SMOOTHING AND PROGNOSIS OF MULTI-FACTOR TIME SERIES OF ECONOMICAL DATA BY MEANS OF LOCAL PROCEDURES (REGRESSION AND CURVATURE EVALUATION)

Alexander Kovaldji, Vladimir Averkiev, Marina Sarkissyan

Abstract: *Smoothing and prognosis belong to the main problems, which we deal with when process various time series. Unlike the typical approaches related with lineal regression on polynomial and trigonometric functions we consider local procedures. We describe algorithms that resolve these problems and present results of experiments using data of the Russian State Statistical Committee. All procedures are realized in the Excel-VBA.*

Keywords: *econometrics, local regression, time series, seasonality*

ACM Classification Keywords: *I.2.m Miscellaneous*

Introduction

The modern econometrics includes many methods of time series (TS) smoothing and prognosis [Kandle, 1981; [Klayner, 2000; Bessonov, 2003; Nosco, 2011]. The majority of these methods take into account integral properties of a given TS. However, in many cases so-called local procedures can provide the better results. This paper describes the original methods of TS analysis: 1) smoothing based on local linear regression 2) revealing seasonal variations based minimization of function curvature. The first method uses the idea of weighted points in the regression model. The second method reduces to the solution of system of 12 linear equations. The similar approaches have been considered in the work [Gubanov, 2001].

Proposed methods

2.1 Smoothing procedure

In the proposed procedure we weight each point using exponential coefficients and then use the weighted least squares method (WLSM). The exponential coefficients are calculated according the formula:

$$w(i) = \frac{1}{2^{\frac{|x(j)-x(i)|}{\lambda}}}, \qquad (1)$$

where *x(j)* is the point which the WLSM is used for.
The WLSM consists in minimization of the values:

$$\sum_{i=1}^{n} \left(w(i) * (y(i) - Ax(i) - B)^2 \right), \qquad (2)$$

To solve this problem we differentiate this sum with respect to A and B and then we obtain the following system of two linear equations:

$$\begin{cases} A * \sum_{i=1}^{n} (w(i) * x^2(i)) + B * \sum_{i=1}^{n} (w(i) * x(i)) = \sum_{i=1}^{n} (w(i) * x(i) * y(i)) \\ A * \sum_{i=1}^{n} (w(i) * x(i)) + B * \sum_{i=1}^{n} w(i) = \sum_{i=1}^{n} (w(i) * y(i)) \end{cases} \quad (3)$$

Having received A and B for each point we calculate:

$$z(i) = A * x(i) + B. \quad (4)$$

One should say that when weights are equal (an extreme case) the local linear regression reduces to the ordinary lineal regression.

2.2 Elimination of season cyclicity

Firstly we introduce the so-called curvature degree according te formula:

$$\sum_{i=2}^{n-1} \left(y(i) - \frac{y(i-1) + y(i+1)}{2} \right)^2 \quad (5)$$

For boundary points we use two additional formulae:

$$(y(1) - 2 * y(2) + y(3))^2, \quad (6)$$

$$(y(n-2) - 2 * y(n-1) + y(n))^2. \quad (7)$$

Therefore the degree of curvature is the sum of (5), (6) and (7)

We consider time series (TS) as a sum of trend and season function. The latter is a periodical function with the period 12 (it is the number of months in a year):

$$y(i) = y^{real}(i) + k(i) \quad (8)$$

So, the season function is a set of 12 season coefficients k(i):

$$k(i) = k(i + 12), \qquad i \in [1, n - 12]. \quad (9)$$

To find these coefficients we minimize the degree of curvature for TS $y^{real}(i)$:

$$\begin{aligned} &[(y(1) - k(1)) - 2 * (y(2) - k(2)) + (y(3) - k(3))]^2 + \\ &+[(y(n-2) - k(n-2)) - 2 * (y(n-1) - k(n-1)) + (y(n) - k(n))]^2 + \\ &+\sum_{i=2}^{n-1} \left((y(i) - k(i)) - \frac{(y(i-1) - k(i-1)) + (y(i+1) - k(i+1))}{2} \right)^2 \xrightarrow[k(1)\ldots k(12)]{} min \end{aligned} \quad (10)$$

To solve this problem we differentiate (10) with respect to *k(i)* having in view (9). Then we can calculate the TS values without season components:

$$y^{real}(i) = y(i) - k(i) \quad (11)$$

Program realization

The proposal methods are realized in the program Prophet. The program is prepared on Excel-VBA.

