MULTIFRACTAL PROPERTIES OF BIOELECTRIC SIGNALS UNDER VARIOUS PHYSIOLOGICAL STATES

Abed Saif Alghawli, Lyudmyla Kirichenko

Abstract: *In the work the results of multifractal analysis of different electrobiological signals are represented. RR-interval's sequences of electrocardiograms obtained from patients before drug's application and after one; electroencephalograms of subjects when they perform any physical action and when they just imagine this and electroencephalograms of laboratory animals for the different phases of wakefulness and sleep were studied. Research was carried out by method of multifractal detrended fluctuation analysis. In all cases, there are significant differences of multifractal characteristics of different physiological states.*

In the work such characteristics of multifractal stochastic processes as a generalized Hurst exponent, scaling exponent, function of multifractal spectrum are discussed. It is shown that it is suitable use the generalized Hurst exponent as the quantitative measure. The results of numerical analysis showed that the estimates of the generalized Hurst exponent have normal distribution that allows to jumping to quantitative interval values. The possible specific values of the generalized Hurst exponent, which should be used in the knowledge bases of decision support systems, were proposed.

Keywords*: multifractal analysis, electrobiological signals, multifractal characteristics, multifractal stochastic processes, Hurst exponent, generalized Hurst exponent.*

ACM Classification Keywords*: G.3 Probability and statistics - Time series analysis, Stochastic processes, G.1 Numerical analysis, G.1.2 Approximation - Wavelets and fractals.*

Introduction

It is now recognized that many information, biological, physical and technological processes have a complex fractal structure. Fractal analysis is used for modeling, analysis and control of complex systems in various fields of science and technology. Fractal analysis is applied to predict seismic activity and tsunami and to determine the age of geological rocks in geology; to study the mutations and changes at the genetic level in biology; to predict the crisis and risk using financial series in economy; to study the turbulence and thermodynamic processes in physics [Mandelbrot, 1983; Feder, 1988; Schroeder, 1991].

Fractal geometry has been used in biology for over a quarter century. Application of fractal method opens up new possibilities in the study of the functional organization of living systems. Stable operation of such a complex, hierarchically organized system is provided by mutual subordination of structures belonging to different spatial scales. Numerous experimental and clinical data provide a basis for concluding that the study of fractal topology of various biological systems it will allow to lay the foundations of fractal diagnostics.

Subjects that exhibit fractal properties can be divided into two groups: self-similar (monofractal) and multifractal. Monofractal subjects are homogeneous in the sense that they have single scaling exponent. Their scaling characteristics remain constant on any range of scales. Multifractal subjects can be expanded into segments with the different local scaling properties. They are characterized by the spectrum of scaling exponents [Mandelbrot, 1983; Feder, 1988; Reidi, 2002].

Signals that are generated by complex self-regulating biological systems have a wide range of properties such as heterogeneity, nonlinearity, nonstationarity, presence of fluctuations and others. It was shown that for many systems biological signals have long-term correlation and fractal (self-similar) properties [Bak, 1987; Shlesinger, 1987; Peng, 1994; Bassingthwaighte, 1994; Goldberger, 2002; May, 2002]. In particular, multifractal properties have been detected in many bioelectric signals.

Review of the literature and problem statement

Heartbeat interval sequences were among the earliest physiological time series that have been discovered properties of self-similarity [Kobayashi, 1992]. In [Kantelhardt, 2002, 2003; McSharry, 2005] fractal and correlation properties of the time intervals between successive heartbeats during light sleep, deep sleep, and rapid eye movement sleep were investigated. In [Al-ani, 2007] the research was presented, which allowed using fractal analysis to automatically classify sleep stage using only the electrocardiogram (ECG) records. In [Kiyono, 2004, 2005, 2008] statistics, correlation and fractal properties of the heart rate for healthy and sick people were considered. In [Hoshiyama, 2008] the fractal exponents of heart rate variability for people practicing yoga and beginners were compared.

To date, it is shown that signals associated with cardiac activity are characterized by not only the self-similarity but also multifractal properties that reflect heterogeneity and nonstationarity of physiological processes. In particular this was a subject of study in [Ivanov, 1999, 2001; Stanley, 1999; Nunes Amaral, 2001] where it was shown that the heart rate had multifractal properties, which were different for healthy people and ones suffering from various diseases. In [Ching, 2007; Kiyono, 2009] multifractal properties of heart rate variability were investigated. In [Abry, 2010] authors suggest the methodology for multifractal analysis of heart rate variability based on wavelet transform.

Numerous investigations devoted to the study of fractal properties of time series of the electroencephalogram (EEG). In [Hwa, 2004] the new method of detecting stroke based on the scaling properties of human EEG time series was offered. In [Shin, 2007] the fractal characteristics of sleep EEG of healthy subjects were studied. In [Figliola, 2007] the EEG signals of birds to characterize the different stage of bird brain maturation were investigated. In [Leea, 2007; Leistedt, 2007] the fractal properties of the sleep EEG in acutely depressed men were analyzed. In [Abasolo, 2008] EEG recordings of patients with Alzheimer's disease were examined. In [Tingting Gao, 2008] the difference in EEG between eyes-closed and eyes-open conditions by fractal analysis was shown. In [Manickam, 2009] the research technique of cross-modal plasticity of blind based on fractal analysis of the EEG was proposed. The work [Tao Zhang, 2011] showed the differences in the multifractal characteristics of EEG for neural activity of epileptic and healthy rats. In [Marton, 2013] multifractal properties of multichannel EEG recordings were investigated. In [Zorick, 2013] multifractal analysis of EEG at different stages of sleep and wakefulness was performed. In [Harikrishnan, 2013] it was shown that the set of parameters characterizing the multifractal spectrum of both EEG and heart rate can distinguish between healthy and pathological states.

This list does not claim to be exhaustive and only shows how physiological fractal signals are widely and variously represented. It is obvious that the fractal characteristics reflect the essential features of the state of the organism. Studies suggest that multifractal methods can be successfully used for the analysis of physiological signals to determinate of functional changes in the organism performance. In most cases, the multifractal characteristics are considered rather qualitatively than quantitatively. However, for the application of the results of fractal analysis as knowledge in medical expert systems we need to use their quantitative interval values.

The purpose of the present work is to develop recommendations for the practical application of the results of multifractal analysis, for possible use as the quantitative characteristics, particularly in the knowledge bases of decision support systems.

Basic definitions and characteristics of fractal stochastic processes

Multifractality is a concept that is able be equally well with some minor modifications applied to functions as well as measures. In the description of the basic concepts and properties of multifractal processes, there are several approaches based on the properties of fractal sets and the moment characteristics of the stochastic processes. To better understand the properties of multifractal characteristics such as scaling exponent and multifractal spectrum, consider the approach based on the study of fractal dimensions of the inhomogeneous sets [Mandelbrot, 1983; Feder, 1988; Shuster, 1988; Reidi, 2002].

Self-similarity of fractal objects is confined in saving object’s structure of zooming. Let consider main characteristics of multifractal set. Suppose that, in general, multifractal attractor occupies some bounded region in d-dimensional Euclidean space and defines set of $N \to \infty$ points. Let divide the entire region into box of side ε and volume ε^d . Let consider the partition function $Z(q,\varepsilon)$ characterized by an exponent q $(-\infty < q < +\infty)$:

$$Z(q,\varepsilon)=\sum_{i=1}^{N(\varepsilon)} p_i^q(\varepsilon)\,, \tag{1}$$

where $p_i(\varepsilon)=\lim_{N\to\infty}\frac{n_i(\varepsilon)}{N}$, $n_i(\varepsilon)$ is number of points into the box with number i, $N(\varepsilon)$ is total number of occupied cells that depends from the size of the box ε. Probabilities p_i characterize relative population of the box.

In general multifractal set is characterized of nonlinear function $\tau(q)$, that determines behavior of partition function $Z(q,\varepsilon)$ with $\varepsilon\to 0$:

$$Z(q,\varepsilon)\propto \varepsilon^{\tau(q)}\,. \tag{2}$$

Function $\tau(q)$ usually is called scaling exponent and defined as

$$\tau(q)=\lim_{\varepsilon\to 0}\frac{\ln Z(q,\varepsilon)}{\ln\varepsilon}\,. \tag{3}$$

In the case of homogeneous fractal set with fractal dimension D all busy boxes have the same number of points that mean $p_i(\varepsilon)=p(\varepsilon)=1/N(\varepsilon)$ and partition function is

$$Z(q,\varepsilon)=N^{1-q}(\varepsilon)=\varepsilon^{-D(1-q)}$$

and function $\tau(q)=(q-1)D$ is linear. If the distribution of points in the boxes isn't the same, the fractal set is heterogeneous, i.e. multifractal, and $\tau(q)$ is a nonlinear function. If $q\to+\infty$, the main contribution to the partition function is made by the boxes that contain the greatest number of particles n_i and, consequently, most likely characterized by the filling p_i. Conversely, if $q\to-\infty$, the main contribution to the partition function is made by the most sparse boxes with small values p_i. Thus, the function $\tau(q)$ shows how heterogeneous set of points is investigated.

