ANALYSIS OF THE PROPERTIES OF ORDINARY LEVY MOTION BASED ON THE ESTIMATION OF STABILITY INDEX

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***Abstract**: The work proposes a method for estimating the stability index of alpha-stable distributions by using moments of fractional order. Provided numerical modeling has fully justified all of the results. Comparative analysis of the efficiency among the proposed method of estimating the stability index and widely used methods was performed. Proposal method is much simpler, far faster and substantially less memory required.*

Estimation of generalized Hurst exponent from time series of the ordinary Lévy process was performed. Multifractal fluctuation analysis method and evaluation based on stability index estimation were compared. The results of numerical modeling showed that proposed method for estimating the fractal properties of the ordinary Lévy process, based on stability index estimation via fractional order moments is a much more accurate.

***Keywords**: alpha-stable variables, stability index estimation, fractional order moments, multifractal stochastic processes, Hurst exponent, generalized Hurst exponent, ordinary Levy motion.*

***ACM Classification Keywords**: G.3 Probability and statistics - Time series analysis, Stochastic processes, G.1 Numerical analysis, G.1.2 Approximation - Wavelets and fractals*

Introduction

Parameters estimation of the random variables is one of the major problems of mathematical statistics. Among a set of different distribution laws the special place is taken by alpha-stable distributions because just and only these laws may limit the distribution of sums of independent identically distributed random variables [Gnedenko, 1954]. Such distributions are widely used in models of stochastic processes describing a wide range of processes and phenomena (e.g., financial and stock market indices, river sinks, medical applications). High peaks, heavy tails and self-similarity are characteristics of such time series [Gnedenko, 1954, Zolotarev, 1986].

In general, case alpha-stable random variable is characterized by four parameters [Zolotarev, 1986], specifies the index of stability $0 < \alpha \leq 2$, offset, scale and symmetry measures. Estimation of these parameters is a difficult task. This is partly caused by those facts that with few exceptions pdf's and cdf's of stable distributions are not expressed in terms of elementary functions.

Despite the variety of methods and algorithms have been developed for solving this problem, none of them provides a statistical efficiency of the resulting estimates (in the sense of reaching Cramer-Rao bound). Furthermore, many of the techniques have a high computational complexity or other drawbacks. Thus, developing new methods for estimating the parameters of alpha-stable distributions remains an actual problem.

Historically, the first groups of methods for estimating the parameters of stable distributions are ones based on order statistics, i.e. quantiles [Fama, 1971; McCulloch, 1986; Garcia, 2011]. These methods are characterized by low computational complexity however their performance (estimation accuracy) is also low, especially applying to indices of stability and symmetry measure estimation. Furthermore, such methods are very sensitive to sample truncation. Nevertheless, due to its simplicity, these methods are widely used both independently and as a parts of other, more complex methods for obtaining an initial approximations of estimates [Borak, 2010].

Another common class of methods for stability index estimating is based on the tails behavior studying [Hill, 1975; Dufour, 2010]. One of the fundamental property of stable distributions is asymptotically power law of the cdf: $P(X > x) \sim x^{-\alpha}$ as $X \to \infty$, $\alpha \neq 2$. The main disadvantage of this method is the bias of the resulting estimates. Furthermore, the effectiveness of such techniques depends essentially on the volume of the sample.

The maximum likelihood method just gives the most accurate estimates of the parameters of stable distributions [Nolan, 2001]. However, its computational complexity is very high, that is caused by both the properties of the method and the computational complexity of calculating the pdf's of stable distributions. That is why this class of methods is not common.

Just methods for estimating the parameters of stable distributions, based on the transition to the frequency domain, are the most common now [Koutrouvelis, 1980; Chenyao, 1999]. Not parameters p of pdf's $f(x;p)$ themselves are estimated by this methods but parameters of their characteristic functions $\varphi(t;p) = M(e^{itx}) = \int_{-\infty}^{\infty} e^{itx} f(x;p)dx$ are. This is because the stable distribution characteristic functions, in contrast to their pdf's, have a relatively simple mathematical form. These methods provide a sufficiently high accuracy of estimation, but are also quite complex computationally.

Stochastic process $X(t)$ is called self-similar, if the process $a^{-H}X(at)$ has the same finite-dimensional distributions, as the original process $X(t)$ has. Parameter H , called Hurst exponent (or Hurst index), is a measure of self-similarity of a stochastic process. There exist a lot of methods for Hurst index estimation based on a time series data, but most of them are designed for processes with finite second-order moments only, so they have substantial errors in estimation the Hurst index [Kirichenko, 2011]. Processes with independent increments which are identically distributed alpha-stable variables have self-similar properties which are completely determined by the stability index α [Cont, 2004]. Therefore, in this case, one should focus on the correct evaluation of stability index using the time series data.

