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APPLICATION OF PARETO OPTIMIZATION APPROACH TO OBSERVABILITY PROBLEM IN LINEAR AERODYNAMIC MODELS

Andriy Zavorotnyy, Veda Kasyanyuk

Abstract: *Application of Pareto optimization based method of solving observability problem in aerodynamics is developed and considered. The error distribution of derivative approximation is investigated. The method takes into account the distribution of derivative approximation error. Systems with linear control are considered. The software for observation problem solving and research of considered method is developed and described. Results of method investigation with different data errors on generated data and on real flight data are given.*

Keywords: *Observability problem, Pareto optimization, aerodynamics, derivative approximation.*

ACM Classification Keywords: *I.6 Simulation and Modeling*

Introduction

Observability problem is the classical problem of control theory and consists in recovering the whole value of state-vector having known only some function of it: $y = f(x)$. $f(x)$ is usually some linear combination of x. The problem can also be formulated as obtaining function $G(y)$, so that $G(y) = G(f(x)) = E(x)$ is some estimate of x if such function G exists.

It is an urgent problem in aerodynamics, because all characteristics of aircraft cannot be measured by sensors during ordinary flight. While during test flights aircraft has more sensors, part of which is removed after testing.

A lot of methods, developed earlier are indeed not applicable because of their complexity and inadequacy in real needs.

The continuous model considered in this work is

$$\dot{x} = Ax + Bu \tag{1}$$

where $x = (x_1(t),\dots,x_n(t))^{\mathrm{T}}$ is the state-vector of object at $t \in [0;T]$, and $u = (u_1(t),\dots,u_n(t))^{\mathrm{T}}$ is the control-vector. A and B are real matrices of comptatible dimensions.

The linear model is considered. Assume it's known some matrix C so that $u = Cx$, thus

$$\dot{x} = \overline{A}x \tag{2}$$

where $\overline{A} = A + BC$.

The classical observation problem for (2) is having at time $\tilde{t} \in [0;T]$ defined first *m* elements of vector *x* to define it's unknown elements.

The Pareto optimization approach gives solution that satisfies several criteria representing different demands [Zavorotnyy, 2004], [Zavorotnyy, 2006].

As in this case it also considers errors in derivative estimation. That's why it was elected for solving observability problem.

Notations

The derivative $\dot{x}$ from (2) has to be estimated by values of x. Let's model the error in estimating $\dot{x}$ considering

$$\dot{x}_{\tilde{t}} = \overline{A}x_{\tilde{t}} + \nu \tag{3}$$

where $\dot{x}_{\tilde{t}}$ and $x_{\tilde{t}}$ are vectors $\dot{x}$ and x at $\tilde{t}$, $\nu = (\nu_1,\dots,\nu_n)^{\mathrm{T}}$ is stochastic variable, characteristics of which are given below.

This vector can also be interpreted as sensor-measurement error in case of sensor-measured $\dot{x}$.

Let the known part of $\dot{x}_{\tilde{t}}$ from (2) is defined as

$$\tilde{\dot{x}}_{\tilde{t}} = \tilde{A}x_{\tilde{t}} + \tilde{\nu} \tag{4}$$

where $\tilde{\dot{x}}_{\tilde{t}} = (\dot{x}_1(t),...,\dot{x}_m(t))^{\mathrm{T}}$, $\tilde{\nu} = (\nu_1(t),...,\nu_m(t))^{\mathrm{T}}$, $\tilde{A} = \begin{pmatrix} \overline{a}_{11} & \overline{a}_{1n} \\ \overline{a}_{m1} & \overline{a}_{mn} \end{pmatrix}$.

Then define

$$y = \tilde{\dot{x}}_{\tilde{t}} - \tilde{A}_m x^m_{\tilde{t}} \tag{5}$$

where $\tilde{A}_m = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{1n} \\ \tilde{a}_{m1} & \tilde{a}_{mm} \end{pmatrix}$ and $x_{\tilde{t}} = (x_1(\tilde{t}),...,x_m(\tilde{t}))^{\mathrm{T}}$.

Derivative approximation

As the method uses the covariance matrix of variable ν, it's distribution has to be studied. In this work we consider system (1) to have oscillating solutions, which is typical case in aerodynamics problems. Thus the distribution is studied for basic trigonometric functions sin(x) and cos(x). For both it appeared to have distribution

$$F_\nu(x) = \frac{1}{\pi}\arcsin(\frac{x}{\alpha}) \tag{6}$$

where α is the absolute maximum of the error.

