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(editors)

New Trends
in
Information Technologies

I T H E A
SOFIA
2010

Krassimir Markov, Vitalii Velychko, Lius Fernando de Mingo Lopez, Juan Casellanos (ed.)

New Trends in Information Technologies

ITHEA®

Sofia, Bulgaria, 2010

ISBN 978-954-16-0044-9

First edition

Recommended for publication by The Scientific Concil of the Institute of Information Theories and Applications FOI ITHEA

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General Sponsor: Consortium FOI Bulgaria (www.foibg.com).

Printed in Bulgaria

ISBN 978-954-16-0044-9

C\o Jusautor, Sofia, 2010

UPPER BOUND ON RATE-RELIABILITY-DISTORTION FUNCTION FOR SOURCE WITH TWO-SIDED STATE INFORMATION

Mariam Haroutunian, Arthur Muradyan

Abstract*: Different models of the source with side information can be considered when side information is known to the encoder, the decoder, both of them, none of them. In this paper, we investigate a generalized model of the discrete memoryless source with two-sided state information introduced by Cover and Chiang in [Cover-Chiang, 2002], which includes the data compression problems mentioned above as special cases. We study the rate-reliability-distortion function, which is understood as the minimum code rate for the encoding of the source messages under the requirement that the decoder reconstructs the messages at a desired distortion level with the error probability exponentially decreasing with the codeword length. In other words, the rate is considered as a function of a fixed distortion level and the error exponent. In this paper the upper bound on the rate-reliability-distortion function is obtained. The upper bounds on rate-reliability-distortion functions of the source with side information are derived as special cases for four possible situations - one of which coincides with known result while the three others were unknown.*

Keywords*: source with side information, rate-reliability-distortion function*

ACM Classification Keywords*: H.0 Information Systems - Conference proceedings*

Introduction

The state information problems were intensively studied. The model when state information is available to the decoder was analyzed by Wyner and Ziv in [Wyner-Ziv, 1976] where the rate-distortion function was derived which shows dependence of minimal rate on a required distortion introduced by Shannon in [Shannon, 1959]. The lossless source coding problem when state information is available at the decoder was investigated by Slepian-Wolf [Slepian-Wolf, 1973] .

The applications of these problems include distributed sensor networks [Xiong-Liveris, 2004], digital upgrade of analog television signals, play-back of the compressed sound in the presence of background noise where decoder is fed with a background correlated signal to improve the quality of decoding. The source coding problem when state information is known to both encoder and decoder is studied in [Viswanathan-Berger, 1997]. The study includes applications in video coding where the pixel value at a given location depends on a pixel at the same location in a previous frame. Here, the previous frame can be considered as a side information for the coding of the present frame.

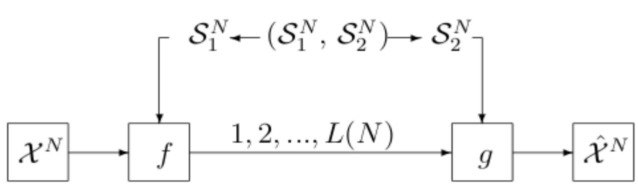


Figure 1. Source with two-sided state information

A generalized model of sources where the encoder and the decoder have correlated state information (Figure 1) was considered by Cover and Chiang in [Cover-Chiang, 2002] where the rate-distortion function was derived. It was proved that rate-distortion functions of the source with state information in four possible situations can be obtained from the generalized formula.

We study the rate-reliability-distortion function introduced by Haroutunian and Mekoush [Haroutunian-Mekoush, 1984] which describes the dependence of rate on reliability and distortion level. The idea was then adopted and extended for multiuser source coding problems [Haroutunian et al, 1998, Haroutunian-Maroutian, 1991, Maroutian-1990, Meulen et al, 2000]. This function is a generalization of the rate-distortion function, since it tends to that function when the error exponent (reliability - *E*) tends to 0. The inverse order dependence of these parameters is studied by Marton in [Marton, 1974].