Problem domain

Moment's method is classical method of point estimation. It is characterized by low computational complexity. However, its application is actually limited by class of distributions subordinated to normal, i.e. having stability index $\alpha = 2$. It's caused by the fact that random variable has no moments of order equal to or higher than α when $\alpha < 2$. In [Zolotarev, 1986] logarithmic moment's method for stable distributions parameters estimation was proposed. This method is simple to implement, but efficiency of estimates obtained by is lower than one obtained by methods based on using the transition to frequency domain.

However, the concept of the moment of a random variable can be generalized to noninteger $s \in \mathbb{R}$. It is known [Uchaikin, 2008] that for any α there exists an infinite set of s ensuring an existence of s-order moment. Thus, stable distributions parameters estimation via fractional moments is a new approach for solving the problem.

This paper is aimed to developing a method for estimating the stability index of α-stable distributions via fractional order moments and applying this method for studying the fractal properties of stochastic processes.

Estimation the stability index of alpha-stable laws via fractional moments method

The absolute moment order s of a random variable with the pdf $f(x)$, considered as function on s , is called as bilateral Mellin transform [Uchaikin, 2008]:

$$\left(\mathcal{M}f(x)\right)(s)=\int_{-\infty}^{\infty}|x|^{s}f(x)dx. \tag{1}$$

It's widely known [Zolotarev, 1986], that α-stable variable has a moments of order $-1 < s < \alpha$. For strictly alpha-stable random variables Mellin transform has a closed form:

$$(\mathcal{M}g(x;\alpha,\rho,\lambda))(s)=\lambda^{s}\frac{\cos(\frac{\pi s}{2}(2\rho-1))\cdot\Gamma(1-s/\alpha)}{\cos(\frac{\pi s}{2})\cdot\Gamma(1-s)}, \tag{2}$$

where $\Gamma(x)$ is the gamma function.

In the case when random variable is symmetric ($\rho = \frac{1}{2}$) and has a unity scale factor ($\lambda = 1$) expression (2) takes the form

$$(\mathcal{M}g(x;\alpha))(s)=\frac{\Gamma(1-s/\alpha)}{\cos(\frac{\pi s}{2})\cdot\Gamma(1-s)}=\frac{\Gamma(1-s/\alpha)}{\chi(s)}, \tag{3}$$

where $\chi(s)=\cos(\frac{\pi s}{2})\cdot\Gamma(1-s)\geq 1$.

Replacing in this relation the theoretical value of the moment $(\mathcal{M}g(x;\alpha))(s)$ with its sample value $Z_n(s)$, we can get an estimate of the stability index α:

$$\hat{\alpha}(n,s)=\frac{s}{1-\Gamma^{-1}(\chi(s)\cdot Z_n(s))}=\frac{s}{1-\Gamma^{-1}(1+Y_n(s))}, \tag{4}$$

where

$$Z_n(s)=\frac{1}{n}\sum_{k=1}^{n}|X_k|^{s}, \quad Y_n(s)=\chi(s)\cdot Z_n(s)-1. \tag{5}$$

Estimator (4), despite the simplicity of mathematical notation, has the obvious disadvantage of using function $\Gamma^{-1}(u)$, which is the inverse of the gamma function $u=\Gamma(x)$. This function not only applies to elementary, but not implemented in any of the known engineering and mathematical packages. Thus, one should numerically solve the nonlinear equation $\Gamma^{-1}(u)=\underset{x\in(0;1)}{sol}\left\{\Gamma(x)=u\right\}$, or (which is slightly simpler) optimization problem $\Gamma^{-1}(u)=\underset{x\in(0;1)}{\mathrm{argmin}}\left\{\left(\Gamma(x)-u\right)^{2}\right\}$ for computing estimate (4) directly. This fact substantially reduces the usefulness of the proposed method and, on the other hand, makes it impossible to analyze the properties of esimate (4) by analytical methods.