Along with the scale exponent $\tau(q)$ for the multifractal characteristics of the set the function of multifractal spectrum (the spectrum of singularities) $f(\alpha)$ is used. The dependence of the probability from the box size $p_i(\varepsilon)$ has an exponential character

$$p_i(\varepsilon)\propto \varepsilon^{\alpha_i}\,, \tag{4}$$

where α_i is some exponent, in general various for the diverse boxes (a measure of the singularity). For the homogeneous fractal all of the exponents α_i are the same and equal to the fractal dimension D.

Function of multifractal spectrum $f(\alpha)$ characterizes a probability distribution for the diverse values α_i. If value $n(\alpha)d\alpha$ is probability of the fact that α_i is in the interval $(\alpha, \alpha+d\alpha)$, i.e. the number of the boxes i that have the same measure $p_i(\varepsilon)$ with $\alpha_i \in (\alpha, \alpha+d\alpha)$, then

$$n(\alpha) \approx \varepsilon^{-f(\alpha)}. \tag{5}$$

So function $f(\alpha)$ is fractal dimension of the some homogeneous fractal subset ξ_α from the original set ξ that is characterized by the same probabilities of the box filling $p_i(\varepsilon) \approx \varepsilon^{\alpha}$.

Taking into account the expressions (1) and (5), the generalized partition function $Z(q,\varepsilon)$ can be written by using function of multifractal spectrum $f(\alpha)$ the next way:

$$Z(q,\varepsilon) = \sum_{i=1}^{N(\varepsilon)} p_i^q(\varepsilon) \approx \int d\alpha\, n(\alpha)\varepsilon^{q\alpha} \approx \int d\alpha\, \varepsilon^{q\alpha - f(\alpha)}.$$

Formally, the transition of variables $\{q, \tau(q)\}$ to the variables $\{\alpha, f(\alpha)\}$ can be made with the help of the next Legendre transformations:

$$\begin{cases} \alpha = \dfrac{d\tau}{dq} \\ f(\alpha) = q\dfrac{d\tau}{dq} - \tau \end{cases} \quad \text{and} \quad \begin{cases} q = \dfrac{df}{d\alpha} \\ \tau(q) = \alpha\dfrac{df}{d\alpha} - f \end{cases}. \tag{6}$$

Figure 1 shows plots of multifractal characteristics $\tau(q)$ and $f(\alpha)$ for monofractal and multifractal stochastic processes. In the case of a monofractal process scaling exponent $\tau(q)$ is a straight line, and the function of multifractal spectrum $f(\alpha)$ is point.

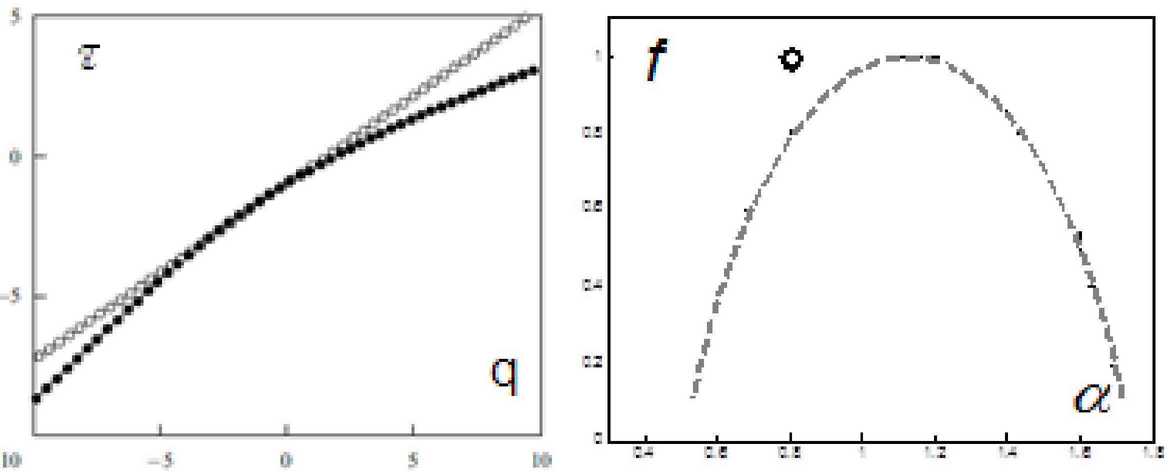


Figure 1. Functions $\tau(q)$ and $f(\alpha)$ for monofractal and multifractal stochastic processes

Now consider the basic concepts of self-similar and multifractal random processes [Calvet, 1997; Reidi, 2002; Kantelhardt, 2008; Abry, 2009].

Stochastic process $X(t)$, $t \geq 0$ with continuous real-time variable is said to be self-similar of index H, $0 < H < 1$, if for any value $a > 0$ processes $X(at)$ and $a^{-H}X(at)$ have same finite-dimensional distributions:

$$\mathrm{Low}\{X(at)\} = \mathrm{Low}\{a^{H}X(t)\}. \tag{7}$$

The notation $\mathrm{Low}\{\cdot\}$ means finite distribution laws of the random process. Index H is called Hurst exponent. It is a measure of self-similarity or a measure of long-range dependence of process. For values $0,5 < H < 1$ time series demonstrates persistent behaviour. In other words, if the time series increases (decreases) in a prior period of time, then this trend will be continued for the same time in future. The value $H = 0,5$ indicates the independence (the absence of any memory about the past) of values of time series. The interval $0 < H < 0,5$ corresponds to antipersistent time series: if a system demonstrates growth in a prior period of time, then it is likely to fall in the next period.

One can show by choosing in (7) $a = 1/t$, that for the self-similar process, the following equality is held:

$$\mathrm{Law}\{X(t)\} = \mathrm{Law}\{\mathrm{a}^{-H}X(at)\} = \mathrm{Law}\left\{\left(\frac{1}{t}\right)^{-H} X(1)\right\} = \mathrm{Law}\{t^{H}X(1)\}. \tag{8}$$

Using (8), the moments of the self-similar random process can be expressed as

$$E\left[|X(t)|^{q}\right] = E\left[|t^{H}X(1)|^{q}\right] = t^{qH}E\left[|X(1)|^{q}\right] = C(q)\cdot t^{qH}, \tag{9}$$

where the quantity $C(q) = E\left[|X(1)|^{q}\right]$.

In contrast to the self-similar processes (7) multifractal processes have more complex scaling behavior:

$$\mathrm{Law}\{X(a\,t)\} = \mathrm{Law}\{\mathcal{M}(a)\cdot X(t)\},\ a > 0, \tag{10}$$

where $\mathcal{M}(a)$ is random function that independent of $X(t)$. In case of self-similar process $\mathcal{M}(a) = a^{H}$.

For multifractal processes the following relation holds:

$$E\left[|X(t)|^{q}\right] = c(q)\cdot t^{qh(q)}, \tag{11}$$

where $c(q)$ is some deterministic function, $h(q)$ is generalized Hurst exponent, which is generally non-linear function. Value $h(q)$ at $q = 2$ is the same degree of self-similarity H . Generalized Hurst exponent of monofractal process does not depend on the parameter q : $h(q) = H$.

The generalized Hurst exponent $h(q)$ is connected with the function $\tau(q)$ by the ratio

$$\tau(q) = qh(q) - 1. \tag{12}$$

Determination of function multifractal spectrum $f(\alpha)$ is carried out according to formulas (6).

Figure 2 shows plot of generalized Hurst exponent $h(q)$ for monofractal and multifractal stochastic processes. In the case of a monofractal process Hurst exponent is a constant..

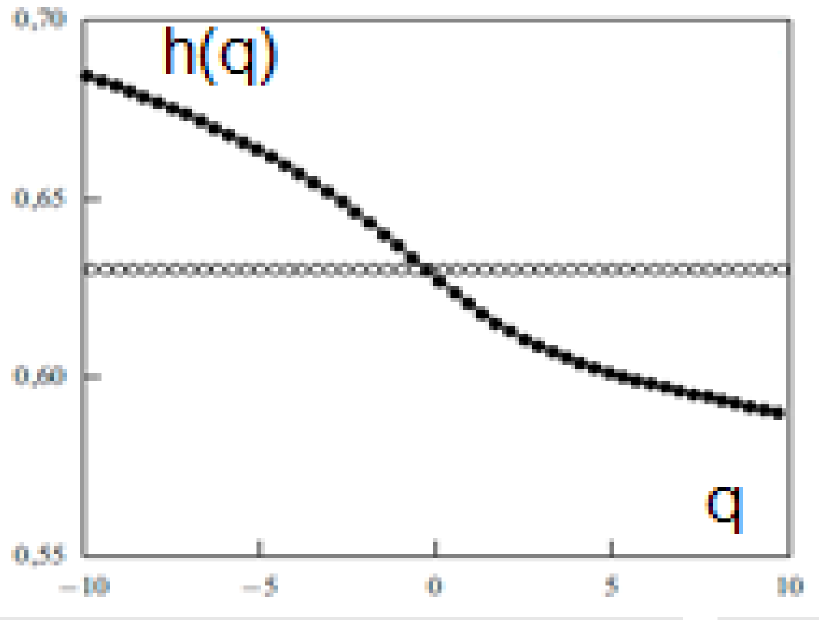


Figure 2. Functions $h(q)$ for monofractal and multifractal stochastic processes

Method of multifractal detrended fluctuation analysis

Two of the most popular research tools multifractal structure of the time series are the method of multifractal detrended fluctuation analysis (MFDFA) [Kantelhardt, 2002a, 2008] that is focused on the processing of non-stationary trended series, and the method of modulus maxima of wavelet transform (WTMM) [Muzy, 1993; Mallat, 1998; Kantelhardt, 2008]. Both methods are powerful tools for statistical processing of time-dependent processes.

