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STORING AND PROCESSING ACCOUNTING INFORMATION BASED ON COLLECT/REPORT PARADIGM

Krassimira Ivanova

***Abstract**: The possibility to store and process accounting information using the Collect/Report Paradigm (CRP) is outlined in this paper. The main idea consists of using CRP to distribute accounting information in multi-dimensional information spaces and stored and processed it in parallel in cloud. Every account may be presented by a separated layer which contains two named sets – Dr and Cr. Storing and reporting may be provided simultaneously without recompilation of the information base.*

***Keywords**: Accounting, Collect/Report Paradigm, Big Data, Cloud computing, BigArM.*

***ACM Keywords:** E.1 Data Structures; Distributed data structures.*

Introduction

The milestone of Collect/Report Paradigm (CRP) [Markov & Ivanova, 2015] is the simple idea that we may use a special kind of organization of the information and this way to develop easy to use information bases and with very high speed for response which enables ***the real-time analytical processing*** (RTAP) [Markov, 2005]. (The RTAP multithreaded processing engine needs to support extremely large volumes of data in real time. The analytics performed are composed of combinations of algorithmic, statistical and logical functions [B-Jensen, 2002]).

RTAP is convenient approach for information service of business activities. Many business concerns have several hundred or even several thousand *business transactions* each day. This is reason to investigate using of Collect/Report Paradigm for storing and processing business information.

In this paper we will discuss the theoretical and practical aspects of implementing CRP in real accounting information service systems. Firstly we will remember the mathematical structures (Named Sets). Then we will outline main characteristics of structures for storing business information and theirs possibilities to be used in the frame of CRP.

Named sets

The concept “Named set” was defined by Mark Burgin. Here we will follow [Burgin, 2011].

Named set **X** is a triple **X** = (X, μ, I) where:

X is the support of X and is denoted by S(X);

I is the component of names (also called set of names or reflector) of X and is denoted by N(X);

μ: X → I is the naming map or naming correspondence (also called reflection) of the named set X and is denoted by n(X).

The most popular type of named sets is a named set **X** = (X, μ, I) in which X and I are sets and μ consists of connections between their elements. When these connections are set theoretical, i.e., each connection is represented by a pair (x, a) where x is an element from X and a is its name from I, we have a *set theoretical named set*, which is binary relation.

Named sets as special cases include:

Usual sets;

Fuzzy sets;

Multisets;

Enumerations;

Sequences (countable as well as uncountable)

etc.

A lot of examples of named sets we may find in linguistics studying semantical aspects that are connected with applying different elements of language (words, phrases, texts) to their meaning [Burgin & Gladun, 1989; Burgin, 2011].

Accounting cycle

Accounting documents contain a reflection of the financial activities of all proprietary information and are a good basis for analysis, and hence - forecasting and planning the future activities of the company [Samuelson & Nordhaus, 1989].

The sequence of accounting procedures used to record, classify, and summarize accounting information is often termed *accounting cycle* [Markov et al, 1993b; Meigs & Meigs, 1989].

The accounting cycle begins with the initial recording of business transactions and concludes with the preparation of formal financial statements summarizing the effects of these transactions upon the assets, liabilities, and owners' equity of the business. The term "cycle" indicates that these procedures must be repeated continuously to enable the business to prepare new, up-to-date financial statements at reasonable intervals.

Many business concerns have several hundred or even several thousand *business transactions* each day. It would not be practicable to prepare a balance sheet after each transaction and it is quite unnecessary to do so. Instead, the many individual transactions are recorded in the accounting records and, at the end of the month or other accounting period, a balance sheet is prepared from these records.

An accounting system includes a separate record for each item that appears in the balance sheet. For example, a separate record is kept for the asset cash, showing all the increases and decreases in cash which result from the many transactions in which cash is received or paid. A similar record is kept for every other asset, for every liability, and for owners' equity.

The form of record used to record increases and decreases in a single balance sheet item is called *an account.* The account is a means of accumulating in one place all the information, about changes in a specific asset, a liability, or owners' equity.

All separate accounts are usually kept in a loose-leaf binder, and the entire group of accounts is called a *ledger.* The electronic variant of the ledger is assumed in this paper.

Structure of the ledger account

➢ Basic types of information items of the transaction

In every transaction there are five main types of information items:

- Integer number;
- Real number;
- String;
- Date;
- Nomenclature.

The first four types are identical with those in programming languages like Pascal. The fifth, the nomenclature Nmcl, is specific named set:

Nmcl: (Nom_code, μ,Value),

where

- Nom_code is a set of nomenclature codes which usually are integer numbers or strings;
- μ - is biunique correspondence between Nom_code and Value;
- Value is a set of any of:
  - Integers;
  - Real numbers;
  - Strings;
  - Dates;
  - Complex records built of these types.

- **The ledger account and transactions**

The ledger account ACC is a specific named set [Markov et al, 1994] of two elements which are named Debit and Credit and which are sets Dr and Cr; i.e.

ACC : ({Debit, Credit}, μ, {Dr, Cr}).

Every element of Dr and Cr is named transaction and is connected with corresponded time point (usually this is any day of the year). This way Dr and Cr are sorted by the time of the transactions.

Every transaction may be described by the relation:

Tr : (<Doc No><Date><Type><Summ><Analitic information>)

where:

- <Doc No> is number of the primary document;
- <Date> is date of the primary document;
- <Type> is type of the primary document;
- <Sum> is the amount of the primary document;
- <Analytic information> is a special record of information items for additional analysis, sorting and searching the transaction record.

- **Correspondence between analytical information items**

The analytic information is the main tool for accounting analysis and preparing secondary documents for the leaders and the managers of the company and theirs decision making. Several additional items are usually connected with every account transaction. This information is needed for more complex analysis of the account or of the ledger as a whole. These items are known as an "analytical features" of the account.

There are three main goals of using the analytical features:

- To analyze the set of the account transactions in Dr or Cr for finding any mistakes or for obtaining any internal correspondences between transactions of the Dr or Cr of the ledger account;
- To analyze together the sets of the account transactions in Dr and Cr for finding any mistakes or for obtaining any internal correspondences between transactions of the ledger account which are included in sets of Dr and Cr;
- To analyze two or more accounts together for obtaining any correspondences between theirs transactions.

- **Time characteristics of the ledger account**

As we said in [Burgin & Gladun, 1989] the time dimension of the account transactions is very important one for the business analysis. The time-period principle is one of the generally accepted accounting principles that guide the interpretation of financial events and the preparation of financial statements. It tells us that the life of a business entity must be divided for accounting purposes into time periods of equal length, so that decision-makers will be informed on current trends within business [Markov et al, 1993b; Meigs & Meigs, 1989].

Time is recorded when:

- The primary document is created;
- The account transaction correspond to the given primary document is entered in the account.

The business analysis may be made:

- For given date;
- For time interval, including or excluding the boundaries of the interval.

➢ **The account balance**

There are several accounting intervals which are used for analyze the set of transactions of the account. Usually these intervals are:

- A day;
- A month;
- A quarter;
- An interval of days;
- An interval of mounts;
- An interval of quarters;
- A year.

The account balance is prepared for every accounting interval which is important for the business. The account balance is amount between total sums of Dr decremented with total sum of Cr. The total sum of Dr or Cr is made only for the transactions belong to the given accounting time interval.

➢ **Subaccounts and batches**

Usually the account contains hundreds of transactions. It is very difficult to analyze so many transactions and prepare report for any analytic condition. Because of this the account may be divided into sub accounts. In this case the account is named set of subaccounts.

There is another kind of grouping the transactions of the account. Using any analytical feature we can group transactions in batches corresponded to every value of given analytical sign. The reports in this case are made for the account sorted and "virtually" divided in batches in accordance with given condition for one or more analytical signs.

➢ **The Ledger**

The ledger LGR is a named set which elements are accounts, i.e.

$$LGR : (NACC, \mu, SACC),$$

where

- NACC is set of ledger accounts' names;
- μ - is biunique correspondence between NACC and SACC;
- SACC is set of ledger accounts.

Names of accounts are strings of digits. It is possible to use any letters but this form is not preferred in the everyday practice.

All accounts in the ledger describe all financial depended activities of the business. Every primary document must be registered in at least of two accounts. This way every account corresponds to some others - there are mappings between accounts. These mappings are very important for business analysis.

The specific of the mappings between accounts is the principle "the domain of mapping always is Dr of an account and co-domain is Cr of the same or other account". Mappings are registered both in corresponded transactions of domain and co-domain. This is an explicit registration of the relationship.

One account may correspond to every other one, but in practice there are clear rules for building accounting correspondence.

So, all financial business activities may be and must be reflected in the ledger by corresponded transactions and relationships between them.

Information operations with the ledger account

The main information operations with the ledger accounts are**:**

- Creating;
- Analyzing.

There are several service information operations - renaming, moving from one place of the ledger to another, deleting, sorting and preparing the batches and etc.

➢ Creating the ledger account

Creating the ledger account is a two steps' process. The first step is defining of the account and establishing its relations with balance sheet and income statement. The second is everyday entering new transactions in the Dr and Cr of the account and preparing work sheet and other financial statements.

➢ Defining the ledger account

The main procedure for the account is its defining. This is sequence of several decisions and activities:

- Naming the account and including it in the hierarchy of the ledger;
- Initializing of the control fields of the account;
- Set up the analytical parameters of the account and connection it with appropriate nomenclatures.

Naming the account is depended with national and international accounting standards. Usually it is clear what name is needed and where in the hierarchy is it.

More complicated is the initializing of the control fields of the account. There are two main types control fields:

- Relations with balance sheet and income statement;
- Initial values in Dr and Cr [Markov et al, 1993b] for the beginning of year and for all preceding months of current year if the account is initialized in the middle of the year.

Setting up the analytical parameters of the account and connection it with appropriate nomenclatures is very important for future analysis of the account and preparing reports for requests which in the stage of initializing could not be expected.

- **Memorial order**

The everyday accounting is based on two main technologies:

- Memorial orders oriented;
- Accounting journals oriented.

In computer based accounting the journals oriented technology is used when accounting cycle contains little number of transactions. In this case a spreadsheet may be used for information service of accounting cycle. So, all correspondences between accounts are made manually and analysis is very difficult.

Memorial orders oriented technology is simpler for working and has great power for computer analysis and preparing the secondary information - all standard financial statements and non standard but very useful reports. Our information model is connected with memorial orders' technology.

Memorial order (MO) contains header and body.

Header is standard information which describes the primary document:

- Type;
- Number;
- Date;
- Total sum;
- Number and name of business partner;
- Type of business operation;

- Income tax if it is included in the sum of the contract; etc.

Body of the MO is specific table used for entering the follow information about transactions:

- Description of the transaction;
- Corresponded accounts;
- Analytical information for each account of the transaction;
- Sum of the transaction.

