COMPUTER PROGRAM FOR SYMULATION OF PRESSURE DISTRIBUTION IN THE HYDRODYNAMIC RADIAL BEARING

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Abstract: The article presents selected numerical methods that are used to solve equations representing the mathematical formulation of engineering problems. The examples of the use of numerical methods in tribology are shown in this paper as well. The way of creating a computer program based on the theory of hydrodynamic lubrication announced by Reynolds is discussed. This program uses the finite difference method to calculate the distribution of hydrodynamic pressure in the radial bearing.

Keywords: Tribology, Computer program, Numerical methods.

ACM Classification Keywords: Numerical Analysis, Software engineering.

Introduction

During tribological studies physical values are often presented as differential equations that describe the laws of physics. These equations can be solved analytically but due to the fact that there may be many of them; they can be complicated and difficult to solve (nonlinear partial differential equation), numerical methods which give approximate required solution are used. Appropriate software allows to obtain numerical solutions of equations representing mathematically formulated engineering problems. The basic methods of calculation used in computer programs are:

- Finite Difference Method (FDM),
- Finite Element Method (FEM),
- Finite Volume Method (FVM).

In brief, these methods rely on the division of the considered continuous area into a finite number of subdivisions (meshing), and then searching and finding approximate solutions in these subdivisions. The solution at any point of space is achieved by interpolation of obtained results. The main differences between these methods are way of finding a solution, defining boundary conditions and method of analysis [Gryboś, 1998].

To calculate the distribution of hydrodynamic pressure in the radial bearing, the finite difference method was used. This method involves approximations that replace the derivatives procured from the differential equations into the finite difference equation, that is approximation of differential equations into difference quotients. These approximations, in algebraic form are associated with each value of the dependent variable in the point of the solution area with values in a number of neighbouring points. These points are selected so as to form a regular grid [Kmiotek, 2008]. A type of grid is usually dependent on the type of coordinate system, suitable for the investigated issue. The most commonly used grid models for two-dimensional problems are presented in Figure 1.

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Fig. 1. The most commonly used grid models for two-dimensional problems: a) rectangular, b) – diagonal, c)-triangular, d) – circular [Kotyński, 2009]

The most important advantages of FDM:

- theoretically it is suitable to solve any type of differential equation (ordinary, partial, linear, nonlinear),
- it allows taking into account the heterogeneity of material,
- it is conceptually simple and uncomplicated to implement on a computer,
- it generates a set of equations which is characterized by the unique matrix that allows using of fast iterative methods for solving this set of equations.

The main disadvantages are:

- difficulties in taking into account the geometry in the case of irregular shapes,
- it can lead to large errors for too coarse grid and to a huge number of nodes and unknowns for too fine mesh leads,
- although there is a possibility of the local density of mesh nodes, it greatly complicates the installation of the
 equations themselves,
- it is probably not suitable for solving the field equations in unlimited areas because it requires the imposition of a finite number of nodes to a grid area [MRS].

The range of use is quite wide: issues in electromagnetic fields, temperature fields, mechanics, hydromechanics. By using FDM as well as other methods, engineering problems can be modeled in the computer memory without the need of building a prototype which greatly simplifies and accelerates the design process.

Discretization of the area on a large number of elements used in each of the methods usually gives more accurate calculation results. However, it requires more FLOPS. In case of very complex systems, solving a problem might be time consuming.

The use of numerical methods in tribological research

The theory of hydrodynamic lubrication announced by Reynolds based on the modification of the physical model of Petrov takes into considerations insights from the experience of Tower. Reynolds took into consideration the eccentric journal position in the bearing which is a prerequisite for the formation of hydrodynamic lift. Mathematical writing of Reynolds theory is based on principles: the conservation of mass (continuity equation) and momentum (Navier-Stokes equation). Reynolds equation is a partial differential equation of second order, inhomogeneous, linear. It's impossible to solve this equation by analytical determination. Reynolds' analyses and his mathematical record of pressure distribution in oil film became the foundation for further development of the theory of hydrodynamic lubrication.