The results of numerical studies show that in the investigation of a time series of unknown complex fractal structure the method MFDFA has to be used in the first place because it is easier to understand and implement [Oswiecimka, 2005, 2006; Kantelhardt, 2002a, 2008; Кириченко, 2011, 2011a]. Most investigations presented in the review were carried out by the method MFDFA.

According to the MFDFA method, for the initial time series $x(t)$ the cumulative time series $y(t)=\sum_{i=1}^{t}x(i)$ is constructed which is then divided into N segments of length τ, and for each segment $y(t)$ the following fluctuation function is calculated:

$$F^2(\tau)=\frac{1}{\tau}\sum_{t=1}^{\tau}(y(t)-Y_m(t))^2 , \qquad (13)$$

where $Y_m(t)$ is a local m-polynomial trend within the given segment. The averaged on the whole of the time series $y(t)$ function $F(\tau)$ has scaling on the segment of length τ:

$$F(\tau)\propto\tau^H .$$

In the study of multifractal properties the dependence of the fluctuation function $F_q(\tau)$ of a parameter q is considered:

$$F_q(\tau)=\left\{\frac{1}{N}\sum_{i=1}^{N}[F^2(\tau)]^{\frac{q}{2}}\right\}^{\frac{1}{q}} . \qquad (14)$$

Since in the case $q=0$ expression (14) contains the ambiguity instead of it the following expression is used:

$$F_q(\tau)=\mathrm{Exp}\left\{\frac{1}{N}\sum_{i=1}^{N}\ln[F^2(\tau)]\right\} .$$

If the investigated series is multifractal and has a long-term dependence, the fluctuation function is represented by a power law

$$F_q(\tau)\propto\tau^{h(q)} , \qquad (15)$$

where $h(q)$ is generalized Hurst exponent.. For monofractal time series the fluctuation function $F_q(\tau)$ is the same for all values q, and the generalized Hurst exponent does not depend on the parameter q: $h(q)=H$.

Figure 3 shows the fluctuation functions $F_q(\tau)$ for monofractal (left) and multifractal (right) processes of parameter values $q=\{-5,-2,0,2,5\}$.

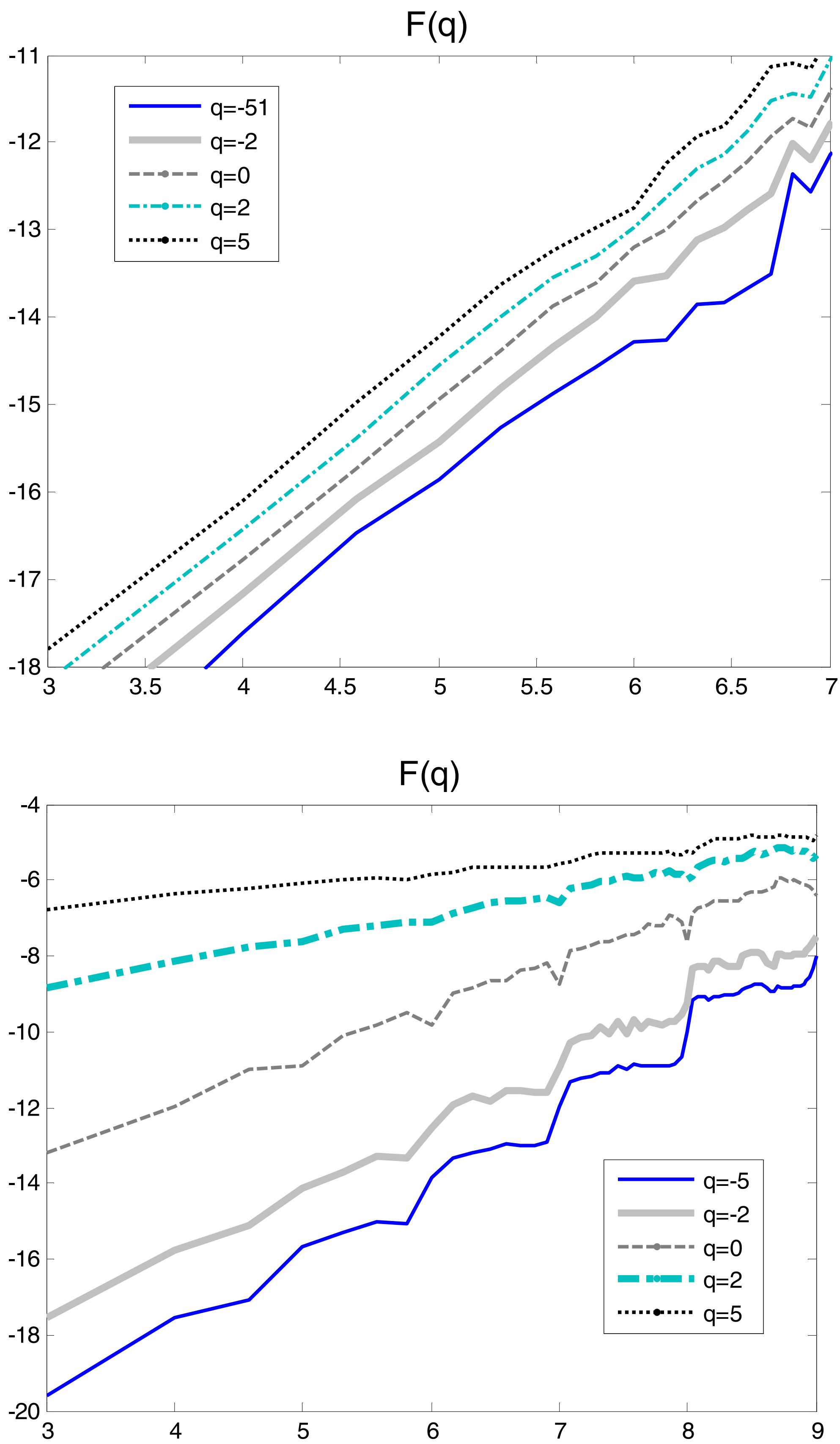


Figure 3. Functions $F_q(\tau)$ for monofractal (left) and multifractal (right) processes

At positive values q major contribution to the function $F_q(\tau)$ is given by segments which show great deviations of $F^2(\tau)$ and in case negative q the segments with small variances of $F^2(\tau)$ dominate. Thus, at negative values q of the generalized Hurst exponent $h(q)$ describes the segments, showing small fluctuations, and at positive q function $h(q)$ characterizes the segments with large fluctuations.

Visualization of inhomogeneity of time series in its multifractal characteristics

One of the most important properties of physiological signals which characterize the state of the organism is their inhomogeneity. Consider as a degree of inhomogeneity of time series which is reflected in its multifractal characteristics.

The simplest model of a multifractal process with the desired properties is a deterministic binomial multiplicative cascade [Feder, 1991; Calvet, 1997; Reidi, 2002]. In its construction, the initial unit interval is divided into two equal intervals, which are assigned weights p_1 and $p_2 = 1 - p_1$, respectively. Then the same procedure is repeated with each of the intervals. As a result, the second step has 4 intervals with weighting coefficients p_1^2, $p_1 p_2$, $p_2 p_1$ and p_2^2. If the number of steps $n \to \infty$ and $p_1 \neq p_2$, we arrive at a limit measure, which is a inhomogeneous fractal set.

Figure 4 shows the time series of values a binomial cascade for values $p_1 = 0.6$ and $p_1 = 0.8$. The number of iterations $n = 10$, i.e. the length of the realization equals 2^{10}. It is obvious that with increasing the weighting coefficient p_1 the inhomogeneity of time series increases also.