➢ Information analysis of the MO

The information analysis of the MO is aimed:

- To make preliminary work sheet in the frame of the MO and to find and to correct any mistakes;
- To prepare two main journals of purchases and sales;
- To compute the income taxes;
- To make information retrieval and to prepare non-standard reports based on the MO description of the primary documents.

It is possible to classify MO in three sets:

- MO for purchases;
- MO for sales;
- MO for other business operations like money transfer between bank account and cash of the company and vice versa.

The information retrieval is needed for preparing reports based on primary documents. The main features of the information retrieval are described below.

➢ Entering new transactions in the Dr or Cr of the accounts

One row of the body of the MO describes one transaction. One primary document may be described by one or more transactions and by one or more MO.

After analysis of the MO all its transactions may be entered in the ledger accounts. As a rule, one transaction is entered in two corresponded accounts. The main rule in this case is: the transaction is entered in Dr of the first account and in Cr of the second. Which account is the first and which is the second is shown in the MO.

✓ Analysis of Dr or Cr

The main goal of accounting is to distribute transactions in accounts for more convenient analysis of the financial information and this way to support decisions making and the control in the business.

✓ Making the work sheet

After every entering the transactions the content of the Dr and Cr of the accounts is changed. The analysis of the current content of a given account is the first step. Usual it is to prepare the total of all sums of the transactions belongs to Dr or Cr respectively.

The totals are very useful for analysis of the dynamic of changes in the accounts. This analysis is based on:

- The total sums from beginning of the given accounting period so cold trial balance (TB);
- The total sum of the transaction belong to the given accounting period or so cold adjustments (ADJ);
- The final result of the getting together of the trial balance and adjustments or so cold adjusted trial balance (ATB).

These three elements (TB, ADJ and ATB) are used for preparing the work sheet of the ledger accounts.

As a rule, the heading of the work sheet consist of three parts:

- The name of the business;
- The title Work Sheet;
- The period of time covered.

In the American and European accounting the body of work sheet contains five pairs of money columns. In Bulgarian accounting the body of the work sheet contains three pairs of money columns (for TB, ADJ and ATB), each pair consisting of a debit and a credit column. The rest two pairs are represented in separate income statement and balance sheet.

Making the work sheet is a standard procedure and may be done automatically.

✓ **Making the journals**

The journals are another form to show the relations between ledger accounts. The journal is a table which visualizes the mapping between given account and all others in the ledger. For every account there are two main mappings - for Dr and Cr respectively.

Making the journals is a standard procedure too and may be done automatically.

✓ **Preparing the batches**

Batch is a subset of the Dr or Cr which corresponds to given condition. The process of preparing the batches is based on additional analytical information in the Dr or Cr of the account. So, there are many variants of grouping the transactions of Dr or Cr in batches. This is main reason for the importance of this information operation for the accounting cycle - it permits the multi lateral analysis in the single account.

Preparing the batches is a standard procedure too and may be done automatically.

- ✓ **Complex requests and reports**

The Dr and Cr may be represented as relations in the Codd relation model. In this case it is possible to use relation algebra or relation calculus for extracting any subsets from Dr or Cr. The resulting subsets may be used for preparing the batches which will contain transactions only from the report of the given relational request. This way number of the elements of the set of batches for analyze may be limited.

Executing the complex requests and preparing the reports and batches based on them in the practice may be done only automatically.

- ➢ **Common analysis of Dr and Cr**

After separate analysis of Dr and Cr it is important to continue with the common view of the two parts of the ledger account. This common view permits to find relationships between elements of Dr and Cr. Only common analysis of Dr and Cr gives total map of the financial operations. It is possible to analyze both Dr and Cr of one account or of the different accounts. Common analysis of Dr's or Cr's of any accounts has little practical effect.

- ✓ **Grouping the transactions and establishing the correspondences between groups**

The main work in common analysis of the Dr and Cr is to reorganize the account in new subsets and establish any correspondences between these subsets. All analytical signs may be used as basis for grouping the transactions of Dr and Cr. After finishing grouping it is possible to integrate transactions in every group to make more general view on the financial operations and processes.

- ✓ **Requests and reports**

As a rule, accounts contain thousands of transactions. In this case grouping is very slow process and in many cases it is impossible to work with so much information. For this goal it is need to have special tool for selecting the appropriate transactions before starting the grouping process. The requests and reports of this kind are similar to the relation algebra or relation calculus but are executed on different relations (Dr and Cr) and the result is again different subsets of Dr and Cr.

Storing and processing accounting information based on CRP

We have used strong hierarchies of named sets to create a specialized mathematical model, for new kind of organization of information bases called Multi-domain Information Model" [Markov, 2004]. The "Information Spaces" defined in the model are kind of strong hierarchies of enumerations (named sets) [Ivanova, 2015]. This permits it to be implemented for storing and processing of accounting information. Let remember that the ledger accounts are kind of named sets.

In the same time, the RDF structures are kind of named sets. This means that the accounting information can be directly represented by RDF triples or quadruples and stored and processed in the frame of CRP.

The advantage of this is the possibility to pass the boundaries of limitations of classical systems and to have freedom of cloud processing.

Conclusion

In the beginning of this paper we have remembered the main structure of the Collect/Report Paradigm. Further we included a short presentation of named sets. After that, the structure and operations with accounting information have been presented in details. In the great companies it may be very large and memory for storing and time processing pass the limitations of classical data bases.

Finally, we outlined the possibility to store and process accounting information using the Collect/Report Paradigm. Following it, the accounting information may be distributed in the cloud multidimensional information spaces and stored and processed in parallel. Every account is presented by a separated layer which contains two named sets – Dr and Cr. Storing and reporting may be provided simultaneously without recompilation of the information base.

More concretely, the advantages of such approach are:

Collecting accounting information may be done for all ledger accounts independently in parallel;

Reporting accounting information may be provided only by the ledger accounts which really contain information related to the request;

Input data as well as results may be in RDF-triple or RDF-quadruple format.

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MODIFIED LYAPUNOV'S CONDITIONS FOR HYBRID AUTOMATA STABILITY

Alexei Bychkov, Eugene Ivanov, Olha Suprun

Abstract: *The problem of stability of stationary states of hybrid automata are considered. Sufficient conditions for stability and instability of trivial stationary states of nonlinear and linear hybrid automata with the help of* s*- and* u*-hybrid Lyapunov's functions are obtained. Also necessary and sufficient condition for existence of solution of Lyapunov's-like equation (copositive matrix), which can be used to construct quadratic Lyapunov's functions on cones are proved.*

Keywords*: dynamical systems, discrete-continuous systems, Lyapunov's equations, Lyapunov's functions, hybrid automaton.*

ACM Classification Keywords*: G.1.7 – Ordinary Differential Equations.*

Introduction

Some dynamical systems combine continuous dynamics and switching between several different discrete states (in certain moments of time or at certain hypersurfaces). Specifically, this property is present in some automatic control systems (for example thermostat, automatic transmission). Difficulties in research of such systems are obvious.

Hybrid automata are used as mathematical descriptions of such systems. Research work on different models of discrete-continuous systems was done by Samoylenko A.M., Martynyuk A.A., Perestyuk N.A., Byslenko N.P., Glushkov V.M., Emel'yanov S.V., DeCarlo R., Branicky M., Pettersson S., Lennartson B. and others [Branicky, 1994]-[Martynyuk, 2002], [Peleties, 1991]- [Ye, 1998].

We will use the variant of method of Lyapunov's functions, based on s- and u- functions [Emel'yanov, 1972] , to investigate stability of stationary states of hybrid automata. Also in the article we propose sufficient conditions for solvability Lyapunov's equations on a some subset of the space of positive define matrices.

For the definition of hybrid automaton we refer the reader to [Bychkov, 2007b], [Alur,1993], [Nicollin, 1993].

We will use the following notation: $|\cdot|$ is Euclidean norm in $\mathbf{R}^n$, $\mathbf{0}$ is the null-vector and $a < b$ ($a \le b$) for vectors is the componentwise comparison.

Definition 1. Trivial stationary state $x = \mathbf{0}$ of hybrid automaton H is called Lyapunov stable, if for any $\varepsilon > 0$ there is $\delta > 0$ such that the inequality $|x(t_0)| < \delta$ implies $|x(t)| < \varepsilon$ for all $t \in \tau$, where τ is a hybrid time.

Main Result

Let's assume, that the hybrid automaton's orbit begins from the first state. Notation $x|_{i \to i+1}$ means that automaton switches from the local state i to the state $i+1$ at the point x. Let's build the following sequence:

$$c^0 \in (0, C),\ \ c^1 = \max_{\substack{x^1|_{1\to 2} \\ V^1(x^1) \le c_0}} V^2(x^1),\ \ c^2 = \max_{\substack{x^2|_{2\to 3} \\ V^2(x^2) \le c_1}} V^3(x^2), \ldots, c^N = \max_{\substack{x^N|_{N\to 1} \\ V^N(x^N) \le c_{N-1}}} V^1(x^N) \quad (1)$$

Also, let's denote by Ω_i the set that describes i-th local state. Let's assume that there exists a set of Lyapunov's functions, defined on sets Ω_i.

Definition 2. An indexed family $V(i,\ x) = \{V^i(x)\},\ i = \overline{1,\ N}$ is said to be a hybrid s-function, if $V^i(x)$ are positively defined and $c^N \le c^0$ for any sequence $\{c^i\},\ i = \overline{0,\ N}$, defined as (1).

We will use a hybrid s-functions to investigate stability of a hybrid automaton's trivial stationary state.

Definition 3. The following indexed family is called the derivative of the s-function $V(i, x)$:

$$\dot{V}(i, x) = \left\{ \frac{dV^i(x)}{dx} f_i(x(t)),\ i = \overline{1,\ N} \right\}.$$

Let's introduce the following notations: $B_r = \{x \in \mathbf{R}^n \mid |x| \le r\}$, $S_r = \{x \in \mathbf{R}^n \mid |x| = r\}$.

Theorem 1. Assume that the hybrid automaton H has trivial stationary state and satisfies conditions $|Q| < \infty\ \ i = \overline{1, N-1},\ Jump(N, x) = (1, x)$. Also assume that the neighborhood of the origin $D \subset X$ is defined and there exists positively defined hybrid s -function $V(i, x)\colon Q\times D\to \mathbf{R}$ such that $\frac{dV^i(x)}{dx} f_i(x(t))\le 0$ for all $x\in D\cap\Omega_i$, $i=\overline{1, N}$.

Then the trivial stationary state is stable.

Proof. Let's assume that $Q=\{1,2\}$ and denote $W_r(i)=\{x\in \mathbf{R}^n \mid V^i(x)\le r\}$.

Let's choose arbitrary $\varepsilon>0$ and show that it is possible to find $\delta>0$ such that for any orbit $x(t)$, the condition $V^2(x(t))<a_2(2)$ for all $t\in\tau$, implies $x(t)\in B_\varepsilon$ for all $t\in\tau$.

