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COMPARATIVE ANALYSIS OF STATISTICAL PROPERTIES OF THE HURST EXPONENT ESTIMATES OBTAINED BY DIFFERENT METHODS

Ludmila Kirichenko, Tamara Radivilova

Abstract: *Estimating of the Hurst exponent for experimental data playas a very important role in research of processes which show properties of self-affinity. There are many methods for estimating the Hurst exponent using time series. The aim of this work is to carry out the comparative analysis of statistical properties of estimates of the Hurst exponent obtained by different methods using short length model fractal time series (the number of values less than 4000). In the work the most common used methods for estimating the Hurst exponent are researched. They are: R/S -analysis, variance-time analysis, detrended fluctuation analysis (DFA) and wavelet-based estimation. The fractal Brownian motion that is constructed using biorthogonal wavelets has been chosen as a model random process which exhibit fractal properties.*

In the work the results of a numerical experiment are represented where the fractal Brown motion was modelled for the specified values of the exponent H. The values of Hurst exponent for the model realizations were varied within the whole interval of possible values 0 < H < 1.The lengths of the realizations were defined as 500, 1000, 2000 and 4000 values. The estimates of H were calculated for each generated time series using the methods mentioned above. Samples of estimates of the exponent H were obtained for each value of H and their statistical characteristics were researched.

The results of the analysis have shown that the estimates of the Hurst exponent, which were obtained for the realisations of short length using the considered methods, are biased normal random variables. For each method the bias depends on the true value of degree of self-similarity of a process and a length of time series. Those estimates which are obtained by the DFA method and the wavelet transform have the minimal bias. Standard deviations of the estimates depend on the estimation method and decrease while the length of the series increases. Those estimates which are obtained by using the wavelet analysis have the minimal standard deviation.

Keywords: *Hurst exponent, estimate of the Hurst exponent, self-similar stochastic process, time series, methods for estimating the Hurst exponent*

ACM Classification Keywords: *G.3 Probability and statistics - Time series analysis , Stochastic processes, G.1 Numerical analysis, G.1.2 Approximation - Wavelets and fractals*

Introduction

Nowadays problems of nonlinear physics, radio electronics, control theory, image processing require development and employment of new mathematical models, methods and algorithms for data analysis. At present time it has been generally accepted, that many of stochastic processes in nature and in engineering exhibit long-range dependence and fractal structure. The most suitable mathematical method for research of dynamics and structure of such processes is fractal analysis.

Stochastic process $X(t)$ is statistically self-similar if the process $a^{-H}X(at)$ show the same second-order statistical properties as $X(t)$. Long-range dependence means slow (hyperbolic) decay in time of the autocorrelation function of a process. The parameter H ($0<H<1$) is called the Hurst exponent and is a measure of self-similarity or a measure of duration of long-range dependence of a stochastic process.

For values $0.5<H<1$ time series demonstrates persistent behaviour. In other words, if the time series increases (decreases) in a prior period of time, then this trend will be continued for the same time in future with the probability which increases the more the Hurst exponent gets greater than 0,5. The value $H=0.5$ indicates the independence (the absence of any memory about the past) of values of time series. The interval $0<H<0,5$ corresponds to antipersistent time series: if a system demonstrates growth in a prior period of time, then it is likely to fall in the next period with the probability which increases the less the Hurst exponent gets below 0.5.

It is obvious that estimation of the Hurst exponent for experimental data plays an important role in study of processes which exhibit properties of self-similarity. There are many methods for evaluation of the Hurst exponent for time series. Sufficient review of these methods is represented in [Willinger, 1996; Clegg, 2005]. Nevertheless, at present time there is no a proper summary research where the results of the estimation of the Hurst exponent H with different methods would be generalized and the comparative analysis of statistical properties of estimations obtained for small amount of sample data would be. The given work is an attempt to carry out such research for the most common used methods of estimation of self-similarity.