As known from mathematical analysis, the function $\Gamma^{-1}(u)$, hence the function

$$x=f(y)=\frac{1}{1-\Gamma^{-1}(1+y)}, \tag{6}$$

are continuous and monotonically decreasing on the range $y\in(0;\infty)$, $x\in(1;\infty)$. Apparently, by approximating (6) with convergent series (on $1/y$) we can get the desired estimate of stability index in a much simpler form than (4), while ensuring any preassigned accuracy. In [Shergin, 2014] it was shown that the linear approximation

$$x^{(1)} \approx a + b/y\ ,\ (a = 1.19236\ ,\ b = 0.64072) \tag{7}$$

provides a relative error not exceeding 3.5%, which is sufficient for practical use. The second order approximation has the form

$$x^{(2)} \approx a + b/y + c/y^2\ ,\ (a = 1.11877,\ b = 0.70107,\ c = -0.012374) \tag{8}$$

Given the expressions (7)-(8) we obtain the approximate estimates of stability index:

$$\hat{\alpha}(n,s) \approx s\left(a + \frac{b}{Y_n(s)}\right) = s\left(a + \frac{b}{\chi(s)\cdot Z_n(s) - 1}\right) \tag{9}$$

$$\hat{\alpha}(n,s) \approx s\left(a + \frac{b}{Y_n(s)} + \frac{c}{Y_n^{\,2}(s)}\right) \tag{10}$$

In [Shergin, 2013] it was shown that on the range $s \in (-1;0) \cup (0;\alpha)$ estimator (4) is consistent and asymptotically unbiased (case $s = 0$ should be excluded as degenerate) and bias value of estimators (9)-(10) caused by an error of series expansion of function (6).

To obtain an asymptotic expression for variance of estimates $D\left[\hat{\alpha}(n,s)\right]$ it was used the fact that $Y_n(s)$ (5) is a cumulative average of independent identically distributed random variables with support $\mathrm{supp}(Y) = [0,\infty)$, hence asymptotic distribution of (5) (as $n \to \infty$) can be described by some infinitely divisible law. As such law the gamma distribution has been chosen. The resulting expression for variance of estimates has the form

$$D[\hat{\alpha}(n,s)] \approx \frac{b^2 D_0(\alpha,s)}{n}, \tag{11}$$

where

$$D_0(\alpha,s) = \frac{s^2\left(\dfrac{\chi^2(s)}{\chi(2s)}\Gamma(1-2s/\alpha) - \Gamma^2(1-s/\alpha)\right)}{\left(\Gamma(1-s/\alpha)-1\right)^4}. \tag{12}$$

From the expression (12) it follows that the variance of the estimates (4), (9)-(10) is finite on the range $s \in (-\frac{1}{2};\frac{\alpha}{2})$. Plot of the function (12) (from [Shergin, 2013]) is shown on Figure 1.

As one can see on Figure 1, for each value of stability index α there exist such values of fractional order s, equal to

$$s_{\min}(\alpha) = \underset{-1/2<s<\alpha/2}{\arg\min}\left(D_0(\alpha,s)\right), \tag{13}$$

which provide a local minimum value of the asymptotic variance of estimates. Thus, despite the fact that the estimates (4) are consistent and asymptotically unbiased for any values of s of range $s \in (-1;0) \cup (0;\alpha)$, estimation accuracy will be the higher as s the closer to $s_{\min}(\alpha)$.

Graphical representation of $s_{\min}(\alpha)$ is shown on Figure 2a, graphs of function (12) $D_{0,\min}(\alpha) = D_0(\alpha, s_{\min}(\alpha))$, corresponding to $s = s_{\min}(\alpha)$ are shown on Figure 2b.