Let's select $r\in(0,\varepsilon)$ such that $B_{r_2}\subseteq D$ and put $a_2(i)=\min\limits_{x\in S_{r_2}} V^i(x)$. Let's choose $b_2(i)\in(0,a_2(i))$. Then $W_{b_2(i)}(i)\subseteq B_{r_2}$. Let's choose $p_2(i)>0$ such that $B_{p_2(i)}\subseteq \Omega_{b_2(i)}(i)$ and put $r_1=\min\limits_{i\in Q} p_2(i)$.

Similarly, the value $b_1(i)\in(0,a_1(i))$ can be defied. Then $W_{b_1(i)}(i)\subseteq B_r$. Let's choose $p_1(i)>0$ such that $B_{p_1(i)}\subseteq W_{b_1(i)}(i)$ and put $\delta=\min\limits_{i\in Q} p_1(i)$.

Let's assume (for distinctness) that the orbit begins in state 1. If the orbit doesn't move from state 1 to state 2, then the theorem degenerates to Lyapunov's theorem. So assume that it moves form state 1 to state 2 at some moment $\tau_0'=\tau_1$.

Then $|x(\tau_0')|=|x(\tau_1)|<r_1$ for all $t\in[\tau_0,\tau_0']$. If the trajectory doesn't move from state 2 to state 1, the theorem is proved. If there is a jump in point $\tau_1'=\tau_2$, then $|x(\tau_1')|=|x(\tau_2)|<r_2$ for all $t\in[\tau_1,\tau_1']$.

By the theorem's statement, there exists a positively defined hybrid s -function. Let's put $c^0=V^1(x(\tau_0))$ in s -function's definition. Then $V^1(x(\tau_2))\le c^2\le c^0=V^1(x(\tau_0))<a_1(1)$, i.e. $\left|V^1(x(\tau_2))\right|<a_1(1)$. Continuing by induction we acquire that $\left|V^1(x(t))\right|<a_1(1)$ for any $t\ge\tau_0$ in the first state, and $\left|V^2(x(t))\right|<a_2(2)$ for any $t\ge\tau_0$ in second state. In both cases $|x(t)|<r_2<\varepsilon$. Theorem are proved.

Note, that for any i, it is sufficient the check in equalities $c^N\le c^0$ and $\frac{dV^i(x)}{dx} f_i(x(t))\le 0$ only on local state Ω_i.

Now let's consider conditions of instability of trivial stationary state. Let's call a local state unstable if it is not Lyapunov stable. Let's give some definitions.

Definition 4. Hybrid time τ is said to be Zeno, if it consists of infinite number of intervals, but $\lim\limits_{i\to\infty} t_i^* <\infty$ (like in Zeno's paradox about Achilles and a turtle).

Let's build a sequence $\{c^i\}$, $i=\overline{0, N}$ as follows:

$$c^0 \in (0,C),\ \ c^1 = \min_{\substack{x^1|_{1\to 2}\\ V^1(x^1)=c^0}} V^2(x^1),\ \ c^2 = \min_{\substack{x^2|_{2\to 3}\\ V^2(x^2)=c^1}} V^3(x^2),\ldots,\ c^N = \min_{\substack{x^N|_{N\to 1}\\ V^N(x^N)=c^{N-1}}} V^1(x^N) \qquad (2)$$

Definition 5. An indexed family $V(i,\ x)=\{V^i(x)\},\ i=\overline{1,N}$ is said to be a hybrid u-function, if $V^i(x)$ are positively defined and $c^N \geq c^0$ for any sequence $\{c^i\},\ i=\overline{0,\ N}$, defined as (2).

Theorem 2. Let H be a hybrid automata with no Zeno orbits. Assume that H has the trivial stationary state and satisfies conditions $|Q|<\infty,\ Jump(i,\ x)=\{(i+1,\ q_i(x))\}$, $i=\overline{1,N-1}$; $Jump(N,x)=(1,q_N(x))$. Also assume that the neighborhood of the origin $D\subset X$ is defined and there exists a hybrid u-function $V(i,\ x)\colon\ Q\times D\to \mathbf{R}$ such that $\frac{dV^i(x)}{dx}f_i(x(t))>0$ for all $x\in D\cap\Omega_i$ and $i=\overline{1,N}$. Then the trivial stationary state is unstable.

Proof. During the live time of a hybrid automaton, there can be two cases:

1) for any $\varepsilon>0$ there exists $x_0\in B_\varepsilon$, such that the orbit, that starts at x_0 performs a finite number of switchings;

2) there exists an $\varepsilon>0$ such that all orbits that start in B_ε perform infinite number of switchings.

In the first case the theorem reduces to Chetaev's theorem about instability.

Let's assume that the second case takes place. For simplicity, let's assume that $Q=\{1,2\}$ and that orbit begins in the first state. Let's assume the contrary: the trivial stationary state is not unstable, i.e. for arbitrarily small $\delta>0$, orbits that begin in B_δ, do not leave the ball B_ε. Then there is some $C>0$, such that $V^i(x(t))<C$ for all $i\in Q$.

From the positive definiteness and continuity of the derivative $\dot V(i,x)=\frac{dV^i(x)}{dx}f_i(\,x(t))$ it follows, that there exists some $v>0$ such that $\dot V(i,x(t))>v$ for all $i\in Q$. Let's compute the difference $V^1(x(t))-V^1(x(t_0))$ (we assume that the orbit is in state 1 at time t after M switchings, where M is an even number):

$$V^1(x(t))-V^1(x(t_0))=\sum_{i=2,4,\ldots,M}\left(\int_{\tau_{i-1}}^{\tau_i}\dot V^1(s)ds+\left[V^1(x(\tau_{i+1}+0))-V^1(x(\tau_i-0))\right]\right)+\int_{\tau_{M+1}}^{t}\dot V^1(s)ds\geq$$

$$\geq\sum_{i=2,4,\ldots,M}\left[V^1(x(\tau_{i+1}+0))-V^1(x(\tau_i-0))\right]+v\left[\sum_{i=2,4,\ldots,M}(\tau_i-\tau_{i-1})+(t-\tau_{N+1})\right].$$

Let's assume the series $\sum_{i=2,4,\ldots,M}(\tau_i-\tau_{i-1})$ to be divergent. Then the expression $v\left[\sum_{i=2,4,\ldots,M}(\tau_i-\tau_{i-1})+(t-\tau_{N+1})\right]$ becomes unbounded with the increase of t. It remains to make sure that the difference $V^1(x(\tau_{i+1}+0))-V^1(x(\tau_i-0))$ is non-negative.

Let's put $c^0=V^1(x(\tau_i))$ in the definition of the u -function. Then $V^1(x(\tau_i))\geq c^1$, and $V^1(x(\tau_{i+1}))\geq c^2\geq c^0$, i.e. inequality $V^1(x(\tau_{i+1}))-V^1(x(\tau_i))\geq 0$ holds.

Thus with the increase of t, the difference $V^1(x(t))-V^1(x(t_0))$ becomes arbitrarily large which contradicts assumption $V^i(x(t))<C$.

If the series $\sum_{i=2,4,\ldots,N}(\tau_i-\tau_{i-1})$ converges, then another series $\sum_{i=3,5,\ldots,N-1}(\tau_i-\tau_{i-1})$ is divergent and the same arguments can be applied to the second state.

Theorem are proved.

Consider a hybrid automaton with the following properties:

1. for each $k\in Q$, the right-hand side of the equation, describing continuous dynamics in k-th local state has the form $f_k(x)=A_k x$, where A_k is $n\times n$-matrix ;

2. each local state is formed by the convex cone $G_k x\geq \mathbf{0}$, where G_k is $m_k\times n$-matrix;

3. switching is cyclic ($1\to 2\to\ldots\to N\to 1$) and occurs on hyper surface $x=B_k y$, where $y\in\mathbf{R}^{n-1}$ and B_k is $n\times(n-1)$-matrix, i.e.

- $Jump(k,x)=\{((k \bmod N)+1,x)\}$, if $x=B_k y$ for some y;
- $Jump(k,x)=\varnothing$, in the other case.

Definition 6. A hybrid automaton, that satisfies properties 1-3 is called linear.

Note, that this definition should not be confused with other definitions of linear hybrid automata which can be found in model checking literature.

Theorem 3. (About piecewise quadratic s-function). Let HA be a linear hybrid automaton. Suppose that there exist positive-definite symmetric $n\times n$-matrices H_k, $k=\overline{1,N}$, such that:

- $a_k=\max\limits_{\substack{G_k x\geq 0\\ x^T x=1}} x^T(A_k^T H_k+H_k A_k)x<0$;
- the matrix $B_k^T(H_\ell-H_k)B_k$ is negative-semidefinite for each switching $k\to(k \bmod N)+1=\ell$.

Then the trivial stationary state is stable.

Proof. Let's define Lyapunov's functions as follows:

$$V_k(x) = x^T H_k x\,,\ k = \overline{1,N}\,.$$

Let's choose arbitrary $x \neq \mathbf{0}$ such that $G_k x \geq \mathbf{0}$. Then conditions $G_k x \geq \mathbf{0}$ and $a_k < 0$ imply the following inequality

$$\dot{V}(x) = x^T(A_k^T H_k + H_k A_k)x \leq x^T(A_k^T H_k + H_k A_k)x < 0\,.$$

Then the following relation describes transition $k \to (k \bmod N) + 1 = \ell$:

$$c^l - c^k = V_\ell(x) - V_k(x) = x^T(H_\ell - H_k)x = y^T B_k^T(H_\ell - H_k)B_k y \leq 0\,,$$

where c^l, c^k denote Lyapunov's functions $V_\ell(x)$ and $V_k(x)$ respectively.

Thus $\{V_k\}_{k=\overline{1,N}}$ form a hybrid s-function, i.e. $c^N \leq c^{N-1} \leq \ldots \leq c^1 \leq c^0$. So according to the Theorem 1, the trivial stationary state is stable. Theorem are proved.