Consequently from (6), $M(\nu) = 0$, and

$$D(\nu) = \frac{\alpha^2\pi}{2} \tag{7}$$

which can now be used in constructing covariance matrix after α having been estimated.

Let covariance matrix is

$$\tilde{\Re}_\nu = \begin{pmatrix} D(\nu_1) & 0 \\ 0 & D(\nu_m) \end{pmatrix} \tag{8}$$

where values $\nu_1 \ldots \nu_m$ are supposed to be independent and $D(\nu_i)$, $i = \overline{1,m}$ is defined in (7).

Pareto estimates

Substituting (4) in (5) we have :

$$y = \tilde{A}_{n-m} x^{n-m}_{\tilde{t}} + \tilde{\nu} \tag{9}$$

where $\tilde{A}_{n-m} = \begin{pmatrix} \overline{a}_{1m+1} & \overline{a}_{1n} \\ \overline{a}_{mm+1} & \overline{a}_{mn} \end{pmatrix}$ and $\tilde{x}^{n-m}_{\tilde{t}} = (x_{m+1}(\tilde{t}),...,x_n(\tilde{t}))^{\mathrm{T}}$. Remind that for $\tilde{\nu}$ expectation is 0 and it's known covariance matrix (8) which is obviously reversible since (7).

In Pareto optimization approach given in [Zavorotnyy, 2004) the followed two-criteria problem applied to (9) is considered:

$$\begin{cases} h(G) = M\|G\tilde{\nu}\|^2 \to \min\limits_G \\ \varphi(G) = \|G\tilde{A}_{n-m} - I\|^2 \to \min\limits_G \end{cases} \tag{10}$$

where first criterion represents noise background of estimate, and second represents operator residual. G is a linear operator, so that $G(y)$ would be the estimate of $\tilde{x}_{\tilde{t}}^{n-m}$.

According to considered method [Zavorotnyy, 2004], we obtain the following continuum of Pareto-optimal estimates from (10):

$$\hat{x}_{\tilde{t}}^{n-m} = \tilde{A}_{n-m}^{T}(\tilde{A}_{n-m}\tilde{A}_{n-m}^{T} + \mu\tilde{\Re}_{\nu})^{-1}y \qquad (11)$$

where μ is the parameter of Pareto-optimization. The increase of μ results in noise background decrease but also in operator residual [Zavorotnyy, 2004]. The criteria for defining value of μ can be found in [Voronin, 1980].

Software

The software implementing described approach (11) has been developed. It allows seeing the method in action both on real data, given in numeric representation as series of known parts of x-vector and on generated by given $\overline{A}$ matrix data (3).

Here is given possibility to compare used method with two other classical methods of solving observability problem, which are also implemented in the software.

In the given examples errors were generated in normal distribution law.

Errors (variance = 0.01) in measurements of known parts of x-vector result in non-smoothness of estimated curve:

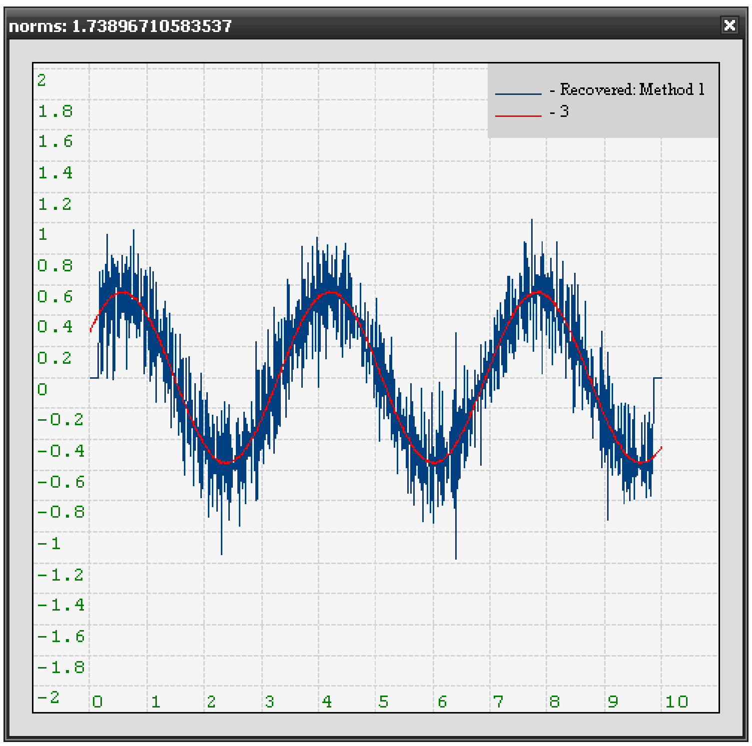

Figure 1: Non-smoothed estimation results.

