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## EVALUATION OF RUNWAY CAPACITY AND SLOTS AT LONDON GATWICK AIRPORT USING QUEUING BASED SIMULATION

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**Abstract:** *The evaluation of the runway capacity and its optimization is one of the core goals of the airports. Most of the time due to infrastructural and regulatory factors it is quite impossible to increase the capacity of the runway. Therefore, it is of utmost priority to optimize its usage. Nowadays, the decision support systems play a very crucial role in defining the threshold capacities at the runway to make it economical and operationally efficient. This fact makes them a crucial factor in the aviation market. The use of the support system for runway evaluation and assessing slots makes the business profitable for both airports and airlines, as they will highly get hurt economically if they do not use the runway as an airside infrastructure efficiently. The main aim of this article revolves around the design of a decision support system, which will help in providing the decisional support to the managers by evaluating the various scenarios for optimization of the runway usage. The evaluation models, which are used in this article, are the queuing based models and they accurately cope with the logic lying behind the runway capacity usage.*

**Keywords:** *Runway Capacity analysis, Slot Management, Queuing Theory and Models, Probabilistic Distribution, Simulation and Optimization*

**ACM Classification Keywords:** *B.2.2 Performance Analysis and Design Aids Simulation, B.4.4 Performance Analysis and Design Aids, F.1.2 Modes of Computation, G.3 Probability and Statistics, G.m Miscellaneous. .*

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### Introduction

The main aim of this article is to examine and evaluate the standard day of operation at the London Gatwick airport. It is important to evaluate the runway capacity as it provides the insight into the number of movements served at the airports [Simpson, Belobaba, 1992], there by directly complying with the slot management. The airport chosen for the evaluation is not by accident – this is the most overloaded single runway airport on the globe. In the process of this evaluation the models of Queuing Theory is used and a special system has been developed and implemented. The queuing theory is also referred as the traffic theory because of the characteristics it possess [Bose, 2002]. The simulation has been applied with adoption of three different models of queue. For this article, only two models will be elaborated. Each model is characterized by the different probability distributions of the time depending on the runway occupancy.

After the simulation models is planned and implemented, it is quite reasonable to form a particular hypothesis that will be examined during the research and will help in final fulfillment of the main goal. The following hypothesis will be investigated:

Hypothesis 1: The busiest period or the peak period takes place in the morning and in the early afternoon what is a consequence of business travel.

Hypothesis 2: The London Gatwick Airport capacity's situation, despite of the fact that it is the busiest single-runway airport, is stable and the probability that the aircraft misses its slot is less than 10%.

Hypothesis 3: Despite of the fact the situation on the researching airport is stable, the small waiting lines might occur. However, the utilization of the runway is optimal and number of aircraft waiting in the queue at a particular moment is smaller than 3 and the probability of that event not taking place is smaller than 20% during the whole period.

Hypothesis 4: The results (L, Lq, W, and Wq) from the main simulation will be very similar to the second long run simulation with more number of observations. The number of events do not influence on the mentioned parameters.

Hypothesis 5: Sum-up of the all aircrafts that missed the slot in investigating period will not be greater than 2 missed slots per hour.

Hypothesis 6: The airlines on the Gatwick airport do not suffer due to additional costs caused by missing the slot or extra fuel consumption.

**Simulation**

The aim of this section is to show the interface and other design aspects of the simulation.. Simulation directly provides reasonable improvement in the application of market mechanisms [Doganis, 1991].

The simulator is named as Runway Examiner, which goes precisely with the task it is accomplishing. All of the steps and actions that occur during the interaction are described here. The simulator uses a detailed scenario that indicates in exact way how the customer works on it. The basic strategy is to identify a so called path through the user case and then to write an exemplary scenario. All the simulations have the generalized characteristics of having an input process, the service mechanism and queue discipline [Cooper, 1981]. Figure 1 shows the runway examiner interface design to evaluate runways.

