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(editors)

Information Models of Knowledge

I T H E A®
K I E V – S O F I A
2010

Krassimir Markov, Vitalii Velychko, Oleksy Voloshin (ed.)
Information Models of Knowledge
ITHEA®
Kiev, Ukraine – Sofia, Bulgaria, 2010
ISBN 978-954-16-0048-1

First edition

Recommended for publication by The Scientific Concil of the Institute of Information Theories and Applications FOI ITHEA
ITHEA IBS ISC: 19.

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General Sponsor: Consortium FOI Bulgaria (www.foibg.com).

Printed in Ukraine

ISBN 978-954-16-0048-1

C\o Jusautor, Sofia, 2010

MODIFIED BRANCH AND BOUND ALGORITHM FOR SOLVING THE HAMILTONIAN RURAL POSTMAN PROBLEM

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***Abstract**: In this paper the Hamiltonian Rural Postman Problem is generalization of the Hamiltonian Travelling Salesman Problem is described. The offered modification of a classical method (Little's method) allows to find exact solution of the Hamilton Rural Postman Problem or to detect that the task is unsolvable.*

***Keywords**: Hamiltonian Rural Postman Problem, Hamiltonian cycle, branch and bound algorithm, Hamiltonian Traveling Salesman Problem*

***ACM Classification Keywords**: G.2.2 - Mathematics of Computing - Discrete Mathematics - Graph Theory - Path and circuit problems*

Introduction

Many results are saved up in research of the Travelling Salesman Problem. They cover the important problems of algorithm design and improvement of methods of combinatorial optimization and their application.

Algorithms with indicators of efficiency which are applicable in real situations aren't known for each applied problem of type of the Travelling Salesman Problem. One of such problems is Rural Postman Problem, formulated as follows [1].

Let *H*=(*V*, *U*) is connected weighed graph, *V* – set of vertices, $|V| = n$, *U* - set of edges. Each edge $\{i, j\} \in U$ has the weight $d_{ij} \in Z_0^+$, $i \neq j$, $i, j = \overline{1,n}$, Z_0^+ - set of non-negative numbers. The symmetric matrix of weights $\left[d_{ij}\right]_n$ completely defines weighted graph *H*. At this matrix: if $\{i, j\} \in U$ then $d_{ij} \in Z_0^+$, else $d_{ij} = \infty$, $i \neq j$, $i, j = \overline{1,n}$, $d_{ii} = \infty$, $i, j = \overline{1,n}$. The nonempty subset of edges $R \subseteq U$ is given. It is required to find a cycle which includes each edge from *R* and has minimum sum of weights of edges.

Let's designate the Hamiltonian cycle of graph *H* which is passing on all edges of set *R* as $z(R)$. Let's name as the Hamilton Rural Postman Problem (HRPP) the problem which consists in determination of the Hamiltonian cycle $z^*(R)$ minimizing a functional

$$C(z(R)) = \sum_{\{k,l\} \in z(R)} d_{kl} . \qquad (1)$$

Interest in solving the HRPP arises when it is required to find an annular route on a transport network of a city or the district, modeled by graph $H = (V,U)$. To each point of departure (arrival) of a network corresponds vertex $i \in V, |V| = n$, and to each edge $\{i, j\} \in U$ corresponds a segment of a road between pair of adjacent points *i* and *j*. Edge $\{i, j\}$ is characterized by weight (cost) d_{ij}. It is equal to expenses for vehicle movement from *i* to *j* or from *j* to *i*.

Algorithm

HRPP is NP-complete, because it is a NP-complete Hamiltonian Travelling Salesman Problem (HTSP) in case when $R = \varnothing$.

In [2] there is the algorithm which correctly finds solution of HTSP if graph *H* is Hamiltonian, or detects its unsolvability. Basis of the offered algorithm is a method of branch and bound scheme. It is fulfilled after check of sufficient conditions of unsolvability HRPP. Clearly, that the complexity of such check should be limited by a polynomial from a problem size.

