NOISE IMMUNITY OF INDUCTIVE MODELING WITH J.FORRESTER'S VARIABLES[3]

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Abstract: *Traditional model of Forrester´s world dynamics contains 5 variables: population, main funds, capital investment in agricultural fraction, pollution and natural resources. In the paper we consider model with the same variables, which is built using inductive modeling technique. We study influence of additive and multiplicative Gaussian noise on the model and test the theoretical results concerning training on noised data.*

Keywords*: inductive modeling, world dynamics, noise immunity.*

ACM Classification Keywords*: 1.6.4. Model validation and analysis.*

Introduction

In 1971 J. Forrester was asked to develop a model of world dynamics. Speaking world dynamics we mean the dynamic interactivity of the main macroeconomical variables. Such models can predict crises and sometimes help to avoid it. J. Forrester in his work [Forrester, 1979] selected five main problems, which could provoke the World Crises. It is overpopulation of our planet, lack of basis resources, critical level of pollution, food shortages and industrialization and the related industrial growth. He tied a single variable with each of these issues. So, we have a five-level system including: population (P), pollution (Z), natural resources (R), fixed capital (K), capital investment agriculture fraction (X). This system is built on the principals of system dynamics and it is presented in the form of five differential equations named the classical Forrester´s model:

Previously we have experimentally studied noise immunity of this [Proncheva, 2014a]. We found out that multiplicative noise, which represents internal system shocks, affects a system less than additive noise that represents external shocks. Also the most sensitive variable was pollution and the most influential variable was natural resources.

In this work we study noise immunity of models built in inductive modeling technique. Here-in-after we will call such models IM-models. Inductive modeling has a long history and many applications [Ivakhnenko, 1968; Madala 1994; Stepashko, 2013]. In the work [Proncheva, 2014b] we shortly describe our experience in building IM-models with J.Forrester's variables but we did not consider noise immunity of these models.

The paper is built by the following way. In section 2 we consider the tools for testing noise immunity of IM-models. Section 3 is devoted to study noise immunity. Section 4 contains conclusions

Tools for Testing Noise Immunity of IM-Models

The classes of predictive models

The models under consideration are supposed to belong to the class of nonlinear difference equations. All data were scaled. So all the values of the variables prove to be in the interval (0, 1), i.e. have the same scale. To do this, the numerical values of population, investment capital and investment capital in agriculture fraction were divided in each year on 10^{10}, the number of remaining natural resources - on 10^{12}.

[3] The work done under support of the British Petroleum grant (BP-RANEPA 2013)

The final models have the following form (1 - 5):

$$P_{t+1} = g_1(P_t, K_t, X_t, Z_t, R_t, P_{t-1}, K_{t-1}, X_{t-1}, Z_{t-1}, R_{t-1}, \dots) \quad (1)$$

$$K_{t+1} = g_2(P_t, K_t, X_t, Z_t, R_t, P_{t-1}, K_{t-1}, X_{t-1}, Z_{t-1}, R_{t-1}, \dots) \quad (2)$$

$$X_{t+1} = g_3(P_t, K_t, X_t, Z_t, R_t, P_{t-1}, K_{t-1}, X_{t-1}, Z_{t-1}, R_{t-1}, \dots) \quad (3)$$

$$Z_{t+1} = g_4(P_t, K_t, X_t, Z_t, R_t, P_{t-1}, K_{t-1}, X_{t-1}, Z_{t-1}, R_{t-1}, \dots) \quad (4)$$

$$R_{t+1} = g_5(P_t, K_t, X_t, Z_t, R_t, P_{t-1}, K_{t-1}, X_{t-1}, Z_{t-1}, R_{t-1}, \dots) \quad (5)$$

In our research we studied two models. The first one contains only given variables with different powers, the second one contains additional pairwise multiplications to consider a combined influence of variables. The variables in the models can use integer, fractional, positive and negative powers.

Checking IM-model

Noise immunity can be checked in two different regimes:

- noise affects on model on the stage of forecast;
- noise can be included to initial data.

