibrations scope $\zeta(x)$, vibrations damping index $\lambda(x)$. Let $(v)_+ = \max\{v, 0\}$ is a cutting function of optional variable v. For watching system with $y(x, \infty) = 1$ direct indexes are determined on formulas:

$$\sigma(x) = \begin{cases} 0, & n_e(x) = 0, \\ [\max_i e_i(x)]_+, & n_e(x) > 0, \end{cases}$$
(10)

$$\zeta(x) = \begin{cases} 0, & n_e(x) = 0, 1, \\ \max_i |e_{2i-1}(x) - e_{2i}(x)|, & n_e(x) > 1, \end{cases} \qquad \lambda(x) = \begin{cases} 0, & n_e(x) = 0, 1, \\ \max_i \{|e_i(x)|/|e_{i-1}(x)|\}, & n_e(x) > 1. \end{cases}$$
(11)

For the stabilization system with $y(x, \infty) = 0$, a process in which has even one extreme $e_1(x)$,

$$\sigma(x) = \max_{i} \left| e_i(x) \right|,\tag{12}$$

and at a calculation $\zeta(x)$ and $\lambda(x)$ in formulas (11) not taken into account $e_1(x)$.

For the calculation of ACS control time the entry times of deviation z(x,t) in the set segment $[-\delta_z, \delta_z]$ of the steady-state value $z(x, \infty) = 0$ are determined by verification of entrance condition:

$$\left|z_{k-1}(x)\right| \ge \delta_z \wedge \left|z_k(x)\right| < \delta_z. \tag{13}$$

At implementation of this condition taking into account denotations (5)–(8) auxiliary values are calculated:

$$u_{i}(x) = \delta_{z} \operatorname{sign} z_{k-1}(x) - z_{k-1}(x), \quad v_{0i}(x) = r_{uk}(x)h, \quad (14)$$

$$s_{i}(x) = h\sqrt{r_{uk}^{2}(x) + 2u_{i}(x)/s_{uk}(x)}, \qquad v_{i}(x) = \begin{cases} v_{0i}(x) + s_{i}(x), v_{i}(x) \le 0, \\ v_{0i}(x) - s_{i}(x), v_{i}(x) > 0. \end{cases}$$
(15)

The moment of time, proper entrance of deviation function z(x,t) in area of steady-state value, is determined:

$$t_i(x) = t_{k-1}(x) + v_i(x) . (16)$$

Control time $t_c(x)$ and its relative value $\tau(x)$ are calculated on formulas:

$$t_c(x) = \max_i t_i(x), \quad \tau(x) = t_c(x)/T_f$$
 (17)

On formulas (3)–(17) for calculation of DQI $\sigma(x)$, $\zeta(x)$, $\lambda(x)$, $t_c(x)$, $\tau(x)$ the algorithms are obtained.

For the synthesis of watching ACS in place of few direct quality indexes it is possible to use their summarizing single index — improved IQE. On the model of kind (1) one can build a transfer function (TF)

$$W(x,s) = \beta(x,s)/\alpha(x,s), \quad \alpha(x,s) = \sum_{i=0}^{n} \alpha_i(x) s^{n-i}, \quad \beta(x,s) = \sum_{i=0}^{m} \beta_i(x) s^{m-i}.$$
(18)

For the watching systems a method is offered for forming of improved IQE I(x) of error e(x,t):

$$I(x) = \int_0^\infty \left[e(x,t) \right]^2 dt \,, \qquad e(x,t) = \sum_{k=0}^l w_k z_t^{(l-k)}(x,t) \,, \tag{19}$$

where *l* is an order of estimation, l < n - m; w_k are weighting coefficients:

$$w_{k} = \mu^{l-k} \gamma_{k}, \quad k = \overline{0, l}; \quad \mu = t_{e}/t_{s}, \quad \gamma(s) = \sum_{k=0}^{l} \gamma_{k} s^{l-k}, \quad w(s) = \sum_{k=0}^{l} w_{k} s^{l-k}.$$
(20)

Here t_e and t_s are control times of etalon and standard processes, $\gamma(s)$ and w(s) are standard and weighting polynomials. On TF (18) Laplace representation of error is formed $E(x,s) = \delta(x,s)/\alpha(x,s)$, where $\delta(x,s) = [\alpha(x,s) - \beta(x,s)w(s)]/s$. On the basis of this representation IQE calculation algorithm is developed [Severin, 2005].