Direct application of branch and bound algorithm from [2] does not allow to solve HRPP. Inclusion of a subset of edges $R \neq \varnothing$ in a required Hamilton cycle turned out so strong restriction that demands other approach to the organization of branching and an evaluation of the lower bound for $C\left(z^{*}\left(R\right)\right)$.

Obviously, if graph H contains suspended vertices HTSP and HRPP has no solutions. Suspended vertices in graph H are finding with complexity $O\left(|V|\right)$. The problems are unsolvable, when graph H has a concatenation point [2, 3]. It is required $O\left(|V|+|U|\right)$ elementary operations to define whether graph H contains a concatenation point [3].

It is easy to see, that HRPP is unsolvable, if in graph H a) the subset of edges of set R forms nonhamiltonian cycle; b) there is a vertex, which is incident three or more edges from R. Therefore, graph H in which the set of edges R does not form a collection of vertex-not crossed chains, does not contain cycle $z\left(R\right)$. Time limited in size $O\left(|V|+|U|\right)$ suffice to check the conditions.

It is known from [4], that HRPP can be transformed to the same task on graph $H=\left(V,U\right)$ in which a) degrees of all vertices are higher 2, b) the set of edges R, which is contain in a required Hamiltonian cycle, organizes matching R.

Let's notice, that $|R|$ limits a solution space so, that it can be empty. Let's name admissible solution $z\left(R\right)$ of HRPP by tour.

Search the solution of the task starts with conversion a matrix of costs of graph H to the reduced matrix [5]. The reason is that there is minimum element $\alpha_i=\min\limits_j d_{ij}$, $i=\overline{1,n}$ in line i which is subtracted from each element of this line. Then in the column j minimum element $\beta_j=\min d_{ij}, j=\overline{1,n}$ which is subtracted from each element of this column is retrieved. Elements α_i and β_j are named as reducing coefficients. From the reduced matrix $\left[d_{ij}\right]_n$ the lower bound of cost of the required solution is searched:

$$\varphi\left(R\right)=\sum_{\{i,j\}\in R}\min\left\{d_{ij},d_{ji}\right\}+\gamma, \tag{2}$$

where $\gamma=\sum_{i=1}^{n}\alpha_i+\sum_{j=1}^{n}\beta_i$.

Generally, the reduced matrix which defines set of all solutions HRPP is not symmetric. To this matrix the weighed multigraph $H'=\left(V,U'\right)$, in which vertices i and j are linked by pair arcs (i, j) and (j, i) if in graph $H=\left(V,U\right)$ they are linked by edge $\left\{i,j\right\}$, corresponds one-to-one. Thus, each edge $\left\{i,j\right\}\in R$ of graph H is presented in multigraph H' by two arcs $\left(i,j\right)\in R'$, $\left(j,i\right)\in R'$, $|U'|=2|U|$, $|R'|=2|R|$.

From the reduced matrix we select elements $\left(i,j\right)$ and $\left(j,i\right)\in R'$ for which $\Delta_{ij}=\min\left\{d_{ij},d_{ji}\right\}>0$ and let

$d_{ij}=d_{ij}-\min\left\{d_{ij},d_{ji}\right\}=d_{ij}-\Delta_{ij}$, $d_{ji}=d_{jji}-\min\left\{d_{ij},d_{ji}\right\}=d_{ji}-\Delta_{ij}$.

Let's name obtained matrix $\left[d_{ij}\right]_n$ completely reduced. The further operations will be fulfilled by means of this matrix.

Conversion of the matrix of costs of graph H to completely reduced matrix has the argued substantiation. HRPP for completely reduced matrix consists in construction the Hamiltonian cycle or detour the minimum cost which includes exactly one arc from each pair $\left(i,j\right)$, $\left(j,i\right)\in R'$, containing at least one arc with zero weight. Thus, the list of arcs of the zero weight which are candidates for inclusion in optimal solution is defined on matrix $\left[d_{ij}\right]_n$.