In this work we consider the first case. We check noise immunity to additive (external shock of the system) and multiplicative (internal shocks) noises. Noise affects only since 2013 year. Before this year the system dynamics is defines by (1-5).

Additive noise that affected on population was simulated by the following way:

$$P_{t+1} = g_1(P_t, K_t, X_t, Z_t, R_t, P_{t-1}, K_{t-1}, X_{t-1}, Z_{t-1}, R_{t-1}, \dots) + level \cdot \xi \cdot \hat{P} \quad (6)$$

here:

ξ - white Gaussian noise (0;1);

$level$ - a level of noise (some fractions);

$\hat{P}$ - an average power of population.

Dynamics of other variables is the same.

Multiplicative noise was simulated by the following way:

$$P_{t+1} = g_1(P_t, K_t, X_t, Z_t, R_t, P_{t-1}, K_{t-1}, X_{t-1}, Z_{t-1}, R_{t-1}, \dots) \cdot (1 + level \cdot \xi) \quad (7)$$

here:

ξ - white Gaussian noise (0;1);

$level$ - the level of a noise (in fractions).

Also we calculate quantitative characteristic of noise immunity. We use the next measure:

$$\sigma_i^{rat} = \frac{1}{15} * \sum_{t=2013}^{2028} \frac{1}{f_i(t)} * \sqrt{\sum_{k=1}^{1000} [(f]_i^k(t) - f_i^{mean}(t))^2 * \frac{1}{1000}} \quad (13)$$

here

σ_i^{rat} - mean-root deviation of variable i;

$f_i(t)$ - the value of variable *i* in moment *t* in un-noised function;

$f_i^k(t)$ - the value of variable *i* in realization *k* in moment *t*;

$f_i^{mean}(t)$ - mathematical expectation of variable *i* in moment *t.*

So, the less σ_i^{rat} is, the more stable a function is.

Software

To build IM-models we used the program package GMDH Shell (GS). GS were developed by GEOS company and it covers problems of extrapolation, approximation and classification [GS, http://www.gmdhshell.com/]. Speaking extrapolation we mean time series prognosis. GS is based on Group Method of Data Handling (GMDH). It is realized in 3 algorithms: combinatorial GMDH, GMDH-type neural networks, GMDH-type decision forest. GS is very fast because of parallel processing and deep optimization of core algorithms. In our previous work we used GMDH-type neural networks [Proncheva, 2014b].

For analysis of noise immunity we use the program "Model-IM" developed in MatLab. This program includes a convenient interface to make this program accessible for end-users.

Experimental Study of Noise Immunity of IM-Models

The simple model

The forecast was made on 15 years. The best model in the class of "simple" model is:

$P_{t+1} = -0.00603058 + 0.00521011 \cdot Z_t^{-3/2} + 0.0255463 \cdot X_{t-7} + 1.22781 \cdot P_t - 0.182304 \cdot P_{t-2}$

$K_{t+1} = 0.00361846 + 1.38141 \cdot K_t - 0.372398 \cdot K_{t-1}$

$X_{t+1} = -0.01487678 - 0.2118857 \cdot X_t + 0.943634 \cdot X_{t-5} + 1.6358 \cdot X_t^{3/2}$

$Z_{t+1} = -0.00551411 + 0.787061 \cdot P_t^{-1/2}$

$R_{t+1} = -0.852927 - 0.0110949 \cdot P_{t-10} - 1.94371 \cdot Z_t^{1/2} + 0.965339 \cdot P_t^{1/2}$

Below (fig. 1) are the results of influence of 20% additive noise. There are 3 lines on the figure: thin uninterrupted line is the initial function, thick line is the forecast, and thin dotted line is the worst function. The mean-root deviation, calculated with (13), is presented in Table 1.

The most sensitive and the most influential variable were also diagnosed. The most sensitive variable reacts the most on shock of other variables. Shock of the most influential variable affects the most the other variable. The most sensitive variable is pollution, as in Forrester's model. The most influential variable is resources. It means that one should pay the main attention on these variables.

Table 1.