The aim of this work is to carry out the comparative analysis of statistical properties of estimates of the Hurst exponent obtained by different methods using short length model fractal time series (the number of values less than 4000). In the work the most common used methods for estimating the Hurst exponent are researched. They are: R/S-analysis (rescaled range method) (see, for instance [Feder, 1988; Peters, 1996; Stollings, 2003; Sheluhin, 2007], variation in time of variance of an aggregate time series (variance-time analysis), see [Stollings, 2003; Sheluhin, 2007], detrended fluctuation analysis (see [Kantelhardt, 2001; Chen, 2002; Gu, 2006; Kantelhardt, 2008]) and estimation using the wavelet analysis (see [Mallat, 1998; Abry, 1998; Abry, 2003]) The fractal Brownian motion has been chosen as a model random process which exhibit fractal properties.

Methods of estimating the Hurst exponent

Rescaled range method. According to this method for the time series $x(t)$ of the length τ the rate $\frac{R(\tau)}{S(\tau)}$ is defined, where $R(\tau)$ is the range of the cumulative deviate series $x^{cum}(t,\tau)$, $S(\tau)$ is standard deviation of the initial series:

$$R/S = \frac{\max(x^{cum}(t,\tau)) - \min(x^{cum}(t,\tau))}{\sqrt{\frac{1}{\tau-1}\sum_{t=1}^{\tau}\left(x(t)-\bar{x}\right)^2}},\ t=\overline{1,\tau}, \tag{1}$$

where $\bar{x}(\tau) = \frac{1}{\tau}\sum_{t=1}^{\tau} x(t)$, $x^{cum}(t,\tau) = \sum_{i=1}^{t} x(i) - \bar{x}(\tau)$.

For a self-similar process and big values of τ this ratio has the following characteristics:

$$M\left[\frac{R}{S}\right] \sim (c\cdot\tau)^H, \tag{2}$$

where c is a constant.

The log-log diagram of dependence of $\frac{R(\tau)}{S(\tau)}$ on τ represents a line approximated by the least square method. Then the estimate of the exponent H is a tangent of angle of slope of the line which represents the dependence of $\log\frac{R(\tau)}{S(\tau)}$ on $\log(\tau)$.

Variance-time analysis. When someone mentions the aggregation of time series $x(t)$ of the length τ on the time scale with the parameter m, they mean the transition to the process $x^{(m)}$, where $x_k^{(m)} = \frac{1}{m}\sum_{t=km-m+1}^{km} x(t)$, $k=\overline{1,\tau/m}$. For self-similar process the variance of the aggregated time series $x^{(m)}$ for big values of m follows the formula:

$$Var(x^{(m)}) \sim \frac{Var(x)}{m^{\beta}}. \quad (3)$$

In this case the parameter of self-similarity $H = 1 - \frac{\beta}{2}$ can be obtained if to generate an aggregated process on different levels of aggregation m and calculate the variance for each level. The dependence diagram of $\log(Var(x^{(m)}))$ on $\log(m)$ will represent a line with a slope equal to $-\beta$.

Detrended fluctuation analysis (DFA). According to the DFA method, for the initial time series $x(t)$ the cumulative time series $y(t) = \sum_{i=1}^{t} x(i)$ is constructed which is then divided into N segments of length τ, and for each segment $y(t)$ the following fluctuation function is calculated:

$$F^2(\tau) = \frac{1}{\tau}\sum_{t=1}^{\tau}(y(t) - Y_m(t))^2, \quad (4)$$

where $Y_m(t)$ is a local *m*-polynomial trend within the given segment. The averaged on the whole of the time series $y(t)$ function $F(\tau)$ depends on the length of the segment: $F(\tau) \propto \tau^H$.

In some interval the diagram of dependence of $\log F(\tau)$ on $\log \tau$ represents a line approximated by the least square method. Estimate of the exponent H is a tangent of the angle of slope of the line which represents the dependence of $\log F(\tau)$ on $\log(\tau)$.

Wavelet-based estimation. According to the discrete wavelet transform, the time series $x(t)$, $(t = 1, 2, ..., n)$ consists of the collection of approximation and detail coefficients. The detail wavelet coefficients $d_x(j,k)$ on the scale level *j* are defined in the following way:

$$d_x(j,k) = 2^{j/2}\sum_{i=1}^{n_j} x(i)\Psi_0(2^{-j}n - k), \quad (5)$$

where $n_j = 2^{-j}n$ is a number of the wavelet coefficients on the level j, Ψ_0 is base wavelet function.