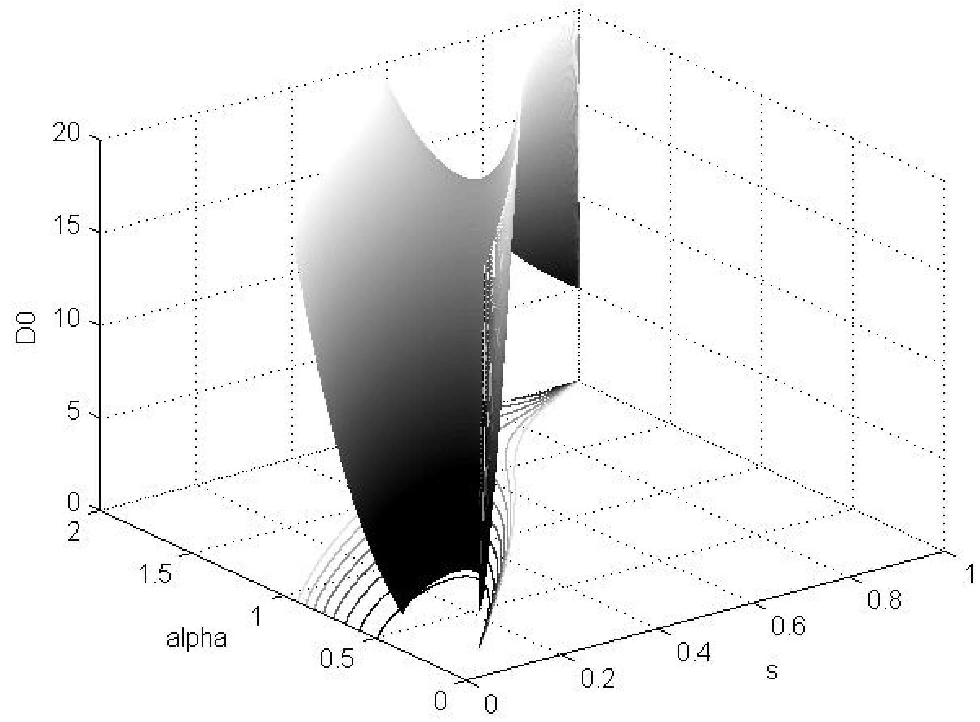
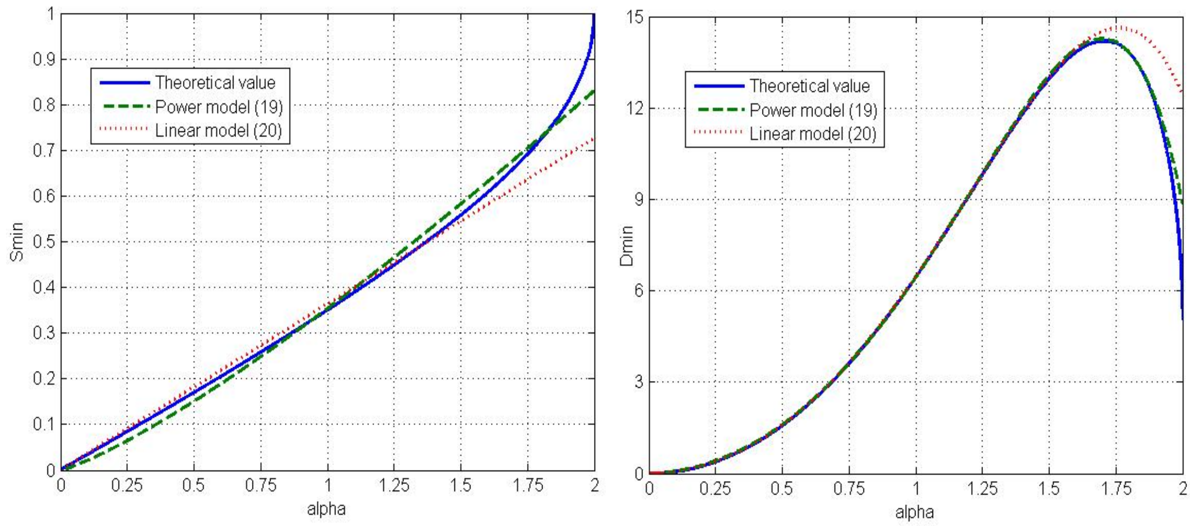


Figure 1. Asymptotic variance of estimates $D_0(\alpha, s)$ by model (12)



a b

Figure 2. Dependencies $s_{\min}(\alpha)$ (a), $D_{0,\min}(\alpha)$ (b), obtained by numerical minimization (13) on s, and by using models (14)-(15)

According to these graphs, function $D_{0,\min}(\alpha)$ peaks at $\alpha^* \approx 1.707$, which corresponds to the value $s_{\min} \approx 0.665$. Thus, the neighborhood of α^* are the least favorable values of stability index to their evaluation by the proposed method.

Dependence (13) can be approximated by

$$s_{\min}^{pow} = 0.35281 \cdot \alpha^{1.2332}, \quad (14)$$

$$s_{\min}^{linear} = 0.3630 \cdot \alpha. \quad (15)$$

From (15) it follows that neighborhood of $s/\alpha \approx 0.3630$ (which corresponds to y_0=0.41007, $x_0 = 2.75482$) is the most reasonable point for Taylor series expansion of (6). That is cause numerical values of coefficients of models (7) – (8) obtained in [Shergin, 2014].

Stability index estimation algorithm

The analysis found that for a given value of the fractional moment order s the stability index α estimators has the form (9) – (10) (with notations (3), (5) and coefficients (7) – (8)). Variance of this estimates will be the least while $s = s_{\min}^{pow}(\alpha)$ (14). Thus, a simple iterative procedure is proposed

$$s^{(m+1)} = s_{\min}^{pow}\left(\hat{\alpha}(n, s^{(m)})\right), \quad (16)$$

where functions $s_{\min}^{pow}(\hat{\alpha})$ and $\hat{\alpha}(n, s^{(m)})$ are calculated according to (14) and (9) or (10) respectively.

An exit condition for the loop (16) has the form $\left|s^{(m+1)} - s^{(m)}\right| \le tol$. Values, equal to 0.25 and 10^{-4} were used as $s^{(0)}$ and tol respectively.