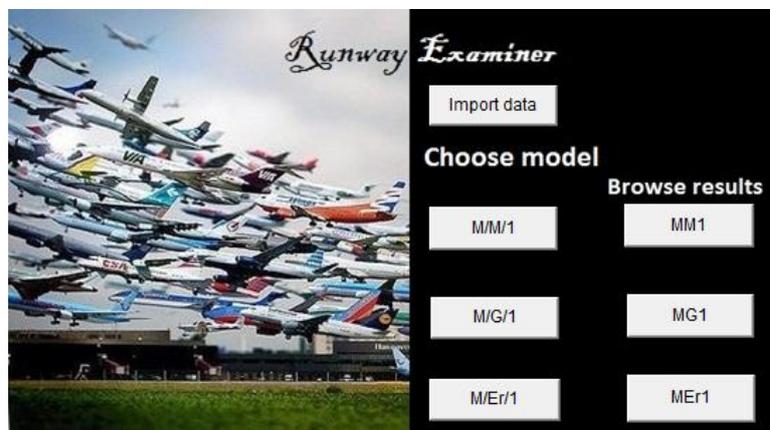
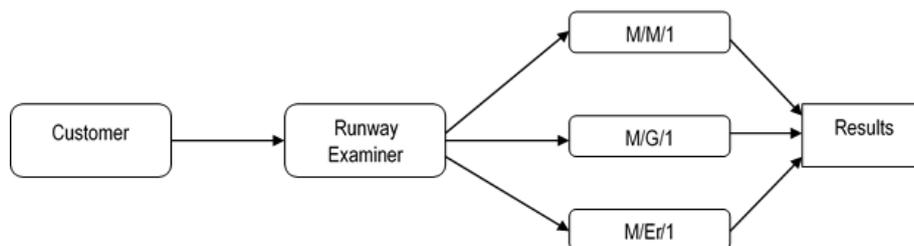


Fig. 1 Runway Examiner interface

The graphical presentation of simulator usage is provide in schema 1.



Schema 1 Simulator Usage

The Runway Examiner for the random customer works as follows:

1. The customer runs the file Runway Examinet.xls and the control panel presented above appears.
2. The user clicks on the button import the data in order to load the desired information about the operation that is scheduled during particular day and time. The imported file has to be a \*.txt type and has to be prepared earlier by the form builder.
3. The next step is choosing the model of queue the customer wants to examine desired airport runway for. The choice has to be made between three considered types of queues: M/M/1, M/G/1, and M/Er/1. After moving the mouse over the button, the short comment including some basic information about the mathematical model of examining the waiting lines is printed.
4. The final step is to press the button Results in order to get the findings of the most important characteristic of the airport runway and browse the figure section.
5. Alternative way is to click the button Browse and observe the results straight from the table printed in the spreadsheet.

Setting the priorities is also an important aspect of the simulation which allows to allocate the appropriate service time [Gross, Donald, Harris, 1998].

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### Simulation results

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This section highlights the results of the simulation. Though we can perform all the three types of simulations using the runway examiner, during this article only results of two simulations is discussed. The simulations, which are discussed in this article, are, namely, M/M/1 queue simulation and M/Er/1 queue simulation.

M/M/1 queue simulation:

The results of this simulation are shown in the table 1:

*Table 1. Characteristics of M/M/1 queue simulation*

Characteristic	Value
Total movements in examining period =	76
Arrival rate $\lambda$ =	0.42
Service rate $\mu$ =	0.44
Occupational rate $\rho$ =	0.95
Number of airplanes by weight class(light;heavy;massive) =	(6,47,23)
Number of movements by the type of movement(land;take-off)	(40,36)
Expected number of users in Queueing system $L$ =	5
Expected time in Queueing system per user $W$ =	12.16
Expected number of users in queue $L_q$ =	2
Expected waiting time in queue per user $W_q$ =	1.01

The formulas used to calculate the parameters are taken from M/M/1 queue simulation [Denardo, 2002]. The total number of movements have not exceeded the 80 that is the maximum possible size of traffic that is allowed by the airport authorities and international regulators at the London Gatwick Airport. That means that, at least theoretically, the Air Traffic Controllers should be able to handle the number the movements that appeared in examining hour. Each queue is described by the arrival rate  $\lambda$  and the service rate  $\mu$  [Jędrzejczyk, Skrzypek, Kukula, Walkosz, 1997]. The arrival rate  $\lambda$  equals 0.4, whereas the service rate  $\mu$  0.45. That means that less that one aircraft appears on the runway every two minutes and respectively roughly 2 minutes is enough for the service station for providing service. The very important characteristic - the occupational rate is 0.89, inhibits that the queuing system in long run is stable, however some waiting lines might appear in some periods of the day depending on the density of the inter-arrivals. Such situation will be examined during the hypothesizes in later section.