For estimation of the Hurst exponent in applied research the method described in [Abry, 1998] is the most used. The mentioned method is based on the statement that the averaged squared values of the wavelet coefficients $\mu_j = \frac{1}{n_j}\sum_{k=1}^{n_j}|d_x(j,k)|^2$ obey the scaling law:

$$\mu_j \sim 2^{(2H-1)j}, \quad (6)$$

where *H* is the Hurst exponent. The following equation represents the practicable method of the estimation of the Hurst exponent:

$$\log_2 \mu_j = \log_2\left(\frac{1}{n_j}\sum_{k=1}^{n_j}|d_x(j,k)|^2\right) \sim (2H-1)j + const. \quad (7)$$

From this formula it can be concluded that if there is the long-range dependence of the time series $x(t)$ then the Hurst exponent H can be obtained by estimating the slope of the graph of the function $\log_2(\mu_j)$ from j.

Modelling of fractal Brownian motion

One of the most known and simple models of stochastic dynamics which exhibits fractal properties is fractal Brownian motion (fBm). It is widely used in physics, chemistry, biology, economics and theory of network traffic.

Gaussian process $X(t)$ is called fractal Brownian motion with the parameter H, $0 < H < 1$, if the increments of the random process $\Delta X(\tau) = X(t+\tau) - X(t)$ are distributed in the following way

$$P(\Delta X < x) = \frac{1}{\sqrt{2\pi}\sigma_0 \tau^H} \cdot \int_{-\infty}^{x} \mathrm{Exp}\left[-\frac{z^2}{2{\sigma_0}^2 \tau^{2H}} \right] dz , \tag{8}$$

where σ_0 is diffusion coefficient.

fBm with the parameter $H = 0.5$ coincides with the classic Brownian motion. Increments of fBm are called fractal Gaussian noise and its dispersion can be described by the formula $D[X(t+\tau) - X(t)] = {\sigma_0}^2 \tau^{2H}$.

There are many methods of construction of fBm for the case of discrete time, which have been considered in [Mandelbrot, 1983; Feder, 1988; Voss, 1988; Cronover, 2000]. These models have some weak sides. One of them is underestimating (overestimating) of the degree of self-similarity of a process for small and big theoretical values of the Hurst exponent and short length of a model realisation [Jeongy, 1998; Cronover, 2000; Sheluhin, 2007].

One of the methods which can help to resolve the mentioned problems is construction of fBm using biorthogonal wavelets [Sellan, 1995; Abry, 1996; Sellan, 1995; Meyer, 1999; Bardet, 2003]. In this case the fBm realization is constructed using discrete wavelet transform where the detail wavelet coefficients on each level are independent normal random values and approximation wavelet coefficients are obtained using fractal autoregression and moving average process FARIMA:

$$B_H(t) = \sum_{k=-\infty}^{\infty} \Phi_H(t-k) S_k^{(H)} + \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} 2^{-jH} \Psi_H(2^j t - k) \varepsilon_{j,k} - b_0 , \tag{9}$$

where Ψ_H is biorthogonal base wavelet function, Φ_H is corresponding Ψ_H scaling function, $S_k^{(H)}$ is stationary Gaussian process FARIMA with the fractal differentiation parameter $d = H - 0.5$, $\varepsilon_{j,k}$ – independent standard Gaussian random values, b_0 – constant where $B_H(0) = 0$.

Results of the investigation

In the work the results of numerical experiment are represented where fractal Brownian motion with the specified value of exponent H have been simulated. The values of H for model realisation were varied within the whole interval $0 < H < 1$. Length of realizations was accepted equal to 500, 1000, 2000 and 4000 values. For each received realization estimates H have been obtained using the methods described above: R/S-analysis $\left(\hat{H}rs\right)$, variance-time analysis $\left(\hat{H}d\right)$, DFA method $\left(\hat{H}fa\right)$ and discrete wavelet transform $\left(\hat{H}w\right)$. For each value of H samples of its estimates have been computed, and their statistical characteristics have been investigated.

Investigation of bias of estimates

Fig. 1 shows dependence of average values of estimates of the Hurst exponent on its theoretical value. Model realizations contained 1000 values. Solid line refers to the theoretical values of H . Obviously, average values of the estimates are biased, where the bias depends on the theoretical values of the Hurst exponent.