Theorem 4 (About exponential attenuation coefficients). Suppose that linear hybrid automaton satisfies conditions of the Theorem 3. Then the following inequality holds (where q_0 is the automaton's initial state):

$$|x(t)| \leq \sqrt{\frac{\lambda_{\max}(H_{q_0})}{\min\limits_{k\in Q}\lambda_{\min}(H_k)}} \cdot |x(t_0)| \cdot \exp\left\{\max_{k\in Q}\frac{a_k}{\lambda_{\min}(H_k)} \cdot \frac{(t-t_0)}{2}\right\}.$$

Proof. The following inequality holds for each $k = \overline{1,N}$:

$$\dot{V}_k(x) = x^T(A_k^T H_k + H_k A_k)x < 0\,.$$

Conditions of the Theorem 3 imply, that $\dot{V}_k(x) \leq a_k\,|x|^2$. From Rayleigh's inequality it follows that

$$\dot{V}_k(x) \leq \frac{a_k}{\lambda_{\min}(H_k)}V_k(x)\,.$$

Now from the Gronwall-Bellman's inequality implies that

$$V_k(x) \le V_k(x(\tau_i)) \cdot \exp\left\{\frac{a_k(t-\tau_i)}{\lambda_{\min}(H_k)}\right\}.$$

This inequality holds only within local state. Let’s write it the current (n -th) segment of hybrid time and for previous segments:

$$V_{q_n}(x(t)) \le V_{q_n}(x(\tau_n)) \cdot \exp\left\{\frac{a_{q_n}(t-\tau_n)}{\lambda_{\min}(H_{q_n})}\right\},$$

$$V_{q_i}(x(\tau_i')) \le V_{q_i}(x(\tau_i)) \cdot \exp\left\{\frac{a_{q_i}(\tau_i'-\tau_i)}{\lambda_{\min}(H_{q_i})}\right\},\ i=1,\dots,n-1.$$

Conditions of the Theorem 3 imply that $V_{q_i}(x(\tau_i)) \le V_{q_{i-1}}(x(\tau_{i-1}'))$, so the following inequality holds:

$$V_{q_n}(x(t)) \le V_{q_n}(x(\tau_n)) \cdot \exp\left\{\frac{a_{q_n}(t-\tau_n)}{\lambda_{\min}(H_{q_n})}\right\} \le$$

$$\le V_{q_n}(x(\tau_{n-1}')) \cdot \exp\left\{\frac{a_{q_n}(t-\tau_n)}{\lambda_{\min}(H_{q_n})}\right\} \le V_{q_n}(x(\tau_{n-1})) \cdot \exp\left\{\frac{a_{q_n}(t-\tau_n)}{\lambda_{\min}(H_{q_n})} + \frac{a_{q_{n-1}}(\tau_{n-1}'-\tau_{n-1})}{\lambda_{\min}(H_{q_{n-1}})}\right\} \le \dots \le$$

$$\le V_{q_0}(x(t_0)) \cdot \exp\left\{\sum_{i=1}^{n-1}\frac{a_{q_i}(\tau_i'-\tau_i)}{\lambda_{\min}(H_{q_i})} + \frac{a_{q_n}(t-\tau_n)}{\lambda_{\min}(H_{q_n})}\right\} \le V_{q_0}(x(t_0)) \cdot \exp\left\{\max_{k\in Q}\frac{a_k}{\lambda_{\min}(H_k)}\cdot(t-t_0)\right\}.$$

Now with the help of Rayleigh’s inequality we obtain the following:

$$|x(t)|^2 \le \frac{\lambda_{\max}(H_{q_0})}{\lambda_{\min}(H_{q_n})}\cdot|x(t_0)|^2 \cdot \exp\left\{\max_{k\in Q}\frac{a_k}{\lambda_{\min}(H_k)}\cdot(t-t_0)\right\} \le$$

$$\le \frac{\lambda_{\max}(H_{q_0})}{\min\limits_{k\in Q}\lambda_{\min}(H_k)}\cdot|x(t_0)|^2 \cdot \exp\left\{\max_{k\in Q}\frac{a_k}{\lambda_{\min}(H_k)}\cdot(t-t_0)\right\}.$$

Theorem are proved.

Let's take a look at the following example. Let $|Q| = 2$, $X = (x_1, x_2)$ and $A_1 = \begin{pmatrix} -0,1 & 1 \\ -2 & -0,1 \end{pmatrix}$, $A_2 = \begin{pmatrix} -0,1 & 2 \\ -1 & -0,1 \end{pmatrix}$.

Let $Inv_1 = \{x_1 > 0\}$, $Inv_2 = \{x_1 < 0\}$, $G_1 = (1,0)$, $G_2 = (-1,0)$, $B_1 = B_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Let's put $H_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$, $H_2 = \begin{pmatrix} 0,5 & 0 \\ 0 & 1 \end{pmatrix}$ and check conditions of the theorem 4:

- $a_1 = \max\limits_{\substack{G_1 x \ge 0 \\ x^T x = 1}} x^T (A_1^T H_1 + H_1 A_1) x = -0,2$; $a_2 = -0,1$;
- $B_1^T (H_2 - H_1) B_1 = 0$, $B_2^T (H_1 - H_2) B_2 = 0$.

Thus trivial stationary state is stable. If we assume, that the first state is initial, then:

$$\lambda_{\max}(H_1) = 2 , \ \lambda_{\min}(H_1) = 1 , \ \lambda_{\min}(H_2) = 0,5 , \ \frac{a_1}{\lambda_{\min}(H_1)} = \frac{a_2}{\lambda_{\min}(H_2)} = -0,2 ,$$

i.e. we obtain the following exponential inequality:

$$|x(t)| \le 2 |x(t_0)| \cdot e^{-0,1(t-t_0)} .$$

Let's investigate conditions that imply existence of a H that satisfies condition 1 of the theorem 3. Let's introduction the following notations:

- $A > 0$ ($A \ge 0$) – matrix A has positive (non-negative) elements;
- $\mathbf{S}^{n \times n}$ – a set of real symmetric $n \times n$-matrices;
- E_n – identity $n \times n$-matrix;
- $(a_1, a_2, .., a_n)$ – horizontal concatenation of column vectors;
- $cl(X)$ – closure of the set $X \subseteq \mathbf{R}^n$ (in usual topology on $\mathbf{R}^n$);
- $\mathrm{Im}_H(X) = \{Hx \mid x \in X\}$, $H \in \mathbf{R}^{n \times n}$, $X \subseteq \mathbf{R}^n$.

The following lemma can be found in convex analysis literature:

Lemma 1. Let $X, Y \subseteq \mathbf{R}^n$ be open convex cones and $cl(X) \cap cl(Y) = \{\mathbf{0}\}$. Then there exists $p \in \mathbf{R}^n$ for which $\forall x \in X, y \in Y : p^T x < 0 < p^T y$.

We obtain the following corollary from this lemma.

Corollary. Let $X,Y \subseteq \mathbf{R}^n$ be convex cones such that $cl(X) \cap cl(Y) = \{\mathbf{0}\}$. Then there exists a nonsingular matrix $C \in \mathbf{R}^{n \times n}$ such that

$$\forall x \in \mathrm{cl}(X) \setminus \{\mathbf{0}\}, y \in \mathrm{cl}(Y) \setminus \{\mathbf{0}\} \;\; Cx < \mathbf{0} < Cy .$$

Proof. Let's assume that $X \neq \{\mathbf{0}\}$, $Y \neq \{\mathbf{0}\}$, because in the other case the corollary is trivial. Let's apply lemma 1 to X and Y. Then

$$\forall x \in \mathrm{cl}(X) \setminus \{\mathbf{0}\}, y \in \mathrm{cl}(Y) \setminus \{\mathbf{0}\} \;\; p^T x < 0 < p^T y$$

for some $p \in \mathbf{R}^n$. Let's denote

$$C(\varepsilon) = \underbrace{(p, p, .., p)}_{n}^T + \varepsilon E_n , \varepsilon \in \mathbf{R} .$$

Then

$$\forall x \in \mathrm{cl}\, X \cap S_1, y \in \mathrm{cl}\, Y \cap S_1 \;\; C(0)x < \mathbf{0} < C(0)y ,$$

where $S_1 = \{x \in \mathbf{R}^n \mid |x| = 1\}$ – unit sphere.

Nonempty sets $\mathrm{cl}\, X \cap S_1$ and $\mathrm{cl}\, Y \cap S_1$ are compact, so there are $a_0, b_0 \in \mathbf{R}$, such that

$$\max_{x \in \mathrm{cl}\, X \cap S_1} \max_{i=1..N} e_i^T C(0)x < a_0 < 0 < b_0 < \min_{y \in \mathrm{cl}\, Y \cap S_1} \min_{i=1..N} e_i^T C(0)y .$$

The function $\max_{i=1..N} e_i^T C(\varepsilon)x$ is continuous in ε, x, so there exists $\varepsilon^* > 0$, such that

$$\forall x \in \mathrm{cl}\, X \cap S_1, y \in \mathrm{cl}\, Y \cap S_1 .$$

Then

$$\max_{i=1..N} e_i^T C(\varepsilon^*)x \le a_0 < 0 < b_0 \le \min_{i=1..N} e_i^T C(\varepsilon^*)y ,$$

whence

$$\forall x \in \mathrm{cl}\, X \setminus \{\mathbf{0}\}, \; y \in \mathrm{cl}\, Y \setminus \{\mathbf{0}\} \;\; C(\varepsilon^*)x < \mathbf{0} < C(\varepsilon^*)y .$$

So we can put $C = C(\varepsilon^*)$. Corollary are proved.

Lemma 2. Let $X \subseteq \mathbf{R}^n$ be a convex cone and $A \in \mathbf{R}^{n \times n}$ be a matrix, such that $\mathrm{cl}\, X \cap \mathrm{Im}_A \mathrm{cl}\, X = \{\mathbf{0}\}$. Then there exists nonsingular matrix $H \in \mathbf{S}^{n \times n}$, such that $\forall x \in \mathrm{cl}\, X \setminus \{\mathbf{0}\} \;\; x^T H x > 0$ and $x^T (A^T H + HA) x \le 0$.

Proof. Let's apply the corollary from the previous lemma to the cones X and $\mathrm{Im}_A X$. Then

$$\forall x \in \mathrm{cl}(X) \setminus \{\mathbf{0}\}, y \in \mathrm{Im}_A \mathrm{cl}(X) \setminus \{\mathbf{0}\} \;\; Cx < \mathbf{0} < Cy$$

for some nonsingular matrix C. Let's put $H = C^T C$. Then for any $x \in \mathrm{cl}\, X \setminus \{\mathbf{0}\}$ the following inequalities hold

$$x^T Hx = (Cx)^T Cx > 0,$$

$$x^T (A^T H + HA)x = 2x^T C^T CAx = 2(Cx)^T (CAx) \le 0,$$

because $Ax \in \mathrm{clIm}_A X$ and $Cx < \mathbf{0} \le C(Ax)$.

Lemma are proved.

Lemma 2 can be used to prove a condition for existence of positive quadratic Lyapunov's function on the cone $\{x \in \mathbf{R}^n \mid x \ge \mathbf{0}\}$.

Theorem 5. Condition $\forall x \ge \mathbf{0}, x \ne \mathbf{0} \;\; Ax \ngeq 0$ is necessary and sufficient for existence of nonsingular matrix $H \in \mathbf{S}^{n \times n}$ such that $H > \mathbf{0}$ and $A^T H + HA < \mathbf{0}$.

