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(editors)

# Information Models of Knowledge

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2010

Krassimir Markov, Vitalii Velychko, Oleksy Voloshin (ed.)

Information Models of Knowledge

ITHEA®

Kiev, Ukraine – Sofia, Bulgaria, 2010

ISBN 978-954-16-0048-1

First edition

Recommended for publication by The Scientific Concil of the Institute of Information Theories and Applications FOI ITHEA

ITHEA IBS ISC: 19.

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Printed in Ukraine

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**ISBN 978-954-16-0048-1**

C\o Jusautor, Sofia, 2010

# DISTURBANCE OF STATISTICAL STABILITY

**Igor Gorban**

***Abstract***: *The survey of author's researches devoted to disturbance of statistical stability of physical quantities and processes is presented. Concepts of statistical stability of random sequences and random processes are formalized. Parameters characterizing their statistical stability in finite observation interval are proposed. It is found that statistical unstable processes are especial class of non-stationary processes. Statistical stability disturbance can be explained by low frequency cyclical fluctuations of expectation. It has been researched statistical stability of a number of physical quantities and processes, in particular line voltage, height of sea waves and their periods, Earth magnetic field fluctuation, and currency rate. It has been found that for all researched quantities and processes, there is statistical stability disturbance on large observation intervals. Obtained results corroborate the hypotheses that are in base of new physico-mathematical theory of hyper-random phenomena: real physical events, quantities, processes, and fields are not statistical stabile and they can be adequately described by hyper-random models taking into consideration their statistical instability.*

***Keywords***: *statistical instability, theory of hyper-random phenomena, uncertainty, probability.*

***ACM Classification Keywords***: *G.3 Probability and Statistics*

## 1. Introduction

Physical basis of probability theory and mathematical statistics is statistical stability of event's frequencies. This fact examined and tested a lot of times by different scientists. It is well known, for instant, coin test led by Buffon and K. Pirson [Гнеденко, 1961]. Buffon flipped a coin 4040 times. K. Pirson led two test series. In the first series a coin was flipped by him 12000 times and in the second series – 24000 times. In all their tossing experiments the head frequencies equaled to near 0.5. Such stable result is typical for any statistical stable conditions.

In statistics and probability theory stability plays very important role. This circumstance was marked by a number of scientists, beginning from Jakob Bernoulli [Bernoulli, 1713]. R. von Mises proposed even to define [Mises, 1964] the concept of probability on the base of event's frequency in fixed conditions. Although Mises's proposition was not supported by the most mathematics and now Kolmogorov's set-theoretic definition of the random event [Колмогоров, 1936, ISO, 2006] is used mainly, importance of statistical stability is not decreased, especially in sphere of applications.

For real physical phenomena (events, quantities, processes, and fields) it is not simple to determine correctly Kolmogorov's probability measure. This fact was marked in many works, for instance, in [Колмогоров, 1986, Скороход, 1990, Тутубалин, 1972, Alimov, Kravtsov, 1992]. The main cause is in difficulty and often impossibility to stabilize statistical conditions.

This stimulated the development of new theories, such as fuzzy logic [Zadeh, Kacprzyk, 1992], neural network [Hagan, Demuth, Beale, 1996], chaotic dynamical systems [Crownover, 1995], ], interval data [Шокин, 1981, Алефельд, Херцбергер, 1987, Shary, 2002, Kreinovich, Berleant, Ferson, Lodwick, 2005, Ferson, Kreinovichy, Ginzburg, Myers, 2003], and others theories.

Author's apprehension in statistic stability of real mass phenomena motivated him to develop physico-mathematical theory of hyper-random phenomena [Горбань, 2007, Gorban, 2008, Gorban, 2009] oriented to description of statistically unstable physical events, quantities, processes, and fields.

The theory of hyper-random phenomena includes two components: mathematical and physical ones. Don't discussing now mathematical part of it, mark that its physical part is based on the hypothesis of existence in real physical world statistically unstable physical phenomena and the hypothesis of hyper-random setting up of the world, essence of which is that statistical unstable phenomena (adequately described by hyper-random models) are mass ones.

Now the basis of new theory is developed in different directions, however, for this time the main question remains disputable: are real phenomena statistically unstable or not?

To obtain the answer to the question a number of experimental researches devoted to disturbance of statistical stability of real physical quantities and processes were led by the author [Горбань, 2010 (1), Горбань, 2010 (2), Горбань, 2010 (3), Горбань, 2010 (4)].

The aim of the article is to present generalized results of these experimental researches.

## 2. Experimental research of line voltage

Research of line voltage is led by computer and simple linkage (consists of transformer and voltage divider). Input voltage was discretized with frequency 5 kHz. Voltage effective values were calculated on the base of every 1024 units and were written in memory. Record was led by session during two months with pause in some days. Duration of every record was 60 hours. For this session time, near $N = 2^{20} \approx 1$ million of effective voltage units were recorded.

Line voltage was changed continuously. In different sessions changes had different character. Time dependences of line voltage for two sessions and according cumulative averages are presented in Fig. 1.