Variable	Mean-root deviation, %
Population	1,4%
Capital investment	1,2%
Capital investment agriculture fraction	1,0%
Pollution	1,3%
Resources	1,2%

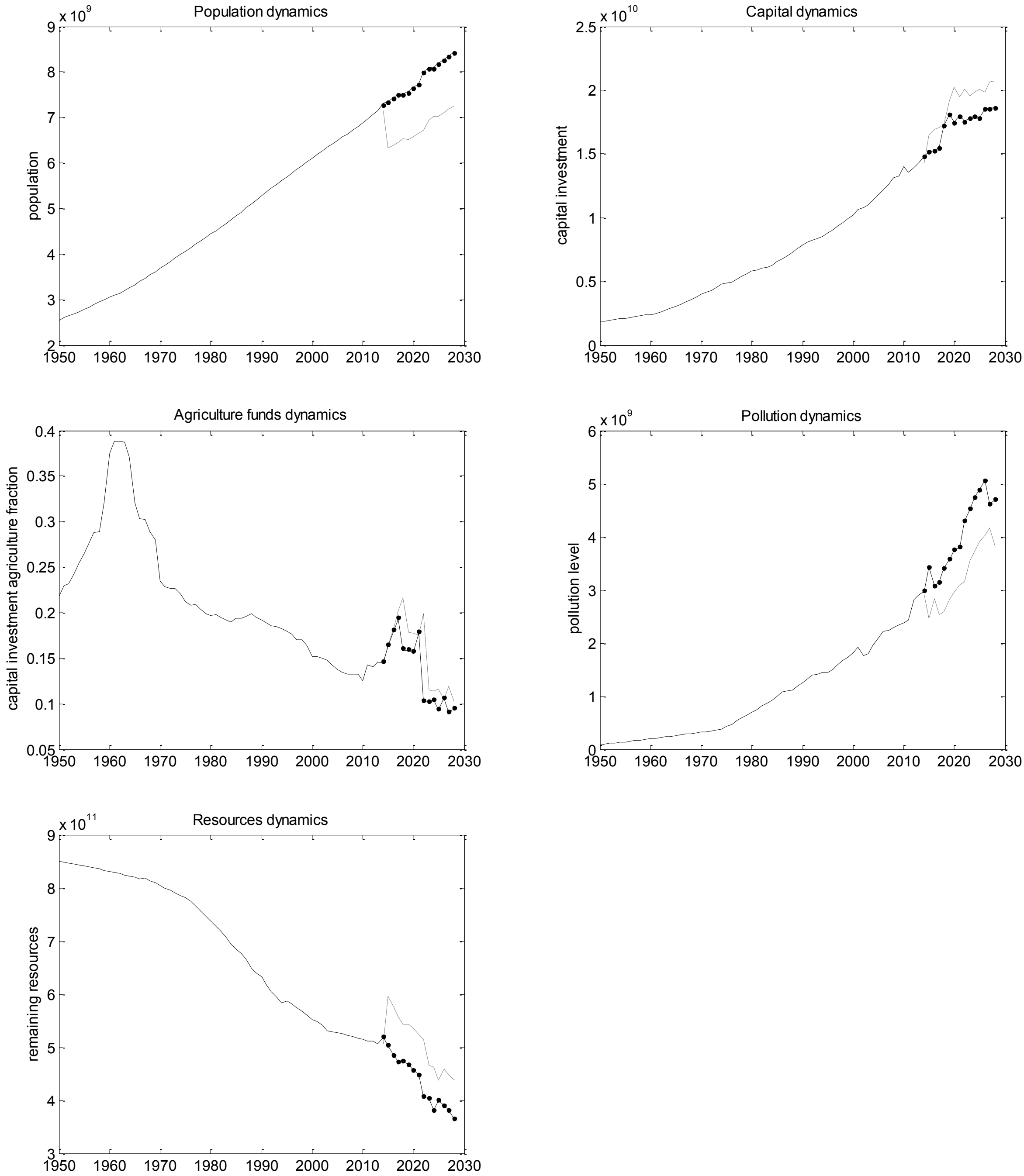


Figure 1. Influence of additive noise on the simple model

The influence of multiplicative noise was also researched. Below (fig. 2) there are the results of influence of 50% additive noise. There are 3 lines on the figure: thin uninterrupted line is the initial function, thick line is the forecast, and thin dotted line is the worst function. The mean-root deviations are presented in table 2. The most sensitive variable is pollution, as in Forrester's model. The most influential variable is resources.