Now, there is a high time to consider the profile of the customers (which are aircrafts in our case) by the weight class and the type of movement. The great majority of the runway system users, taking into consideration the historical data, are the heavy class aircrafts. Completing the profile – less than 10% of total number of aircrafts are light aircrafts flying on the regional lines mainly. The distribution of the traffic by the type of represents equilibrium. Almost the same number of planes land and start their journey at the London Gatwick.

The next step was to, using Queuing Theory formulas to get the expected number of users in queuing system  $L$ , expected time in queuing system per user  $W$ , expected number of users in queue (Average number of airplanes in the queue)  $L_q$  and expected waiting time in queue per user  $W_q$ . The values shown in the table indicate that the queuing system is rather stable. The average number of clients in the system is 6. That number may seem high, but it should be kept in mind that some flights have been scheduled at the same time, which is why the short queue may occur, (only one aircraft on average is expected to stay in waiting line). The total time is very likely resulting from this fact. The average number spend in queue per user is equal 6.73.

Finally, the distribution of service and arrival time per user is provided by the service time and it balances between 1 and 3 minutes, almost 98% of all examining movements are in this range. Considering the inter-arrival time of the aircraft it is between one and 5 minutes.

M/Er/1 queue simulation:

The results of this simulation are shown in the table 2:

Table 2. Characteristics of M/Er/1 queue simulation

Characteristic	Value
Total movements in examining period =	76
Arrival rate $\lambda$ =	0.39
Service rate $\mu$ =	0.43
Occupational rate $p$ =	0.91
Number of airplanes by weight class(light;heavy;massive) =	(6,47,23)
Number of movements by the type of movement(land;take-off)	(40,36)

Expected number of users in queuing system $L =$	5
Expected time in queuing system per user $W =$	12.16
Expected number of users in queue $L_q =$	2
Expected waiting time in queue per user $W_q =$	1.01

The formulas used to calculate the parameters are taken from M/Er/1 queue simulation [Tijms, 2003]. The values that are different from the first sight are arrival rate  $\lambda$  and service rate  $\mu$ . The distinction between them is the same as in M/M/1; however, their proportion, which is also the occupational rate, is the lowest from all the models. The expected number of users in queuing system  $L$  is equaled to 5, the value of expected number of users in queue –  $L_q$  is equaled to 2, what in the case of investigation insinuate that the system will face greater problems with the queues that form. Generally, the results are pretty close that might indicates congenital distribution of time General and Erlang. The time each aircraft on average spent in queue is around one minute and in system 12 minutes. Considering the arrival distribution of time, it is similar as in model M/M/1 – the distribution time in both cases is in Poisson process and it has been normal that they will differs only slightly. The occupation rate ( $\rho = \lambda r / \mu$ ) which is required to be less than one [Adan, Resing, 2002] is also up to the mark. The more detailed interpretation of the results characterizing this model is presented with particular hypothesizes.

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### Hypothesizes testing

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This section will analyze the hypothesizes defined at the beginning of the article. Hypothesizes provide more comprehensive treatment to increase the optimality of the results [Lehmann, Erich L., Romano, Joseph P., 2005]. The results are presented in the form of the table. After that, each outcome is interpreted in harmony with the mathematical and statistical formulas. Though we can test all the hypothesis based on the result, in this article on hypothesis 1,2,3 and 5 is tested.

#### Hypothesis 1 – Peak period

The first hypothesis has opened the issue of choosing peak period, because according to the literature this is the time when it is the most probable that the airport will be congested. The congestion will automatically create a waiting line that disturbs the flight schedule plan, often for many hours.