In this paper, an upper bound of the rate-reliability-distortion function is derived for the source with two sided state information. The limit of this bound for $E \rightarrow 0$ coincides with the rate-distortion function, obtained in [Cover-Chiang, 2002]. As a special case, we derive the upper bounds on the rate-reliability-distortion functions for four possible situations of the source with side information, one of them coincides with the rate-reliability-distortion function of the DMS [Haroutunian et al, 2008], while the three others were unknown.

In the next section we give the definitions of the concepts extended for the considered generalized model. Description of the main theorem along with corollaries is given in section 3. Proof of the theorem is given in section 4.

Notations and Definitions

Capital letters are used for random variables $\mathrm{S}_1, \mathrm{S}_2, \mathrm{U}, \mathrm{X}, \hat{\mathrm{X}}$ taking values in the finite sets $S_1, S_2, U, X, \hat{X}$, respectively, and lower case letters $s_1, s_2, u, x, \hat{x}$ for their realizations. Small bold letters are used for N -length vectors $\mathbf{x} = (x_1, .., x_N) \in X^N$.

A generalized model representing the source with two-sided state information is depicted in Figure 1.

S_1 and S_2 are the state information, known to the encoder and the decoder taking values from the set S_1 and S_2, X is an i.i.d. random variable taking values in the finite set X (the alphabet of messages of the source). The finite set $\widehat{X}$ different from the set X, represents the reproduction alphabet of the receiver, in general case.

The generating probability distribution of the source with two-sided state information is given as

$$P^* = P_1^* \circ P_2^* = \{P^*(x, s_1, s_2) = P_1^*(x, s_1) P_2^*(s_2 \mid x, s_1), x \in X, s_1 \in S_1, s_2 \in S_2\}.$$

We consider the memoryless source, which means that the probability of N-length vector of message $\mathbf{x} = (x_1, .., x_N) \in X^N$ and state information vectors $\mathbf{s}_1 = (s_{11}, .., s_{1N}) \in S_1^N$, $\mathbf{s}_2 = (s_{21}, .., s_{2N}) \in S_2^N$ is defined as the product of component probabilities

$$P^{*N}(\mathbf{x}, \mathbf{s}_1, \mathbf{s}_2) = \prod_{n=1}^{N} P^*(x_n, s_{1n}, s_{2n}).$$

Let

$$d : X \times \hat{X} \rightarrow [0, \infty)$$

be the given distortion between the source and the reconstructed message. The distortion measure for the vectors $\mathbf{x} \in X^N$ and $\hat{\mathbf{x}} \in \hat{X}^N$ is defined as the average of the components' distortions

$$d(\mathbf{x}, \hat{\mathbf{x}}) = \frac{1}{N} \sum_{n=1}^{N} d(x_n, \hat{x}_n).$$

The code (f_N, g_N) is a pair of the encoding and the decoding functions

$$f_N : X^N \times S_1^{\ N} \to \{1, 2, ..., L(N)\}$$

and

$$g_N : \{1, 2, ..., L(N)\} \times S_2^{\ N} \to \hat{X}^N$$

where $L(N)$ is the *volume* of the code.

The task of the system is to ensure reconstruction of the source messages at the receiver at a given distortion level Δ and with a small error probability. Our problem is estimating of the minimum of the code volume.

To define error probability for given code (f_N, g_N) and Δ ≥ 0 consider the following set of triples:

$$A = \{\mathbf{x} \in X^N, \mathbf{s}_1 \in S_1^{\ N}, \mathbf{s}_2 \in S_2^{\ N} : g_N(f_N(\mathbf{x}, \mathbf{s}_1), \mathbf{s}_2) = \hat{\mathbf{x}}, d(\mathbf{x}, \hat{\mathbf{x}}) \le \Delta\}.$$

The error probability of the code (f_N, g_N), for the source probability distortion P^*, Δ and *N* is defined as:

$$e(f_N, g_N, P^*, \Delta, N) \le 1 - P^{*N}(A).$$

A positive number *R* is called the (E, Δ)-*achievable rate* for a given P^*, E > 0 and Δ ≥ 0 if there exists a code (f_N, g_N) such that

$$\frac{1}{N} \log L(N) \le R + \varepsilon$$

and error probability is exponentially small

$$e(f_N, g_N, P^*, \Delta, N) \le \exp\{-N(E - \delta)\}.$$

for every ε > 0, δ > 0 and sufficiently large *N*. The minimum (E, Δ)-achievable rate is denoted by *R(E, Δ,* P^**)* and is called the *rate-reliability-distortion* function.