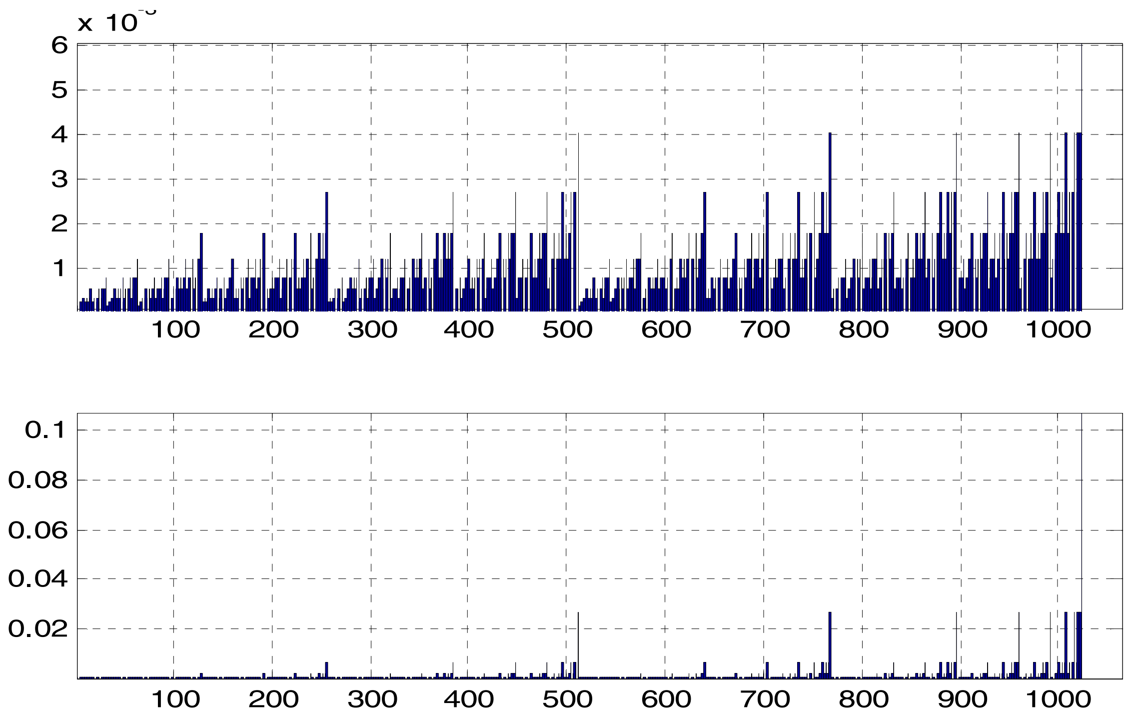


Figure 4. Time series of binomial cascade for values $p_1 = 0.6$ (top) and $p_1 = 0.8$ (bottom).

In this case multifractal characteristics of the binomial process depend only on the weighting coefficient p_1 and are calculated analytically:

$$\tau(q) = \frac{-\ln(p_1^q + p_2^q)}{\ln 2}, \quad h(q) = \left(\frac{1}{q} - \frac{\ln(p_1^q + p_2^q)}{q \ln 2}\right),$$

$$\begin{cases} \alpha = -\dfrac{p_1^q \ln(p_1) + p_2^q \ln(p_2)}{\ln 2\,(p_1^q + p_2^q)} \\ \\ f(\alpha) = -\dfrac{q}{\ln 2}\dfrac{p_1^q \ln(p_1) + p_2^q \ln(p_2)}{(p_1^q + p_2^q)} + \dfrac{\ln(p_1^q + p_2^q)}{\ln 2} \end{cases}.$$

Figure 5 shows plots of the generalized Hurst exponent $h(q)$, the scaling exponent $\tau(q)$ and the function of multifractal spectrum $f(\alpha)$ for time series of binomial cascade for values of weighting coefficient $p_1 = \{0.6, 0.7, 0.8, 0.9\}$. It should be noted that with increasing inhomogeneity of series the value $\Delta h = h(q1) - h(q2)$ increases, the scaling exponent $\tau(q)$ becomes more convex and the range of multifractal spectrum $f(\alpha)$ becomes a wider.

Investigation results

1. Research of multifractal characteristics of RR-interval's sequences.

It's known that for the diagnosis and detection of diverse heart's diseases analysis of the electrocardiogram has an important place. ECG is a recording of electrical heart's activity. The slightest deviation from the norm may indicate the violation of the cardiac rhythm and the presence of diverse diseases. One of the methods of diagnosing heart diseases is analysis of the series constructed by the RR-intervals.

RR-interval is the time interval between adjacent teeth of electrocardiogram and it equals to the duration of the cardiac cycle. These intervals are very important in determining the heart rate and diagnosis of diverse types of cardiac arrhythmias. Figure 6 [PhysioNet] shows the construction of RR-interval's sequences. It's known, that these types of series have chaotic structure [Shuster, 1988; Hoyer, 1997], so it's possible to analyze them using multifractal methods.

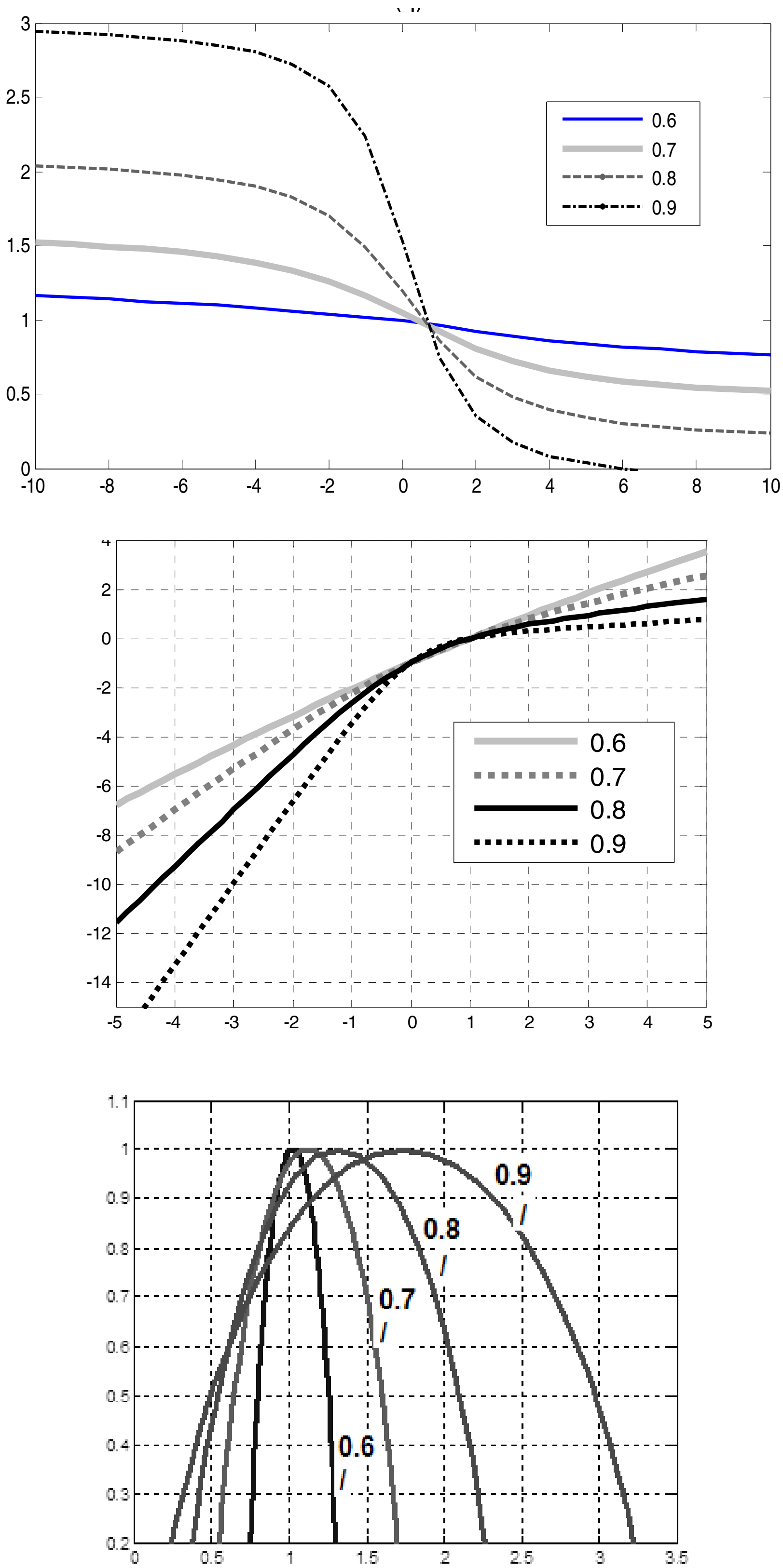


Figure 5. Functions $h(q)$, $\tau(q)$ and $f(\alpha)$ of binomial cascade for different values p_1