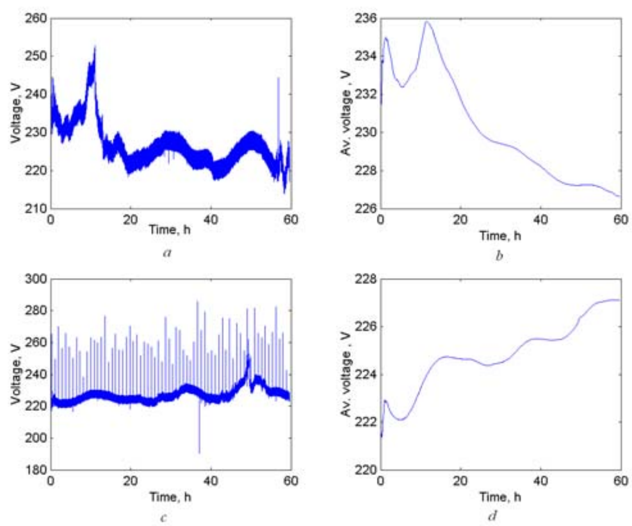



Fig. 1. Time dependences of line voltage for two sessions (in hours) (a, c) and according cumulative averages (b, d)

Analyze of a number records showed that all cumulative averages are not decayed (Fig. 1 *b*, *d*). This strange on the first view fact, sharply contrasts with well known theoretical results, demonstrating time decay of cumulative averages when the observation time is increased. Such decay is typical, for instance, for white Gaussian noise (model 1) and harmonic oscillation (model 2) presented in Fig. 2.

To clear causes of such difference let us appeal to the theory.

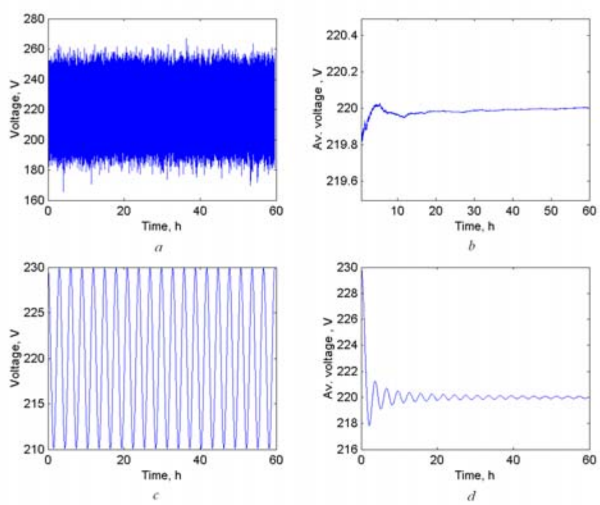



Fig. 2. White Gaussian noise (model 1) (a), harmonic oscillation (model 2) (c), and according cumulative averages (b, d)

## 3. Some theory questions

Sequence $X_1, X_2, ...$ of random variables (random sample) will be called a statistically stable one [Горбань, 2010 (1)], if expectation of sample variance $\bar{D}_{Y_N} = \frac{1}{N}\sum_{n=1}^{N}(Y_n - \bar{m}_{Y_N})^2$ of average $Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ ($n = \overline{1,N}$) tends to zero when sample size $N$ tends to infinity where $\bar{m}_{Y_N} = \frac{1}{N}\sum_{n=1}^{N} Y_n$ is sample mean of average. Sequences that don't accord to this clause will be called statistically unstable ones.

The type of convergence is not essential hear but to obtain necessary mathematical rigor we shall mean probability convergence.

It is well known Chebyshev theorem that presents the law of large numbers states [Гнеденко, 1961]: average $Y_N = \frac{1}{N}\sum_{n=1}^{N} X_n$ of sequence $X_1, X_2, ...$ of independent in pair random variables, that have finite variances and expectations $m_{x_1}, m_{x_2}, \ldots$, tends in probability to mean $m_{y_N} = \frac{1}{N}\sum_{n=1}^{N} m_{x_n}$ when sample size $N$ tends to infinity.

Draw attention on one nuance unnoticed by the most: this theorem does not say about convergence of neither average $Y_N$, nor mean $m_{y_N}$. It states the convergence of these variables to each other, or otherwise, the convergence of their difference to zero. This means that average $Y_N$ and mean $m_{y_N}$ can have not the limit. They can fluctuate around the constant. In the process of such fluctuation they are changed without fail by synchronously manner.

It follows from Chebyshev theorem, that random sequence is statistically stable if and only if sample variance of mean $m_{y_N}$ tends to zero when sample size $N$ tends to infinity.

Random process will be called a statistically stable one, if expectation of the integral $\frac{1}{T}\int_0^T (Y(t) - \bar{m}_{y_T})^2 \mathrm{d}t$ tends to zero when $T \to \infty$ where $Y(t) = \frac{1}{t}\int_0^t X(t_1)\mathrm{d}t_1$ is average of $X(t_1)$ in the interval $(0,t)$, $\bar{m}_{y_T} = \frac{1}{T}\int_0^T Y(t)\mathrm{d}t$ is mean of average $Y(t)$. Random processes that don't accord to this clause will be called statistically unstable ones.

Mark, that determinate quantity $x_0$ can be regarded in rough as a particular case of a random quantity that has in the distribution function a unit step in the point $x_0$ [Горбань, 2007, Gorban, 2009]. Analogues, determinate function $x_0(t)$ can be regarded in rough as a particular case of a random function with distribution function $F(x;t) = \mathrm{rect}(x - x_0(t))$. Therefore statistically stable and statistically unstable concepts are suitable for sequence of determinate quantities and for sequence of determinate functions.

Examples of statistically stable sequences are uniform random sample with finite two first moments and also nonuniform random sample with finite two first moments, for which mean of expectations has a limit.

The sequence is statistically unstable one if mean of expectations has not a limit (for instance, fluctuates) when sample size tends to infinity. Numerical divergent sequence is statistically unstable one too.

Underline, for random processes, concept of nonstationarity and concept of statistical instability are not identical.