Further work on the development of the theory of hydrodynamic lubrication focused on as exact solutions as possible of the Reynolds' equations for various geometric and dynamic characteristics of the lubricating film and took into account the actual conditions in the lubricating film [Lawrowski, 2008]. The authors [Korzyński, 2007] have analyzed the use of numerical techniques in tribological studies in Poland which shows that the precursor of the numerical calculations of Reynolds' full equation for isothermal laminar flow in slide journal bearing of finite

length was J. Burcan who described the results in the article [Burcan, 1971]. The same author in [Burcan, 1973] presented a numerical calculation of the bearing with hyperboloid bearing liner, while the solution for the nonisothermallubricating film was presented in [Burcan, 1975]. It is worth to mention another article [Krzeminski-Freda, 1972] from that period, in which the elasto-hydrodynamic lubrication problem has been solved numerically as well. An interesting study on the issue of flat oil flow in slide radial- bearing is shown in [Wierzcholski, 1974]. The complete solution of Reynolds' equation for the pseudoplastic Reinara-Rivlina power model, was presented in [Wierzcholski, 1980] [Wierzcholski, 1978] who used the numerical calculation technique. Numerical solution of the problem of hydrodynamic lubrication when the oil film is cavitated shown in [Kicinski, 1985] who modifies and improves the classical Reynolds' equation. In those years presented works were pioneering because the technique of numerical calculation was not developed that much and so easy to use as today. Currently, it is possible to make intensively developed research in the field of friction, lubrication and wear among them thanks to tremendously advanced computer technology and methods of calculation.

An example of work that uses advanced computer software is the work of [Sęp, 2006] in which the threedimensional adiabatic flow of oil in the radial hydrodynamic bearing was modeled. This model assumes that in the tested bearing fluid friction occurs and oil completely separates cooperating elements. The oil flow system is described by Navier-Stokes equations together with the energy equation. Oil characteristics and boundary conditions for the test model were determined, and then were the flow equations solved using the finite element software package ADINA 8.1. The results of computer simulation enabled the determination of hydrodynamic pressure distribution, temperature and flow velocity in the oil film. In order to verify the computer simulation the author conducted experimental study. He constructed a test stand to measure the relative displacement of journal and bush; the pressure and temperature in the oil film.



Fig. 2. Comparison between experimental and calculated circumferential pressure distribution [Sęp, 2006]

Figure 2 shows the comparison of pressure distribution obtained through computer simulations and in the experiment. Comparison of experimental results and numerical calculations showed that the results are not identical but they have very high level of compatibility. The differences among results exist due to both shortcomings in the methods of measurement and computer modeling imperfections (approximations in the mathematical description of phenomena, errors of the calculation method) [Sep 2006].

The authors [Ronen, 2001] have studied the effect of surface microstructure, (made as a form of micro pores on the element simulating the piston ring) on the tribological properties of a simplified model of friction pair ringcylinder. By means of numerical methods they have solved the system of equations consisting of the Reynolds equation and the equation of motion for the simplified model of a ring-cylinder in which the surface of the element of a model imitating ring was textured. They showed that surface texturing can effectively influence the maintenance of the hydrodynamic effect, even in the parallel surfaces and also that optimally chosen surface texture can significantly reduce the frictional losses in the reciprocating automotive components.

The authors of the publication [Kligerman, 2005] performed an analytical model as well, in order to study the possibility of reducing friction between the piston ring and cylinder liner. In their research they took into account the piston rings with specially prepared laser surface structure. In this model the distribution of hydrodynamic pressure and time-dependent clearance between the piston ring and cylinder liner are received from the simultaneous solution using numerical methods of Reynolds equation and equations of motion of the ring in the radial direction. Using a created model, they analyzed the influence of texturing parameters such as dimples depth, texture density area, and textured portion of the nominal contact surface on the average friction force between a piston ring and cylinder liner at different ring widths and operating conditions.

In [Raeymaekers, 2007] there were made attempts to show that the coefficient of friction between magnetic tape and the guide at which the tape moves can be reduced through the implementation of micro dimples on the guide. Micro dimples were made by laser texturing. The microstructure in the form of micro dimples conduces the formation of the air cushion between the tape and the guide, which helps to decrease the friction coefficient between the cooperating elements. Model was created in order to find optimal parameters for the geometry of texture made on the surface of the guide, by which it is possible to reach a maximum pressure of the air cushion and thus it will decrease the friction coefficient between the two cooperating elements. Presuming additional simplifying assumptions concerning, among others, lack of flexibility of the tape and the assumption that the full lubrication fluid is formed in the model, Reynolds equation was used to describe the pressure distribution, reigning in the space between the tape and roll. Also the authors of [Arregui, 2008] conducted the simulation of head - magnetic tape storage device. In this work to describe the distribution of pressure in the forming air cushion between the tape and head Reynolds equation was used as well. To reflect the specificity of the system accurately, additional mathematical model describing the movement of the tape was introduced. To solve the mathematical equations they introduced a new numerical method which is based on the finite element method.