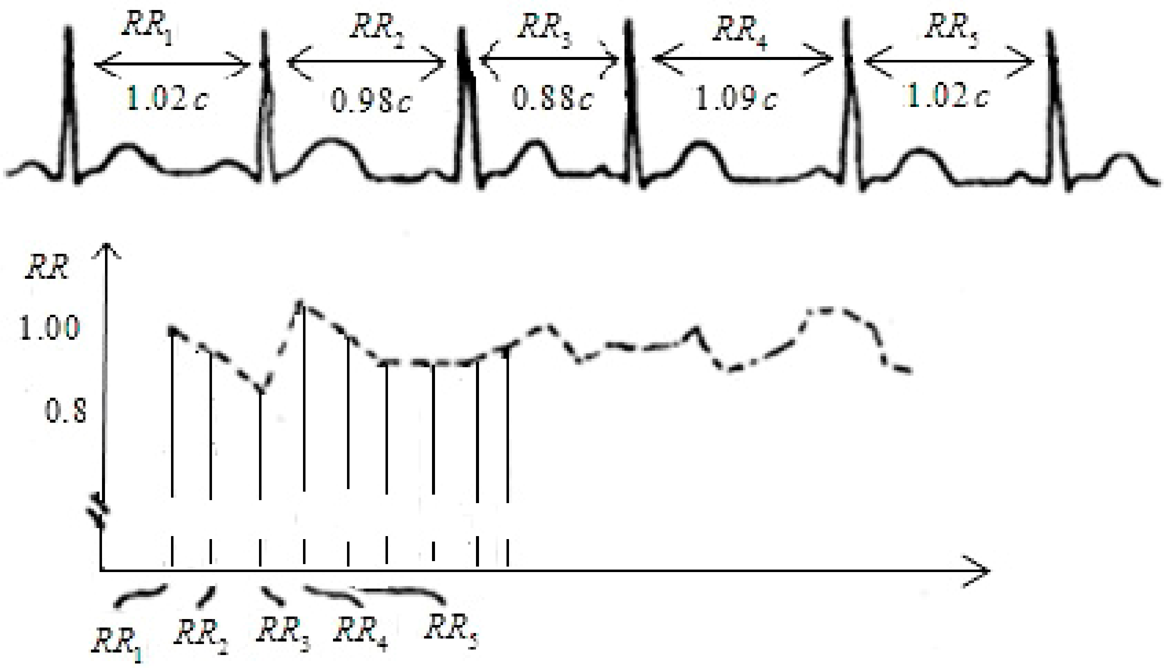


Figure 6. Image of normal ECG-signal with RR-intervals and constructing of RR-interval's sequence

Initial data for the research were obtained on the particularized website [PhysioNet] containing an extensive medical database. Figure 7 shows RR-interval's sequence, generalized Hurst exponent $h(q)$, scaling exponent $\tau(q)$ and multifractal spectrum $f(\alpha)$ which are typical for RR-intervals of the person, who has no heart diseases.

The database contains cardiogram records of the patients involved in medication trials. The medical investigation included patients belonging to the age group from 45 to 69 years who have a heart arrhythmia. The data of RR-intervals before and after taking medication were used to treat and prevent tachycardia by increasing heart rate. Figure 8 shows the RR-interval's sequences, generalized Hurst exponents $h(q)$, scaling exponents $\tau(q)$ and multifractal spectrums $f(\alpha)$ of patient that were a typical for the majority of patients before and after drug application.

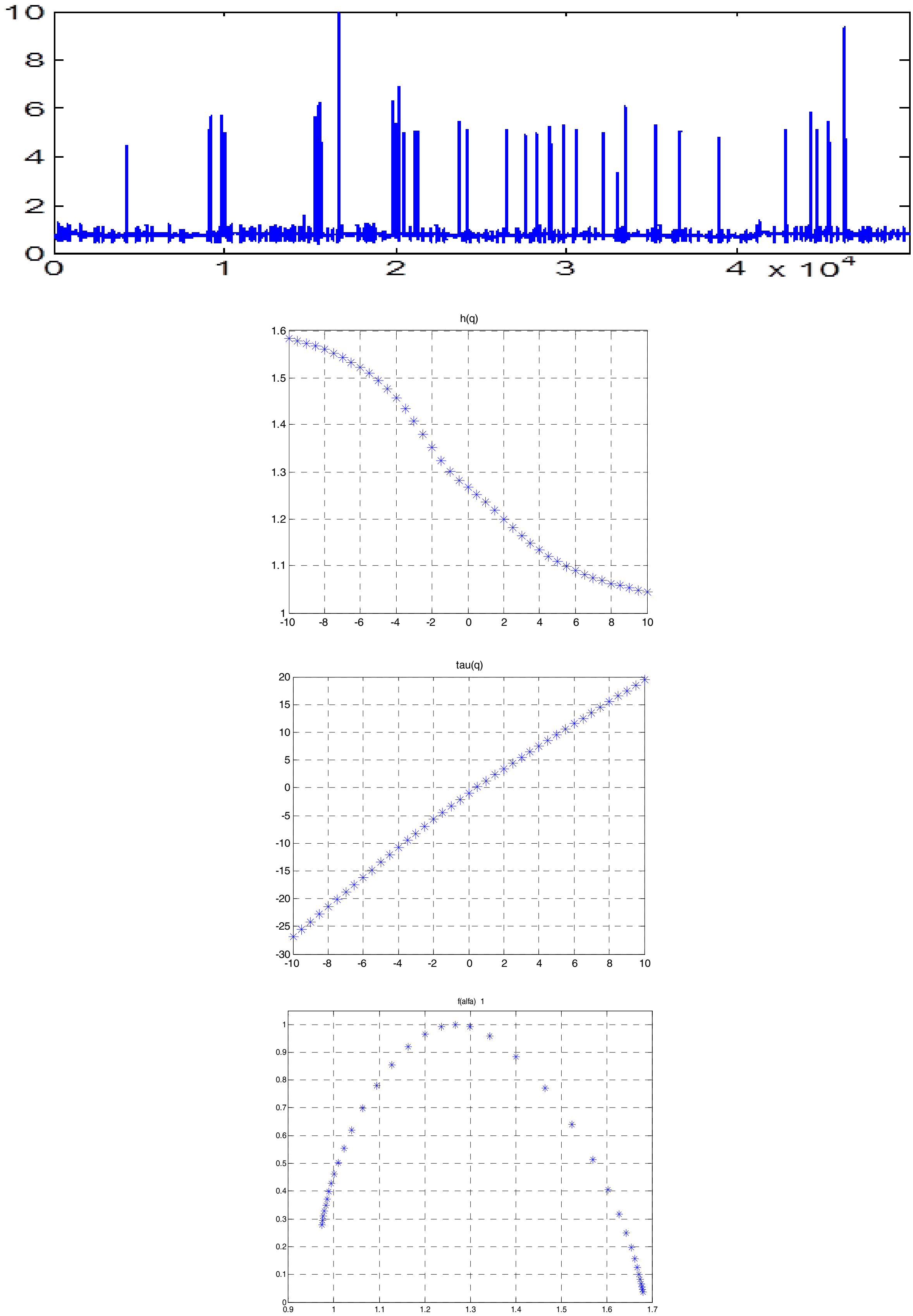


Figure 7. RR-intervals, generalized Hurst exponent, scaling exponent and multifractal spectrum of healthy person