Stationary ergodic (in expectation) processes are statistically stable ones. Among non stationary processes there are as statistically stable as statistically unstable processes. Hence, statistically unstable processes are especial class of non stationary processes.

The fact of statistical stability or statistical instability of real sequence with finite sample size or the same fact of real process on the finite interval observation cannot be established in principle, because for establishing such facts the sequence or the process must be infinite ones.

However, concept of statistical stability on finite interval may be formalized. The bases for such formalization may be detection the tendency of stabilization of the average level or the tendency of stabilization of the mean of expectations when sample size increases. These tendencies may be the quality indicators of statistic stability of the process. Before going to quantity characteristics let us analyze possible causes led to disturbance of statistical stability.

## 4. Possible causes led to disturbance of statistical stability of random processes

Any non stationary random process $X(t)$ can be presented in the following form:

$$X(t) = m_{x(t)} + \overset{\circ}{X}(t)$$

where $m_{x(t)}$ is expectation of the process $X(t)$ and $\overset{\circ}{X}(t)$ is random process with zero expectation.

Expectation $m_{y(t)}$ of average $Y(t)$ is determined by expectation $m_{x(t)}$:

$$m_{y(t)} = \frac{1}{t}\int_0^t m_{x(t_1)}\mathrm{d}t_1 .$$

Hence statistic stability of the process $X(t)$ depends from particularities of expectation $m_{x(t)}$.

Examine random processes with different changes of expectation $m_{x(t)}$, in particular periodic, intermittent, and aperiodic types.

Let $m_{x(t)}$ is a periodic function with period $T$. Then its Fourier expansion

$$m_{x(t)} = \sum_{k=-\infty}^{\infty} \dot{a}_k \exp\left(\frac{\mathrm{j}2\pi}{T}kt\right) \tag{1}$$

where $\dot{a}_k = a_k \exp(\mathrm{j}\varphi_k)$ is complex expansion coefficient.

Then average expectation

$$m_{y(t)} = a_0 + \sum_{k=1}^{\infty} a_k \frac{\sin \pi tk/T}{\pi tk/T}\cos(\pi tk/T + \varphi_k) . \tag{2}$$

It follows from expression (2) that variable part of average expectation is described by harmonic functions subsided on the low of $\sin x / x$. Subside speed of these functions is determined by value of the period $T$: it decreases with rising the period and tends to zero when $T \to \infty$.

Minimum subside speed has the first term of series (according to $k = 1$). In average expectation (2) harmonics of high orders are suppressed. Suppress level increases with increasing of harmonic order.

Changing of average expectation is not significant if observation interval $t$ essentially less than the period $T$. This is area of statistical stability. However, situation gradually changes when value $t$ approaches to the period $T$. As follow from expression (2), average expectation is changed significantly in the interval $t \in [0, T]$. It means that in the interval $t \in [0, T]$ statistical stability is disturbed.

Mark, essential changes of average expectation and disturbances of statistic stability may be registered in the observation intervals that larger than period $T$ too, in particular, when amplitudes of high order harmonics are large and numbers of these harmonics are not high. Fig. 3 demonstrates these particularities (models 3, 4).

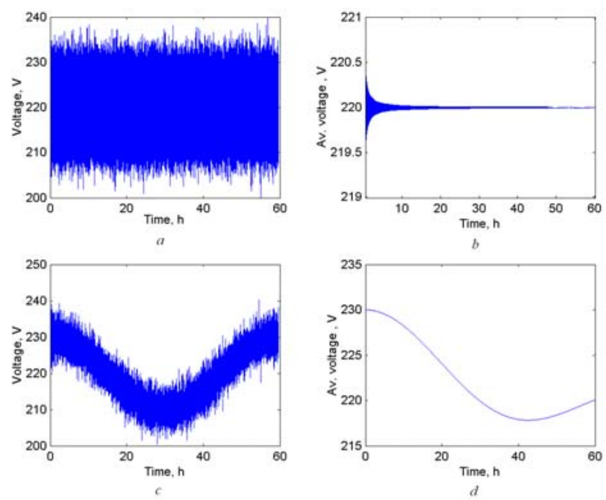



Fig. 3. Models of random processes with high (model 3) (a) and low (model 4) (c) oscillation frequency of expectation and according averages (b, d)

It follows from expression (2), that fluctuation of average expectation $m_{y(t)}$ tends to zero when $t \to \infty$. This means that, although random process with periodic oscillation of expectation has in some observation intervals disturbances of statistical stability, it is statistically stable in infinite interval.

Let $X(t)$ is a random process consists of $Q$ processes $X_q(t)$ ($q = \overline{1, Q}$) with near equal levels and periodic expectations. Periods of these expectations are $T_1, T_2, \ldots, T_Q$. The period $T_{q+1}$ of each process $X_{q+1}(t)$ is essentially larger than the period $T_q$ of previous process $X_q(t)$.

In observation interval from zero to $t$ that is essentially less than $T_1$, there are no essential changes of expectations and therefore the process $X(t)$ is statistically stable, in practice. With approaching $t$ to the period $T_1$ the process $X_1(t)$ (and therefore the process $X(t)$) becomes more statistically unstable. With further rising of observation time statistical properties of the process $X_1(t)$ appear and the process $X(t)$ becomes similar to statistically stable one.

With approaching $t$ to the period $T_2$ the process $X_2(t)$ becomes more statistically unstable again. This leads to disturbance of statistical stability of the process $X(t)$, and so on. When $Q \to \infty$ and $T_Q \to \infty$ interchange of statistically stable and unstable states reaches to infinite observation interval.