An example of use of numerical methods in tribological studies of human hip joint is the work of [Wierzcholski, 2002]. In this work the authors carried out the numerical analysis of the friction coefficient for asymmetric isothermal steady flow of non-Newtonian synovial fluid in human hip joint including variable viscosity dependent on changes in speed deformation during the lubrication. This work presents the results of numerical values representing the coefficients of friction in the human hip joint obtained for the real working conditions, taking into consideration non-conventional lubrication conditions. Numerical calculations were performed using the finite differences method in the integration of partial differential equations in the areas of joint that were lubricated by synovial fluid. A very important aspect of this work is the fact that the numerically obtained values of the friction coefficients occurring in the gap of human hip joint are used in orthopedic therapy. The follow-up of the same author's research on the tribological issues taking place in the human hip joint is the work of [Wierzcholski, 2007] in which the numerical values of variables like pressure, compressive stress and load occurring on cartilage that is situated on the spherical head bone of the hip joint of the man were assigned. In these studies the case of hydrodynamic lubrication of the hip joint carried out during the rotation of the head bone of the joint was taken into account. Synovial fluid layer completely separates the cartilage on the surface of the head bone and the acetabulum. For the purposes of research non-Newtonian properties of synovial fluid for which viscosity decreases with increasing of the speed of deformation were included. Constant density of the synovial fluid was adopted. The model takes into account that during the operation the height of the gap of the hip joint which is restricted by the area of the joint cartilage changes its value. In numerical calculations the method of finite differences was used.

Modeling pressure distribution in the hydrodynamic radial bearing

The theory of hydrodynamic lubrication is based on the following assumptions adopted by Reynolds:

- The lubricant is a Newtonian liquid this assumption is fulfilled by lubricating oils and most of other liquids used as lubricants,
- There is a laminar flow of fluids most of the hydrodynamically lubricated units fulfill this assumption,
- The inertia forces caused by acceleration of the flow are excluded. The forces of inertia are small in comparison with tangential forces working on the liquid,
- It is assumed that the liquid is incompressible meaning that the volume of liquid passing through each cross-section the oil gap per unit time is constant.

For many practical engineering applications it is assumed that the viscosity throughout the gap lubricant is constant i.e., η = const. This approach is known in the literature as "isoviscous" model, where thermal effects in hydrodynamic film are omitted. Under this assumption, and many others described in details in [Kicinski, 1994] [Hebda, 1980] the pressure distribution p(x, z) in the oil film of statically loaded, lubricated by incompressible liquid radial bearing is described by the equation of Reynolds [Hebda, 1980] :

$$\frac{\partial}{\partial x}\left(h^{3}\cdot\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^{3}\cdot\frac{\partial p}{\partial z}\right) = 6\cdot U\eta\frac{\partial h}{\partial x} \tag{1}$$

where:

p - is the pressure [Pa],

- *h* is the hydrodynamic film thickness [m],
- U the peripheral velocity of journal on bearing [m/s],
- η is the dynamic viscosity of the bearing [Pas],
- x, z are hydrodynamic film co-ordinates [m].

In order to obtain the numerical solution of a model of radial bearing, according to the theory of hydrodynamic lubrication it is necessary to determine the geometry of the oil gap. The schematic drawing N° 3 shows the basic geometrical correlations occurring in the radial bearing. For easier readability a big difference between the radius of the journal and bearing was adopted. In fact, this difference is not so big. In comparison to the dimensions of the bearing, thickness of formed oil film is very small. The angle β is equal to 2π for a full bearing, if β is less than 2π it is known as partial bearing. We will only be considering the case where β is equal to π .



Fig. 3. The basic geometrical correlations in the radial bearing: D - diameter of the bearing, d = 2 * r-diameter of the journal, r - radius of the journal, c - radial clearance, e - absolute eccentricity, h0 - the minimum oil film thickness, [HBT]

Numerical implementation

For the purposes of getting numerical solution of Reynolds equation, it was brought to dimensionless form by adopting the following substitution (based on the work [Hebda, 1980]):

$$\bar{x} = \frac{x}{d}$$
 $\bar{z} = \frac{z}{L}$ $\bar{h} = \frac{h}{2c}$ $\bar{p} = \frac{p}{\omega \eta} \left(\frac{c}{r}\right)^2$ (2)

where:

 \bar{x} , \bar{z} - are non-dimensional hydrodynamic film co-ordinates [m],

L - is the bearing axial length [m],

d - diameter of the journal [m],

 $ar{p}$ - is the non-dimensional pressure,

 \overline{h} - is the non-dimensional hydrodynamic film thickness,

c - is the bearing radial clearance [m],

 η - is the dynamic viscosity of the bearing [Pas],

 ω – angular velocity [rad/s],

and taken as: $U = \omega * d / 2$ and appropriate transformation was made so that it took the form of the equation:

$$\frac{\partial}{\partial \bar{x}} \left(\bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{x}} \right) + \left(\frac{d}{L} \right)^2 \cdot \frac{\partial}{\partial \bar{z}} \left(\bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{z}} \right) = 3 \frac{\partial \bar{h}}{\partial \bar{x}}$$
(3)

Following the numerical solution of equation (3), i.e. after specifying the dependence of dimensionless pressure distribution upon dimensionless coordinates, obtained quantity was expressed back in the standard pressure forms. In order to facilitate the numerical implementation the surface of bearing has been spread and nodal points have been set aside in the surface area (m + 1) * (n + 1).



Fig. 4 Separation of the set of nodal points for the finite difference method from the surface of the bearing a) schematic drawing showing the grid points used, b) view of a single set of five adjacent points

In the point with the indexs *i*, *j* (Fig. 4) individual units of the Reynolds equation are brought closer, replacing the derivatives into differential quotients:

$$\frac{\partial}{\partial \bar{x}} \left(\bar{h}^{3} \frac{\partial \bar{p}}{\partial \bar{x}} \right) = \frac{\bar{h}^{3}_{i,j+\frac{1}{2}} \left(\frac{\bar{p}_{i,j+1} - \bar{p}_{i,j}}{\Delta \bar{x}} \right) - \bar{h}^{3}_{i,j-\frac{1}{2}} \left(\frac{\bar{p}_{i,j} - \bar{p}_{i,j-1}}{\Delta \bar{x}} \right)}{\Delta \bar{x}}$$

$$\frac{\partial}{\partial \bar{z}} \left(\bar{h}^{3} \frac{\partial \bar{p}}{\partial \bar{z}} \right) = \frac{\bar{h}^{3}_{i+\frac{1}{2},i} \left(\frac{\bar{p}_{i+1,j} - \bar{p}_{i,j}}{\Delta \bar{z}} \right) - \bar{h}^{3}_{i-\frac{1}{2},i} \left(\frac{\bar{p}_{i,j} - \bar{p}_{i-1,j}}{\Delta \bar{z}} \right)}{\Delta \bar{z}}$$

$$\frac{\partial}{\partial \bar{z}} \left(\bar{h}^{3} \frac{\partial \bar{p}}{\partial \bar{z}} \right) = \frac{\bar{h}^{3}_{i+\frac{1}{2},i} \left(\frac{\bar{p}_{i+1,j} - \bar{p}_{i,j}}{\Delta \bar{z}} \right) - \bar{h}^{3}_{i-\frac{1}{2},i} \left(\frac{\bar{p}_{i,j} - \bar{p}_{i-1,j}}{\Delta \bar{z}} \right)}{\Delta \bar{z}}$$

$$\frac{\partial}{\partial \bar{x}} = \frac{\bar{h}_{i,j+\frac{1}{2}} - \bar{h}_{i,j-\frac{1}{2}}}{\Delta \bar{x}}$$
(4)

After the substitution into (3) the difference quotients (4) and organizing them, the following expression about $\bar{p}_{t,f}$ is obtained:

$$\bar{p}_{l,j} = \frac{\frac{3(\bar{h}_{l,j-\frac{1}{2}} - \bar{h}_{l,j+\frac{1}{2}})}{\Delta \bar{x}} + \left(\frac{d}{L}\right)^2 \left(\frac{\bar{h}_{l-\frac{1}{2};j}^3 \cdot \bar{p}_{l-1,j} + \bar{h}_{l+\frac{1}{2};j}^3 \cdot \bar{p}_{l+1,j}}{\Delta \bar{x}^2}\right) + \left(\frac{\bar{h}_{l,j-\frac{1}{2}}^3 \cdot \bar{p}_{l,j-1} + \bar{h}_{l,j+\frac{1}{2}}^3 \cdot \bar{p}_{l,j+1}}{\Delta \bar{x}^2}\right)}{\left(\frac{\bar{h}_{l,j+\frac{1}{2}}^3 + \bar{h}_{l,j-\frac{1}{2}}^3}{\Delta \bar{x}^2} + \left(\frac{d}{L}\right)^2 \frac{\bar{h}_{l+\frac{1}{2};j}^3 + \bar{h}_{l-\frac{1}{2};j}^3}{\Delta \bar{x}^2}\right)}$$
(5)