The explained reasons open possibility to adapt the classical algorithm of branch and bound described in [6], for searching the solution of the HRPP. The mode of branching of admissible solutions $z(R)$ keeps within the known scheme of construction of a binary search tree [6].

Let's put in correspondence to the root of a tree of search $\{z(R)\}$ completely reduced matrix $\left[d_{ij}\right]_n$ with bound $\varphi(R)$, define the arc of multigraph H' which initiates branching. To realize it, as well as at [6], each element (p,q) in $\left[d_{ij}\right]_n$, if $d_{pq}=0$, let's estimate as value

$$\gamma(p,q)=\min_{i\neq q} d_{pi}+\min_{j\neq p} d_{jq}, \tag{3}$$

also find the element (p,q) which has the greatest value

$$\Gamma(k,l)=\max\{\gamma(p,q)\,|\,d_{pq}=0\}, \tag{4}$$

In multigraph H', the arc (p,q) corresponds to element (p,q). This arc initiates a partition of set of all detours to two subsets and, in the conditions of HRPP, generates two cases: $\{k,l\}\notin R$ and $\{k,l\}\in R$.

In case of $\{k,l\}\notin R$ the set of all solutions of the problem is divided into subsets $\{\{k,l\}\notin R\}$ and $\{(\overline{k,l})\}$. The first subset includes all detours which contain arc (k,l), the another one includes all detours which do not contain this arc.

The matrix by which the low bound $\varphi((k,l)\in R')$ of cost of all detours of set $\{\{k,l\}\notin R'\}$ is calculated, is provided as in [6] according to the following rule.

If the set *R* contains the edge $\{x,k\}$, the required detour together with the arc (k,l) joins the arc (x,k). Similarly, if the set *R* contains the edge $\{y,l\}$, the arc (k,l) joins the arc (l,y). Inclusion of the arc (x,k) or the arc (l,y) in a subset of solutions $\{(k,l)\notin R'\}$ means, that the matrix which defines it, does not contain the line *k* and the column *l*, as well as the line and the column, numbers of which are the beginning and the end of the joined arc. In case of $\{x,k\}$, $\{y,l\}\in R$, the arcs (x,k) and (l,y) join to arc (k,l), and in the matrix, which defines the set $\{(k,l)\notin R'\}$, the lines *x*, *k*, *l* and the columns *k*, *l*, *y* exclude.

Let's designate the lower bound of branching vertex as φ. For an arc $(k,l)\notin R'$ which initiates branching, let

$$\mu_k=\begin{cases} d_{xk}, \text{ if } \{x,k\}\in R, \\ 0, \text{ else.}\end{cases},\quad \mu_l=\begin{cases} d_{ly}, \text{ if } \{l,y\}\in R, \\ 0, \text{ else.}\end{cases}$$

Here d_{xk}, d_{ly} - the elements of a matrix which corresponds to the bound φ. In case $\varphi=\varphi(R)$ they are the elements of the completely reduced matrix $\left[d_{ij}\right]_n$. Then, the cost of all detours of the set $\{(k,l)\notin R'\}$ is bounded below by the value

$$\varphi((k,l)\notin R)=\varphi+\mu_k+\mu_l+\sum\alpha_i'+\sum\beta_j', \tag{5}$$

where α_i' and β_j' - the reduction factors. Those coefficients are obtained as a result of transformation of a matrix, which to the bound φ corresponds, to a matrix, which defines the set $\{(k,l)\notin R\}$.

The matrix which defines the set $(\overline{k,l})$, and the bound $\varphi(\overline{k,l})$ are presented by the value $\varphi(\overline{k,l})=\varphi+S(k,l)$, where $S(k,l)$ is the sum of reduction factors which is obtained as a result of reduction of a matrix of costs.