Optimization Tasks of Linear Automatic Control Systems

Taking into account the high scope values σ_m , ζ_m , λ_m for DQI $\sigma(x)$, $\zeta(x)$, $\lambda(x)$ and requirements of maximal ACS response speed the system optimization task can be formulated as task of the constrained optimization which requires minimization of control time at implementation of limits on the other indexes:

$$\min_{x} \tau(x), \qquad \sigma(x) \le \sigma_m, \quad \zeta(x) \le \zeta_m, \quad \lambda(x) \le \lambda_m.$$
(21)

Using the improved integral estimation the task of control system optimization consists in minimization IQE:

$$\min_{x} I(x)$$
 (22)

However, statements of optimization control system tasks (21) and (22) take into account neither priority of direct indexes nor limitation of their definitional domain and definitional domain of integral estimation.

The analysis of automatic control system requirements allows to set the following preference order of direct quality indexes: $\sigma(x)$, $\zeta(x)$, $\lambda(x)$, $\tau(x)$. The feature of these indexes as private quality criteria of the automatic control systems is the limitation of their definitional domain by stability conditions. On Routh criterion for stability of linear ACS with transfer function (18) there are necessary and sufficient conditions:

$$\alpha_i(x) > 0, \quad i = \overline{0, n}; \qquad \rho_k(x) > 0, \quad k = \overline{2, n-1},$$
(23)

where $\rho_k(x)$ are elements of the first column of Routh table. The analysis of Routh criterion and research of properties of functions $\rho_k(x)$ justify the stepwise scheme of passage to the stability domain: if some from elements $\rho_k(x)$ is not positive, it is suggested to increase first of them to the positive value by the change of parameters values vector x, and then to increase subsequent elements. To simplify the scheme of passing to the stability domain and to meet the conditions of direct quality indexes (21), the parameter space R^p is divided into three domain sequences. The inequalities (23) and (21) are satisfied on the following domains of limitations:

$$\Omega_1 = \{ x \mid \alpha_i(x) > 0, i = 0, n \}, \quad \Omega_k = \{ x \mid \rho_k(x) > 0 \}, \quad k = \overline{2, n-1},$$
(24)

$$\Omega_n = \{ x \mid \sigma(x) \le \sigma_m \}, \quad \Omega_{n+1} = \{ x \mid \zeta(x) \le \zeta_m \}, \quad \Omega_{n+2} = \{ x \mid \lambda(x) \le \lambda_m \}.$$
(25)

On these m = n + 2 domains the derived intersection domains D_k and domains of limitations levels H_k are formed:

$$D_1 = \Omega_1; \quad D_k = D_{k-1} \cap \Omega_k, \quad k = \overline{2, m};$$
(26)

$$H_0 = R^p \setminus D_1; \qquad H_k = D_k \setminus D_{k+1}, \quad k = \overline{1, m-1}; \qquad H_m = D_m.$$
(27)

The domains of levels divide parameters space into the sequence of disjoint domains. The degree of violation of the first group of inequalities (23) is presented by penalty function

$$P(x) = \sum_{i=0}^{n} [-\alpha_i(x)]_+.$$
(28)

Stepwise principle of transferring to the stability domain and satisfaction of all limitations of direct quality indexes is based on the following: from any point x of parameters space R^p it is necessary to pass consistently to the level domain with greater index by minimizing in the current level domain using its corresponding penalty function. Taking into account the amount of levels domains there will be no more such steps of transition than the number of limitations m. For realization of stepwise principle of satisfaction of limitations in the task of ACS synthesis with optimization of direct quality indexes on the basis of levels domains, a vector objective function is introduced

$$F(x) = \begin{cases} (0; P(x)), & x \in H_0; \\ (k; -\rho_{k+1}(x)), & x \in H_k, & k = \overline{1, n-2}; \\ (n-1; \sigma(x) - \sigma_m), & x \in H_{n-1}; \\ (n; \zeta(x) - \zeta_m), & x \in H_n; \\ (n+1; \lambda(x) - \lambda_m), & x \in H_{n+1}; \\ (n+2; \tau(x)), & x \in H_{n+2}. \end{cases}$$
(29)

Denote the first coordinate of this function as the function of level $F_1(x)$ and the second coordinate as the function of penalty $F_2(x)$. The vector objective function (29) can be calculated algorithmically.

Algorithm for calculation of the vector objective function for direct quality indexes optimization.

Input parameters: x is a vector of variable parameters, T_f is the upper limit of integration interval, L is a number of steps of integration, σ_m , ζ_m and λ_m — maximum acceptable values of DQI. Output parameter: F is a value of vector objective function. 1. On model (1) calculate TF (18) with the characteristic polynomial $\alpha(s) = \alpha(x, s)$ of degree n. 2. If the necessary stability conditions are violated, calculate penalty function (28), let F = (0; P) and go to 12. 3. Let k = 1. 4. On Routh chart calculate $\rho_{k+1} = \rho_{k+1}(x)$. 5. If $\rho_{k+1} \leq 0$ let $F = (k; -\rho_{k+1})$ and go to 12. 6. If k < n-2 let k = k + 1 and go to 4. 7. On formulas (2)–(17) by numerical integration with quadratic interpolation calculate values of DQI $\zeta = \zeta(x)$, $\lambda = \lambda(x)$, $t_c = t_c(x)$, $\tau = \tau(x)$. 8. If $\sigma > \sigma_m$ let $F = (n-1; \sigma - \sigma_m)$ and go to 12. 11. Let $F = (n+2; \tau)$. 12. Exit the algorithm. Like function (29) a vector objective function is built for minimization of the improved IQE (19):

$$F(x) = \begin{cases} (0; P(x)), & x \in H_0; \\ (k; -\rho_{k+1}(x)), & x \in H_k, \\ (n-1; I(x)), & x \in H_{n-1}. \end{cases}$$
(30)