Obviously, the estimates of the Hurst exponent are biased in a region of persistence as well as in a antipersistence one. Since most of fractal processes have the long-range dependence, we will be considering results only for the interval $0,5 < H < 1$. From Fig. 1 we can see that the estimates obtained by the methods of R/S - analysis and variance-time analysis are the most biased.

Let us consider the results of estimation of the exponent H by the method of R/S-analysis. The method of rescaled range proposed by Hurst is, perhaps, the most popular one and used in all fields of scientific research. Its main merit is its robustness. Actually, this method works even on non-stationary data. But also, as it was noticed by Hurst, the estimates of H below $H \approx 0,75$ obtained by R/S-method are overestimated, and the estimate of H over $H \approx 0,75$ are understate.

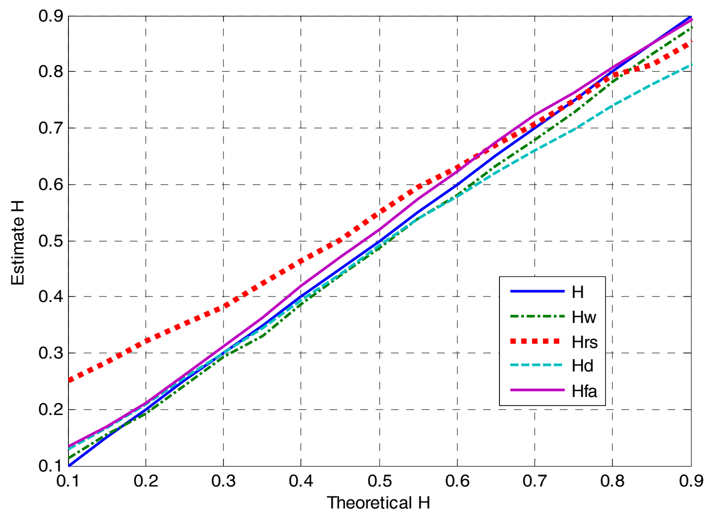

Figure 1. Dependence of average values of estimates obtained by various methods on theoretical values of H

Fig. 2 represents dependence of average values of estimates $\hat{H}rs$ on theoretical values of H for model series of different length. Obviosly, average values of estimates can be approximates quite well with lines $\hat{H}_N = k_N H + b_N$, where coefficients k_N and b_N depend on realization N where the estimation is done. This lines cross the line of theoretical values of H at around $H \approx 0,75$; and are overestimated below this values and are underestimated over this value. The results of the performed research confirm the results obtained by analysing the estimation another models [Feder, 1988; Jeongy, 1998; Кириченко, 2005; Sheluhin, 2007]. With the increase of the realisation length N the angle of slope k_N of the approximated line increases slowly and approaches the theoretical value $\pi / 4$.

Due to its simplicity and easy understanding of its results the method of variance-time analysis is the most common used for assessment of self-similarity of the information network traffic. Nevertheless, for processes with long-range dependence this method gives undervalued estimates. [Jeongy, 1998; Кириченко, 2005; Sheluhin, 2007]. This can be unacceptable, for instance, in case of assessment of the network load during the transmission of the self-similar traffic. [Stollings, 2003].

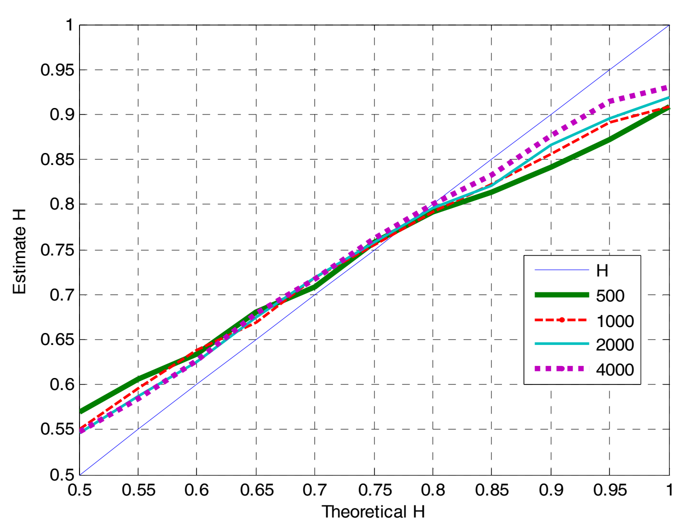

Figure 2. Dependence of average values of estimates $\hat{H}rs$ on theoretical value of H