It was performed comparative analysis of the efficiency among the proposed method of estimating the stability index and well known methods based on quantile parameter estimates and regression parameter estimates STABCULL and STABREG [MFE Toolbox for MATLAB]. Plot of the error variance of the stability index estimates on the sample length is shown on Figure 3.

As one can see, error variance provided by all of these methods is about the same. However proposal method is much simpler, about 12 times faster than STABREG and substantially less memory required.

Basic definitions and characteristics of fractal stochastic processes

Stochastic processes that exhibit fractal properties can be divided into two groups: self-similar (monofractal) and multifractal. Monofractal processes are homogeneous in the sense that their scaling characteristics remain constant at any range scale. Monofractal processes have the single scaling exponent. Multifractal processes can be expanded to ranges with different local scaling properties. Multifractal processes have the spectrum of scaling exponents. Consider the basic concepts of self-similar and multifractal random processes [Feder, 1991; Calvet, 1997; Reidi, 2002; Kantelhardt, 2008].

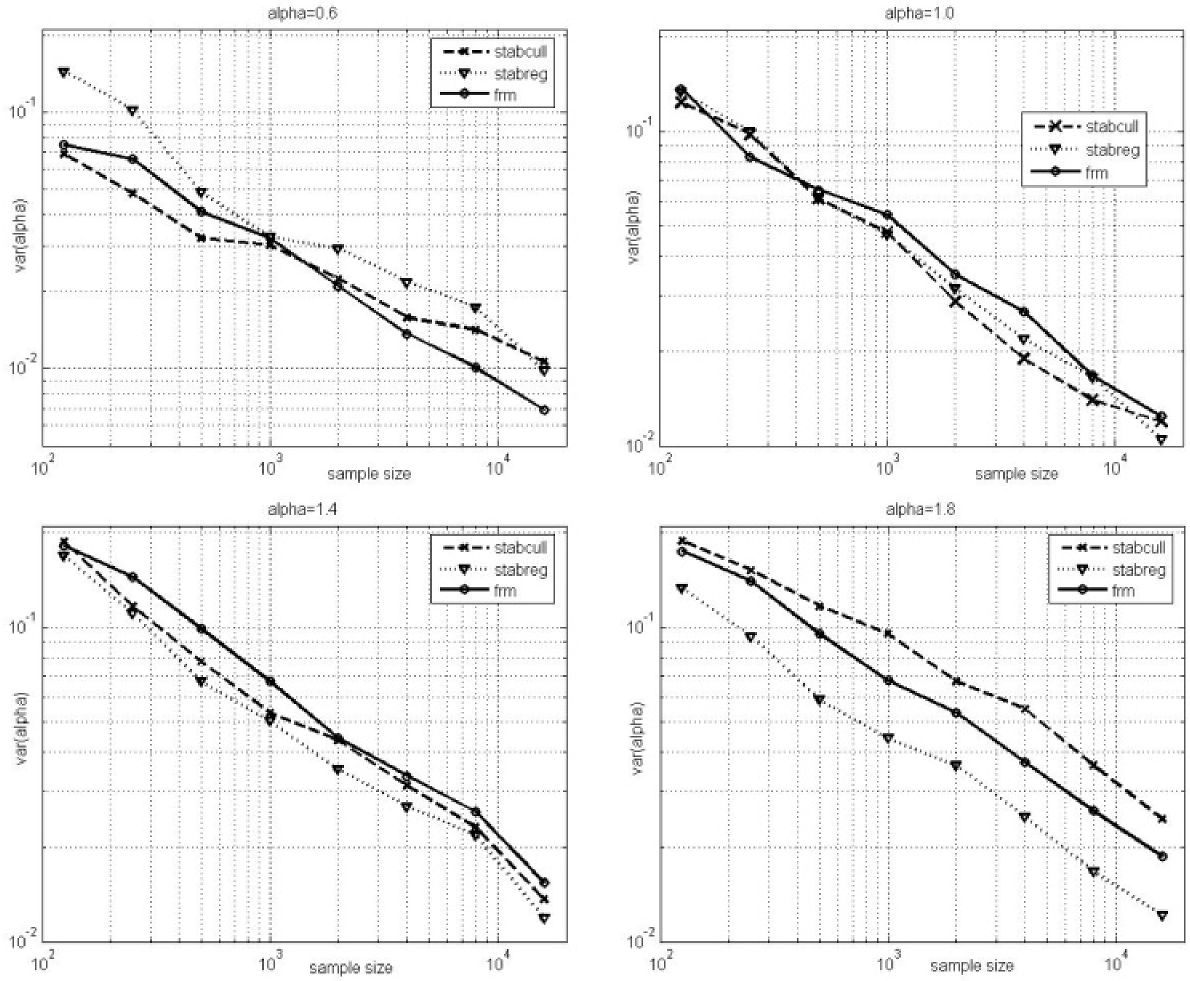


Figure 3. Error variance of the stability index estimates on the sample length via proposal method (Frm) and widely used Stabreg and Stabcull