For notion of *types*, *mutual information* $I_{P,Q_1}(U \wedge X)$, *divergence* $D(P \| P^*)$, we refer to [Cover-Thomas, 1991, Csiszа́r- K□rner, 1981, Csiszа́r-1998].

We introduce the following probability distributions with some auxiliary finite set *U*:

$$Q_1 = \{Q_1(u \mid x, s_1), x \in X, s_1 \in S_1, u \in U\},$$
$$Q_2 = \{Q_2(\hat{x} \mid u, s_2), \hat{x} \in \hat{X}, u \in U, s_2 \in S_2\},$$
$$P = P_1 \circ P_2 = \{P(x, s_1, s_2) = P_1(x, s_1) P_2(s_2 \mid x, s_1), x \in X, s_1 \in S_1 s_2 \in S_2\},$$
$$PQ(x, \hat{x}) = \sum_{s_1, s_2, u} P(x, s_1, s_2) Q_1(u | x, s_1) Q_2(\hat{x} | u, s_2).$$

Following estimates [Cover-Thomas, 1991, Csiszа́r- K□rner, 1981] are used in the paper. For any type $P_1 \in P_N(X, S_1)$

$$(N+1)^{-|X||S_1|} \exp\{NH_{P_1}(X, S_1)\} < |T_{P_1}^{\ N}(X, S_1)| \le \exp\{NH_{P_1}(X, S_1)\} \quad (1)$$

and any conditional type Q1 and $\mathbf{u} \in T_{P,Q_1}(U)$

$$(N+1)^{-|X||S_1||U|} \exp\{NH_{P,Q_1}(X, S_1 \mid U)\} < |T_{P,Q_1}^{\ N}(X, S_1 \mid \mathbf{u})| \le \exp\{NH_{P,Q_1}(X, S_1 \mid U)\}. \quad (2)$$

The number of probability distributions on X , S_1 , S_2 is upper estimated as follows:

$$|P_N(X, S_1, S_2)| < (N+1)^{|X||S_1||S_2|}. \quad (3)$$

Formulation of Results

Let $\alpha(E, P^*) = \{P : D(P \| P^*) \le E\}$ and

$$R'(E, \Delta, P^*) = \max_{P \in \alpha(E,P^*)} \min_{Q_1, Q_2 \in Q(P,\Delta)} \left[I_{P,Q_1}(U \wedge S_1, X) - I_{P,Q_1}(U \wedge S_2) \right],$$

where the minimization is carried under following distortion constraint

$$Q(P, \Delta) = \{(Q_1, Q_2) : \sum_{x,\hat{x}} d(x, \hat{x}) PQ(x, \hat{x}) \le \Delta\}.$$

Theorem. *R' is the upper bound of the rate-reliability-distortion function for any $E > 0$, $\Delta > 0$ and P^**

$$R(E, \Delta, P^*) \le R'(E, \Delta, P^*).$$

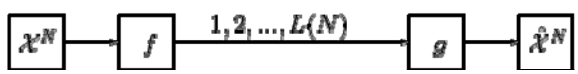
The proof of the theorem is given in the next section.

Corollary 1. We obtain the rate-distortion function $R(\Delta, P^*)$ established in [Cover-Chiang, 2002] when $E \to 0$ for any $\Delta \ge 0$ and probability distribution P^*

$$\lim_{E \to 0} R'(E, \Delta, P^*) = \min_{Q_1, Q_2 \in Q(P^*,\Delta)} \left[I_{P^*,Q_1}(U \wedge S_1, X) - I_{P^*,Q_1}(U \wedge S_2) \right].$$

Corollary 2. We obtain the upper bounds on the rate-reliability-distortion functions for four possible situations of the source with side information as special cases.

