TESTING STABILITY OF THE CLASSICAL FORRESTER MODEL TO INITIAL DATA AND ADDITIVE NOISE

Olga Proncheva, Mikhail Alexandrov, Sergey Makhov

Abstract: *The classical Forrester model of world dynamics is a system of 5 differential equations related with 5 macro-economical variables (population, resources, etc.). This model was developed at 1970-1971 but by the moment its stability to noise was not studied. The plan of experiments is described and the results of modeling are presented. It proved that a) noise affects stronger initial data then the model during its functionality b) change of resources is the most critical value in comparison with the other system variables. All experiments have been made by means of the program WorldDyn developed on MatLab.*

Keywords*:* Forrester model, word dynamics noise immunity, numerical analysis

Introduction

At 1970 year the Roman Club (nongovernmental organization of politicians and scientists) asked John Forrester (professor of MTI) to create a model, which could predict the development of our world. In 2 weeks he presented his model “World-1” but this model proved to be too crude. At 1971 Forrester presented his second model called “World-2” and just this model is considered in the paper.

In his work Forrester selected five main problems, which could provoke World Crisis in the future. It is an overpopulation of our planet, a lack of basis resources, a critical level of pollution, food shortages and industrialization. Each of these problems was reflected in the corresponding variable:

- Population (P)
- Pollution (Z)
- Natural resources (R)
- Capital investment (fixed assets) (K)
- Proportion of funds invested in agriculture (X)

All variables were united in one system of differential equations. Forrester developed the principles of system dynamics and these principles defined the structure of the mentioned system of equations.

Forrester has many followers: Vladimir Egorov [Egorov, 1980], Sergey Makhov [Machov,2003], David Meadows [Meadows, 2007], et al. But we do not know papers where noise immunity of the model was studied. We associate the noise with inaccuracy of initial data and tuned model parameters. In the paper we study this problem under the essential restriction: the noise is considered as the additive one.

The paper consists of 3 chapters. The second chapter deals with planning experiments, the third chapter describe the experiments, and finally we give a short description of the software we used for experiment implementation.

Planning experiments

2.1 Noised model and methods of its calculation

The noised Forrester model looks like the following system:

$$\frac{dP}{dt} = P\left(c_B B_C B_P B_F B_Z - c_D D_C D_P D_F D_Z\right) + \vartheta \tag{1}$$

$$\frac{dK}{dt} = c_K P K_C - \frac{K}{T_K} + \theta \tag{2}$$

$$\frac{dX}{dt} = \frac{X_F X_Q - X}{T_X} + \mu \tag{3}$$

$$\frac{dZ}{dt} = P Z_K - \frac{Z}{T_Z} + \sigma \tag{4}$$

$$\frac{dR}{dt} = -P R_C + \tau \tag{5}$$

Here: ϑ, θ, μ, σ, τ are stationary white noise.

To calculate the model we used two subprograms from MatLab package. These programs realize the well-known Runge-Kutta method and Adams method [Petrov, 2006]. Many variables in the model, such as B_C, were presented in a tabulated form. To use them we applied the linear interpolation. The same way was used by Forrester.

Before the experiments we tested the influence of time step on the results of calculation. It proved that such an influence was inessential. For this reason we used one-year step.

2.2. Analysis of Forrester model stability

The middle value of each noise realization was equal 0. The noise dispersion ε^2 was calculated by the following way:

a) Forecast period

The dispersion is equal $\varepsilon^2 = \alpha\ [\frac{1}{71}\int_{1900}^{1970} f^2(t)dt]$. Here: α is a coefficient of proportionality, the value in parenthesis is a middle value of a power for the correspondent variable. The dispersion did not depend on the variable itself. Naturally, the absolute values of noise were different for different variables.

b) Initial data

The dispersion is equal $\varepsilon^2=\alpha f$. Here: α is a coefficient of proportionality, f is a value of the correspondent variable

We calculated the model 100 times for various noise realizations and fixed the number of cases with convergent processes.