Let's describe a case $(k,l)\in R$. Solution space HRPP $\{z(R)\}$ is divided into two subsets $\{(k,l)\}$ and $\{(l,k)\}$. The first subset contains all detours which include the arc (k,l), the second one - all detours which include the arc (l,k).

To the branching vertex boundary φ and matrix *D* which generates arc (k,l) or (l,k), $\{k,l\}\in R$, with a maximum estimation let correspond. The matrix which defines subset $\{(k,l)\}$, is a result of elimination from *D* a line *k* and a column *l*, appropriations the value ∞ to element d_{lk} and reduction of the obtained shorted matrix. Similarly, for determination a matrix which defines subset $\{(l,k)\}$, it is necessary in *D* to suppose $d_{kl}=\infty$, to remove a line *l* and a column *k* and to fulfill the reduction of the obtained shorted matrix. The lower boundaries of cost of detours for subset $\{(k,l)\}$ and $\{(l,k)\}$ are calculated as follows:

$$\varphi(k,l)=\varphi+d_{kl}+\sum_{i\neq k}\alpha_i'+\sum_{j\neq l}\beta_j', \tag{6}$$

$$\varphi(l,k)=\varphi+d_{lk}+\sum_{i\neq k}\alpha_i'+\sum_{j\neq l}\beta_j', \tag{7}$$

where α_i', β_j' - reduction factors which are obtained as a result of transformation of a matrix *D*.

The situation when either $d_{kl}=\infty$, or $d_{lk}=\infty$ in D is possible. It follows from (1), that if $d_{kl}=\infty$, $\{(k,l)\}=\varnothing$, if $d_{lk}=\infty$, $\{(l,k)\}=\varnothing$.

Modified Little's method of solving the Hamiltonian Rural Postman Problem has the following appearance.

0. $H=(V,U)$ - the nonoriented weighed graph with degrees of vertices higher than 2, presented by a matrix of costs with power $n=|V|$; R − matching of graph *H* which contains in required Hamiltonian cycle $z^*(R)$ of the minimum cost. Let $C^*=\infty$.

1. The matrix of costs of graph *H* is transformed to the reduced matrix. From it, the lower bound $\varphi(R)$ of costs for all solutions of a problem is calculated. From the reduced matrix completely reduced matrix $D=\left[d_{ij}\right]_n$ which corresponds to the oriented weighed multigraph $H'=(V,U')$, $|U'|=2|U|$, $|R'|=2|R|$ is defined. In H' it is required to find the Hamiltonian cycle (detour) which contains exactly the one arc from each pair of arcs (i,j), $(j,i)\in R'$ and has the minimum cost.

2. In matrix *D* under formulas (3) and (4) an element (k,l) is calculated. In a multigraph it is presented by an arc (k,l) which generates the branching vertex in a search tree.

3. If $\{k,l\}\notin R$, then go to step 7.

4. If $d_{lk}=\infty$, then a subset of detour $\{(l,k)\},\{k,l\}\in R$, which contains an arc (l,k) is empty; go to step 6.

5. To define the subset of detours $\{(l,k)\},\{k,l\}\in R$ which contain an arc (l,k), the reduced matrix in which $d_{kl}=\infty$ and with excluded line *l* and the column *k* is considered. After matrix reduction under the formula (7), the lower boundary $\varphi(l,k)$ of cost of arcs of all detours of subset $\{(l,k)\}$ is calculated. Subset $\{(l,k)\}$ generates in a search tree the current active vertex $\{(l,k)\}$. If $C^*>\varphi(l,k)$ then vertex $\{(l,k)\}$ with reduced matrix *D* and bound $\varphi(l,k)$ joins branching vertex, else it is not considered further. Go to step 9.