When $T_{q+1} < 2T_q$ unstable areas are joined and process $X(t)$ is statistically unstable in all observation interval $[T_1, T_Q)$.

Forming process of statistically unstable areas is illustrated by the models 5 and 6 (Fig. 4).

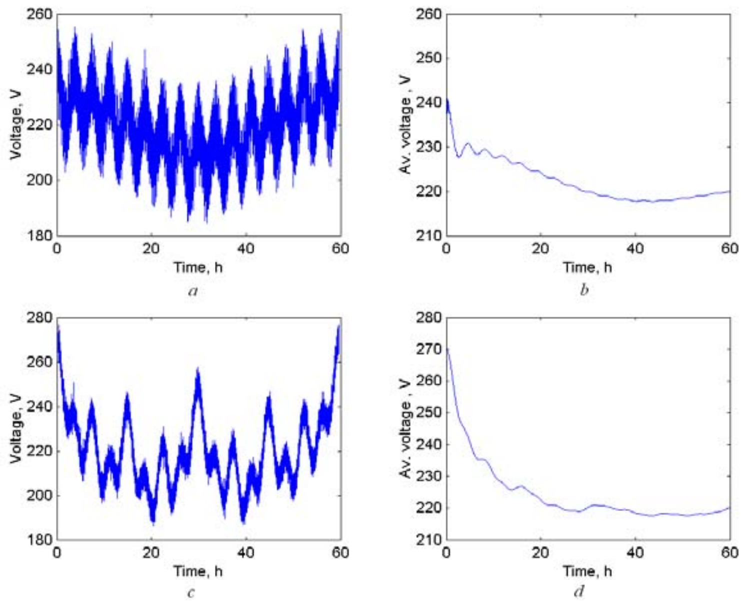



Fig. 4. Model of random process with expectation that has tree essentially different on frequency harmonics (model 5) (a), model of random process with five close to each other on frequency harmonics (model 6) (c), and according averages (b, d)

Described additive model explains interchanging of statistically stable and unstable states in real processes.

Random processes with intermittent changing of expectation are not interesting very much because distinguished samples create changes in average, however these changes are smoothed out fast (model 7, Fig. 5, *a*, *b*).

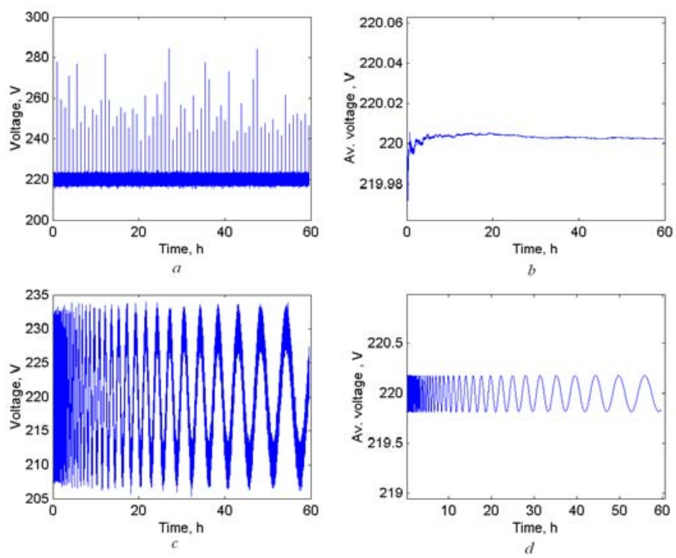



Fig. 5. Model of random process with intermittent change of expectation (model 7) (a), model of random process with aperiodic change of expectation (model 8) (c), and according averages (b, d)

Random processes with aperiodic changing of expectation are more interesting. Mark, process $X(t)$ with expectation, which changes periodically with the period $T$, may be regarded in the observation interval $t < T$ as aperiodic process. So, aperiodic processes may be statistically unstable.

If expectation $m_{x(t)}$ may be presented as expansion in a Taylor series

$$m_{x(t)} = \sum_{k=0}^{\infty} a_k t^k \ , \tag{3}$$

then

$$m_{y(t)} = \sum_{k=0}^{\infty} \frac{a_k t^k}{k+1} \ . \tag{4}$$

It follows from expressions (3) and (4), if expectation $m_{x(t)}$ is changed on low $t^k$ expectation $m_{y(t)}$ is changed on the same low too. So, if $m_{x(t)} = t^k$, the process $X(t)$ is not statistically stable in any observation interval.

Mark, in compare with expansion (3), expansion (4) has addition coefficient $(k+1)^{-1}$. Therefore, in general, changing low of expectation $m_{y(t)}$ doesn't repeat changing low of expectation $m_{x(t)}$ and so, processes with aperiodic changing of expectations are not always statistically unstable ones.

The process $X(t)$, expectation of which is changed periodically in logarithmic scale with period $T$, has interesting property. In this case, expectation of the process

$$m_{x(t)} = \sum_{k=-\infty}^{\infty} \dot{a}_k \exp\left( \frac{\mathrm{j}2\pi}{T} k \ln t \right).$$

According average expectation

$$m_{y(t)} = a_0 + 2\sum_{k=1}^{\infty} \frac{a_k}{\sqrt{1 + 4\pi^2 k^2 / T^2}} \sin(2\pi k \ln t/T + \varphi_k + \mathrm{arctg}(T / 2\pi k)).$$

As follow from last expression, for the process $X(t)$, expectation of which is changed periodically in logarithmic scale, average expectation $m_{y(t)}$ is described by harmonic non subsided functions. This mean that the process is not statistically stable in the interval $[0, \infty)$. Such process (model 8) and according average are presented in Fig. 5, *c*, *d*.