2.3. Software development

To make the experiments we developed the program WorldDyn. This program has 3 options for study the noise influence on the Forrester model: noise affects initial data, noise affects all variables simultaneously on the stage of forecast, and noise affects each variable separately. The program has a convenient graphic interface. It allows to see: initial (un noised) function, all realizations of this function with a noise, and the worst realization (it has the maximal root-mean-square deviation). A help system is a part of interface. Figure 1 presents some elements of the program interface

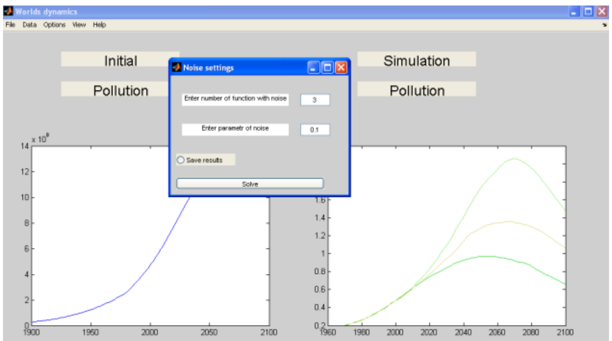

Fig. 1. Elements of WorldDyn interface

Experiments

3.1. Noise affects all variables on the stage of forecast

As an example we consider the case with the 20% noise. Figure 2 presents the results of modeling for all macro-economical variables. There are 3 lines on the figure: thin uninterrupted line is the initial function, thick line is the forecast, and thin dotted line is the worst function.

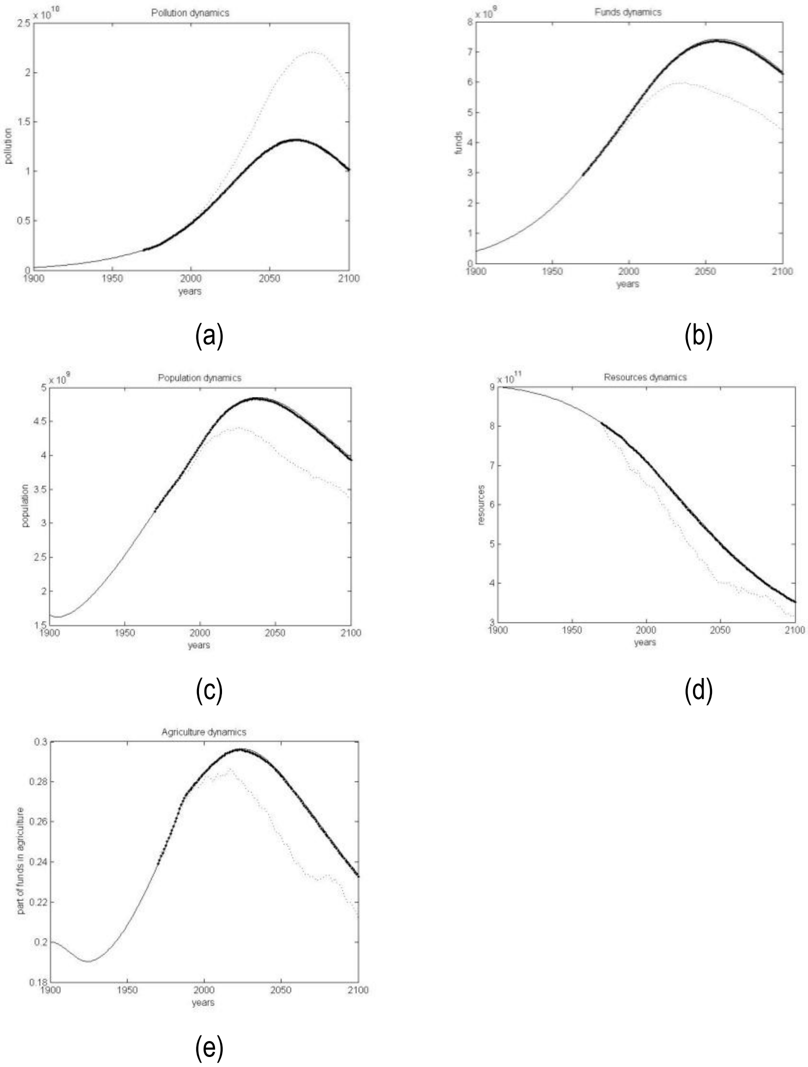

Fig. 2: Results of modeling (a) pollution dynamics; (b) funds dynamics; (c) population dynamics; (d) resources dynamics; (e) dynamics of capital investment in agriculture fraction.