6. To determinate the subset of detours $\{(k,l)\},\{k,l\}\in R$ which contain an arc (k,l), the reduced matrix in which $d_{lk}=\infty$ and both the line *k* and the column *l* are excluded is considered. Under the formula (6) the lower

bound $\varphi(k,l)$ of cost of all detours of subset $\{(k,l)\}$ which initiates in a searching tree the current active vertex $\{(k,l)\}$ is calculated. If $C^* > \varphi(k,l)$, then a vertex $\{(k,l)\}$ with a reduced matrix *D* and a bound $\varphi(k,l)$ joins the branching vertex, otherwise the vertex is excluded from the further reviewing; go to step 9.

7. In determination of the subset of detours $\{(\overline{k,l})\}, \{k,l\} \notin R$ which is not containing arcs (k,l), the matrix in branching vertex is reduced to matrix *D* after replacement $d_{kl} \neq \infty$ on $d_{kl} = \infty$. If $C^* > \varphi(\overline{k,l})$ then in a searching tree a vertex $\{(\overline{k,l})\}$ is added to the branching vertex together with a reduced matrix *D* and a lower bound $\varphi(\overline{k,l})$ of cost of detours $\{(\overline{k,l})\}$, otherwise it has no further prolongation. The bound $\varphi(\overline{k,l})$ is equal to the lower bound of vertex of branching increased by the sum of reduction factors.

8. For determination of the subset of detours $\{(k,l) \notin R'\}$ which contain arc (k,l), in a matrix at branching vertex the line *k* and a column *l* is excluded*;* lines *s*, *k* and columns *k*, *l*, if $\{s,k\} \in R$, are excluded; lines *k*, *l* and columns *p*, *l*, if $\{l,p\} \in R$, are excluded; lines *s*, *k*, *l* and columns *k*, *l*, *p*, if $\{s,k\},\{l,p\} \in R$,'are excluded. Matrix *D* is searched by reduction of the obtained matrix. After an evaluation of the lower bound $\varphi((k,l) \notin R')$ of cost of detours $\{(k,l) \notin R'\}$ which is defined under formulas (5), a vertex $\{(k,l) \notin R'\}$ with the matrix *D* and a bound $\varphi((k,l) \notin R')$ at performance of condition $C^* > \varphi((k,l) \notin R')$ joins branching vertex, otherwise it is excluded from review.

9. If dimension of matrix *D* is equal to 1, admissible solution of HRPP is obtained. If the estimation of vertex of a branching tree is less than $C(z(R))$, it is remembered obtained admissible solution $z(R)$. We appropriate an estimation of current vertex as $C(z(R))$. We exclude from the further reviewing the vertices which have estimation more or equal C^*. If there is no remained vertex after elimination, go to step 10. Otherwise, it's necessary to find a vertex of searching tree with the minimal estimation. If there are such vertices, chose vertex which has the greatest depth.

10. If no admissible solution is found, then HRPP is unsoluble. Otherwise $z^*(R)$ - a required Hamiltonian cycle, and C^* - its weight.

Example

Graph $H = (V,U)$ is defined by the matrix of costs

$$
\left[d_{ij}\right]_6 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix}
\infty & 10 & \infty & 5 & 7 & 18 \\
10 & \infty & 23 & 15 & \infty & 8 \\
\infty & 23 & \infty & 25 & 24 & \infty \\
5 & 15 & 25 & \infty & \infty & 23 \\
7 & \infty & 24 & \infty & \infty & 11 \\
18 & 8 & \infty & 23 & 11 & \infty
\end{bmatrix}
\end{array}.
$$

Subset of edges $R = \{\{3,4\},\{1,5\},\{2,6\}\}$. It is required to find a solution of HRPP or to detect that it has no solution.