The most logical way of defending such a sentence is to take one randomly chosen day of the airport operation and investigate it hour by hour by the known methods. It is important to mention that the British authorities and international aviation institutions allow the airport due to its location to operate during the nighttime; however, the operations between midnight and 6.00 am are limited to 25 movements. In the regular hour of the operation, the airport is allowed to serve 40 arrivals or departures on its single-runway. The results of the event is presented below in table 3:

Table 3. Peak period investigation

Period	No. of movements	Possible movements	Landings	Taking-off	% of usage
6.00 - 7.00	35	40	14	21	88%
7.00 - 8.00	38	40	16	22	95%
8.00 - 9.00	38	40	18	20	95%
9.00 - 10.00	36	40	16	20	90%
10.00 - 11.00	38	40	14	24	95%
11.00 - 12.00	38	40	15	23	95%
12.00 - 13.00	35	40	15	20	88%
13.00 - 14.00	38	40	14	24	95%
14.00 - 15.00	37	40	16	21	93%
15.00 - 16.00	38	40	20	18	95%
16.00 - 17.00	38	40	16	22	95%
17.00 - 18.00	33	40	12	21	83%
18.00 - 19.00	30	40	18	12	75%
19.00 - 20.00	36	40	18	18	90%
20.00 - 21.00	40	40	22	18	100%
21.00 - 22.00	24	40	13	11	60%
22.00 - 23.00	15	40	7	8	38%
23.00 - 24.00	16	40	8	8	40%
24.00 - 1.00	18	25	6	12	72%
1.00 - 2.00	12	25	5	7	48%
2.00 - 3.00	15	25	4	11	60%
3.00 - 4.00	14	25	5	9	56%
4.00 - 5.00	16	25	8	8	64%
5.00 - 6.00	24	25	9	15	96%

The above table unambiguously shows that the distribution of the movements at the London Gatwick airport during its everyday operation. It indicates the total number of movements each hour and the contribution of arrivals and departures to that number. Additionally, there is a column showing the total allowed number of movement per hour and the percentage of its utilization by scheduled movements.

From the analysis it is quite clear to observe that the periods indicated in the hypothesis are one of the busiest, however the higher number of movements occurs between 7 pm and 8 pm. The first hypothesis was not completely correct so its status become disapproved.

Hypothesis 2 – probability of missing the slot

The second hypothesis highlights the problem of missing the assigned slots. The concept of this hypothesis has an operating approach. The exact formulation of the hypothesis is that the London Gatwick Airport capacity's situation, despite of the fact it is the busiest single-runway airport, is stable and the probability that the aircraft misses its slot is less than 10%.

For defending this hypothesis, the research outcome is presented below in the table 4.

*Table 4. Percentage of missed slots by the models*

Attempt	M/M/1 model	M/Er/1 model	Attempt	M/M/1 model	M/Er/1 model
1	7.5%	7.3%	26	13.5%	0.0%
2	9.7%	5.2%	27	14.2%	6.1%
3	1.4%	8.4%	28	2.6%	1.8%
4	11.0%	4.8%	29	9.2%	7.0%
5	6.1%	12.0%	30	12.3%	0.4%
6	14.2%	12.4%	31	2.1%	2.1%
7	7.4%	2.8%	32	3.3%	0.3%
8	3.3%	6.6%	33	2.2%	4.6%
9	9.1%	0.9%	34	13.8%	2.5%
10	13.6%	12.0%	35	0.5%	1.0%
11	11.3%	1.8%	36	14.8%	4.3%
12	7.8%	3.0%	37	8.0%	3.4%
13	7.0%	6.2%	38	9.9%	2.2%
14	3.0%	4.3%	39	3.6%	0.9%
15	14.8%	0.9%	40	13.1%	1.2%
16	9.1%	11.8%	41	4.6%	0.6%
17	0.9%	3.5%	42	9.4%	7.4%
18	14.8%	12.7%	43	8.9%	10.3%

19	0.0%	1.4%	44	11.2%	1.1%
20	8.0%	9.1%	45	0.6%	2.4%
21	11.7%	12.1%	46	2.4%	7.3%
22	7.4%	3.9%	47	1.5%	2.9%
23	2.1%	0.5%	48	5.1%	0.8%
24	14.8%	11.3%	49	10.2%	10.0%
25	8.3%	3.2%	50	4.2%	9.0%
Mean	8.2%	6.3%		7.2%	3.6%

The table compares the probabilities of missing the slot, as different models were considered; the different probability distributions of service time are taken into account. The results differ while taking into consideration each model. 10% is the threshold that should not be exceeded in any model; this will indicate the overall stable situation in the investigating period.