One can see that the forecast is very close to the initial line, which reflects un noised function. It means that our model is stable to the 20% noise. In this case all functions converge and the relative root-mean-square deviation for every variable is equal:

- 0,31% for population
- 0,51% for funds
- 0,26% for agriculture
- 1,27% for pollution
- 0,46% for resources

Generally speaking, the Forrester model is very stability to the noise, which acts on the forecast period. Even when the noise level reaches 50% we have 69% convergent functions. This result also can be considered as the very good one.

3.2. Noise affects isolated variables on the stage of forecast

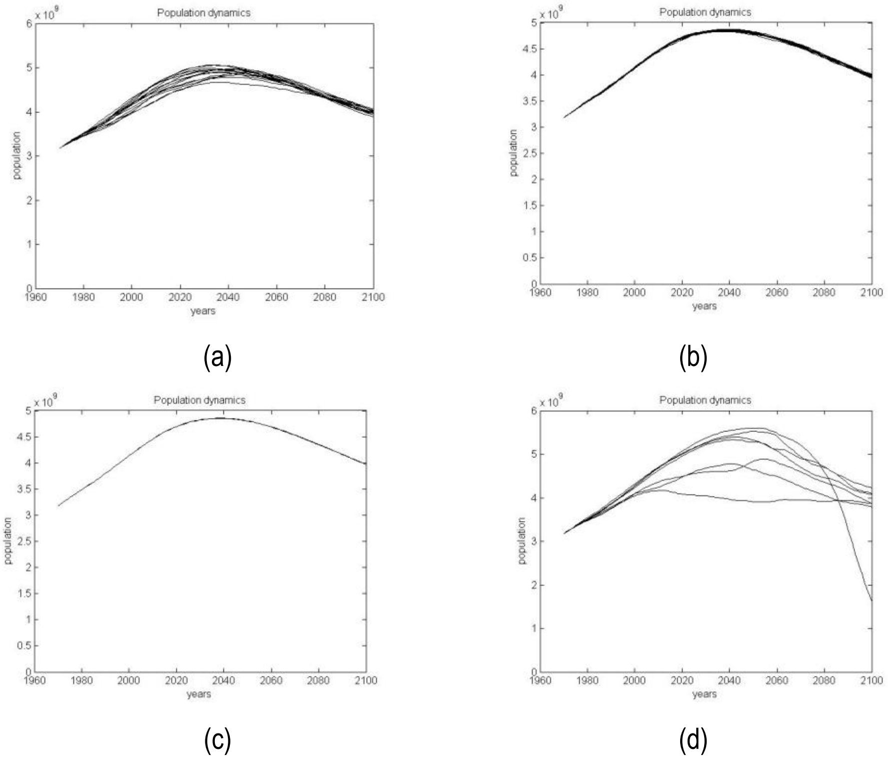

Fig. 3: Results of modeling pollution dynamics (a) noise affects only funds; (b) noise affects only agriculture; (c) noise affects only pollution; (d) noise affects only resources.

When noise affects only one variable the results of forecast change weakly. For that reason we used 50% noise to observe a significant difference. Our purpose is to reveal the most influential variable. Figure 3 illustrates the population dynamics given the influence of different variables.

It is easy to see that resources are the most influential variable.

3.3. Noise affects initial values of all variables

As an example we consider the case with the 20% noise as we have done it in the section 3.1. Figure 4 presents the results of modeling for all macro-economical variables.