The goal of control systems optimization using vector objective functions (29) and (30) can be presented as minimization of the function of penalty $F_2(x)$ with the priority condition of maximization of function of level $F_1(x)$, which in turn can be presented as a single task of vector optimization:

$$\min_{\mathbf{x}} F(\mathbf{x}) \tag{31}$$

Unlike the tasks of scalar optimization (21) and (22) the task of vector optimization (31) takes into account the stability conditions and order of preference of limitations. The process of optimal synthesis of ACS is grounded by minimization $F_2(x)$ with priority maximization $F_1(x)$ as optimization of vector functions (29) and (30) on the basis of comparison of their two arbitrary values $U = (U_1; U_2)$ and $V = (V_1; V_2)$ by the binary operations:

$$U < V = \begin{cases} 1, & U_1 > V_1 \lor U_1 = V_1 \land U_2 < V_2, \\ 0, & U_1 < V_1 \lor U_1 = V_1 \land U_2 \ge U_2, \end{cases} \qquad U > V = \begin{cases} 1, & U_1 < V_1 \lor U_1 = V_1 \land U_2 > V_2, \\ 0, & U_1 > V_1 \lor U_1 = V_1 \land U_2 \le U_2, \end{cases}$$
(32)

$$U \leq V = \begin{cases} 1, & U_1 > V_1 \lor U_1 = V_1 \land U_2 \leq V_2, \\ 0, & U_1 < V_1 \lor U_1 = V_1 \land U_2 > U_2, \end{cases} \quad U \geq V = \begin{cases} 1, & U_1 < V_1 \lor U_1 = V_1 \land U_2 \geq V_2, \\ 0, & U_1 > V_1 \lor U_1 = V_1 \land U_2 < U_2. \end{cases}$$
(33)

These operations, allowing to determine which of the two values of vector objective function is «better», «worse», «not worse», or «not better», can be used in the numerical methods of unconstrained optimization.

Calculation of Quality Indexes of Nonlinear Control Systems

For nonlinear models the state vector and control coordinate will depend nonlinearly on the value of input influence u = u(t). Unlike the linear model of ACS in state space (1), the nonlinear model can be presented as:

$$\frac{\partial X(x,u,t)}{\partial t} = f[X(x,u,t),u,t], \qquad y(x,u,t) = C(x,u)X(x,u,t). \tag{34}$$

For the stable watching system at an input step signal $u(t) = u_s 1(t)$ with magnitude $u_s \in [u_{\min}; u_{\max}]$ the output matrix C(x, u) scales an output coordinate $y(x, u, \infty) = 1$. At a fixed value of parameters vector x let's build transient processes in model (34) on the quantum of time $[0, T_f]$ and calculate the Jacobian matrix of vector function of equation (34) by differentiating it on state vector coordinates:

$$A(x,u) = \partial f[X(x,u,t),u,t] / \partial X(x,u,t) \Big|_{x=0, u=0, t=0}$$
(35)

Let's introduce notation similar to (2) and (3), but taking into account system nonlinearity:

$$\Phi(x,u) = \int_0^h e^{A(x,u)\tau} d\tau \,. \tag{36}$$

Transient process in the control system on a model (34) it is possible to build on recurrent formulas for k = 1, L:

$$X_{k}(x,u) = X_{k-1}(x,u) + \Phi(x,u)f[X_{k-1}(x,u), u, t_{k-1}].$$
(37)

As a result of application of formulas, similar to formulas (4)–(17) but with functions depending both on x and u, we can calculate the direct indexes of quality $\sigma(x,u)$, $\zeta(x,u)$, $\lambda(x,u)$, $t_c(x,u)$, $\tau(x,u)$. Unlike linear ACS for the nonlinear systems an integral estimation (19) can be calculated only by numerical integration of the nonlinear system of differential equations (34) together with differential equation of estimation:

$$\partial I(x,u,t) / \partial t = \sum_{k=0}^{l} w_k z_t^{(l-k)}(x,u,t) \,. \tag{38}$$

For the extended system Jacobian matrix (35) and integral of matrix exponent (36) are calculated. The improved IQE $I(x, u, T_f)$ will be a result of integration of such system of differential equations using formula (37) on the span of time $[0, T_f]$ required for the convergence of improper integral.