So, we got the analogy with Forrester's model. Multiplicative noise affects the model much less than the additive one, and in both cases the most sensitive variable is pollution and the most influential one is resources.

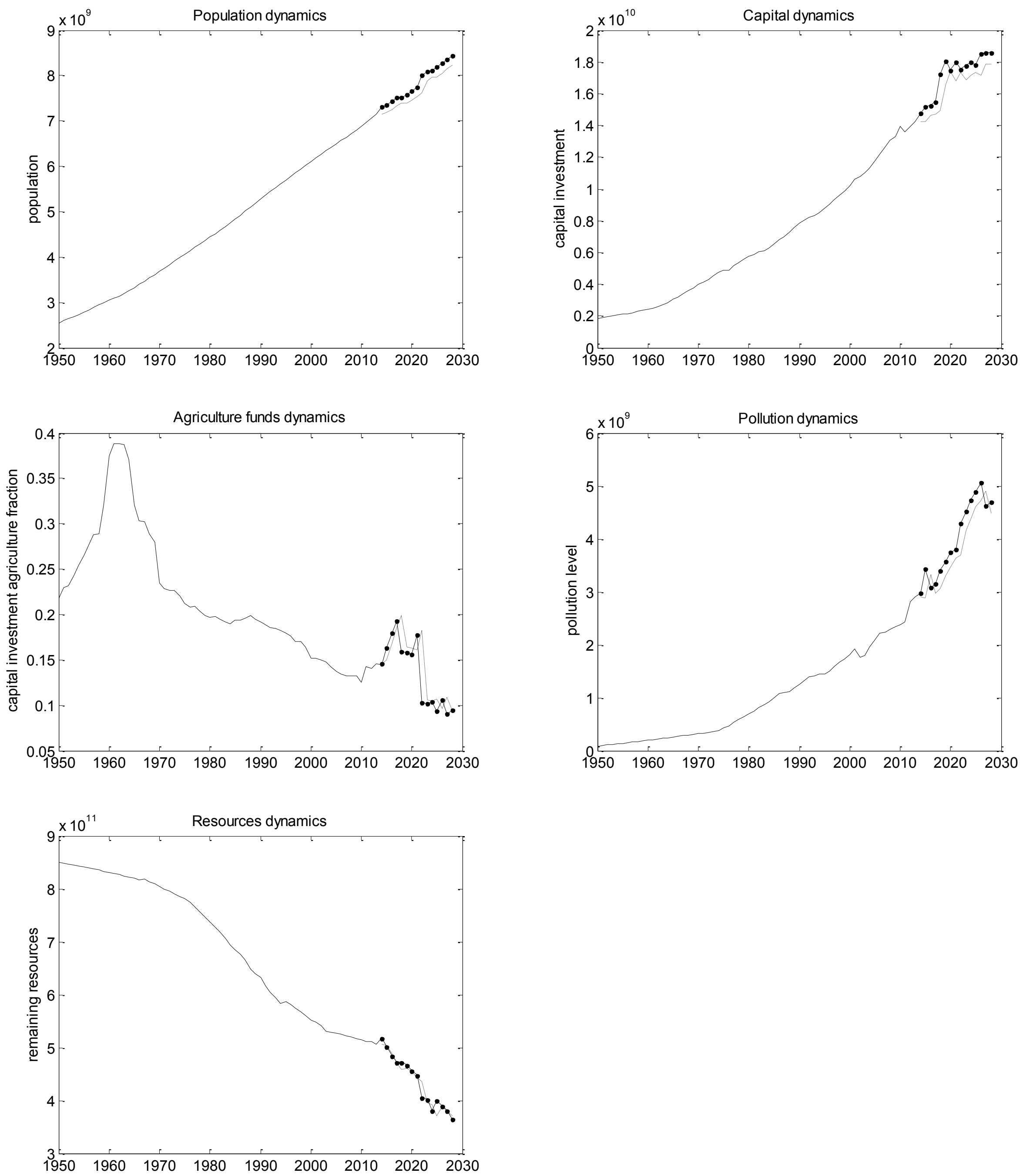


Figure 2. Influence of multiplicative noise on the simple model

Table 2

Variable	Mean-root deviation, %
Population	0,4%
Capital investment	0,5%
Capital investment agriculture fraction	0,3%
Pollution	0,4%
Resources	0,1%