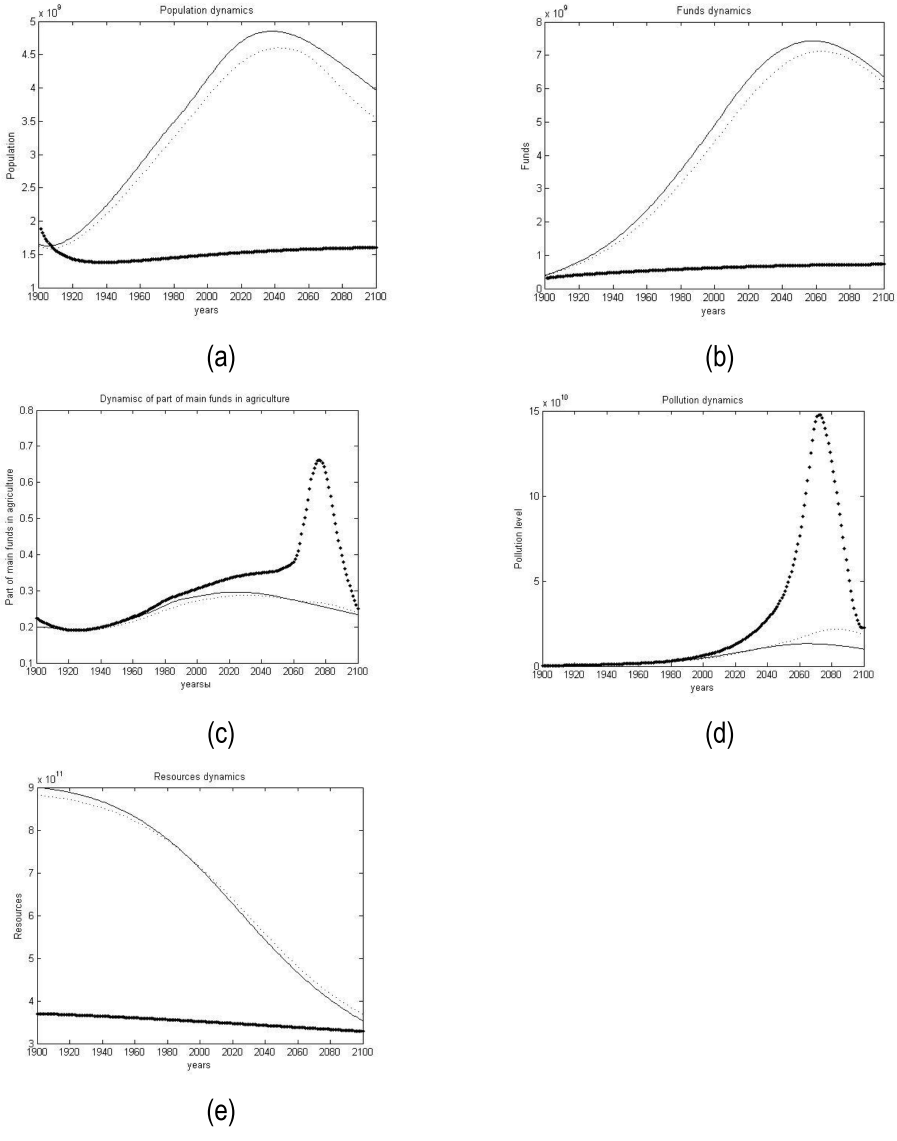

Fig. 4: Results of modeling (a) population dynamics; (b) funds dynamics; (c) agriculture dynamics; (d) pollution dynamics; (e) resources.

There are 3 lines on the figure: thin uninterrupted line is the initial function, thick line is the forecast, and thin dotted line is the worst function.

Let us compare the Figure 2 and the Figure 4. One can see the essential difference between the noise influence on the stage of forecast and the noise influence on the initial data. Namely, the Forrester model is more stable to noise on the stage of forecast. The related roof-mean-square deviations are:

- 14,58% for pollution
- 24,29% for funds
- 11,41% for agriculture
- 52,74% for pollution
- 12,97% for resources

3.4 Dependence on the level of noise.

The experiments described in the sections 3.1 and 3.3 were completed with the 20% noise. We repeated these experiments with the 10% noise to reveal the dependence of results on the level of noise. Table 1 jointed together the results of both experiments.

Table 1. Related roof-mean-square deviation of results

	Forecast 10%	Forecast 20%	Initial data 10%	Initial data 20%
Population	0,22%	0,31%	7,45%	14,58%
Funds	0,35%	0,51%	13,92%	24,29%
Agriculture	0,18%	0,26%	5,21%	11,41%
Pollution	0,82%	1,27%	31,05%	52,74%
Resources	0,33%	0,46%	7,34%	12,97%

It is easy to see that when the level of noise increases in 2 times then the deviation of results for the case of forecast increases in 1.5 times and for the initial data in 2 times.

4. Conclusion

In the paper we have studied the stability of the classical Forrester model to additive noise. The experiments show that

- the model is completely stable that is all its functions (variables) converge for the level of noise less than 22%;

2/3 of all functions converge for the level of noise 50%; here noise affects all variables on the stage of forecast

- noise in initial data causes the essentially stronger effect then the same noise on the stage of forecast
- the most influential variable is resources; its changes provoke the strongest reaction of the model

In future we plan to continue our research in two directions:

- to get the theoretical assessments of noise influence on the model for the case of stable mode of model functionality
- to analyze the noise influence on Egorov's and Makhov's models.

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