<u>Case 1:</u> No state information at the sender and receiver: $S_1 = \varnothing, S_2 = \varnothing$



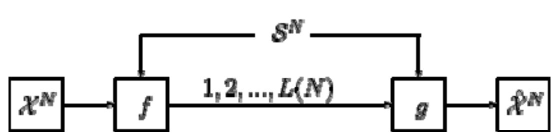
Here $(S_1, X) = X, I(U \wedge S_2) = 0,$ with distributions $P(x), Q_1(u \mid x)$ and $Q_2(\hat{x} \mid u),\ X \to U \to \hat{X}$ forms Markov chain. Therefore

$$\min_{Q_1(u|x)} I_{P,Q_1}(U \wedge X) \ge \min_{Q(\hat{x}|x)} I_{P,Q_1}(\hat{X} \wedge X), \text{ with equality iff } U = \hat{X}.$$

For this case we get the formula established in [Haroutunian et al, 2008]:

$$R'(E, \Delta, P^*) = \max_{P \in \alpha(E,P^*)} \min_{Q_1(u|x) Q_2(\hat{x}|u)} I_{P,Q_1}(U \wedge X) = \max_{P \in \alpha(E,P^*)} \min_{Q(\hat{x}|x)} I_{P,Q_1}(\hat{X} \wedge X).$$

<u>Case 2:</u> State information on both sides is the same: $S_1 = S_2 = S$



With distributions $P(x \mid s), Q_1(u \mid x,s)$ and $Q_2(\hat{x} \mid u,s)$, $X \to U \to \hat{X}$ forms Markov chain conditioned on S. Therefore

$$\min_{Q_1(u|x,s)} I_{P,Q_1}(U \wedge X \mid S) \geq \min_{Q(\hat{x}|x,s)} I_{P,Q}(\hat{X} \wedge X \mid S), \text{ with equality iff } U = \hat{X}.$$

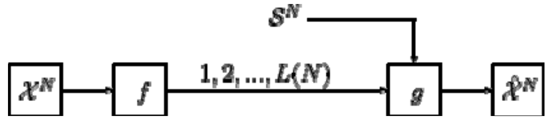
We obtain the upper bound of rate-reliability-distortion function

$$R'(E,\Delta,P^*) = \max_{P \in \alpha(E,P^*)} \min_{Q_1(u|x,s) Q_2(\hat{x}|u,s)} \left[I_{P,Q_1}(U \wedge S, X) - I_{P,Q_1}(U \wedge S) \right] =$$

$$= \max_{P \in \alpha(E,P^*)} \min_{Q_1(u|x,s) Q_2(\hat{x}|u,s)} \left[I_{P,Q_1}(U \wedge S) + I_{P,Q_1}(U \wedge X \mid S) - I_{P,Q_1}(U \wedge S) \right] =$$

$$= \max_{P \in \alpha(E,P^*)} \min_{Q(\hat{x}|x,s)} I_{P,Q_1}(\hat{X} \wedge X \mid S).$$

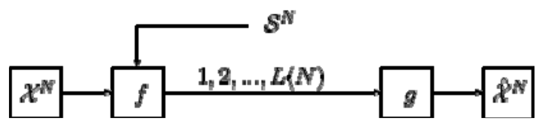
<u>Case 3</u>: State information on the receiver: $S_1 = \varnothing, S_2 = S$



Here $(S_1, X) = X$, and with distributions $P(x \mid s), Q_1(u \mid x)$ and $Q_2(\hat{x} \mid u,s)$, the upper bound of rate-reliability-distortion function is:

$$R'(E,\Delta,P^*) = \max_{P \in \alpha(E,P^*)} \min_{Q_1(u|x) Q_2(\hat{x}|u,s)} \left[I_{P,Q_1}(U \wedge X) - I_{P,Q_1}(U \wedge S) \right].$$