The model with pairwise multiplication

The best model in the class of models with pairwise multiplications is:

P_{t+1}=-0.001603058-0.0468069$\cdot P_{t-2}$+0.0255463$\cdot X_{t-7}$+0.09195$\cdot Z_t^{-3/2}$

K_{t+1}= - 0.00187356 - 0.165445$\cdot K_t \cdot X_t$+1.00087$\cdot K_t$

X_{t+1} = -0.000409618 - 0.132082$\cdot t^{1/2}$- 3.34978$\cdot X_t^{1/2}$+1.08047 $X_t^{3/2}$

Z_{t+1} = -1.98521e-16 + 0.787061$\cdot Z_t \cdot P_t^{-1/2}$

R_{t+1} = 0.852927- 0.0110949$\cdot P_{t-}$10-1.94371$\cdot Z_t^{1/2}$+$Z_t^{1/2} \cdot P_t^{1/2}$

Below (fig.3) there are the results of influence of 20% additive noise on the model with pairwise multiplications. The mean-root deviations, calculated with (29), are presented in table 3. In this case the most sensitive and influential variables are also pollution and resources respectively.

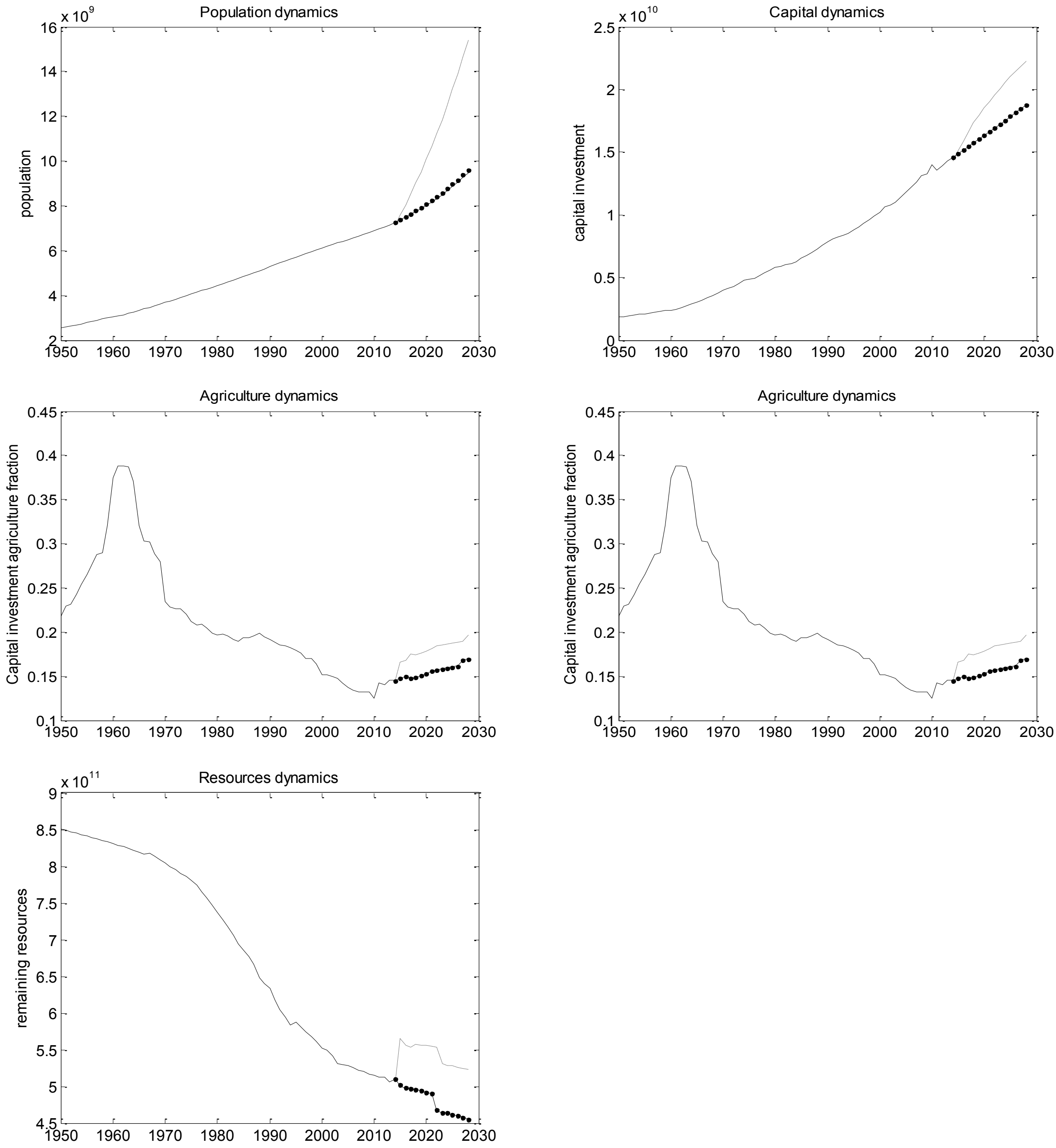


Figure 3. Influence of additive noise on the model with pairwise multiplications

Table 3.

Variable	Mean-root deviation, %
Population	2,4%
Capital investment	2,2%
Capital investment agriculture fraction	2,1%