1. We reduce given matrix of costs and obtain reduced matrix

$$\left[d_{ij}\right]_6 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & \infty & 5 & \infty & 0 & 1 & 13 \\ 2 & 2 & \infty & 0 & 7 & \infty & 0 \\ 3 & \infty & 0 & \infty & 2 & 0 & \infty \\ 4 & 0 & 10 & 5 & \infty & \infty & 18 \\ 5 & 0 & \infty & 2 & \infty & \infty & 4 \\ 6 & 10 & 0 & \infty & 15 & 2 & \infty \end{array} .$$

We calculate the lower bound of costs of all HRPP solutions:

$$\varphi(R) = \min\{d_{34}, d_{43}\} + \min\{d_{15}, d_{51}\} + \min\{d_{26}, d_{62}\} + \sum_{i=1}^{6}\alpha_i + \sum_{j=1}^{6}\beta_j =$$

$$= \min\{2,5\} + \min\{1,0\} + \min\{0,0\} + 56 + 16 = 74 .$$

$\Delta_{34} = \min\{d_{34}, d_{43}\} = d_{34} = 2 > 0$, therefore let $d_{34} = 0$, $d_{43} = 3$, we obtain completely reduced matrix

$$\left[d_{ij}\right]_6 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & \infty & 5 & \infty & 0 & 1 & 13 \\ 2 & 2 & \infty & 0 & 7 & \infty & 0 \\ 3 & \infty & 0 & \infty & 0 & 0 & \infty \\ 4 & 0 & 10 & 3 & \infty & \infty & 18 \\ 5 & 0 & \infty & 2 & \infty & \infty & 4 \\ 6 & 10 & 0 & \infty & 15 & 2 & \infty \end{array} .$$

2. We calculate estimations for each zero elements of completely reduced matrix:

$\gamma(1,4) = 1, \gamma(2,3) = 2, \gamma(2,6) = 4, \gamma(3,2) = 0, \gamma(3,4) = 0, \gamma(3,5) = 1, \gamma(4,1) = 3,\ \gamma(5,1) = 2, \gamma(6,2) = 2$.

Arc $(2,6)$ of multigraph H' initiates branching.

3. $\{2,6\} \in R$.

4. $d_{26} \neq \infty$. The set of all solutions is divided into the subset of detours $\{(6,2)\}$, which contains arc $(6,2)$ and a subset of detours $\{(2,6)\}$, which contains arc $(2,6)$.

5. We calculate a matrix which defines a subset of detours $\{(6,2)\}$. We exclude the arc $(2,6)$, let $d_{26} = \infty$, also we exclude line 6 and column 2 . We reduce the obtained matrix and obtain new matrix

$$\left[d_{ij}\right]_5 = \begin{array}{c|ccccc} & 1 & 3 & 4 & 5 & 6 \\ \hline 1 & \infty & \infty & 0 & 1 & 9 \\ 2 & 2 & 0 & 7 & \infty & \boxed{\infty} \\ 3 & \infty & \infty & 0 & 0 & \infty \\ 4 & 0 & 3 & \infty & \infty & 14 \\ 5 & 0 & 2 & \infty & \infty & 0 \end{array} .$$

This matrix gives estimation

$$\varphi(6,2) = \varphi(R) + d_{62} + \sum_{i \neq 6}\alpha_i' + \sum_{j \neq 2}\beta_j' = 74 + 0 + 4 = 78 .$$

6. We obtain a matrix which defines the subset of detours $\{(2,6)\}$. To the arc $(6,2)$ we assign the weight $d_{62} = \infty$, exclude line and column 6. We reduce the obtained matrix:

$$\left[d_{ij}\right]_5 = \begin{array}{c} \\ 1 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} \infty & 5 & \infty & 0 & 1 \\ \infty & 0 & \infty & 0 & 0 \\ 0 & 10 & 1 & \infty & \infty \\ 0 & \infty & 0 & \infty & \infty \\ 8 & \boxed{\infty} & \infty & 13 & 0 \end{bmatrix} \end{array} .$$

We find an estimation:

$\varphi(2,6) = \varphi(R) + d_{26} + \sum_{i \neq 2} \alpha_i' + \sum_{j \neq 6} \beta_j' = 74 + 0 + 2 + 2 = 78$.