<u>Case 4:</u> State information on the sender: $S_1 = S, S_2 = \varnothing$



Here $I(U \wedge S_2) = 0,$ with distributions $P(x \mid s), Q_1(u \mid x,s)$ and $Q_2(\hat{x} \mid u)$ $X \to U \to \hat{X}$ forms Markov chain and hence

$$\min_{Q_1(u|x,s)} I_{P,Q_1}(U \wedge X) \geq \min_{Q(\hat{x}|x,s)} I_{P,Q}(\hat{X} \wedge X), \text{ with equality iff } U = \hat{X}.$$

Since $U = \hat{X}$ and $\hat{X}$ is independent of S we also have $I_{P,Q_1}(U \wedge S \mid X) = 0$. Taking into account these properties we get the upper bound of rate-reliability-distortion function:

$$R'(E,\Delta,P^*) = \max_{P \in \alpha(E,P^*)} \min_{Q_1(u|x,s) Q_2(\hat{x}|u)} I_{P,Q_1}(U \wedge S, X) =$$

$$= \max_{P \in \alpha(E,P^*)} \min_{Q_1(u|x,s) Q_2(\hat{x}|u)} \left[I_{P,Q_1}(U \wedge X) + I_{P,Q_1}(U \wedge S \mid X) \right] =$$

$$\max_{P \in \alpha(E,P^*)} \min_{Q_1(u|x,s) Q_2(\hat{x}|u)} I_{P,Q_1}(U \wedge X).$$

$$= \max_{P \in \alpha(E,P^*)} \min_{Q(\hat{x}|x,s)} I_{P,Q}(\hat{X} \wedge X) =$$

$$= \max_{P \in \alpha(E,P^*)} \min_{Q(\hat{x}|x)} I_{P,Q}(\hat{X} \wedge X).$$

Proof of the Theorem

Let $J(P,Q_1)=\exp\{N[I_{P,Q_1}(X,S_1\wedge U)+\varepsilon]\}$ for types P,Q_1 and $\varepsilon>0$.

Lemma. *For every type* P and conditional type Q_1 there exists a collection of vectors

$$\{\mathbf{u}_j\in T^N_{P,Q_1}(U),\ j=1,..,J(P,Q_1)\},$$

Such that for *N* large enough

$$T^N_{P_1}(X,S_1)\subset\bigcup_{j=1}^{J(P,Q_1)}T^N_{P,Q_1}(X,S_1\,|\,\mathbf{u}_j).$$

The proof of the lemma is similar to the covering lemma from [Haroutunian et al, 2008].

The proof of the theorem is based on the construction of a code (f_N,g_N) based on the idea of *importance* of source vectors of messages of type P not farther from P^* (in sense of divergence). It is shown that (E,Δ) achievable rate of the constructed code satisfies (4).

The triple of sets for source messages and state information (available at encoder and decoder) of length N can be represented as a union of all disjoint types of vector triples:

$$X^N\times S_1^N\times S_2^N=\bigcup_{P\in P_N(X,S_1,S_2)}T^N_P(X,S_1,S_2).$$

For $\delta>0$ and for N large enough the probability of appearance of vector triples of types beyond $\alpha(E+\delta,P^*)$ can be estimated in the following way:

$$P^{*N}\Big(\bigcup_{P\notin\alpha(E+\delta,P^*)}T^N_P(X,S_1,S_2)\Big)=\sum_{P\notin\alpha(E+\delta,P^*)}P^{*N}(T^N_P(X,S_1,S_2)\le$$
$$\le(N+1)^{|X||S_1||S_2|}\exp\{-N\min_{P\notin\alpha(E+\delta,P^*)}D(P\,||\,P^*)\le$$
$$\exp\{|X||S_1||S_2|\log(N+1)-N(E+\delta)\}\le\exp\{-N(E+\delta/2)\}.$$

The first 2 inequalities follow from the definition of $\alpha(E,P)$ and type properties.