Proof. Sufficiency follows from lemma 2, so let's prove necessity. Let's assume that there is matrix $H \in \mathbf{S}^{n \times n}$, such that $H > \mathbf{0}$ and $A^T H + HA < \mathbf{0}$, and prove that $\forall x \ge \mathbf{0}, x \ne \mathbf{0}$, $Ax \ngeq 0$. Let's suppose contrary: there exists vector $x_0 \ge \mathbf{0}, x_0 \ne \mathbf{0}$, such that $Ax_0 \ge \mathbf{0}$. Then $x_0^T (A^T H + HA)x_0 = 2(Hx_0)^T Ax_0 > 0$ because $Hx_0 \ge \mathbf{0}$, $Ax_0 \ge \mathbf{0}$ and $Hx_0 \ne \mathbf{0}$, $Ax_0 \ne \mathbf{0}$. The inequality $x_0^T (A^T H + HA)x_0 > 0$ contradicts assumption $A^T H + HA < \mathbf{0}$. So $\forall x \ge \mathbf{0}, x \ne \mathbf{0}$, $Ax \ngeq 0$.

Theorem are proved.

Let's show how to use theorem 5. Let's consider the system that describes local dynamics of linear hybrid automaton:

$$\dot{x} = Ax, \; Gx \ge \mathbf{0}, \tag{3}$$

where $A, G \in \mathbf{R}^{n \times n}$, and A is nonsingular. Let's apply a nonsingular change of variables $y = (y_1, .. y_n)^T = Tx$, such that for some indices i, j, $0 \le i < j \le n+1$, the condition $Gx \ge \mathbf{0}$ is equivalent to $y_1, .. y_i \ge 0, y_j, .. y_n = 0$.

Without loss of generality we can assume that equations $y_j,..y_n = 0$ are not present. Then system (3) will be equivalent to the system like

$$\dot{y} = TAT^{-1}y = \overline{A}y\ ,\ y_1,..y_n \geq 0\ , \tag{4}$$

where all variables y_i are non-negative. Then we can apply theorem 5: if $\forall y \geq \mathbf{0}, y \neq \mathbf{0}\ \ \overline{A}y \ngeq \mathbf{0}$, then there exists quadratic form $y^T Hy, H \in \mathbf{S}^{n\times n}$, such that $y^T Hy > 0$ and $y^T(\overline{A}^T H + H\overline{A})y < 0$ for any $y \geq \mathbf{0}, y \neq \mathbf{0}$. Then the following conditions are satisfied for any x, such that $Gx \geq \mathbf{0}$:

$$x^T T^T HTx > 0;$$

$$x^T T^T((TA^T T^{-1})^T H + HTAT^{-1})Tx = x^T(A^T T^T HT + T^T HTA)x < 0\ .$$

Then the quadratic form $x^T T^T HTx$ is the Lyapunov's function for the system (3). So it can be used to construct hybrid s-function.

Conclusion

In the article we considered the problem of stability of stationary states of hybrid automata. We obtained sufficient conditions for stability and instability of trivial stationary states with the help of s- and u-hybrid Lyapunov's functions. Also in the article we obtained sufficient condition for stability of stationary states of linear hybrid automata, and necessary and sufficient condition for existence of solution of Lyapunov's-like equation, which can be used to construct quadratic Lyapunov's functions on cones (theorem 5).

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CODE DESCRIPTIONS OF CLASSES METHOD FOR PATTERN RECOGNITION WITH MULTIPLE CLASSES

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Abstract: *A new method for solving pattern recognition tasks with multiple classes is proposed that is based on the standard ECOC approach. The main modification involves so-called code description of the classes. Unlike class code of ECOC the code descriptions represent multisets of codes of classes' training objects. Another modification takes advantage of optimization of the initial set of binary subtasks. The method's theoretical substantiation is based on the ideas of algebraic and logical approach to pattern recognition. Its advantage is demonstrated with the model data set.*

Keywords: *pattern recognition, multiple classes, ECOC, correctness, algebraic approach, logical approach, code description of class.*

ACM Classification Keywords: *I.5 Pattern Recognition — I.5.0 General.*

MSC: *68T10.*

Introduction

A pattern recognition task with multiple classes is considered hereinafter. The problem statement of its standard form from Zhuravlev, [1977a] is used.

Definition 1. *The pattern recognition task Z is defined as follows. Let $\tilde{S}_t(Z) = \{S_1, \ldots, S_m\}$ be a training sample described by real vectors $S_i = (a_{i1}, \ldots, a_{in})$, $i = 1, \ldots, m$. The sample is divided into l classes $K_1, \ldots, K_l$. The classification of the training sample objects is defined by information vectors $\alpha_i = (\alpha_{i1}, \ldots, \alpha_{il})$, where α_{ij} is a value of the predicate "$S_i \in K_j$". It is required to construct an algorithm A for calculating the classification of a new object S.*

If the classes do not overlap the classification of an object can be described by a single number $\alpha_i \in \{1, \ldots, l\}$ that is used further.

A pattern recognition task is called a task with multiple classes when $l > 2$. The case is distinguished by the fact that not all recognition methods are able to solve such tasks directly. Unlike nearest neighbors method or estimates calculating algorithm Zhuravlev, [1977a,b] such methods as support vector machine Cortes et al., [1995] or statistically weighed syndromes Kuznetsov et al., [1996] require additional stages. Firstly, a set of binary subtasks is solved directly and then their results are combined and interpreted in terms of initial set of classes. Some of the multistage approaches are quite obvious. They are one-vs-all Cortes et al., [1995] and one-vs-one Knerr et al., [1990]. Other examples can be found in Rocha et al., [2014]. There are also more general approaches. For example in ECOC (Error Correcting Output Codes) method Dietterich et al., [1995] arbitrary subdivisions of initial set of classes are used. Each class then achieves a binary code as well as each object. The decision on objects classification is made depending on closeness of its code to classes' codes. This method was further generalized in Allwein et al., [2000]. Binary subdivisions in that case consist only of a subset of initial classes and codes became ternary that allows including one-vs-one into general approach.

Thus, three general steps can be distinguished in the approach. Firstly, a set of binary subtasks is constructed. Random subdivisions are often used in that stage in general methods. Secondly, a recognition method is trained for each of the subtasks. Finally, the recognition results of a new object by the set of trained algorithms are interpreted in terms of initial classes.

A novel recognition method is described in the present article in which the first and the third steps are modified. The initial set of subtasks is reduced by solving an optimization task taking into account their recognition quality. The final interpretation is performed based on code descriptions of the classes (CDC), i. e. multisets of codes of its training objects instead of a single class' code.

The next chapter is devoted to the theoretical basis of the method. And the following one to the formal description of the method and experimental results.

Theoretical basis

The two main questions are answered in the present chapter. What requirements are to be satisfied by the reduced set of binary subtasks? And how to process CDCs taking into account importance of its elements and avoiding additional training?

The requirements for the set of subtasks are derived from considerations of correctness. Correctness of the final recognition method is required at least in the case of correctness of its first level algorithms.

Correctness is one of the key qualities considered considered in the algebraic recognition theory developed by academician Yu. I. Zhuravlev in 1970s Zhuravlev, [1977a,b]. It is understood as the ability of the algorithm to recognize given reference sample without errors. A great number of theoretical research of his students is devoted to the problem. Here are some formal definitions.

Definition 2. *Let's consider recognition task Z and reference set $\tilde{S}_r(Z) = \{S^1, \ldots, S^q\}$ with known classification $\alpha^t \in \{1, \ldots, l\}$, i. e. the predicate «$S^t \in K_{\alpha^t}$», $t = 1, \ldots, q$ holds. An algorithm A is called correct for the task Z and the reference set $\tilde{S}_r(Z)$ if $A(S^t) = \alpha^t$ for all $t = 1, \ldots, q$. Here $A(S^t) \in \{1, \ldots, l, \Delta\}$ is the algorithm's answer on S^t classification that is its class number or rejection Δ.*

Definition 3. *Let's consider recognition task Z and two disjoint subsets of the set of its classes $K^0 \in \{K_1, ..., K_l\}$, $K^1 \in \{K_1, ..., K_l\}$, $K^0 \cap K^1 = \emptyset$. A binary subtask of the task Z is defined as a recognition task Z' with the following properties: $\tilde{S}_t(Z') = \tilde{S}_t(Z) \cap (K^0 \cup K^1)$, $\tilde{S}_r(Z') = \tilde{S}_r(Z) \cap (K^0 \cup K^1)$, classes correspond to K^0 and K^1. The class K_i is called active in the binary subtask Z' if $K_i \in (K^0 \cup K^1)$. The binary subtask is called full if all initial classes are active in it. The number of active classes in the subtask $r(Z_i)$ is called its rank.*

The main condition of the existence theorem for the correct recognition algorithm Zhuravlev, [1977a,b] is pairwise non-isomorphicity of reference objects, i. e. existence of a training object for each pair of reference ones that distances of the reference objects to it differ in some feature subset: $\forall S^i, S^j \in \tilde{S}_r(Z)$, $\exists S_k \in \tilde{S}_t(Z)$, $p \in \{1, \ldots, n\}$, such as $|a_{kp} - a_p^i| \neq |a_{kp} - a_p^j|$. It and pairwise inequality of classes are the sufficient conditions for the existence of the correct algorithm in the algebraic closure of ECA (estimates calculating algorithms) family Dokukin, [2001]. Two-stage recognition scheme is defined as follows.

Definition 4. *Let's consider recognition task Z and W of its binary subtasks $Z_1, \ldots, Z_W$. An algorithm A_i solving the task Z_i, $i = 1, \ldots, W$ is called a first-stage algorithm. An algorithm A solving the task Z over outputs of the first-stage ones is called a second-stage algorithm.*

At that the vector $\gamma(K_i)$, there $\gamma(K_i)_j = 1$ if $K_i \in K_j^0$; $\gamma(K_i)_j = -1$ if $K_i \in K_j^1$; $\gamma(K_i)_j = 0$ otherwise; is called class K_i code, $i = 1, \ldots, l$, $j = 1, \ldots, W$. Class K_i rank $r(K_i)$ is the number of binary subtask in which it is active $r(K_i) = |\{\gamma(K_i)_j \mid \gamma(K_i)_j \neq 0, j = 1, \ldots, W\}|$.

Object's code $\gamma(S^t)$ is defined in a similar manner: $\gamma(S^t)_j = 1$ if $K_{A_j(S^t)} \in K_j^0$; $\gamma(S^t)_j = 1$ if $K_{A_j(S^t)} \in K_j^1$; $\gamma(S^t)_j = 0$ otherwise, $t = 1, \ldots, q$, $j = 1, \ldots, W$.

Let's consider recognition task Z with multiple classes and its binary subtasks $Z_1, \ldots, Z_W$. Evidently, if classes' codes are different in the set of subtasks and binary subtasks are full the ECOC algorith is correct. Indeed, reference vector codes are equal to their classes' codes due to correctness of algorithms $A_1, \ldots, A_k$, and since the codes are different there is no collisions possible.

The case of nonfull tasks is more difficult since objects of ignored classes can get arbitrary estimates. If then both full and nonfull subtasks are allowed the disadvantage can be fixed easily by adding to every nonfull subtask a full one in which all active classes of the former one are included into one metaclass and all inactive ones into another. Thus, the most difficult case is the case of nonfull subtasks. Let's describe the sufficient conditions for that case, but first let's change recognition scheme a little.