Fig. 3 shows the dependence of the average values of estimates $\hat{H}d$ obtained by the method of variance-time analysis. These dependence can also be approximated by the lines $\hat{H}_N = k_N H + b_N$ where coefficients k_N and b_N depend on the length of realization N. In this case the approximating lines cross the line of theoretical values at around $H \approx 0,5$ (see Fig. 1) and actually the estimates of the exponent H are undervalued within the whole interval of persistence. The bias increases with the growth of the Hurst exponent, particularly for $H > 0,9$. It can be noted that the bias of the estimates $\hat{H}d$ are greater that the appropriate bias of $\hat{H}rs$. With the increase of the realization length N the bias slowly decreases.

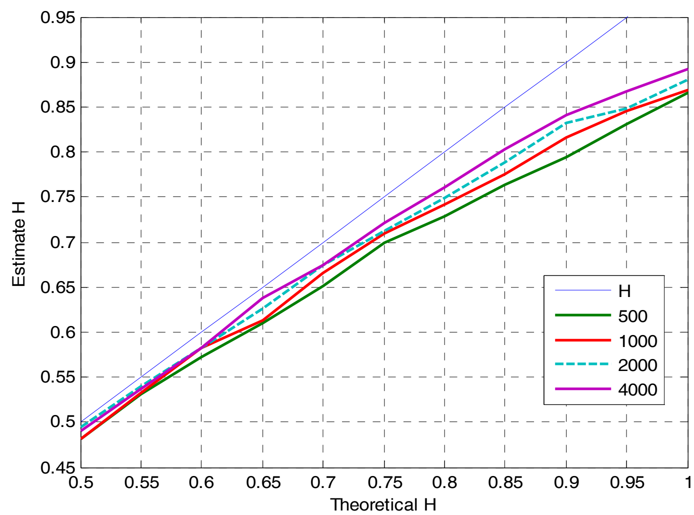

Figure 3. Dependence of average values of estimates $\hat{H}d$ on theoretical value of H

Method DFA is based on the ideology of one-dimensional random walk and is widely used in analysis of bioelectric signals. Estimates $\hat{H}fa$ obtained by DFA method can be characterised by small bias within the interval $0,5 < H < 0,9$ (see Fig. 4) even for realizations of short length. The sign of this bias reverses and increases for $H > 0,9$. It should be brought into focus that the most of natural and information fractal processes have the degree of self-similarity less than 0,9.

Methods of estimation of the Hurst exponent by the use of wavelet analysis are the most recent and still have not been commonly used. Nevertheless, their merits are obvious. In the work [Abry, 1998] it has been shown that the estimates are asymptotically unbiased if base wavelets are chosen in a proper way. Fig 5 represents dependence of the average values of $\hat{H}w$ obtained by the method of discrete wavelet expansion with the base wavelet function of Daubechies D4. It is obvious that with the increase of the time series length N the bias decreases and is actually equal to 0 for $N \approx 4000$.

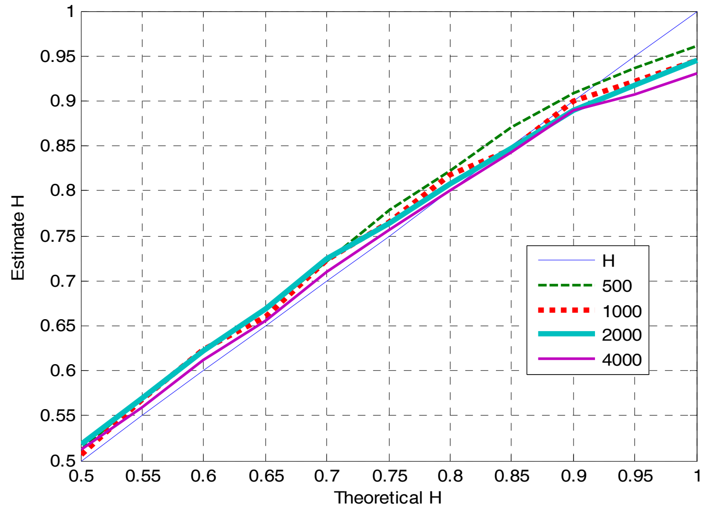

Figure 4. Dependence of the average values of the estimates $\hat{H}fa$ on the theoretical value of H