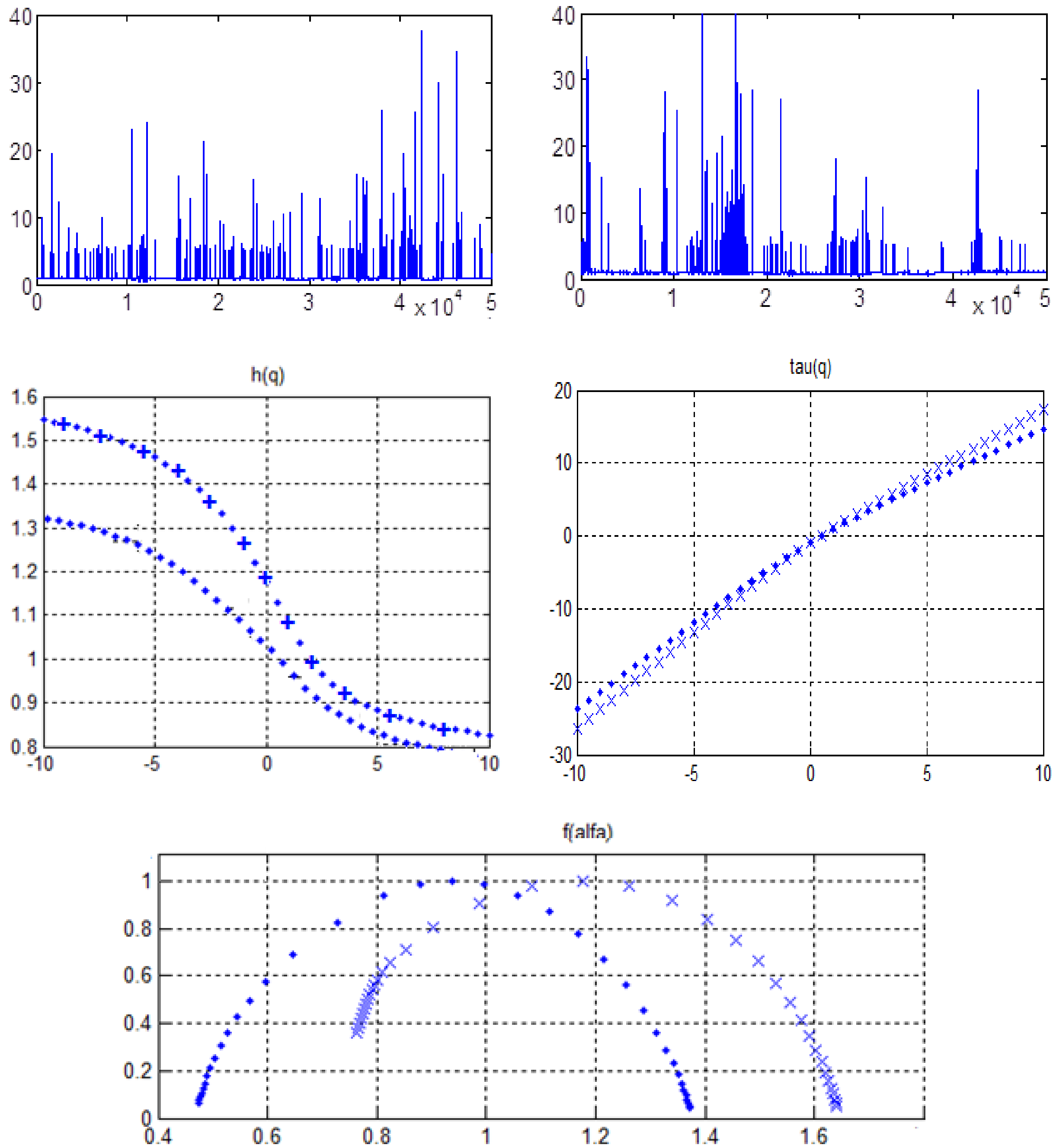


Figure 8. RR-intervals, generalized Hurst exponent, scale exponent and multifractal spectrum before drug's application (points) and after application (crosses).

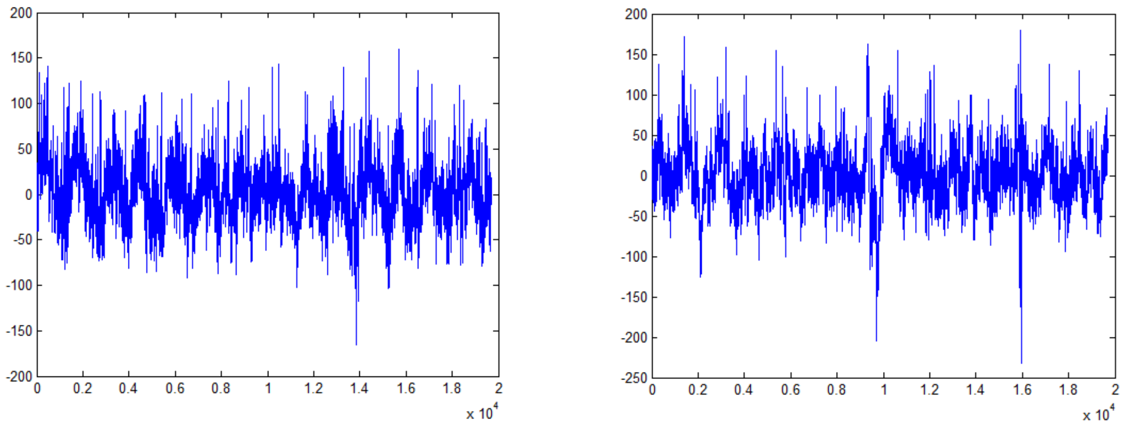
Researchers have shown that drug's application causes changes of multifractal characteristics of RR-interval's sequence. More visual characteristics that distinguishes time series before and after medication are function of multifractal spectrum $f(\alpha)$ and generalized Hurst exponent $h(q)$. Almost all patients have shift to the right of $f(\alpha)$ or shift up of $h(q)$ after medication, i.e. values of these functions has increased and became closer to the characteristics of healthy persons.

2. Research of multifractal characteristics of EEG records for real and imagined actions.

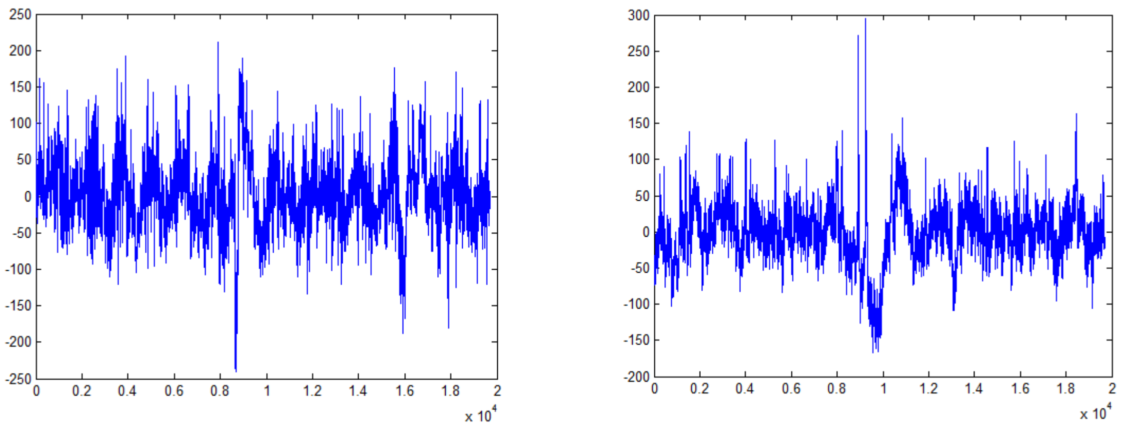
In the work investigation is conducted as multifractal characteristics of EEG of person change when he performs any physical action, and when he just imagines that he does. As experimental data the EEG signals of subjects performing certain actions were taken [PhysioNet]. Each subject performed the following complex of actions:

- In the right (left) corner of the computer screen a circle appeared and the subject clenched respectively his right (left) hand into a fist;
- In the right (left) corner of the computer screen a circle appeared and the subject just imagined that he clenched his hand into a fist, although in fact hand remained motionless.

In Figure 9 the plots of initial EEG time series for the two subjects who performed described above tasks are given. To the left EEG records when the subject clenches his hand into a fist are presented. To the right we can see EEG records when the subject just imagines that he does.



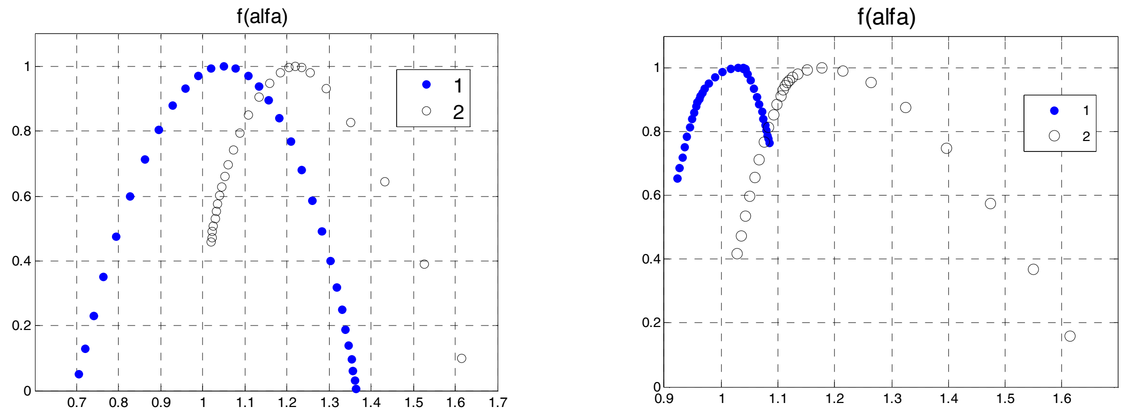
Subject 1



Subject 2

Figure 9. EEG records: subject clenches his hand into a fist (left) and subject just imagines that he does (right)

The multifractal characteristics of the corresponding time series were studied. The function of multifractal spectrum $f(\alpha)$ demonstrates the differences between the two states most graphically. Figure 10 presents functions of multifractal spectrum corresponding to the EEG records shown in Figure 9. Line 1 corresponds to the state when subject clenches his fists, and Line 2 corresponds when subject just imagines that he does.



Subject 1 Subject 2

Figure 10. Functions of multifractal spectrum of EEG

Line 1: subject clenches his fists; **Line 2:** subject just imagines that he does.

Thus, for this experiment function under the imagined action is significantly shifted to the right that makes it possible to distinguish between two states of the subject. During the study EEG records obtained with different electrodes were examined. It was found that EEG records of a number of electrodes are more sensitive to variations in the physical activity of the person and multifractal characteristics obtained from these records for the real and imagined actions differ significantly. In the investigation of EEG records of other electrodes the explicit relationship between the multifractal characteristics has not been identified.

They also the EEG records when subjects performed (imagined) other actions were considered. In the case of the movements of one hand the function of multifractal spectrum was shifted to the right as to whether when a subject was just imagined motion data. For the task with the movement of both hands the function of multifractal spectrum was contrary shifted to the right. Moreover, the shift of multifractal spectrum depends on the selected electrode.

3. Research of multifractal characteristics of EEG records for the different phases of wakefulness and sleep.

In the work we investigated EEG records of laboratory animals, which were divided into phases of wakefulness (AWAKE), slow-wave sleep (SWS) and rapid eye movement sleep (REM). Figure 11 shows typical realizations of the EEG for the different phases of wakefulness and sleep.