5. Random processes in finite observation interval: characteristics of statistical instability

In a finite observation interval statistical instability of random sequence $X_1, X_2, ..., X_N$ or random process $X(t)$, both with limited first two moments, can be characterized by parameters describing fluctuation of average $Y_N$ or fluctuation of expectation $m_{y_N}$.

Fluctuation of average $Y_N$ is characterized by sample variance $\bar{D}_{Y_N}$ of average. To characterize statistical instability of random sequence, the parameter of statistical instability $\gamma_{1N} = \dfrac{\mathrm{M}\left[\bar{D}_{Y_N}\right]}{D_{x_N}}$ may be used where $\mathrm{M}[\bullet]$ is expectation operator and $D_{x_N} = \dfrac{1}{N}\sum_{n=1}^{N} D_{x_n}$ is mean of variances $D_{x_n}$ of random variables $X_n$.

Fluctuation of expectation $m_{y_N}$ is characterized by fluctuation variance $\bar{D}_{m_{y_N}} = \dfrac{1}{N}\sum_{n=1}^{N}(m_{y_n} - \bar{m}_{m_{y_N}})^2$ of expectation of average $Y_N$, where $\bar{m}_{m_{y_N}} = \dfrac{1}{N}\sum_{n=1}^{N} m_{y_n}$ is fluctuation average of expectation $m_{y_n}$ of average $Y_N$.

Therefore another parameter of statistical instability may be the parameter $\gamma_{2N} = \frac{\bar{D}_{m_{y_N}}}{D_{x_N}}$ that is fluctuation variance of average expectation $\bar{D}_{m_{y_N}}$ normalized on mean $D_{x_N}$ of variances of random variables $X_n$.

It follows from Chebyshev theorem, that parameter of statistical instability $\gamma_{1N}$ tends to parameter of statistical instability $\gamma_{2N}$ when $N \to \infty$. If the sequence is statistically stable, the value of these parameters tends to zero (because $\mathrm{M}\left[\bar{D}_{Y_N}\right] \to 0$ and $D_{x_N}$ is a finite quantity) and if the sequence is statistically instable, the value of them tends to some positive number, fluctuates, or tends to infinite.

The values of statistical instability $\gamma_{1N}$ and $\gamma_{2N}$ depend from value of expectation of sample variance of average $\mathrm{M}\left[\bar{D}_{Y_N}\right]$, variance of average expectation $\bar{D}_{m_{y_N}}$, and mean $D_{x_N}$ of variances of random variables $X_n$. With reducing of fluctuation parameters $\mathrm{M}\left[\bar{D}_{Y_N}\right]$ and $\bar{D}_{m_{y_N}}$, parameters of statistical instability are reducing too, and with reducing of mean $D_{x_N}$ of variances, parameters of statistical instability are rising.

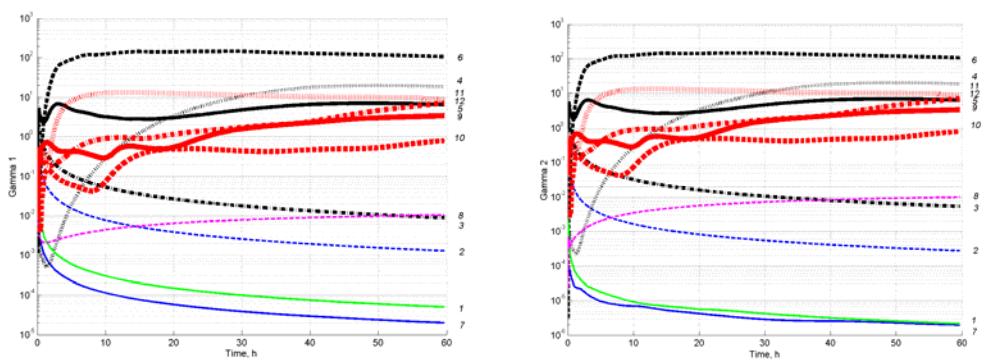
Sometimes more useful are parameters of statistically instability $\mu_{1N}$, $\mu_{2N}$ linked with parameters $\gamma_{1N}$ and $\gamma_{2N}$ by the following expressions: $\mu_{1N} = \sqrt{\frac{\gamma_{1N}}{1+\gamma_{1N}}}$, $\mu_{2N} = \sqrt{\frac{\gamma_{2N}}{1+\gamma_{2N}}}$. Less values of parameters $\mu_{1N}$, $\mu_{2N}$ more statistically stable is according sequence.

Parameters $\gamma_{1N}$ and $\gamma_{2N}$ have only lower bound (that is zero) and parameters $\mu_{1N}$, $\mu_{2N}$ have as lower as upper bounds: lower bound equals to zero and upper bound – to unit.

Calculating results of parameters $\gamma_{1N}$ and $\gamma_{2N}$ for described eight models and also four obtained records of line voltage are presented in Fig. 6. According calculating results of parameters $\mu_{1N}$ and $\mu_{2N}$ are viewed in Fig. 7.