Let S be a reference object. Its estimate for the class K_j, $j = 1, \ldots, l$, is calculated as

$$\Gamma_j(S) = \left| \left\{ t \,|\, K_j \in K_t^d,\ A_t(S) = d,\ d \in \{0, 1\},\ t = 1, \ldots, W \right\} \right| . \tag{1}$$

I. e. each algorithm A_i by assigning object S into one of the metaclasses increases its estimate for each of containing classes by one. Object S is than assigned to a class with maximum estimate. If there are multiple classes with the maximum estimate the object is rejected.

Statement 1. *The described scheme is equivalent to ECOC.*

Proof. Indeed, let's consider an arbitrary reference object S^t and an arbitrary class K_j. Distance between object's code $\gamma(S^t)$ and class' code $\gamma(K_j)$, $d(S^t, K_j)$, calculated as a number of different positions is equal to a number of subtasks in which the class is inactive or it is active but the algorithm is incorrect. At the same time number of votes achieved by $\gamma(S^t)$ for $\gamma(K_j)$, $v(S^t, K_j)$, is equal to a number of binary subtasks in which the class is active and the algorithm is correct. Thus, $d(S^t, K_j) = W - v(S^t, K_j)$, and minimum of the former is achived simultaniously with the maximum of the latter. The statement is proved. □

Definition 5. *The number $d(K_i, K_j)$ of subtasks in wich both classes K_i and K_j are active but belong to different metaclasses is called distance between the classes.*

$$d(K_i, K_j) = \left| \{ t \in \{1, \ldots, W\} \mid \gamma(K_i)_t \neq \gamma(K_j)_t,\ \gamma(K_i)_t \neq 0,\ \gamma(K_j)_t \neq 0 \} \right| . \tag{2}$$

Let's consider recognition task Z and its nonfull binary subtasks $Z_1, \ldots, Z_W$ of equal rank $r < l$. Let all first-stage algorithms $A_1, \ldots, A_W$ be correct for the corresponding binary subtasks.

Theorem 1. *If for any two classes difference of their ranks is less than distance between them, i. e.*

$$r(K_j) - r(K_i) < d(K_j, K_i),\ \forall i, j = 1, \ldots, l,\ i \neq j \,,$$

then the second-stage algorithm A is correct.

Proof. Let's consider an arbitrary object $S^t \in K_i$. Since all the first-stage algorithms are correct S^t gets a vote for its class in all the binary subtasks where it is active, i. e. $\Gamma_i(S^t) = r(K_i)$. Let's consider another arbitrary class K_j. S^t gets a vote for K_j in two cases: if K_j is active and K_i is not, and if both classes are active and belong to the same metaclass. Thus, $\Gamma_j(S^t) \leq r(K_j) - d(K_i, K_j)$. Consequently, $\Gamma_i(S^t) - \Gamma_j(S^t) \geq r(K_i) - r(K_j) + d(K_i, K_j)$. By conditions of the theorem $r(K_i) - r(K_j) + d(K_i, K_j) > 0$ so $\Gamma_i(S^t) > \Gamma_j(S^t)$. Q.E.D. □

In case first-stage algorithms are incorrect there are two possibilities. If there exist only few errors the Theorem 1 can be modified to correct those by requiring greater distances between classes. But if errors are numerous the Theorem 1 becomes irrelevant and correctness can be achieved only by considering individual codes of the objects. And nonfull tasks become not very useful too. Indeed, if an algorithm solves some binary subtask incorrectly it nevertheless assigns objects of inactive classes to one of the metaclasses. The reference objects of an inactive class will most probably be assigned to the metaclasses unequally. Thus, by including K_j into the binary subtask the algorithms quality can be decreased a little or even improved.

Considering the initial question the following conditions will be required from the reduced set of subtasks. Firstly, only the full subtasks will be considered. Secondly, the distances between classes' codes will be maximized.

The second question refers to a method for combining first-stage results. It can be made for example by training new algorithm in an objects' codes feature set. But the additional training would require a separate sample and would complicate an algorithm. On the other hand the combination of a binary training information and the lack of a training stage is typical to the logical approach to recognition and to production expert systems Giarratano et al., [2007] in particular. At that the information can be represented either as description of objects in form of logical formulas and rules, or by precedents as an enumeration of objects and their classes. In the former case resolution methods are used and in the latter one recognition methods, for example the one described in Krasnoproshin et al., [1998]. There are also tasks in which both approaches are used simultaniously, such as medical diagnostics tasks Ablameiko et al., [2011]. At that object resolution method allows using precedent information for logical inference and fuzzy object resolution allows weighing the precedents.

The rest of the chapter is devoted to describing theoretical basis of the method starting with redefinition of the recognition task Z using terms customary to the logical approach Ablameiko et al., [2011]:

Definition 6. *Let X be a subset of objects of arbitrary nature. Let subsets $X_1, \dots, X_l$ called classes be defined in the set as well as the initial information I_0 of classes $X_1, \dots, X_l$. It is required to find an algorithm A defined at the whole set X that calculates a result in terms of belonging to a classes $X_1, \dots, X_l$ for an arbitrary object $x \in X$ using the information I_0.*

Let $S = \{s_1, \dots, s_n\}$ be a set of all features in universe of discourse of the task Z, where $n < \infty$; D_j be a set of its values $s_j \in S$. Without loss of generality let's say that $D_j = \{0, 1, \dots, |D_j| - 1\}$ and denote

$$D = \left\{0, 1, \dots, \max_j \{|D_j| - 1\}\right\} = \{0, 1, \dots, k-1\}.$$

It is supposed that all features possess values from the set D, where $k \neq 1$.

Definition 7. *The object is defined as correspondence*

$$p(s_1, \dots, s_n) = (D_1^p, \dots, D_n^p),$$

where $D_j^p \subset D$ is a set of values of feature $s_j \in S$ of object p, and $D_j^p \neq \varnothing$. Objects are called equal if $\forall j\ D_j^p = D_j^q$. If $|D_j^p| = 1$ the feature values is called known. If instead $|D_j^p| > 1$ the object p is considered a set of objects such that s_j enumerates D_j^p and the rest features coincide with corresponding features of p. A set of objects is also called a collection.

Definition 8. *An object is called normalized if all its features are known. A collection is called normalized if all its objects are normalized. Let's denote X^{norm} the set of all normalized objects of universe of discourse of the task Z: $X^{norm} = D^n$. An object for which one feature is known and the rest are undefined is called feature-object:*

$$p^j(s_1, \dots, s_n) = \left(D, \dots, D, \left\{d_j^{p^j}\right\}, D, \dots, D\right).$$

Let $V \subset X$, $W \subset X$ be some arbitrary collections. Product of objects p and q is defined by its features $D_j^{pq} = D_j^p \cap D_j^q$. Let's consider the following operations over objects and collections:

1. negation: $\overline{V} = X^{norm} \backslash V$;
2. multiplication: $V \wedge W = \bigcup\limits_{p \in V, q \in W} \{pq\}$, where objects with $D_j^{pq} = \varnothing$ are not included;
3. addition: $V \vee W = V \bigcup W$.

Statement 2. *The set of operation $\{\neg, \wedge, \vee\}$ over collections is full.*

The proof is described in Shut, [2012, 2014].

Thus, an algebra of objects $G =< \rho(X), \{\neg, \wedge, \vee\} >$ is defined where $\rho(X)$ is set of all possible collections. An algebra of normalized objects $G^{norm} =< \rho(X^{norm}), \{\neg, \wedge, \vee\} >$ is defined also, where $\rho(X^{norm})$ is the collection of normalized objects. Let's consider object resolution method for the task Z.

Definition 9. *An object r is called object resolvent for objects p and q, if feature values of r satisfy the condition:*

$$D_j^r = \begin{cases} D_j^p \bigcup D_j^q, j = h \\ D_j^p \bigcap D_j^q, j \neq h \end{cases},$$

where h is index of an arbitrary feature $s_h \in S$. The operation of constructing object resolvent is denoted $r = Or_h(p,q)$.

The appropriateness of object resolution method for constricting new objects based on precedent information is shown in Shut, [2012, 2014]. And the algorithm looks as follows. Let's consider class X_i and determine whether an object x belongs to it. Let's denote $X_i^0 = X^0 \bigcap X_i$ and A_1 the algorithm itself:

Step 1. Set $Y_i = X_i^0$.

Step 2. If $x \in Y_i$ go to step 6, otherwise go to step 3.

Step 3. Get from Y_i an unconsidered triplet (p, q, h), where p and q are objects and h is feature index. If there are no unconsidered triplets go to step 6.

Step 4. Calculate $r = Or_h(p,q)$. If there exists such index j $D_j^r = \varnothing$ go to step 3.

Step 5. If $r \notin Y_i$ set $Y_i := Y_i \bigcup \{r\}$. Go to step 2.

Step 6. Stop.

The algorithm A_1 is applicable for both direct and reverse inference. Direct inference means that if the algorithm has stopped by achieving the object x it implies that the collection $Norm(X_i^0)$ contains it. Thus, $x \in X_i$. Reverse inference states the opposite. If the algorithm has stopped by achieving object o it implies that $Norm(Y_i) = X$, i. e. Y_i potentially contains all objects of X. That is why $x \notin X_i$. If either result is not achieved it means that conclusions about objects x belonging to the class X_i or not can't be achieved with the algorithm.

The next step is to apply the resolution method to the multistage scheme. Let's consider the new task of W binary features where codes of the initial objects formed by the set of first-stage algorithms become objects. The objects are included to the set with their copies so that objects weights can be calculated as corresponding share of the code in class' code description. It allows applying a fuzzy object resolution method to the problem, i. e. object resolution method in case I_0 is described by fuzzy logic functions.

Let E be an arbitrary set. Let's define characteristic function $\mu_E(x)$ which value describes membership function of an element x to the set E: $\mu_E(x) \in [0,1]$. Let E_1, E_2 be fuzzy subsets of E. Let's consider the following fuzzy logic operations Kofman, [1982]:

1. addition: $\mu_{\overline{E_1}}(x) = 1 - \mu_{E_1}(x)$,
2. intersection: $\mu_{E_1 \bigcap E_2}(x) = \min\{\mu_{E_1}(x), \mu_{E_2}(x)\}$,
3. union: $\mu_{E_1 \bigcup E_2}(x) = \max\{\mu_{E_1}(x), \mu_{E_2}(x)\}$.

Every collection $V \subset X$ is assigned a characteristic function $\mu_V(p)$ that describes membership function of an object p. It is defined as the corresponding share of the objects. Let N^p denote total number of instances of p in X^0 and N_i^p denotes number of instances of p in X_i^0. Thus, $N^p = \sum_{i=1}^{l} N_i^p$. Let's define $\mu_{X_i}(p)$ as

$$\mu_{X_i}(p) = \frac{N_i^p}{N^p}.$$

so that $\mu_{X_i}(p) \in [0,1]$.

There are fuzzy logic analogues of the resolution method. One of them is described in Lee, [1972]. Let's describe an algorithm of solving the problem Z by using fuzzy object resolution method.