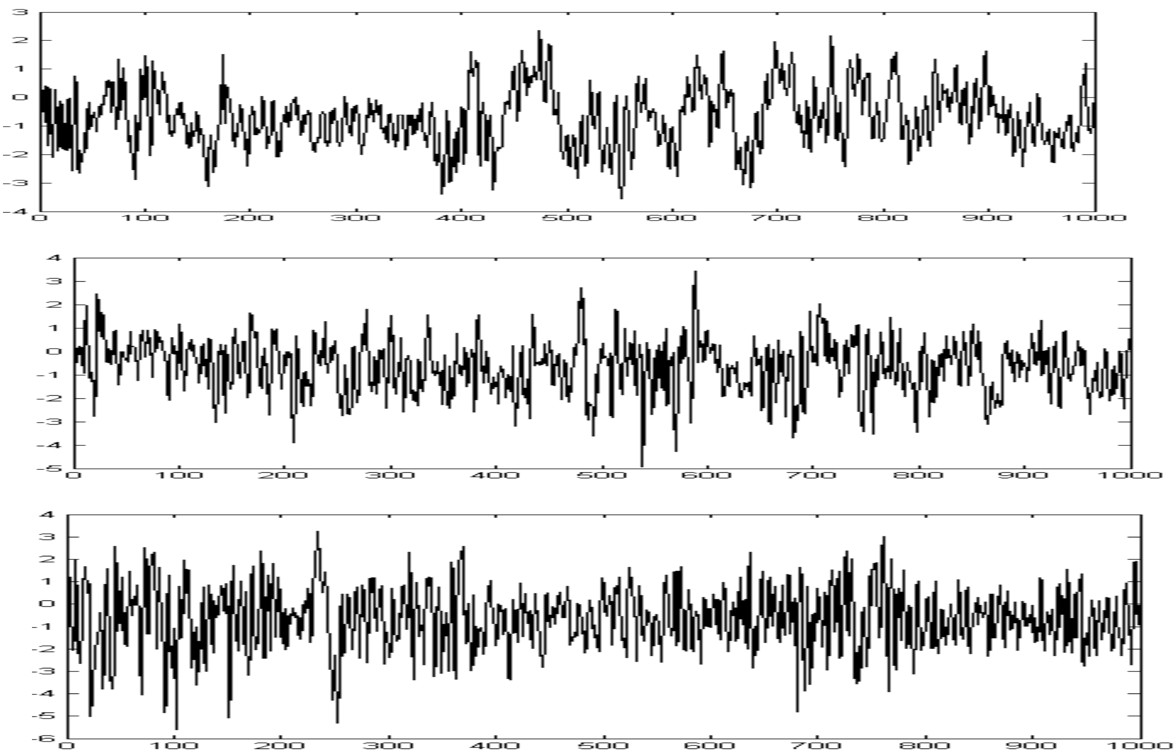


Figure 11. EEG records: AWAKE (top), SWS (middle) and REM (down)

The conducted multifractal analysis showed significant differences in the characteristics of the EEG records of phases of wakefulness and sleep. Figure 12 shows generalized Hurst exponent $h(q)$, scaling exponent $\tau(q)$ and multifractal spectrum $f(\alpha)$ which corresponding to the EEG records shown in Figure 11.

The analysis also shows that there is undoubted long-term dependence for EEG of wakefulness: in this case Hurst exponent $H = h(2)$ appreciably more than 0.5. The phase of slow-wave sleep is characterized by antipersistence, in this case the Hurst exponent H takes values in the range of less than 0.5. For REM sleep estimates of the Hurst exponent are close to the value 0.5, they have the meanings and larger and less than 0.5. It characterizes a very weak dependence EEG autocorrelation in this case.

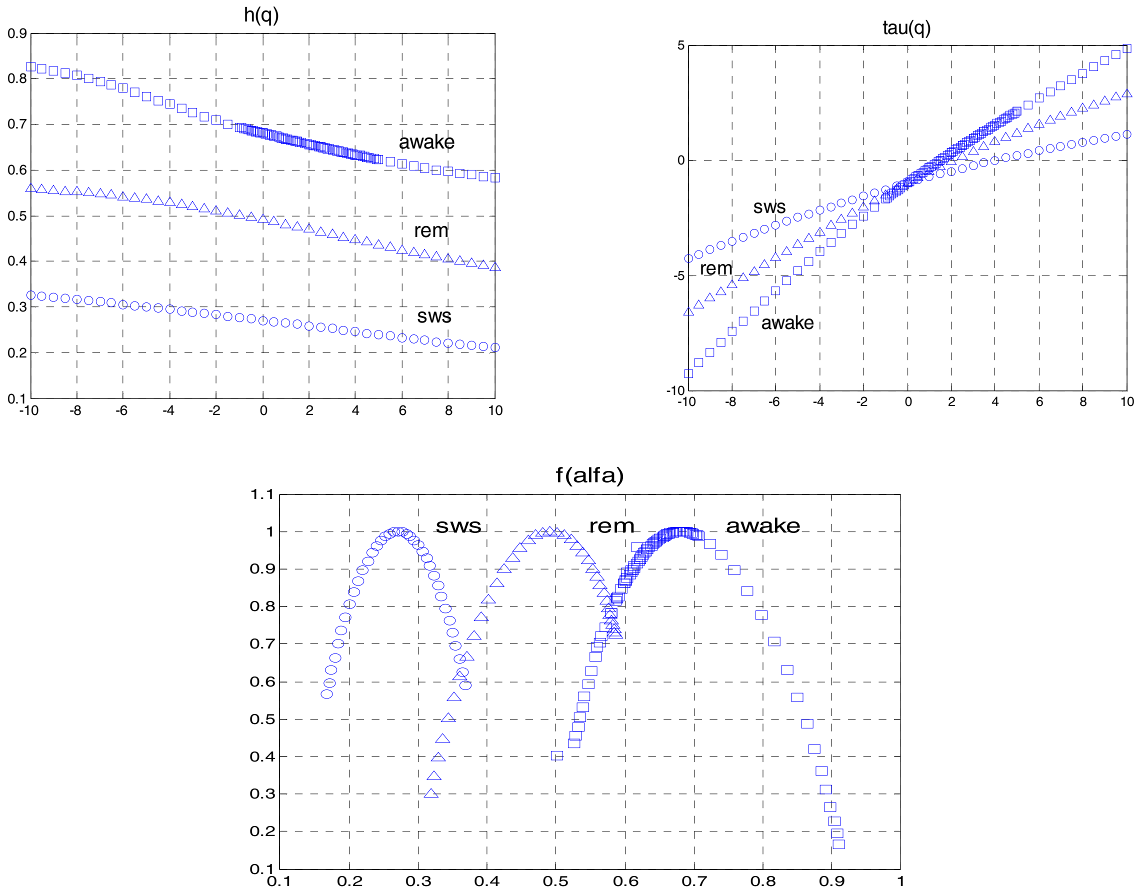


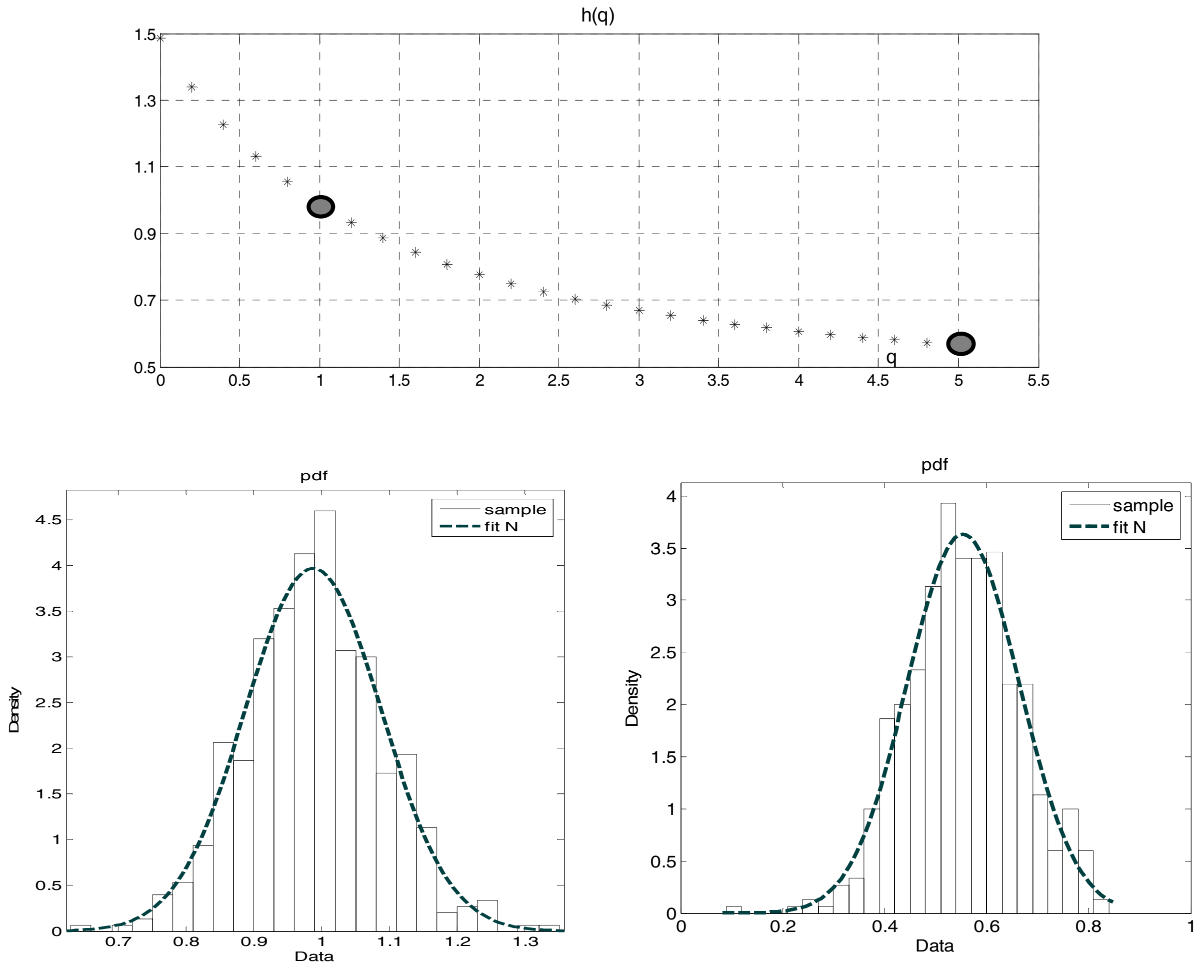
Figure 12. Generalized Hurst exponent, scale exponent and multifractal spectrum of the EEG records: AWAKE (boxes), SWS (bubbles) and REM (triangles)