The quality of curve is obviously much better after smoothing:

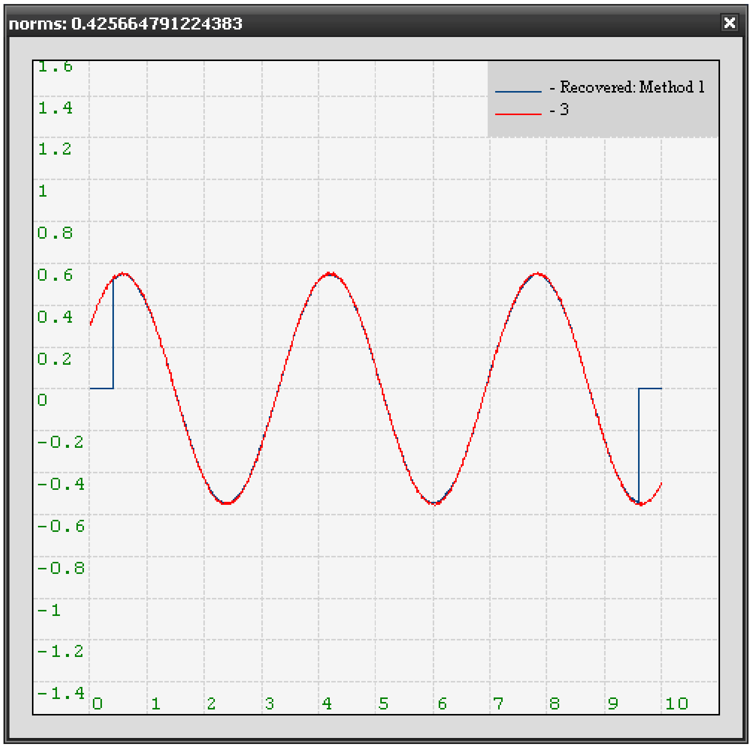

Figure 2: Smoothed estimation result.

Smoothness can substantially increase the quality of estimation because of zero expectation error of derivative estimation.

In the case of data measurement errors (variance = 0.0001), when classical methods are inconsistent, considered method gives perfect results:

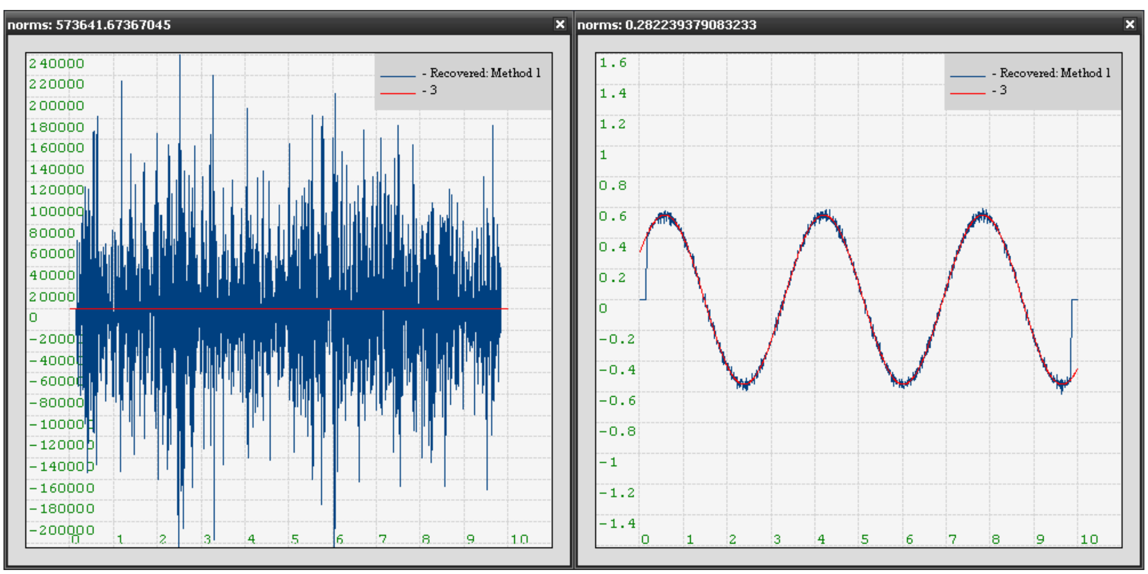

Figure 3: Non-smoothed estimation result of 2 methods.