Stochastic process $X(t)$, $t \geq 0$ with continuous real-time variable is said to be self-similar of index H, $0 < H < 1$, if for any value $a > 0$ processes $X(at)$ and $a^{-H}X(at)$ have same finite-dimensional distributions:

$$\text{Low}\{X(t)\} = \text{Low}\left\{a^{-H}X(at)\right\}. \tag{17}$$

The notation $\text{Low}\{\cdot\}$ means finite distribution laws of the random process. Index H is called Hurst exponent. H is a measure of self-similarity of a stochastic process. Ordinary moments of self-similar process can be can be expressed by

$$\mathcal{M}\left[|X(t)|^q\right] = \mathcal{M}\left[\left|t^H X(1)\right|^q\right] = t^{qH}\mathcal{M}\left[|X(1)|^q\right] = C(q)\cdot t^{qH}, \tag{18}$$

where value $C(q) = \mathcal{M}\left[|X(1)|^q\right]$.

In contrast to the self-similar processes (17) multifractal processes have more varied scaling behavior:

$$\mathrm{Law}\{X(at)\} = \mathrm{Law}\{\mathcal{M}(a)\cdot X(t)\},\ a>0, \tag{19}$$

where $\mathcal{M}(a)$ is random function that independent of $X(t)$.

In case of self-similar process $\mathcal{M}(a) = a^H$. Hurst exponent of multifractal processes is a random function of the argument a: $H(a) = \log_a \mathcal{M}(a)$. Relation (19) can be reformulated as follows:

$$\mathrm{Law}\{X(at)\} = \mathrm{Law}\{a^{H(a)}\cdot X(t)\}. \tag{20}$$

Defining characteristic of multifractal processes: process $X(t)$ is multifractal, if the following relation holds:

$$\mathcal{M}\left[|X(t)|^q\right] = c(q)\cdot t^{qh(q)}, \tag{21}$$

where $c(q)$ is some deterministic function, $h(q)$ is generalized Hurst exponent, which is generally non-linear function. Value $h(q)$ at $q=2$ is the same degree of self-similarity H. For monofractal processes generalized Hurst exponent does not depend on the parameter q: $h(q)=H$.

There are many methods for estimating the parameters of self-similar and multifractal processes from time series. [Clegg, 2005; Kantelhardt, 2008].When estimating the Hurst exponent in practice most commonly used methods are R/S -analysis, variance-time analysis and detrended fluctuation analysis (DFA). When estimating multifractal characteristics one of the most popular methods is multifractal detrended fluctuation analysis (MFDFA) [Kantelhardt, 2002].

According to the MFDFA method, for the initial time series $x(t)$ the cumulative time series $y(t) = \sum_{i=1}^{t} x(i)$ is constructed which is then divided into N segments of length τ, and for each segment $y(t)$ the following fluctuation function is calculated:

$$F^2(\tau) = \frac{1}{\tau}\sum_{t=1}^{\tau}(y(t) - Y_m(t))^2, \tag{22}$$

where $Y_m(t)$ is a local m-polynomial trend within the given segment. The averaged on the whole of the time series $y(t)$ function $F(\tau)$ depends on the length of the segment: $F(\tau) \propto \tau^H$.

In the study of multifractal properties the dependence of the fluctuation function $F_q(s)$ of a parameter q is considered: $F_q(s) = \left\{\frac{1}{N}\sum_{i=1}^{N}[F^2(s)]^{\frac{q}{2}}\right\}^{\frac{1}{q}}$. If the investigated series is multifractal and has a long-term dependence, the fluctuation function is represented by a power law

$$F_q(s) \propto s^{h(q)}, \tag{23}$$

where $h(q)$ is generalized Hurst exponent. For monofractal time series the fluctuation function $F_q(s)$ is the same for all segments, and the generalized Hurst exponent does not depend on the parameter q: $h(q)=H$. For multifractal series $h(q)$ s a nonlinear function.