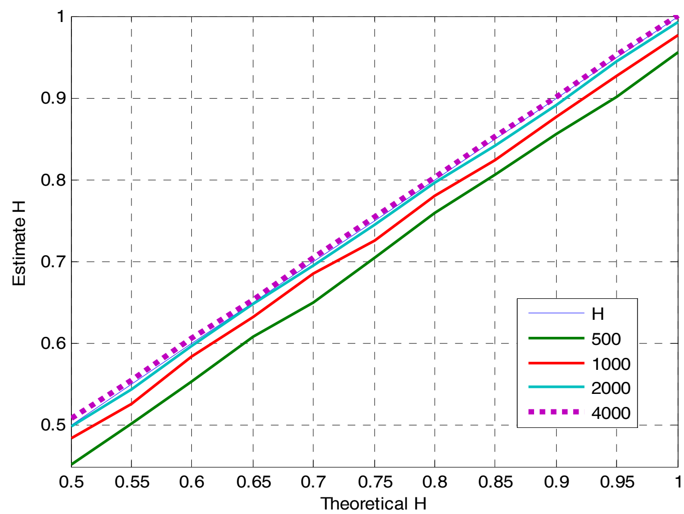

Figure 5. Dependence of the average values of the estimates $\hat{H}w$ on the theoretical values of H

Research of standard deviations of the estimates

In the work the dependence of standard deviations of estimates of the Hurst exponent on the values of H and length of the model fractal series has been investigated for each method. In Table 1 the values of standard deviations of the estimates of the Hurst exponent which have been received for the series of length of 1000 values are represented.

Table 1. Standard deviations of the estimates of the Hurst exponent

Estimation method	Range $S_{\hat{H}}$	Dependence on H
R/S-analysis	$0.03 \le S_{\hat{H}} \le 0.08$	Increases along with H
Variation of dispersion	$S_{\hat{H}} \approx 0.06$	No obvious trend
DFA	$S_{\hat{H}} \approx 0.07$	No obvious trend
Wavelet analysis	$S_{\hat{H}} \approx 0.045$	No obvious trend

Table 2 demonstrates how standard deviations obtained during the estimation of $\hat{H}$ (rounded value) decrease with the increase of the length of time series. In this specific case the model Hurst exponent $H = 0.8$.

Table 2. Standard deviations of the estimates depending on length of time series

$S_{\hat{H}}$	500	1000	2000	4000
S_{Hrs}	0.08	0.06	0.05	0.04
S_{Hd}	0.07	0.06	0.05	0.045
S_{Hfa}	0.085	0.07	0.055	0.045
S_{Hw}	0.065	0.045	0.03	0.02

The problem of distribution law of the estimates of H was considered in number of works where it was shown, numerically and analytically, that the estimates are normal for a specific method or specific values of the Hurst exponent (see, for instance [Feder, 1988, Peters, 1996; Abry, 1998]). In this work the distribution of the estimates $\hat{H}$ have been researched for each method and different values of the parameter. For all considered methods the hypothesis of normal distribution of sample values of the estimates with parameters $N(\bar{H}, S_{\hat{H}})$ have been suggested. Nearly for all sample data the hypothesis has been accepted with the confidence level $\alpha = 0.05$ by few fitting criterions.

Conclusion

Thus, the estimates of the Hurst exponent which are obtained by the methods considered above are biased normal random variables. For each method the bias depends on a true value of degree of self-similarity and the length of a time series. Standard deviations of the estimates depend on the estimation method and decrease with the growth of the series length. Summarizing the results of the numerical research we can make a conclusion that the estimates with the least bias and standard deviation can be given by the method which uses wavelet analysis. Also, other methods have some merits which can be significant in relation to some aims and ways of the research. For instance, R/S -analysis allows to estimate the degree of self- similarity of a time series for which the wavelet estimation nearly cannot be applied, DFA method gives the best results for short series. Thus, in the most cases for the estimation of the Hurst exponent it makes sense to use various methods and comparison of the results provides an extra information.

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