Quantitative characteristic of multifractality degree

In practice, we are dealing with estimates of multifractal characteristics obtained by the realizations of finite length. In addition to variations in characteristics associated with features and uniqueness of organism, the variations related to inhomogeneity and lengths of the physiological signal have essential value. The accuracy of sample characteristics strongly depends on the length of signal realization.

Consider which of the multifractal characteristics: generalized Hurst exponent $h(q)$, scaling exponent $\tau(q)$ and multifractal spectrum $f(\alpha)$ is the most appropriate to apply for quantitative estimating. Since the functions $h(q)$, $\tau(q)$ and $f(\alpha)$ have one-to-one correspondence between themselves it is sufficient to use only one function as the basic characteristic. It is suitable use the generalized Hurst exponent $h(q)$ for this because the scaling exponent $\tau(q)$ in many cases is visual poorly informative (see for example Fig. 8) and the values of function of multifractal spectrum $f(\alpha)$ essentially depend on length of the of realization [Oswiecimka, 2006; Кириченко, 2011a] and need of more complex description.

For multifractal processes, the question of the distribution law of generalized Hurst exponent estimates was considered in a number of works, where it was shown numerically and analytically that the estimates are normal random variables. In the [Kantelhardt, 2002a; Павлов, 2007] it is shown that the large values parameter q lead to large errors. In [Kantelhardt, 2002a; Oswiecimka, 2006; Кириченко, 2011] the laws of distribution estimates $h(q)$ at different values of the parameter q were investigated. The analysis of the sample distribution laws $h(q)$ has shown that at $q > 0$ estimates has normal distribution the parameters of which depend on the value q . At $q < 0$ the sample values of the generalized Hurst exponent $h(q)$, in general, are not normally distributed. Figure 13 shows the values of the function $h(q)$ ($0 \le q \le 5$) and histograms of the estimates $h(q)$ received from realizations of the stochastic binomial cascade of length 1024 at $q = 1$ and $q = 5$.



a) b)

Figure 13. Generalized Hurst exponent $h(q)$ ($0 \le q \le 5$) and histograms of the estimates $h(q)$ at $q = 1$ (a) and $q = 5$ (b)

Numerical analysis showed that a random variable $\Delta h = h(q1) - h(q2)$ at $q > 0$ has normal distribution $N(m_h, s_h)$ the parameters of which depend on the realization length and values q. As the basic values $h(q)$ on which it is possible to carry out numerical comparison of multifractal properties we can use values $\hat{h}(0.1)$, $\hat{h}(5)$, $\Delta\hat{h} = \hat{h}(0.1) - \hat{h}(5)$, $\hat{H} = \hat{h}(2)$ (see Figure 14).

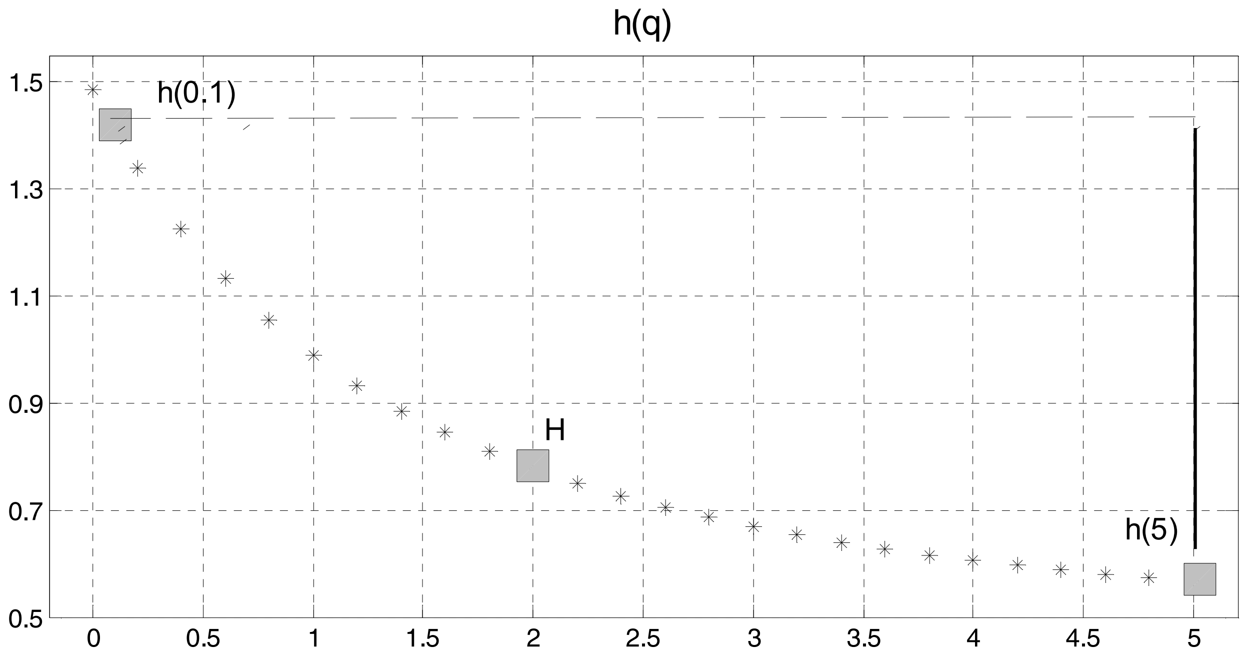


Figure 14. The basic values $h(q)$ for comparison

Then, when constructing confidence intervals it is need to use the following inequalities:

$$\overline{h}(q) - t_\alpha s_h(N) \le h(q) \le \overline{h}(q) + t_\alpha s_h(N), \quad q = 0.1,\ 2,\ 5.$$

where N is the length of investigated series, $\overline{h}(q)$ and $s_h(N)$ are mean and standard deviation of the estimate of generalized Hurst exponent $h(q)$ that was obtained from set of time series length N, α is the significance level, t_α is quantile of the standard normal distribution.

Conclusion

In the work the results of multifractal analysis for three different cases electrobiological signals are represented: RR-interval's sequences of electrocardiograms obtained from patients before drug's application and after one; electroencephalograms of subjects when they perform any physical action, and when they just imagine this and electroencephalograms of laboratory animals for the different phases of wakefulness and sleep. Research was carried out by method of multifractal detrended fluctuation analysis. In all cases, there are significant differences of multifractal characteristics.

In the work such characteristics of multifractal stochastic processes as a generalized Hurst exponent, scaling exponent, function of multifractal spectrum are discussed. It is shown that it is suitable use the generalized Hurst exponent as a quantitative measure. The results of numerical analysis showed that the estimates of the generalized Hurst exponent have normal distribution for positive values of parameter that allows to jumping to quantitative interval values. The possible specific values of the generalized Hurst exponent, which should be used in the knowledge bases of decision support systems, were proposed.

The research results presented in the review and carried out in this work show that distinct changes of fractal characteristics of physiological signals become apparent at various diseases, at change physical and mental loadings on an organism, during functional changes in the brain, etc. This would suggest that multifractal techniques can be successfully used in the analysis of various physiological signals, in particular electrobiological ones to determine the changes in functional activity of an organism. Multifractal analysis of electrocardiogram and electroencephalogram records can be basis for the statistical studies that will enable to form diagnostic methods that are relevant to clinical practice.

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