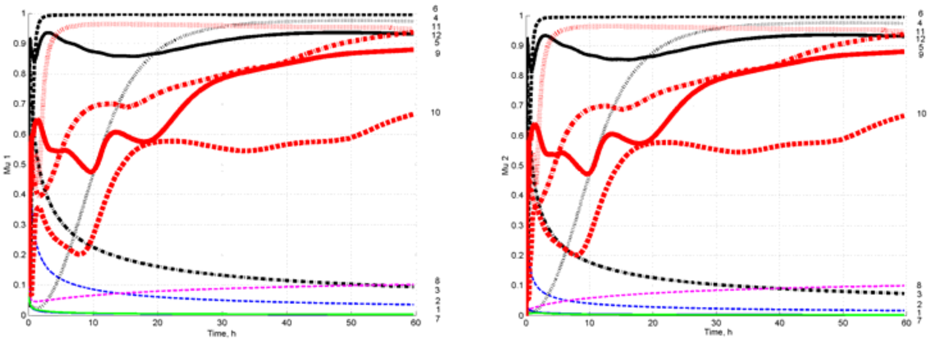
Thin lines 1 – 2 and 7 – 8 present calculating results for models 1 – 2 and 7 – 8 accordingly, bold lines 3 – 6 present calculating results for models 3 – 6 accordingly, and heavy lines 9 – 12 present calculating results for obtained records of line voltage.

In calculations, sample variances were used instead of variances.




a b

Fig. 6. Parameter of statistical instability of average $\gamma_{1N}$ (a) and parameter of statistical instability of expectation of average $\gamma_{2N}$ (b)



a b

Fig. 7. Parameter of statistical instability of average $\mu_{1N}$ (a) and parameter of statistical instability of expectation of average $\mu_{2N}$ (b)

It follows from the curves, in the area of high observation time for all models and real processes, parameter of statistical instability of average $\gamma_{1N}$ practically equals to parameter of statistical instability of expectation of average $\gamma_{2N}$ ($\gamma_{1N} \approx \gamma_{2N} = \gamma_N$) and parameter of statistical instability of average $\mu_{1N}$ – to parameter of statistical instability of expectation of average $\mu_{2N}$ ($\mu_{1N} \approx \mu_{2N} = \mu_N$). So, any of these parameters may be used to describe statistical instability of the process.

Real processes can be presented as a mixture of statistical stable and statistical unstable components. Value of parameter $\mu_N$ characterizes approximately percentage composition in the researched process of statistically unstable components. Time changes of value of this parameter give information about observation intervals on which the process can be regarded as statistically stable or statistically unstable.

For models 1 – 3 and 7, that accord to statistically stable processes, parameter value $\mu_N$ steadily reduces with rising of observation time $t$; for models 4 – 6 and 8, that accord to statistically unstable processes, value of this parameter increases. In the area of large observation times, for all real voltage processes the value of the parameter or steadily increases or oscillates on high level.

In the area of large observation times, values of parameters $\gamma_{1N}$, $\gamma_{2N}$ and $\mu_{1N}$, $\mu_{2N}$ are higher for statistically unstable processes than for statistically stable ones. This fact corroborates usefulness of parameters $\gamma_{1N}$, $\gamma_{2N}$ and $\mu_{1N}$, $\mu_{2N}$ for detection of statistical stability disturbance.

In the area of large observation times, values of instability parameters $\gamma_{1N}$, $\gamma_{2N}$ are in dozens times more for all experimental obtained records (not only presented in Fig. 6, 7) than for researched statistically stable models. It follows from this result that line voltage fluctuation has evidently statistically instable properties.

The interval on which parameters of statistical instability have high values, begins from some hours and follows to end of the records. So, the area of statistical instability is continues one, and coves range from some hours to not less 60 hours.

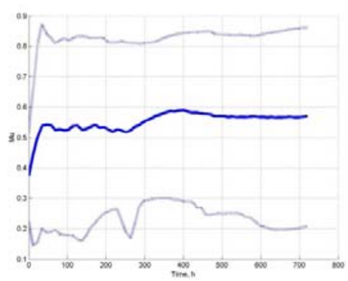
Stable character of disturbances of statistical stability of line voltage allows to assume that there are analogous disturbances of statistical stability in case of other physical (and may be not only physical) phenomena. Statistical stability of some others physical quantities are researched in the next sections.

## 6. Experimental research of statistical stability of height of sea waves and of their periods

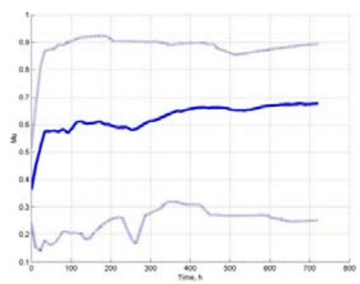
Statistical stability of height of sea waves and of their periods was experimentally researched on the basis of statistical data obtained by Shirshov Oceanology Institute, RAS [ESIMO, 2010] from 2001 to 2003 years over a period of 15 months. Data was registered by wave station with interval from an hour to some hours.

Statistical instability parameters $\mu_N$ were calculated for height of sea waves and for their periods (Fig. 8).

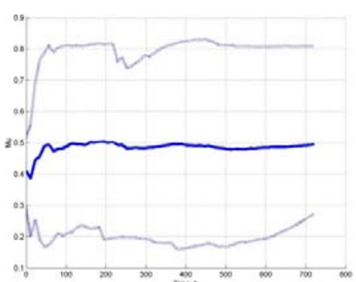
It follows from the figure that statistical instability parameters $\mu_N$ have high value beginning from the zero reading, according the first ten observation hours. This means that in all observation interval, time dependences of height of sea waves and time dependences of periods of these waves have statistically unstable character. Therefore in time intervals, that are more ten hours statistical prediction of value of these parameters is practically impossible.