Pollution	2,1%
Resources	2,4%

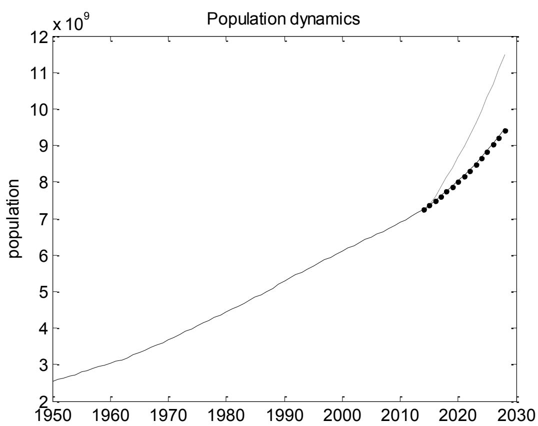
The final experiment was completed with 50% multiplicative noise. Its results are presented on figure 4. Table 4 contains the mean-root deviations.

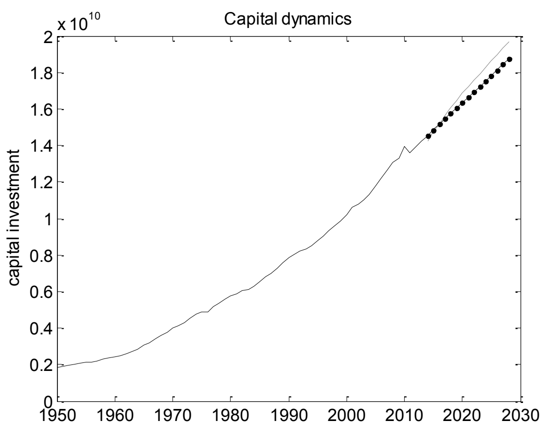
Experiments showed that the most sensitive variable is pollution, and the most influential one is resources. In case of the model with pairwise multiplications it was also detected that multiplicative noise affects the model less than the additive one.

Table 4.

Variable	Mean-root deviation, %
Population	1,0%
Capital investment	0,5%
Capital investment agriculture fraction	0,4%
Pollution	0,4%
Resources	0,5%

In addition one can say that simple model better adapts to noise than the model with pairwise multiplications. This result confirms the well-known theoretical fact that simple model is more stable to noise but worse approximates real data [Stepashko, 2008]. By the way other our experiments show that the model with pairwise multiplications gives forecast, which almost coincides with real data.