Equation 5 can be presented in general form:

$$\bar{p}_{i,f} = a_0 + a_1 \cdot \bar{p}_{i+1,f} + a_2 \cdot \bar{p}_{i-1,f} + a_3 \cdot \bar{p}_{i,f+1} + a_4 \cdot \bar{p}_{i,f-1}$$
(6)

where a_0 , a_1 , a_2 , a_3 , a_4 - constant data for each grid point.

Pressure $\bar{p}_{t,f}$ is a function of these constants and a function of the four nearby pressure data (in the grid). For n^*m grid points (Fig. 4) n^*m equations were obtained and further fixed in a program written in Matlab environment. Finally, the way of pressure distribution p = p(x, z) was obtained. For points lying on the edge of the acetabulum (black ones in Figure 3) there is no need to write the equation, since the pressure at these points is known.

If the pivot axis and the acetabulum are parallel, the thickness of the oil gap *h* is only a function of one variable angle φ :

$$h(\varphi) = \sqrt{\left(\frac{D}{2}\right)^2 + eD\sin(\alpha + \varphi) + e^2} - r \tag{7}$$

The variable x is a linear function of radius D/2 of the form:

$$x = \varphi \cdot \frac{D}{2} \tag{8}$$

Following transformation of derivative of *h* with respect to *x* is given by:

$$\frac{dh}{dx} = \frac{eD\cos(\alpha+\varphi)}{2\sqrt{\left(\frac{D}{2}\right)^2 + eD\sin(\alpha+\varphi) + e^2}}$$
(9)

As before, at the point with the indexes i, j (Fig. 3), with respect of the above formulas each word of the Reylnods equation (3) are brought closer by replacing pressure derivatives into differential quotients:

$$\frac{\partial}{\partial \bar{x}} \left(\bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{x}} \right) = 3\bar{h}^2 \frac{d\bar{h}}{d\bar{x}} \cdot \frac{\partial \bar{p}}{\partial \bar{x}} + \bar{h}^3 \frac{\partial^2 \bar{p}}{\partial \bar{x}^2} = 3\bar{h}_f^2 \frac{d\bar{h}}{d\bar{x}} (j) \left(\frac{\bar{p}_{l,l+1} - \bar{p}_{l,l-1}}{2\Delta \bar{x}} \right) + \bar{h}_f^3 \left(\frac{\bar{p}_{l,l-1} - 2\bar{p}_{l,l} + \bar{p}_{l,l+1}}{\Delta \bar{x}^2} \right)
= \frac{\partial}{\partial \bar{z}} \left(\bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{z}} \right) = \bar{h}_f^3 \frac{\partial^2 \bar{p}}{\partial \bar{z}^2} = \bar{h}_f^3 \left(\frac{\bar{p}_{l-1,l} - 2\bar{p}_{l,l} + \bar{p}_{l+1,l}}{\Delta \bar{z}^2} \right)$$
(10)

After the substitution of members (10) to (3) and organizing them, the equation was obtained:

$$\frac{\partial}{\partial \bar{x}} \left(\bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{x}} \right) = 3\bar{h}^2 \frac{d\bar{h}}{d\bar{x}} \cdot \frac{\partial \bar{p}}{\partial \bar{x}} + \bar{h}^3 \frac{\partial^2 \bar{p}}{\partial \bar{x}^2} = 3\bar{h}_f^2 \frac{d\bar{h}}{d\bar{x}} (j) \left(\frac{\bar{p}_{l,l+1} - \bar{p}_{l,l-1}}{2\Delta \bar{x}} \right) + \bar{h}_f^3 \left(\frac{\bar{p}_{l,l-1} - 2\bar{p}_{l,l} + \bar{p}_{l,l+1}}{\Delta \bar{x}^2} \right)$$

$$\frac{\partial}{\partial \bar{z}} \left(\bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{z}} \right) = \bar{h}_f^3 \frac{\partial^2 \bar{p}}{\partial \bar{z}^2} = \bar{h}_f^3 \left(\frac{\bar{p}_{l-1,l} - 2\bar{p}_{l,l} + \bar{p}_{l+1,l}}{\Delta \bar{z}^2} \right)$$
(11)