Input data includes: initial data set; parameter of smoothing; hypothesis about data generation model (additive or multiplicative season component); period of prognosis.

The program presents in graphical form: TS without outliers; TS without seasonality; smoothed TS without outliers and seasonality (trend); extrapolated trend; forecasted TS. One can see the program Prophet Interface on the figure 1.

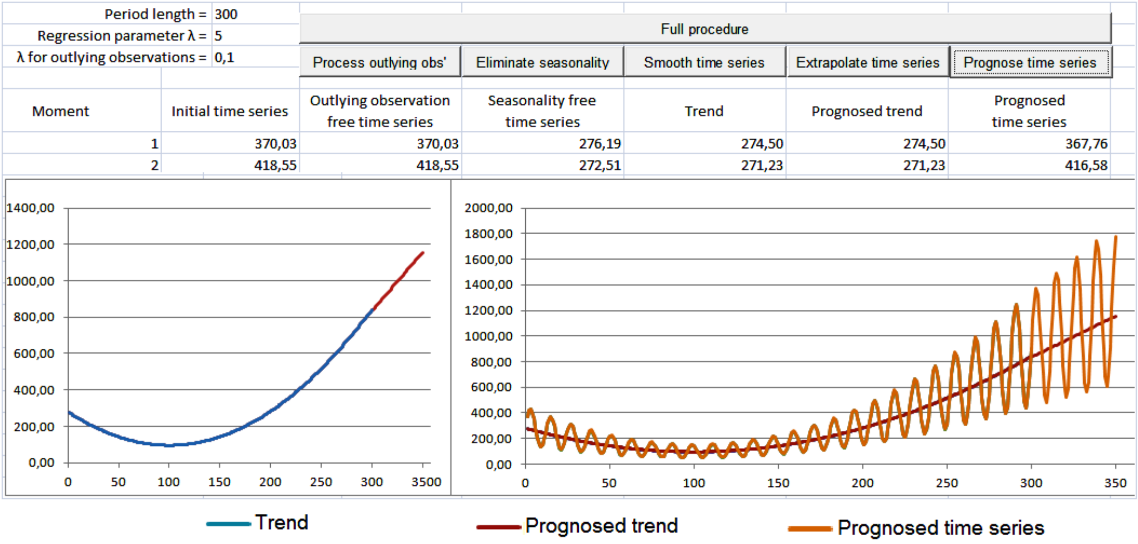

Fig.1. Program Prophet Interface

Experiments

4.1 Local regression

Figure 2 presents the result of smoothing by local lineal regression. TS is a set of oil prices (brand Brent) given on the interval of 25 years. The parameter of smoothing here is equal 500.

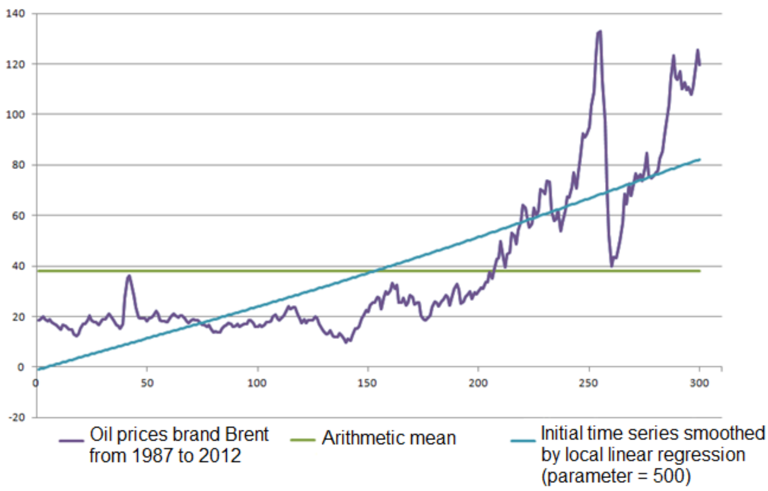

Fig. 2. Smoothing by local linear regression

We should note that the local linear regression does not inverse local minimums and maximums unlike moving average method with odd number of points in the window. We illustrate such an effect on figure 3.