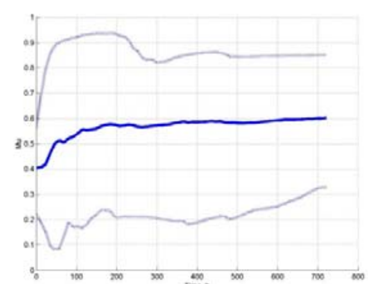
a



b



c



d

Fig. 8. Parameter of statistical instability $\mu_N$ averaged on 15 months (solid lines) and bounds of the parameter (dotted lines): a – for maximum height of waves, b – for 10% height of waves, c – for periods of maximum height of waves, d – for periods of 10% height of waves

## 7. Experimental research of statistical stability of Earth magnetic field

Magnetic field of the Earth is changed in time and space. Observations of its fluctuation are led for a lot years in different Earth points.

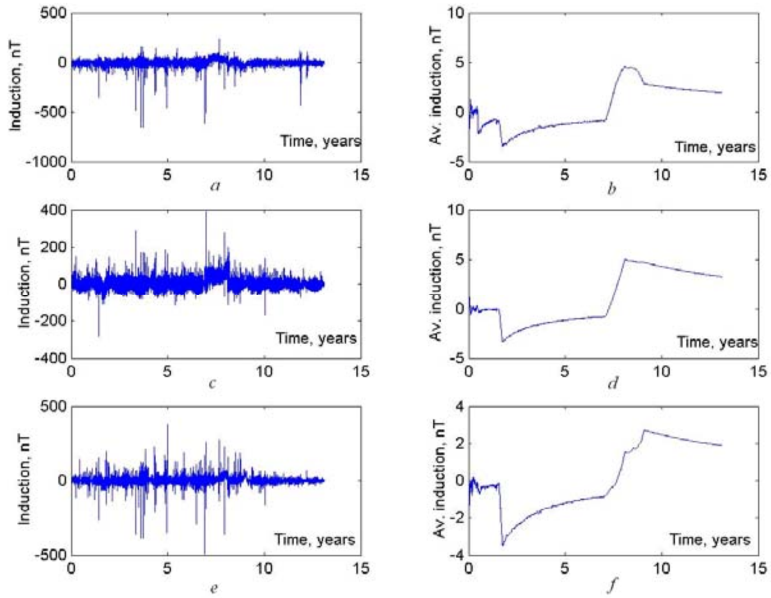



Fig. 9. Time dependencies of $x$ , $y$ , and $z$ components of Earth magnetic induction for13 years in Moscow region (*a*, *c*, *e*) and according time dependencies of averages (*b*, *d*, *f*)

Time dependencies of $x$ , $y$ , and $z$ components of Earth magnetic induction obtained on the basis of IZMIRAN data [IZMIRAN, 2010] are presented in Fig. 9 *a*, *c*, *e*. According time dependencies of averages are shown in Fig. 9 *b*, *d*, *f*. Statistic instability parameters $\mu_{1N}$ , $\mu_{2N}$ calculated on the described technique are presented in Fig. 10.

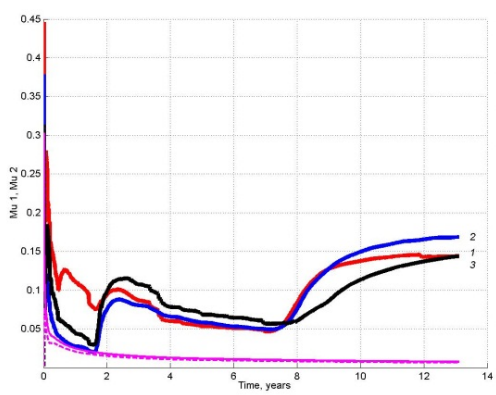
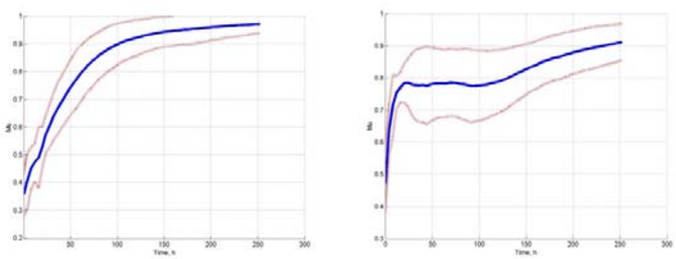



Fig. 10. Time dependencies of statistical unstable parameters $\mu_{1N}$ , $\mu_{2N}$ for $x$ , $y$ , and $z$ components of Earth magnetic induction for 13 years in Moscow region (according *1*, *2*, and *3* curves) and also for control white Gaussian noise (curves without number). Solid lines accord to parameters $\mu_{1N}$ and dotted lines – to parameters $\mu_{2N}$

Analyze of curves shows that Earth magnetic induction field is not statistically stable, in general, although there are some time intervals where magnetic induction is practically stable. Duration of these intervals is not regular and fluctuates from some months to some years. Hence, statistical prediction of Earth magnetic induction field in the interval that is more some months is problematic and in the interval that is more some years is practically impossible.

## 8. Experimental research of statistical stability of currency rate

Imaging about statistical instability of currency rate gives curves in Fig. 11, obtained on the basis of FOREXITE data [FOREXITE, 2010]. It follows from curves, statistical instability parameter has high values beginning from the first observation hours and continuously rises. Hence, currency rate is statistically unstable quality and its statistical prediction is practically impossible.