Basic definitions and characteristics of ordinary Levy motion

Consider the basic concepts of self-similar and multifractal random processes [Cont, 2004]. A stochastic process $X(t),\ t \geq 0$ with real values is called Levy process, if it possesses the following properties:

- Process is right-continuity and left limits;
- Process starts at zero ($X_0 = 0$);
- At every time interval $t_0,...,t_n$ increments $X(t_0), X(t_1) - X(t_0),..., X(t_n) - X(t_{n-1})$ are independent random variables;
- Increments are stationary;
- Stochastic continuity is performed : $\forall \varepsilon > 0 \ \lim\limits_{h \to 0} P\left(\left|X(t+h) - X(t)\right| \geq \varepsilon\right) = 0.$

A stochastic process $X(t),\ t \geq 0$ with real values is called α – stable Levy process (ordinary Levy motion) , if it possesses the following properties:

- $X(t)$ is Levy process;
- For every $a > 0$, $t \geq 0$ the following relationship holds:

$$\text{Law}\{X(at)\} = \text{Law}\{a^{1/\alpha} X(t)\}. \tag{24}$$

Comparing the expressions (17) and (24), it is clear, that α – stable Levy processes have the property of self-similarity. Shown [Nakao, 2000; Oswiecimka, 2006] that such processes are multifractal. In this case the function of generalized Hurst exponent $h(q)$ takes the form:

$$h(q) = \begin{cases} 1/\alpha & q \leq \alpha \ , \\ 1/q & q > \alpha \ ; \end{cases} \tag{25}$$

where α is stability index.

Obviously, the function of generalized Hurst exponent of ordinary Levy motion is completely determined by the stability index α .

Investigation Results

In this work the results of a numerical experiment are represented where realizations of ordinary Levy motion have been simulated. The length of realizations was accepted equal to 250, 500, 1000 and 2000. For every received realization, generalized Hurst exponent have been obtained using two methods: directly by MFDFA and on based of the estimating stability index by formulas (8). The obtained estimation values of generalized Hurst exponent then were averaged over a set of realizations. Value of the parameter q changed in the range $-5 \leq q \leq 5$.

Figure 4 (above) shows typical realization of ordinary Levy motion. Its increments (below) are independent stable random variables X_i with stability index $\alpha = 1.2$, ie $X_i \sim S_{1.2}(1,0,0)$. Figure 5 shows the estimates of $h(q)$, that obtained by the realizations of varying lengths by MFDFA method. The realizations of such a process are self-similar of Hurst exponent $H = \dfrac{1}{\alpha} = \dfrac{5}{6}$. Dashed line on the graph shows the theoretical values of the function $h(q)$.

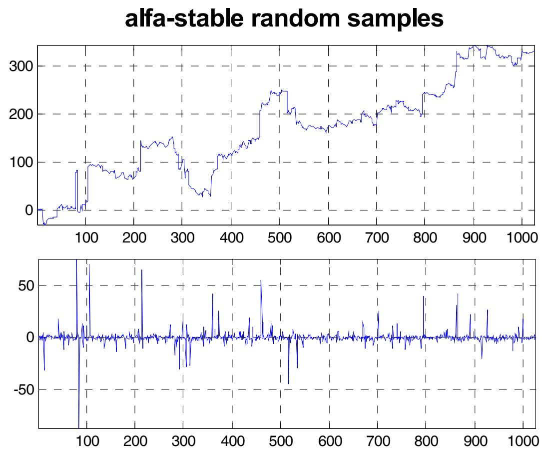


Figure 4. Levy motion realization (above), its increments (below)

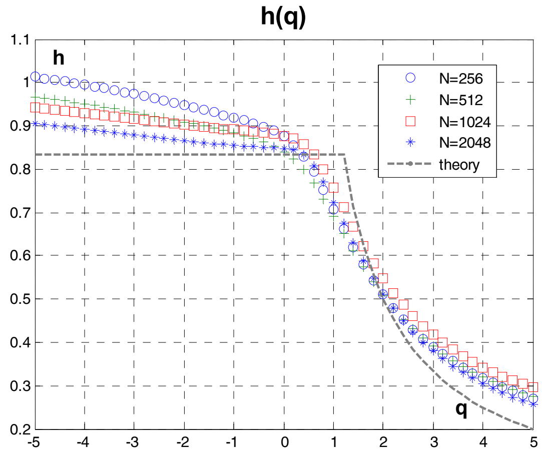


Figure 5. Function $h(q)$ of Levy motion realizations

Figure 6 shows the estimation results of generalized Hurst exponent $h(q)$ by MFDFA method (left) and on based of the estimating stability index using the method of fractional moments (right). The length of realizations in these cases was equal to 5000 values. Continuous line on the graphs shows the theoretical values of the function $h(q)$