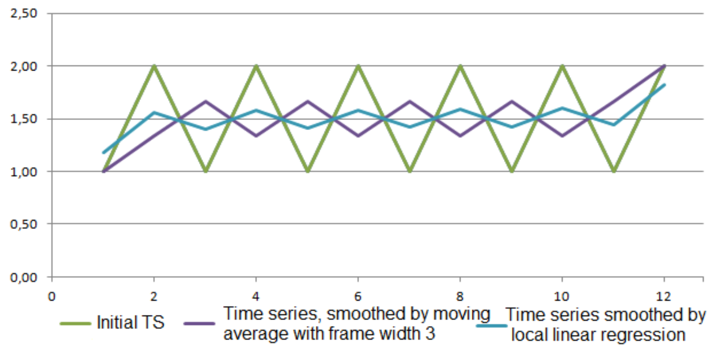

Fig. 3. Smoothing TS with local linear regression and with moving average (odd number of points in the window)

4.2 Elimination of season cyclicity

We prepared artificial TS by summing a given trend and a cyclic season wave. See this TS on figure 4

We could recover the initial trend having applied our method. The results are presented on figure 5.

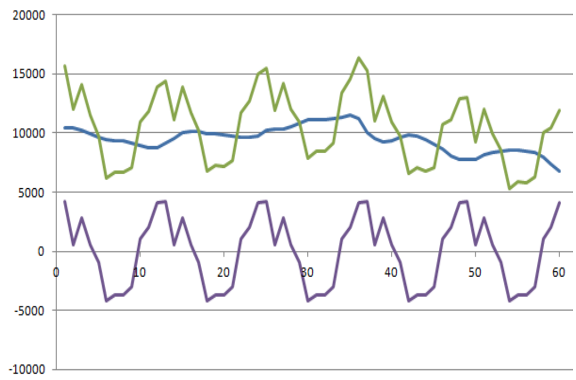

Fig. 4. Artificial TS as a sum of a given trend and a cyclic season wave

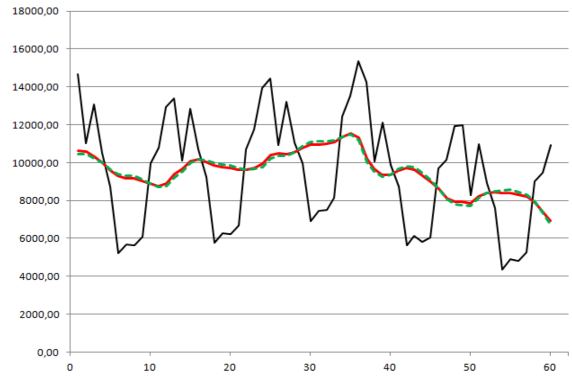

Fig. 5. Initial data, data with a season wave, and recovered data

One can see that the TS without a season wave proved to be close to the real data. Table `1 presents the quantitative values related with seasonality.

Table 1. Characteristics of seasonality

Month number	Initial seasonality	Obtained coefficients
Seas(1)	4205,18	4022,54
Seas(2)	560,18	425,95
Seas(3)	2858,11	2766,15
Seas(4)	527,54	518,81
Seas(5)	-892,84	-821,73
Seas(6)	-4205,19	-4076,85
Seas(7)	-3658,88	-3506,85
Seas(8)	-3650,12	-3510,47
Seas(9)	-3045,01	-2950,52
Seas(10)	1073,28	1097,07
Seas(11)	2071,83	2023,09
Seas(12)	4155,93	4012,82

Conclusions

The main results of the paper are:

1. New method of identification of non-parametric short- and long- run trend of time series by weighted local linear regression was developed.
2. New method for eliminating non-parametric season fluctuations of time series by minimization curvature of residual time series was suggested.
3. Both methods were realized in the computer program on Excel-VBA and their efficiency was demonstrated on real data.

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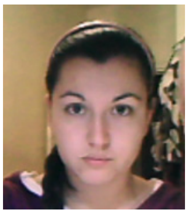

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