The algorithm A_1^f:

Step 1. The algorithm A_1 is applied to X. Let Y_i be the set of objects that are considered belonging to X_i, $i = 1, ..., l$ by applying it, i. e. $\forall p \in Y_i$, $\mu_{X_i}(p) > t$, t being a given threshold $t \in [0, 1]$.

Step 2. For every object $p \in X \backslash \left(\bigcup_{i=1}^{l} Y_i \right)$ the steps 3–4 are applied.

Step 3. For each class X_i the value of $\mu_{X_i}(p)$ is calculated.

Step 4. Let $\{\mu_{Y_v}(p)\} = \max_i \{\mu_{Y_i}(p)\}$, $w = \max_v \{v\}$. If $\mu_{X_w}(p) > t$ the object p is added to Y_w:

$$Y_w = Y_w \bigcup p.$$

Step 5. Stop.

The results of the algorithm A_1^f are interpreted as this. If $p \in Y_i$ when it stops then $p \in X_i$. Thus, the algorithm A_1^f assignes the object p to the class with maximum membership function.

The difference between fuzzy object resolution method and direct comparison of object's and class' codes can be seen in the following example.

Example 1. *Let's consider recognition task with 4 classes and let's define the binary subtasks as every pair of classes against every other (see. Fig. 1).*

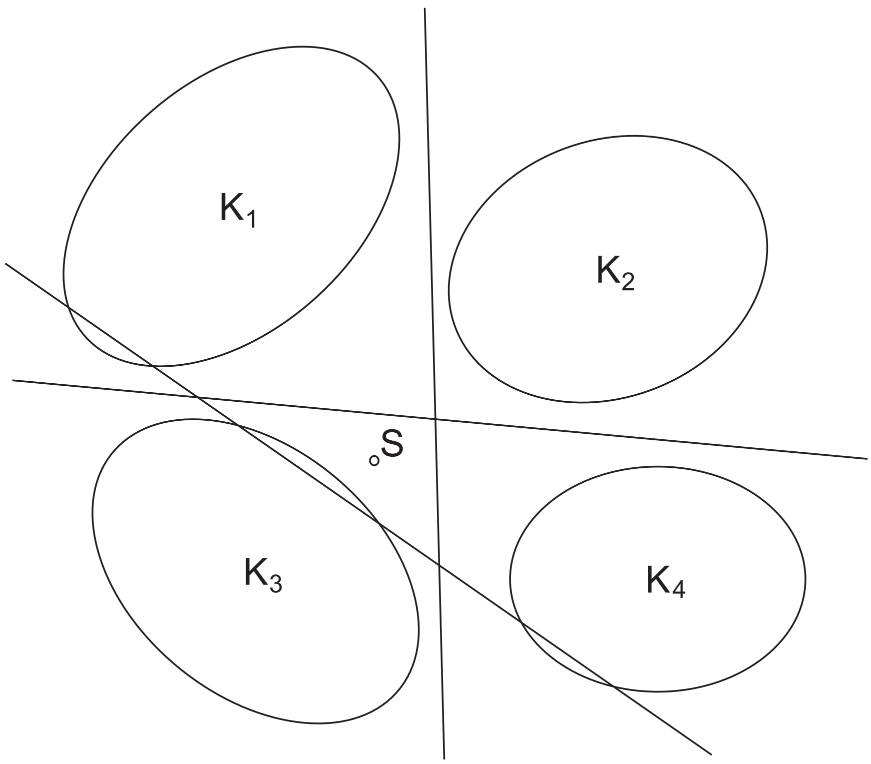

Figure 1: Interpretation of the first-stage results.

There are three of those tasks $\{1, 2\} - \{3, 4\}$, $\{1, 3\} - \{2, 4\}$, $\{1, 4\} - \{2, 3\}$. Consequently the classes are assigned the following codes: $\gamma(K_1) = (1, 1, 1)$, $\gamma(K_2) = (1, 0, 0)$, $\gamma(K_3) = (0, 1, 0)$, $\gamma(K_4) = (0, 0, 1)$. Lines in the Fig. 1 demonstrate trained linear recognition algorithms solving those binary subtasks. The first two tasks are solved correctly but the third one contains a vast number of errors. For example the whole second class is assigned a wrong metaclass and in the other three classes a part of objects (let's say 10 % for clarity) is treated wrong too.

Now let's consider the object S. It's code is $\gamma(S) = (0, 1, 1)$ and it is equally distant from the three nearest class codes: $\gamma(K_1) = (1, 1, 1)$, $\gamma(K_3) = (0, 1, 0)$, $\gamma(K_4) = (0, 0, 1)$. Thus, the object's class is impossible to tell though it is likely to be class 3.

However, the object resolution method will assign S to the third class. Indeed, first class precedents get codes $(1,1,1)$ (90%) and $(1,1,0)$ (10%), second class ones are $(1,0,1)$, third class ones are $(0,1,0)$ (90%), $(0,1,1)$ (10%), and fourth class ones are $(0,0,1)$ (90%), $(0,0,0)$ (10%).

The algorithm has its drawbacks. For example if a code is not presented in the training set it will not be recognized. To compensate for the said effect a modification of it is proposed. An exact description is given in the following chapter but the idea is to use a monotonically decreasing function in the neighborhood of each code to produce estimates for the missing ones.

Implementation

This chapter is devoted to the formal description of the proposed Code Description of Classes Method as well as the set of experiments demonstrating its quality. The method is based on the results of the previous chapter as well as other researches of the ECOC approach. In Berger, [1999] the probability of mixing of results of different classes was analyzed in case of random binary subtasks. It was shown that the algorithm is inclined to err if the codes of corresponding classes are close. In Dietterich et al., [1995] it is also stated that good performance of the algorithm requires both separability of metaclasses and codes. We will maximize distances between class codes that covers the mentioned conditions as well as conditions of the Theorem 1 in case of full subtasks.

The initial set of binary subtasks is generated randomly. After that an optimization task is solved to maximize distances between initial codes. The modification of the code set is made by weighing its components. Let $\|\alpha_{ij}\|_{l\times W}$ be a code matrix where l is the number of initial classes and W is the number of the binary subtasks. Then the task is described by the formula

$$\sum_{j=1}^{W} |\alpha_{\nu j} - \alpha_{\mu j}| x_j \geq y;\ \forall \nu,\ \mu;\ \nu > \mu;\ \nu,\ \mu = 1, \ldots, l\,,$$

$$\sum_{j=1}^{W} x_j = W\,,$$

$$y \to \max\,.$$

It should be noted that the optimal weights are often zeroed that allows reducing number of subtasks. The second role of the weights is to modify distance function by considering their importance

$$d(S^t, K_j) = d(\gamma(S^t), \gamma_j) = \sum_{j=1}^{W} |\alpha_{ij} - \beta_j| x_j\,, \tag{3}$$

where β is code of the object S^t, $\gamma(S^t) = \beta_j$.

After the subtasks are generated and first-stage algorithms are trained, code class descriptions are formed. They are multisets of codes of training objects calculated by the same first-stage algorithms. Let class K_j be described by a set of pairs $\{\gamma_{ji}, \nu_{ji}\}$, $i = 1, \ldots, W_j$, where $\gamma_{ji} = \gamma(S)$, $S \in K_j \cap \tilde{S}_t(Z)$ are codes of objects of class K_j; ν_{ji} is the share of code γ_{ji} in the description of class K_j, $\nu_{ji} = \frac{|\{S | S \in K_j \cap \tilde{S}_t(Z), \gamma(S) = \gamma_{ji}\}|}{|K_j \cap \tilde{S}_t(Z)|}$, W_j is the number of different codes in the description of class K_j. The estimate of an arbitrary object S for the class K_j is then calculated by formula

$$\Gamma_j(S) = \sum_{i=1}^{W_j} \nu_{ji} \frac{1}{(1 + d(\gamma(S), \gamma_{ji}))^2}\,,$$

where $d(\gamma_1, \gamma_2)$ is either Hamming distance between codes γ_1 and γ_2, or the distance defined in (3).

For the experimental purposes the two modifications were tested against simple ECOC separately and in combination. Thus, four methods were involved in tests. The tests were performed with a model data set of 12 classes. At that the random sets of binary subtasks of given cardinality were generated and the same ones were used with each of the methods. It should be mentioned that binary subtasks were solved with SVM implementation from scikit-learn package Pedregosa et al., [2011].

The model task is specially designed to complicate separation of the classes. 20 normally distributed samples were generated on the flat with centers ordered in five columns and four rows. After that some pairs or triplets of the said samples were joined into total of 12 classes (see Fig. 2).

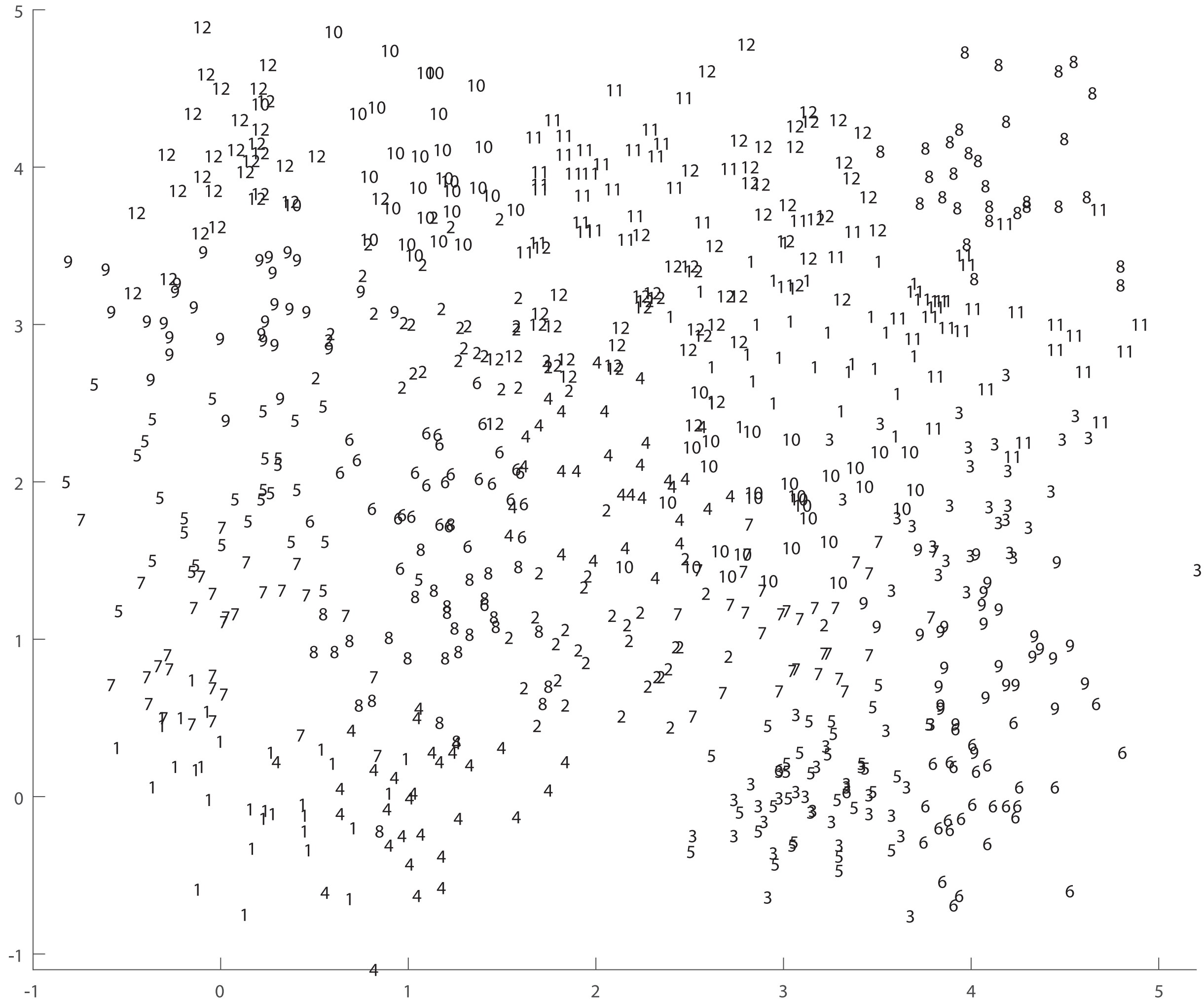

Figure 2: The model sample of 12 classes.