Encoding

For type P and conditional type Q_1 denote

$$C(P,Q_1,j)=T^N_{P,Q_1}(X,S_1\,|\,\mathbf{u}_j)-\bigcup_{j'<j}T^N_{P,Q_1}(X,S_1\,|\,\mathbf{u}'_j),\ j=1,..,J(P,Q_1).$$

Step 1

Let us fix the type $P\in\alpha(E+\delta,P^*)$ and conditional types $(Q_1,Q_2)\in Q(P,\Delta)$.

From the definition of $C(P,Q_1,j)$ and from the lemma we have

$$\bigcup_{j=1}^{J(P,Q_1)}C(P,Q_1,j)=\bigcup_{j=1}^{J(P,Q_1)}T^N_{P,Q_1}(X,S_1\,|\,\mathbf{u}_j)\supset T^N_{P_1}(X,S_1).$$

Step 2

Let $L(P,Q_1)=J(P,Q_1)/\exp\{NI_{P,Q_1}(U\wedge S_2)\}$. Randomly chosen indices of $\mathbf{u}_j\in T^N_{P,Q_1}(U)$ are uniformly distributed to $L(P,Q_1)$ bins. Denote by $B(i)$ the set of indices assigned to bin i.

Step 3

Considering $\mathbf{x}, \mathbf{s}_1$ encoder sends number i such that $(\mathbf{x}, \mathbf{s}_1) \in C(P, Q_1, j)$ and $j \in B(i)$.

Decoding

Step 4

By receiving number i and $\mathbf{s}_2$ state information the decoder looks for $\mathbf{u}_k$ such that $k \in B(i)$ and $\mathbf{u}_k \in T^N_{P,Q_1}(U \mid \mathbf{s}_2)$. If there is such k, the decoder selects $\hat{\mathbf{x}}_i \in T^N_{P,Q_1}(\hat{X} \mid \mathbf{u}_k, \mathbf{s}_2)$. If there is no such k, or more than one k,decoder chooses preliminary fixed reconstruction vector $\hat{\mathbf{x}}_0$. If decoder receives i_0, again $\hat{\mathbf{x}}_0$ is chosen.

The distortion between $\mathbf{x}$ and $\hat{\mathbf{x}}_i (i = 1,.., L(P, Q_1))$ can be calculated in the following way:

$$d(\mathbf{x}, \hat{\mathbf{x}}_i) = N^{-1} \sum_{x,\hat{x}} d(x, \hat{x}) n(x, \hat{x} | \mathbf{x}, \hat{\mathbf{x}}_i) = \sum_{x,\hat{x}} d(x, \hat{x}) PQ(x, \hat{x}) = E_{P,Q_1,Q_2} d(X, \hat{X}) \leq \Delta .$$

So number of used vectors $\hat{\mathbf{x}}$ for fixed P and corresponding conditional types Q_1, Q_2 is equal

$$L(P, Q_1) = \exp\{N[I_{P,Q_1}(X, S_1 \wedge U) - I_{P,Q_1}(U \wedge S_2) + \varepsilon]\} .$$

The number of vectors $\hat{\mathbf{x}}$ reconstructed under allowed distortion constraint for all P is not more than $P_N(X, S_1, S_2) L(P, Q_1)$. From the definition of (E, Δ)-achievable rate and from (3) we obtain:

$$\frac{1}{N} \log[P_N(X, S_1, S_2) L(P, Q_1)] - \varepsilon =$$
$$= I_{P,Q_1}(X, S_1 \wedge U) - I_{P,Q_1}(U \wedge S_2) + N^{-1} |X||S_1||S_2| \log(N+1) \leq$$
$$\leq \max_{P \in \alpha(E, P^*)} \min_{Q_1, Q_2 \in Q(P, \Delta)} \left[I_{P,Q_1}(X, S_1 \wedge U) - I_{P,Q_1}(U \wedge S_2) \right].$$

The theorem is proved.

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