Based on the conducted research and obtained findings, it can be claimed that the raised hypothesis is correct. The London Gatwick, despite of the fact it is the busiest single runway airport, represents the stability of the operations in examined period. The number of aircrafts missing their slots is in each investigated model is smaller than 10%

Hypothesis 3 – likelihood of queue occurrence

The hypothesis number three actually has been answered by the data collected for the purpose of previous one. Despite of the fact the situation on the researching airport is stable, the small waiting lines might occur. The second part of the hypothesis gave specific numbers describing the queue and for those objectives the models has been tested. The second part of raised hypothesis standpoints “the utilization of the runway is optimal and number of aircrafts waiting in the queue at a particular moment is smaller than 3 and the probability of that events’ absence is smaller than 20% during the whole period”. To defend this statement the formulas from the Queuing Theory are quite adequate. Those formulas were Q, which calculates the number of aircrafts on average waiting in queue, and P (n) which as a result will give the probability that more than 3 aircrafts waits for the runway. The results of this analysis is provided in table 5. The calculations from the peak period have been reproduced, as they are based on the results that consists randomized number in itself. The standard deviation has had a small positive value that is the reason that the trail of 20 attempts is enough to perform this research. The table 5 is shown below:

Table 5. Results on Q and P (n=3) by the models

Attempt	Formula	M/M/1 model	M/Er/1 model	Attempt	Formula	M/M/1 model	M/Er/1 model
1	Q	0	0	11	Q	1	2
	P(n=3)	0.13	0.08		P(n=3)	0.06	0.18
2	Q	2	1	12	Q	1	1
	P(n=3)	0.19	0.20		P(n=3)	0.24	0.19

3	Q	3	1	13	Q	3	2
	P(n=3)	0.22	0.22		P(n=3)	0.13	0.10
4	Q	3	3	14	Q	1	1
	P(n=3)	0.14	0.19		P(n=3)	0.17	0.18
5	Q	1	2	15	Q	3	0
	P(n=3)	0.14	0.02		P(n=3)	0.03	0.23
6	Q	3	1	16	Q	0	1
	P(n=3)	0.16	0.15		P(n=3)	0.11	0.11
7	Q	3	0	17	Q	2	3
	P(n=3)	0.01	0.01		P(n=3)	0.15	0.20
8	Q	0	3	18	Q	0	3
	P(n=3)	0.11	0.00		P(n=3)	0.08	0.22
9	Q	1	0	19	Q	1	0
	P(n=3)	0.22	0.08		P(n=3)	0.14	0.19
10	Q	1	2	20	Q	0	1
	P(n=3)	0.03	0.01		P(n=3)	0.13	0.20
Average	Q	2	1	Average	Q	1	1
	P(n=3)	0.13	0.10		P(n=3)	0.12	0.18

The table calculates the quantity of the aircraft waiting on average in the line and it is rounded to the nearest integer. The second value in the each attempt calculates the probability that the number of aircraft waits in forming queue is greater than 3. Both formulas provide an appropriate view to check if the raised hypothesis has been proved or disproved.

The summary of above outcomes gives a clear answer for the raised hypothesis. The number of the aircraft waiting on average in forming waiting line in each model is lower than 3. The average from the M/M/1 model is equal to 2 aircrafts, whereas in the model with Erlang distribution model is even lower and just one airplane on average has to wait for its access to the runway. Only in 11 attempts the number of investigating flights has equaled 3 and there has been no observation of the number greater than 3

#### Hypothesis 5 – slots missed in total

The fifth hypothesis highlights the similar topic as a second one – the missed slots. The defending however takes a different approach – it sums-up the total number of aircrafts that missed the slot by the column of leaving time. It counts the time of leaving by summing the service time with the time of leaving of

the predecessor. After that the next column compares that time with the slot range and prints the information “ok” for hit or “not ok” for missed one. The experiment counts and sum up the cells with the string “not ok”. The trail of 40 attempts is sufficient to conduct the test. After the test, it will be possible to compare the results with those attained from the second hypothesis. The formulated hypothesis is as, “Sum-up of the all aircrafts that missed the slot in investigating period will not be greater than 2 missed slots per hour” The table 6 presents the findings of the research.