Optimization Tasks of Nonlinear Control Systems

In the first approaching stability of the nonlinear control system can be defined on a linearized model. For this purpose we differentiate the vector function of equation (34) on input action:

$$B(x,u) = \partial f[X(x,u,t), u, t] / \partial u \Big|_{x=0, u=0, t=0}$$
(39)

Taking into account matrix (35) let's present the linearized model of the nonlinear system (34):

$$\partial X(x,u,t)/\partial t = A(x,u)X(x,u,t) + B(x,u)u, \qquad y(x,u,t) = C(x,u)X(x,u,t). \tag{40}$$

On this model let's build a transfer function

$$W(x,u,s) = \beta(x,u,s)/\alpha(x,u,s), \quad \alpha(x,u,s) = \sum_{i=0}^{n} \alpha_i(x,u) s^{n-i}, \quad \beta(x,u,s) = \sum_{i=0}^{m} \beta_i(x,u) s^{m-i}.$$
(41)

On a characteristic polynomial $\alpha(x, u, s)$ let's define the penalty function P(x, u) of kind (28) and elements of the first column of Routh table $\rho_k(x, u)$. Vector objective functions for quality criteria optimization (29) and (30) also will depend both on the vector of the varied parameters x and on input action u. Let's designate these

functions through F(x, u) and introduce n_u input step signals $u_i(t) = u_{si}1(t)$, $i = 1, n_u$ with magnitudes $u_{si} \in [u_{\min}; u_{\max}]$. Changing the value of input action u_{si} at the fixed value of vector x, we will get different values of vector function $F^{(i)}(x) = F[x, u_{si}1(t)]$ and using comparison operations (32) find the worst value

$$G(x) = \max_{i} F^{(i)}(x)$$
(42)

By analogy with task (31) for linear ACS the task of optimization of the nonlinear systems can be presented as:

$$\min_{x} G(x) \tag{43}$$

The solution of this task gives the optimal vector of the varied parameters, resulting to the best quality of transient processes for the specified set of input actions.

Modification of Genetic Algorithms for Vector Optimization of Control Systems

For optimization of vector objective functions we offer modifications of genetic algorithms. Initial population from M individuals is generated by introducing a set of random vectors $x^{(j)}$, $j = \overline{1, M}$ with real coordinates in the space of parameters R^p of the control system or vectors of binary values. In the second case it is necessary to represent every binary vector in space R^p and convert them to the vectors $x^{(j)}$, $j = \overline{1, M}$. Usually for this purpose a binary-to-decimal code or Gray code is used. The values of vector objective functions (29), (30) or (42) $F^{(j)} = F(x^j)$, $j = \overline{1, M}$ are calculated for all individuals using systems models equations (1), (34), (35), (39)–(41), quality indexes calculation formulas (2)–(20), (36)–(38), and defining expressions of vector functions (23)–(28). To rank individuals by the degree of fitness it is suggested to use the vector objective function sorting algorithms on the basis of operations of comparison of its values (32)–(33). The fitness level of individuals is subsequently scaled by the inverse square root $1/\sqrt{j}$ of their rank j in the sorted sequence. The scaled fitness level is used in the casual mechanism of selection. Application of genetic operators to the paternal individuals and generation of descendants is made, as well as in scalar genetic algorithms, with the use of different types of crossover, mutation, inversion. For all got descendants the values of vector objective function are calculated, and their ranks are obtained, similarly to the stage of forming the initial population. The new population is formed based on the results of sorting.

Conclusion

The researches results allow to formulate next conclusions.

1. The calculation methods of direct quality indexes and improved integral quadratic estimations have been studied for the linear automatic control systems. These quality indexes are defined only in stability domain of the systems.

2. The optimization tasks of quality indexes of the linear automatic control systems are presented as the tasks of optimization of vector objective functions, taking into account the conditions of stability of the systems, requirements to the quality indexes and priority of system requirements. For modification of optimization methods the set of comparison operations for vector objective functions is introduced.

3. The methods of quality indexes calculation have been also considered for the nonlinear automatic control systems. These quality indexes are the functions of not only varying parameters but also input action of control system.

4. Through dependence of quality indexes on input action of nonlinear control systems for one value of vector of varying parameters the several vector functions values are calculated at different values of input action. By the choice from these vector values the worst value of the vector objective function of nonlinear system is determined.

5. Vector modifications of genetic algorithms for optimization of vector objective functions, allowing to solve the tasks of synthesis for the linear and nonlinear control systems, have been developed.

The efficiency of the proposed application of genetic algorithms for the vector optimization of quality indexes of control systems has been confirmed by computational experiments on the test and applied tasks.

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