9. Dimension of the reduced matrix is more than 1. Any of vertices $\{(2,6)\}$, $\{(6,2)\}$ of branching tree can be chosen as the active. We choose the vertex $\{(2,6)\}$.

2. We calculate estimations for each zero elements of the matrix: $\gamma(1,4) = 1$, $\gamma(3,2) = 5, \gamma(3,4) = 0, \gamma(3,5) = 0, \gamma(4,1) = 1, \gamma(5,1) = 0, \gamma(5,3) = 1,\ \gamma(6,5) = 8$.

Arc $(6,5)$ initiates branching.

3. $\{6,5\} \notin R$; the set of detours $\{(2,6)\}$ is divided into the subset of detours $\{(6,5) \notin R'\}$ which includes arc $(6,5)$, and a subset of detours $\{(\overline{6,5})\}$ which does not contain this arc.

7. In the matrix which defines a subset of detours $\{(2,6)\}$, let $d_{65} = \infty$ and reduce obtained matrix. Outcome of such operations is the matrix

$$\left[d_{ij}\right]_5 = \begin{array}{c} \\ 1 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} \infty & 5 & \infty & 0 & 1 \\ \infty & 0 & \infty & 0 & 0 \\ 0 & 10 & 1 & \infty & \infty \\ 0 & \infty & 0 & \infty & \infty \\ 0 & \infty & \infty & 5 & \boxed{\infty} \end{bmatrix} \end{array}$$

and estimation $\varphi(\overline{6,5}) = \varphi(2,6) + S(6,5) = 78 + 8 = 86$ which limits from below costs of all detours $\{(\overline{6,5})\}$. Set $\{(\overline{6,5})\}$ includes arc $(2,6)$ and does not contain arc $(6,5)$.

8. In a matrix which corresponds to the lower bound $\varphi(2,6)$, we exclude line 6 and column 5. As edges $\{6,5\} \notin R$ and $\{5,1\} \in R$ are adjacent, we exclude line 5 and column 1 and we forbid arc $(1,2)$ assigning $d_{12} = \infty$ to exclude subcycles. We obtain a matrix

$$\left[d_{ij}\right]_3 = \begin{array}{c} \\ 1 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \boxed{\infty} & \infty & 0 \\ 0 & \infty & 0 \\ 9 & 0 & \infty \end{bmatrix} \end{array}$$

and estimation $\varphi((6,5) \notin R') = \varphi(2,6) + d_{51} + \sum \alpha_i' + \sum \beta_j' = 78 + 0 + 1 = 79$.

9. Dimension of this matrix is more than 1, we select the vertex $\varphi(6,2)$ which has the minimal lower estimation.

2. We calculate an estimation for each zero element:

$\gamma(1,4)=1, \gamma(2,3)=4, \gamma(3,4)=0, \gamma(3,5)=1, \gamma(4,1)=5, \gamma(5,1)=0, \gamma(5,6)=9$.

The further branching is fulfilled by means of an arc $(5,6)$.

3. $\{5,6\} \notin R$, the set of detours $\{(6,2)\}$ is presented by a division into subset $\{(5,6) \notin R'\}$ and subset $\{(\overline{5,6})\}$.

7. To vertex $\{(\overline{5,6})\}$ corresponds the matrix

$$\left[d_{ij}\right]_5 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \infty & \infty & 0 & 1 & 9 \\ 2 & 0 & 7 & \infty & \infty \\ \infty & \infty & 0 & 0 & \infty \\ 0 & 3 & \infty & \infty & 14 \\ 0 & 2 & \infty & \infty & \boxed{\infty} \end{bmatrix} \end{array}$$

and an estimation $\varphi(\overline{5,6}) = \varphi(6,2) + S(5,6) = 78 + 9 = 87$.