а б

Fig. 11. Statistical instability parameter $\mu_N$ averaged on 16 decades (solid line) and bounds of its changes determined by STD (dotted lines) for currency rate of AUD relative to USD for 2001 (a) and 2002 (b) years

## Conclusion

The materials generalizing results of author's researches in the field of disturbance of statistical stability have been presented. The concepts of statistical stability of random sequences and random processes are formalized. Parameters characterizing their statistical stability in finite observation interval are proposed. It is found that statistical unstable processes are especial class of non-stationary processes. Statistical stability disturbance can be explained by low frequency cyclical fluctuations of expectation. It has been researched statistical stability of a number of physical quantities and processes, in particular line voltage, height of sea waves and their periods, Earth magnetic field fluctuation, and currency rate. It has been found that for all researched quantities and processes, there is statistical stability disturbance on large observation intervals. Obtained results corroborate the hypotheses that are in base of new physico-mathematical theory of hyper-random phenomena: real physical events, quantities, processes, and fields are not statistical stabile and they can be adequately described by hyper-random models taking into consideration their statistical instability.

## Bibliography

[Alimov, Kravtsov, 1992] Yu.I.Alimov, Yu.A.Kravtsov. Is probability a "normal" physical quantity? // Physics-Uspekhi. – Vol. 5. – No 7, 1992. – P. 606 – 622.

[Bernoulli, 1713] Jakob Bernoulli. The art of conjecturing. – 1713.

[Crownover, 1995] R.M.Crownover. Introduction to fractals and chaos. Jones and Bartlett Pub., Inc., Boston – London. – 1995.

[ESIMO, 2010] Единая государственная система информации об обстановке в мировом океане ЕСИМ. Данные Института океанологии им. П.П. Ширшова РАН. http://ias.ocean.ru/esimo.

[Ferson, Kreinovichy, Ginzburg, Myers, 2003] S.Ferson, V.Kreinovichy, L.Ginzburg, D.S.Myers. Constructing probability boxes and Dempster-Shafer structures. SAND report SAND2002-4015. – 2003. – 143 p.

[FOREXITE, 2010] FOREXITE. www.forexite.com.

[Gorban, 2008] I.I.Gorban. Hyper-random Phenomena: Definition and Description // International Journal "Information Theories and Applications". – No 15, 2008. – P. 203 – 211.

[Gorban, 2009] I.I.Gorban. Cognition horizon and the theory of hyper-random phenomena. // International Journal "Information Theories and Applications". – 2009.

[Hagan, Demuth, Beale, 1996] M.T.Hagan, H.B.Demuth, M.H. Beale. Neural network design. Boston. MA: PWS Publishing. – 1996.

[ISO, 2006] International standard ISO 3534-1:2006(E/F). Statistics. Vocabulary and symbols. Part I: General statistical terms and terms used in probability. – 2006.

[IZMIRAN, 2010] Данные о вариации индукции магнитного поля в районе Москвы. Институт земного магнетизма, ионосферы и распространения радиоволн им. Н.В. Пушкова РАН. http://forecast.izmiran.rssi.ru/bankr.htm.

[Keller, 1986] J.B. Keller. The probability of heads // Am. Math. Monthly. Vol. 93, 1986. – P. 191.

[Mises, 1964] R. von Mises. Mathematical theory of probability and statistics / Edited and complemented by H. Geiringer. – N.Y. and London: Acad. Press. – 1964. – 232 p.

[Zadeh, Kacprzyk, 1992] L.A.Zadeh, J.Kacprzyk (Eds.). Fuzzy logic for the management of uncertainty. John Wiley & Sons. – New York. – 1992.

[Алефельд, Херцбергер, 1987] Г.Алефельд, Ю.Херцбергер. Введение в интервальные вычисления. – М.: Мир. – 1987. – 356 с.

[Гнеденко, 1961] Б.В.Гнеденко. Курс теории вероятностей. – М.: Изд-во физ.-мат. литературы. – 1961. – 406 с.

[Горбань, 2007] И.И.Горбань. Теория гиперслучайных явлений. К.: ИПММС НАН Украины. – 2007. – 184 с. (эл. версия на сайте http://ifsc.ualr.edu/jdberleant/intprob/).

[Горбань, 2010 (1)] И.И.Горбань. Нарушение статистической устойчивости физических процессов // Математические машины и системы. – №1, 2010. – С. 171 – 184.

[Горбань, 2010 (2)] И.И.Горбань. Эффект статистической неустойчивости в гидрофизике. Труды десятой Всероссийской конференции «Прикладные технологии гидроакустики и гидрофизики» 25 – 27 мая 2010, Санкт-Петербург.

[Горбань, 2010 (3)] И.И.Горбань. Статистическая неустойчивость магнитного поля Земли. Труды 6 дистанционной конференции “Системы поддержки принятия решений. Теория и практика. СППР '2010” 7 июня 2010 г., Киев.

[Горбань, 2010 (4)] И.И.Горбань. Исследование статистической устойчивости курса валют. Труды пятой конференции «Математическое и имитационное моделирование систем» 22 – 26 июня 2010, Киев.

[Колмогоров, 1936] А.Н.Колмогоров. Основные понятия теории вероятностей. – М.: ОНТИ. – 1936. – 175 с.

[Колмогоров, 1986] А.Н.Колмогоров. О логических основаниях теории вероятностей // Теория вероятностей и математическая статистика. – М.: Наука. – 1986. С. 467 – 471.

[Скороход, 1990] Предисловие А.В.Скорохода к монографии В.И.Иваненко, В.А.Лабковский. Проблема неопределенности в задачах принятия решения. – К.: Наукова думка. – 1990. – 134 с.

[Тутубалин, 1972] В.Н.Тутубалин. Теория вероятностей. – М.: Из-во Московского университета. – 1972. – 230 с.

[Шокин, 1981] Ю.И.Шокин. Интервальный анализ. – Новосибирск: Наука. – 1981. – 112 с.

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