Replacing the matrix of unknowns at elements of $\bar{p}_{t,f}$ (*i* = 1: *n*, *j* = 1: *m*) on a single column vector \bar{p} of \bar{p}_k elements given by:

$$k = (i-1)m + j \tag{12}$$

equation (11) can be written as:

$$a_{k,k-1}\bar{p}_{k-1} + a_{k,k}\bar{p}_k + a_{k,k+1}\bar{p}_{k+1} + a_{k,k-m}\bar{p}_{k-m} + a_{k,k+m}\bar{p}_{k+m} = b_k$$
(13)

where:

$$a_{k,k-1} = \frac{\bar{h}_{j}^{3}}{\Delta \bar{x}^{2}} - \frac{3\bar{h}_{j}^{2}}{2\Delta \bar{x}} \frac{d\bar{h}}{d\bar{x}}(j) , a_{k,k} = -\left(\frac{2\bar{h}_{j}^{3}}{\Delta \bar{x}^{2}} + 2\left(\frac{d}{L}\right)^{2} \frac{\bar{h}_{j}^{3}}{\Delta \bar{z}^{2}}\right) , a_{k,k+1} = \frac{\bar{h}_{j}^{3}}{\Delta \bar{x}^{2}} + \frac{3\bar{h}_{j}^{2}}{2\Delta \bar{x}} \frac{d\bar{h}}{d\bar{x}}(j)$$

$$a_{k,k-m} = \left(\frac{d}{L}\right)^{2} \frac{\bar{h}_{j}^{3}}{\Delta \bar{z}^{2}} , \qquad a_{k,k+m} = \left(\frac{d}{L}\right)^{2} \frac{\bar{h}_{j}^{3}}{\Delta \bar{z}^{2}} , \qquad b_{k} = 3\frac{d\bar{h}}{d\bar{x}}$$
(14)

The coefficients $a_{k,k-1}$, $a_{k,k}$, $a_{k,k+1}$, $a_{k,k-m}$, $a_{k,k+m}$ are elements different from zero in the matrix A, the b_k coefficient is the element of one-column vector B. As a result a system of linear equations are given in a form of:

$$A\bar{p} = b \tag{15}$$

with the unknowns \bar{p} .

By solving numerically set of equations (15) according to the formula: $\bar{p} = A^{-1}b$ pressure values were obtained.

The visualization of results

Calculations were carried out in a rectangular system *x*, *y*, *z* (Fig. 5), where the x-axis was the length of the arc on the surface of the bearing, and the z-axis was the length of the bearing itself. Dimensionless pressure distribution on the surface of the bearing in a spatial configuration *X*, *Y*, *Z* is presented in Figure 6. Calculations are performed for n = 20, m = 60. Figure 7 shows the differences between dimensionless pressure $p\overline{1}$ (calculated

according to formula 11) and \bar{p} (calculated according to formula 5). Figure 8 shows the distribution of dimensionless pressure $\bar{p}1$ and p for one section z = 23.8095 and the difference $\bar{p}1-\bar{p}$ along the x axis.



Fig. 5. The dimensionless pressure distribution in the oil film of a bearing in the rectangular system x,y,z



Fig. 6. The distribution of dimensionless pressure in the oil film of a bearing in spatial layout X, Y,Z



Fig. 7. The difference between dimensionless pressure p1 and p in the rectangular system x, y, z



Fig. 8. The distribution of dimensionless pressure $p\bar{1}$ (denoted by "o") and $p\bar{p}$ (denoted by "*") and the difference $(p\bar{1}-p\bar{p} - \text{denoted by solid line})$ along the x axis for the cross section z = 23.8095

Conclusions

The complexity of tribological phenomena is so large that without the use of computer technology it is difficult to make any research in this area. Computer technology that uses advanced numerical methods is a supportive tool, in many cases it allows to carry out modelling and simulation of complex tribological occurrence which take place in different nodes and different working conditions. This paper presents only a few examples which, to some extent, reflect the use of numerical methods in tribological studies. The finite difference method used in this work makes it possible to solve the Reynolds equation for arbitrary input parameters. It cannot be forgotten that in order to determine the real condition of the bearing, there is a need for specifying the parameters under which hydrodynamic oil film that is able to move the load inflicted (not considered by the Reynolds equation) will be generated. Program prepared by the authors and presented in this paper allows to understand both the way of the implementation of numerical calculations and the very essence of the theory of hydrodynamic lubrication.

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