8. In the matrix which describe set of detours $\{(5,6) \notin R'\}$, we assign $d_{21} = \infty$ to exclude subcycles:

$$\left[d_{ij}\right]_3 = \begin{array}{c} \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} \boxed{\infty} & 0 & 7 \\ \infty & \infty & 0 \\ 0 & 3 & \infty \end{bmatrix} \end{array}.$$

We calculate an estimation of vertex: $\varphi((5,6) \notin R') = \varphi(6,2) + d_{15} = 78 + 1 = 79$.

9. For branching we choose vertex $\{(6,5) \notin R'\}$ with the minimal lower bound.

2. We calculate estimations for each zero element of the matrix which corresponds to current vertex:

$\gamma(1,4)=0, \gamma(3,2)=9, \gamma(3,4)=0, \gamma(4,3)=9$.

Branching is continued by an arc $(3,2) \notin R'$.

7. Matrix

$$\left[d_{ij}\right]_3 = \begin{array}{c} \\ 1 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \infty & \infty & 0 \\ \boxed{\infty} & \infty & 0 \\ 0 & 0 & \infty \end{bmatrix} \end{array}$$

defines set of detours $\{(\overline{3,2})\}$ with lower bound $\varphi(\overline{3,2}) = \varphi(6,5) + S(3,2) =$

$= 79 + 9 = 88$

8. In a matrix which corresponds to vertex $\{(\overline{6,5}) \notin R'\}$, we assign $d_{32} = \infty$ and exclude lines 3, 4 and columns 2, 3.

9. As a result we obtain a matrix of the one element (1, 4) and, $\{(2,6)\}$. Therefore we obtain detour (2,6), (6,5), (5,1), (1,4), (4,3), (3,2) with cost $\varphi((3,2) \notin R') = \varphi((\overline{6,5}) \notin R') + d_{43} = 79 + 0 = 79$.

All vertices of the branching tree have estimations, greater or equal 79, go to step 10.

10. The obtained admissible solution of problem

$z(R)=\{(2,6),(6,5),(5,1),(1,4),(4,3),(3,2)\}$ with cost $C(z(R))=79$ is optimum.

The Fig. 1 illustrates the solution tree of this example.

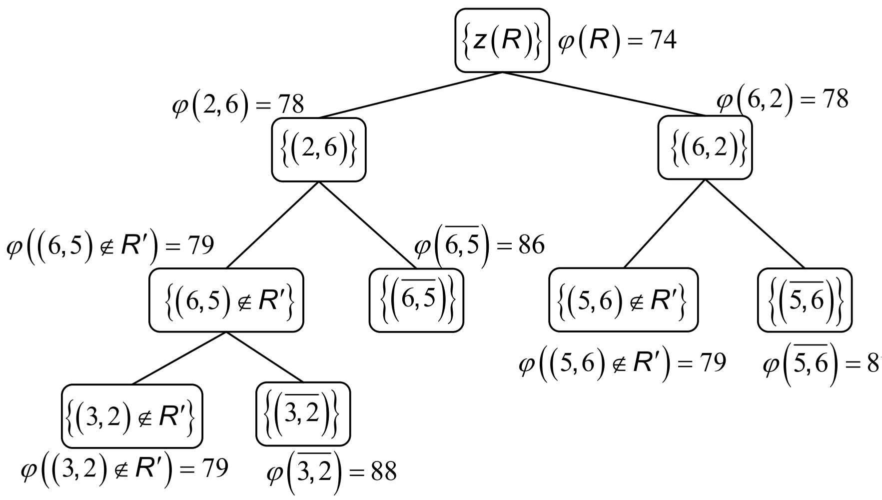

Fig. 3. The Solution Tree

Conclusion

The method has been implemented in C# programming language. For performance tests we have used Intel Core 2 Duo 1,6GHz/2Gb RAM PC. Time of solving the Travelling Salesman Problem for dimension 50 occupies an average of 5 minutes.

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