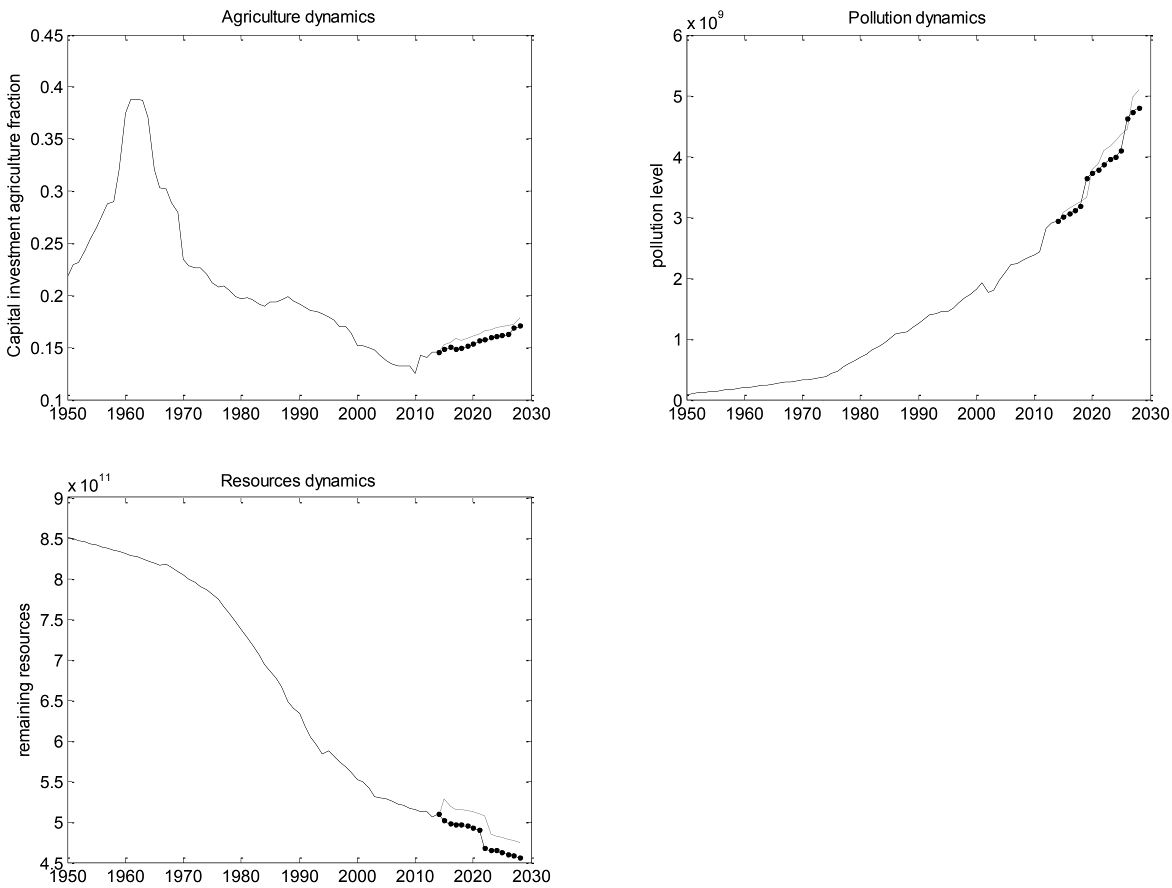


Figure 4. Influence of multiplicative noise on the model with pairwise multiplications

Conclusion

In the paper we studied noise immunity of models built in GMDH technique. The simple model with individual variables and the model with pairwise multiplications were considered. Our results are the following:

- The most influential variable in the model is resources and the most sensitive is pollution. This result coincides with that for Forrester's model [Proncheva, 2014a];
- Additive noise affects on both models more than the multiplicative one. This result coincides with that for Forrester's model [Proncheva, 2014a];
- The simple model better adapts to noise independently whether it is the addivive one or the multiplicative one.

Acknowlegements

The author is grateful to Dr. Alexandrov, Dr. Koshulko, Dr. Makhov and Dr. Stepashko for their interest to my work and advices.

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