Although after few steps of smoothing classical method gives acceptable results, it cannot be compared with results of the developed method, which perfectly fits the curve:

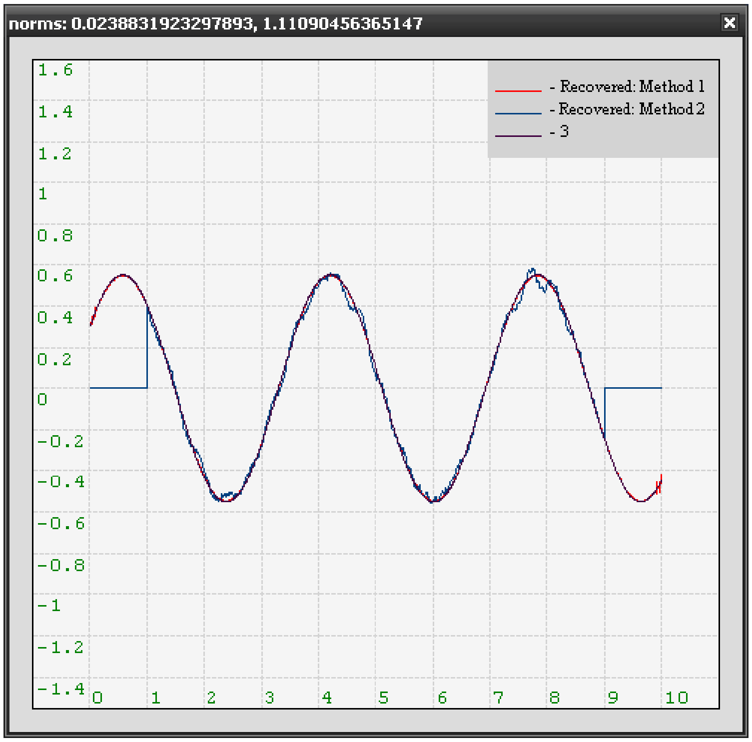

Figure 4: Smoothed estimation result of 2 methods.

In cases of larger measurement errors (variance=0.1) both methods give unacceptable results without smoothing, but this is what we have after smoothing (the best smoothing parameters for each method were selected):

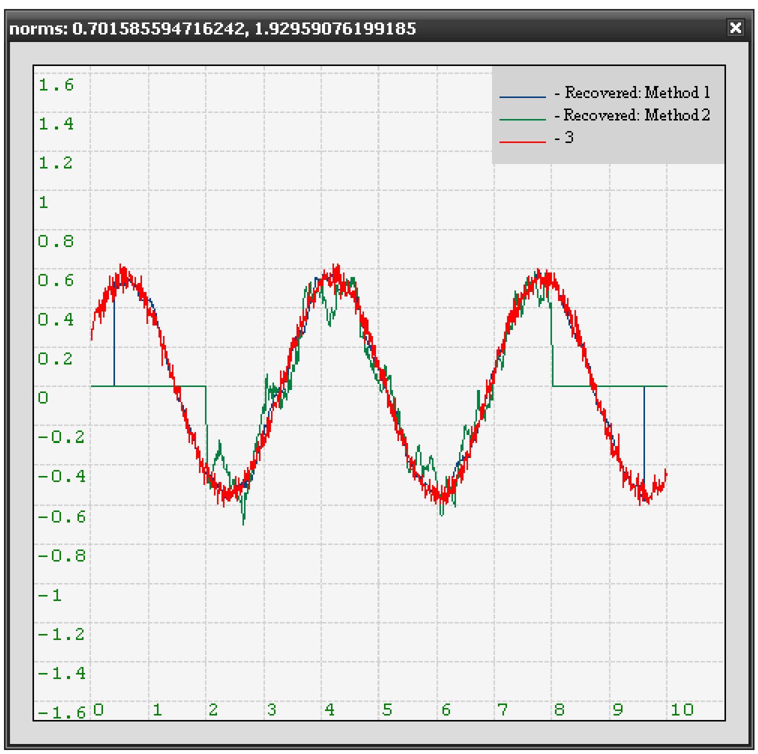

Figure 5: Smoothed estimation result of 2 methods

The order of derivative approximation method in (4) can also vary, influencing estimation result.

Conclusion

We have developed observability problem solving method, which is based on Pareto optimizations in the case of linear model in aerodynamics.

The method has been tested on generated data and showed the best results in comparison with classical methods.

As the problem has been investigated only in case of linear control, we shall continue work on considering wider range of models. Also we plan to run the method on real flight data and integrate received results with identification problem in case when some elements of matrix A are unknown.

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