Table 6. Number of missed slots by model

Attempt	M/M/1 model	M/Er/1 model	Attempt	M/M/1 model	M/Er/1 model
1	2	0	21	1	2
2	1	1	22	0	2
3	3	3	23	2	3
4	1	0	24	3	2
5	2	3	25	0	0
6	0	1	26	2	1
7	2	3	27	3	3
8	2	3	28	1	3
9	3	0	29	2	2
10	2	2	30	1	3
11	2	3	31	2	5
12	1	1	32	0	4
13	1	4	33	1	1
14	3	4	34	1	2
15	0	3	35	2	4
16	2	3	36	0	3
17	1	2	37	2	3
18	1	3	38	1	3
19	3	1	39	2	4
20	3	1	40	0	2
Mean/h	2	2		1	3
Percent	4.2%	3.8%		3.2%	6.3%
Hypothesis 2	4.9 %	7.7 %		4.9 %	7.7 %

The investigating period considered here is two hours. From the mathematical point of view, the results from the simulation and chosen simulation models show unambiguously that the London Gatwick Airport deals with the runway operations in satisfactory way. The aircraft appearing on the runway in the great majority catch the slots and even if they have to wait, the waiting time is not very long. The outcomes are rounded to the nearest integer. The results in M/M/1 presents the range between 0 and 3 that gives the mean 2 missed slots per one hour of operation during peak period. In reality, such a score is considered close to perfect and highlights the good runway organization at London Gatwick Airport. The model with the general distribution of service time has a wider range, affecting the mean – the number of aircrafts missing the slot in 40 attempts during the period of two hours is equal to 3. The last model – M/Er/1 range is from 0 and 5 missed slots that gives a mean 3 in 40 attempts. The table additionally includes the percentage of the number of missed slots to the total number of slots and compares the results from the second hypothesis. The results are very close and the trend attained from the Hypothesis 2 results' is maintained. Backing the hypothesis, in some attempts, the number of missed slots has been greater than 2 but on average in two models the statement is proved.

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## Conclusion

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In relation to the conducted research, the following conclusions are formulated:

- The Queuing theory has its application in runway investigation and simulation concept.
- The busiest period at the London Gatwick, while examining the normal day of operation, occurs between 7 pm and 9 pm. It is not the expected peak period that is formed in the hypothesis, which was based on business traffic and nominated around 8 am and 4 pm.
- The slot situation on the London Gatwick airport is stable even in the peak period. The authorities do not exceeds the regulated number of hourly slots and this number is sufficient to face the demand.
- Despite of the fact that situation is stable small waiting lines have occurred in the simulation. However, the number of aircrafts staying in the queue at a particular moment has been lower than 3 and the probability that it will be greater was less than 15%.
- The results from the simulations in short or long run do not possess large differences. All of examining parameters have acted similar way with no heeding to number of observations.
- The number of aircrafts that miss the slot every hour, accordingly to the simulation, is relatively low and do not affect the stability of operations on the runway.
- The airlines using London Gatwick Airport for their operations, do not suffer a significant financial penalties from delays in the operation.
- Comparing all models, if the arrival rate  $\lambda$  and the service rate  $\mu$  are constant the Markovian distribution of time gives the largest values for examining total time and number of customers in queue and in the whole system.
- The distribution of movements on investigating airport is balanced by the type of movement, which is characterized by the majority of heavy aircrafts and little number of light planes, while considering the weight classes.
- Visual Basic Applications for this particular simulation has been found as simple, user friendly and sufficient programming language for building the user interface for the purpose of presenting the results of the research.

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