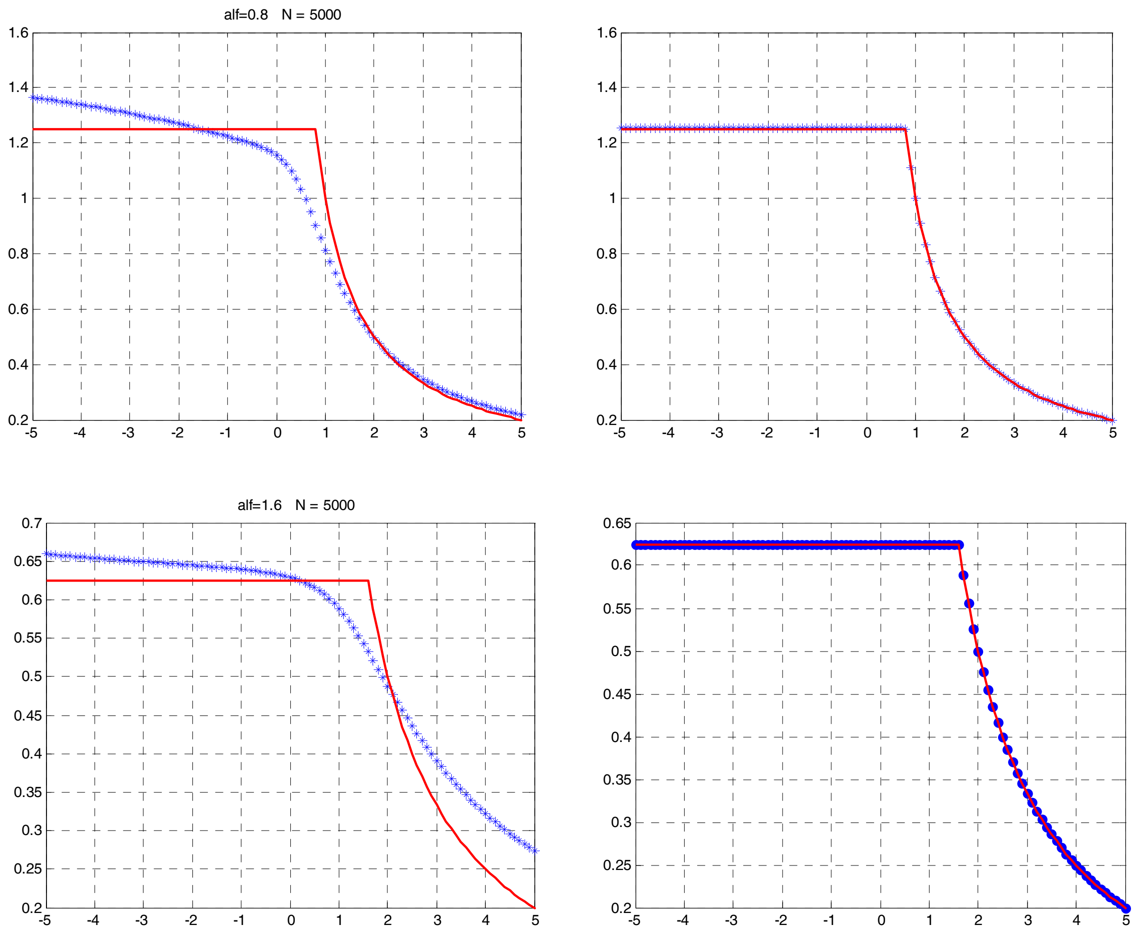


Figure 6. Estimates of $h(q)$ for Levy motion realizations of α =0.8 (above) and α =1.8 (below) by MFDFA (left) on based of the estimating α (right).

Obviously, the evaluation function of the generalized Hurst exponent on based of the estimating stability index provides much more accurate results. However, the use of such an approach is possible only under condition of acceptance of the hypothesis that the process under study is ordinary Levy motion.

Conclusion

The problem of estimating the stability index (alpha) of $S\alpha S$-distributions via fractional order moments was considered. The required estimate was obtained. The consistency and asymptotic unbiasedness of this estimate were proved, and their asymptotic variance was estimated. As it was found, for any admissible $0 < \alpha \le 2$ there exists a fractional order moment value $s_{\min}(\alpha)$, which minimize the asymptotic variance of estimates of α. Dependence $s_{\min}(\alpha)$ was obtained and approximated in a polynomial form.

The provided numerical modeling has fully justified all of the results. It was performed comparative analysis of the efficiency among the proposed method of estimating the stability index and wide known methods based on quantile parameter estimates and regression parameter estimates. Proposal method is much simpler, far faster and substantially less memory required.

Estimation of generalized Hurst exponent from time series of the ordinary Lévy process was performed. Multifractal fluctuation analysis method and evaluation based on stability index estimation were compared. The results of numerical modeling showed that proposed method for estimating the fractal properties of the ordinary Lévy process, based on stability index estimation via fractal order moments is a much more accurate.

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