The results are show in Table 1. Though the experimental set is small it demonstrates one particular advantage of the CDC method. Its result in that task is better than that of ECOC regardless the number of initial subtasks. Even though separate application of its two modifications can provide better results, the same cannot be told about them. Indeed, optimization tend to provide better results with a greater number of subtasks while the code descriptions work better with less of them.

Table 1: The experimental results.

No. of subtasks	ECOC	Optimization	Code descriptions	CDC
20	68.9	66.7	70.8	70.0
40	69.9	68.4	71.6	70.9
60	69.5	71.1	71.2	72.5
80	69.6	71.3	70.0	71.1

As a conclusion it can be stated that the article gives only a first idea of the method. Its implementation is yet far from ideal. For example the form of monotonically decreasing function used in the algorithm is pure heuristic with no theorerical or experimental basis to support the choice. Although it seem to work in some cases.

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MULTIFRACTAL PROPERTIES OF BIOELECTRIC SIGNALS UNDER VARIOUS PHYSIOLOGICAL STATES

Abed Saif Alghawli, Lyudmyla Kirichenko

Abstract: *In the work the results of multifractal analysis of different electrobiological signals are represented. RR-interval's sequences of electrocardiograms obtained from patients before drug's application and after one; electroencephalograms of subjects when they perform any physical action and when they just imagine this and electroencephalograms of laboratory animals for the different phases of wakefulness and sleep were studied. Research was carried out by method of multifractal detrended fluctuation analysis. In all cases, there are significant differences of multifractal characteristics of different physiological states.*

In the work such characteristics of multifractal stochastic processes as a generalized Hurst exponent, scaling exponent, function of multifractal spectrum are discussed. It is shown that it is suitable use the generalized Hurst exponent as the quantitative measure. The results of numerical analysis showed that the estimates of the generalized Hurst exponent have normal distribution that allows to jumping to quantitative interval values. The possible specific values of the generalized Hurst exponent, which should be used in the knowledge bases of decision support systems, were proposed.

Keywords*: multifractal analysis, electrobiological signals, multifractal characteristics, multifractal stochastic processes, Hurst exponent, generalized Hurst exponent.*

ACM Classification Keywords*: G.3 Probability and statistics - Time series analysis, Stochastic processes, G.1 Numerical analysis, G.1.2 Approximation - Wavelets and fractals.*

Introduction

It is now recognized that many information, biological, physical and technological processes have a complex fractal structure. Fractal analysis is used for modeling, analysis and control of complex systems in various fields of science and technology. Fractal analysis is applied to predict seismic activity and tsunami and to determine the age of geological rocks in geology; to study the mutations and changes at the genetic level in biology; to predict the crisis and risk using financial series in economy; to study the turbulence and thermodynamic processes in physics [Mandelbrot, 1983; Feder, 1988; Schroeder, 1991].

Fractal geometry has been used in biology for over a quarter century. Application of fractal method opens up new possibilities in the study of the functional organization of living systems. Stable operation of such a complex, hierarchically organized system is provided by mutual subordination of structures belonging to different spatial scales. Numerous experimental and clinical data provide a basis for concluding that the study of fractal topology of various biological systems it will allow to lay the foundations of fractal diagnostics.

Subjects that exhibit fractal properties can be divided into two groups: self-similar (monofractal) and multifractal. Monofractal subjects are homogeneous in the sense that they have single scaling exponent. Their scaling characteristics remain constant on any range of scales. Multifractal subjects can be expanded into segments with the different local scaling properties. They are characterized by the spectrum of scaling exponents [Mandelbrot, 1983; Feder, 1988; Reidi, 2002].

Signals that are generated by complex self-regulating biological systems have a wide range of properties such as heterogeneity, nonlinearity, nonstationarity, presence of fluctuations and others. It was shown that for many systems biological signals have long-term correlation and fractal (self-similar) properties [Bak, 1987; Shlesinger, 1987; Peng, 1994; Bassingthwaighte, 1994; Goldberger, 2002; May, 2002]. In particular, multifractal properties have been detected in many bioelectric signals.

Review of the literature and problem statement

Heartbeat interval sequences were among the earliest physiological time series that have been discovered properties of self-similarity [Kobayashi, 1992]. In [Kantelhardt, 2002, 2003; McSharry, 2005] fractal and correlation properties of the time intervals between successive heartbeats during light sleep, deep sleep, and rapid eye movement sleep were investigated. In [Al-ani, 2007] the research was presented, which allowed using fractal analysis to automatically classify sleep stage using only the electrocardiogram (ECG) records. In [Kiyono, 2004, 2005, 2008] statistics, correlation and fractal properties of the heart rate for healthy and sick people were considered. In [Hoshiyama, 2008] the fractal exponents of heart rate variability for people practicing yoga and beginners were compared.

To date, it is shown that signals associated with cardiac activity are characterized by not only the self-similarity but also multifractal properties that reflect heterogeneity and nonstationarity of physiological processes. In particular this was a subject of study in [Ivanov, 1999, 2001; Stanley, 1999; Nunes Amaral, 2001] where it was shown that the heart rate had multifractal properties, which were different for healthy people and ones suffering from various diseases. In [Ching, 2007; Kiyono, 2009] multifractal properties of heart rate variability were investigated. In [Abry, 2010] authors suggest the methodology for multifractal analysis of heart rate variability based on wavelet transform.

Numerous investigations devoted to the study of fractal properties of time series of the electroencephalogram (EEG). In [Hwa, 2004] the new method of detecting stroke based on the scaling properties of human EEG time series was offered. In [Shin, 2007] the fractal characteristics of sleep EEG of healthy subjects were studied. In [Figliola, 2007] the EEG signals of birds to characterize the different stage of bird brain maturation were investigated. In [Leea, 2007; Leistedt, 2007] the fractal properties of the sleep EEG in acutely depressed men were analyzed. In [Abasolo, 2008] EEG recordings of patients with Alzheimer's disease were examined. In [Tingting Gao, 2008] the difference in EEG between eyes-closed and eyes-open conditions by fractal analysis was shown. In [Manickam, 2009] the research technique of cross-modal plasticity of blind based on fractal analysis of the EEG was proposed. The work [Tao Zhang, 2011] showed the differences in the multifractal characteristics of EEG for neural activity of epileptic and healthy rats. In [Marton, 2013] multifractal properties of multichannel EEG recordings were investigated. In [Zorick, 2013] multifractal analysis of EEG at different stages of sleep and wakefulness was performed. In [Harikrishnan, 2013] it was shown that the set of parameters characterizing the multifractal spectrum of both EEG and heart rate can distinguish between healthy and pathological states.

This list does not claim to be exhaustive and only shows how physiological fractal signals are widely and variously represented. It is obvious that the fractal characteristics reflect the essential features of the state of the organism. Studies suggest that multifractal methods can be successfully used for the analysis of physiological signals to determinate of functional changes in the organism performance. In most cases, the multifractal characteristics are considered rather qualitatively than quantitatively. However, for the application of the results of fractal analysis as knowledge in medical expert systems we need to use their quantitative interval values.

The purpose of the present work is to develop recommendations for the practical application of the results of multifractal analysis, for possible use as the quantitative characteristics, particularly in the knowledge bases of decision support systems.

Basic definitions and characteristics of fractal stochastic processes

Multifractality is a concept that is able be equally well with some minor modifications applied to functions as well as measures. In the description of the basic concepts and properties of multifractal processes, there are several approaches based on the properties of fractal sets and the moment characteristics of the stochastic processes. To better understand the properties of multifractal characteristics such as scaling exponent and multifractal spectrum, consider the approach based on the study of fractal dimensions of the inhomogeneous sets [Mandelbrot, 1983; Feder, 1988; Shuster, 1988; Reidi, 2002].

Self-similarity of fractal objects is confined in saving object’s structure of zooming. Let consider main characteristics of multifractal set. Suppose that, in general, multifractal attractor occupies some bounded region in d-dimensional Euclidean space and defines set of $N \to \infty$ points. Let divide the entire region into box of side ε and volume ε^d . Let consider the partition function $Z(q,\varepsilon)$ characterized by an exponent q $(-\infty < q < +\infty)$:

$$Z(q,\varepsilon)=\sum_{i=1}^{N(\varepsilon)} p_i^q(\varepsilon)\,, \tag{1}$$

where $p_i(\varepsilon)=\lim\limits_{N\to\infty}\frac{n_i(\varepsilon)}{N}$, $n_i(\varepsilon)$ is number of points into the box with number i, $N(\varepsilon)$ is total number of occupied cells that depends from the size of the box ε. Probabilities p_i characterize relative population of the box.

In general multifractal set is characterized of nonlinear function $\tau(q)$, that determines behavior of partition function $Z(q,\varepsilon)$ with $\varepsilon\to 0$:

$$Z(q,\varepsilon)\propto \varepsilon^{\tau(q)}\,. \tag{2}$$

Function $\tau(q)$ usually is called scaling exponent and defined as

$$\tau(q)=\lim_{\varepsilon\to 0}\frac{\ln Z(q,\varepsilon)}{\ln\varepsilon}\,. \tag{3}$$

In the case of homogeneous fractal set with fractal dimension D all busy boxes have the same number of points that mean $p_i(\varepsilon)=p(\varepsilon)=1/N(\varepsilon)$ and partition function is

$$Z(q,\varepsilon)=N^{1-q}(\varepsilon)=\varepsilon^{-D(1-q)}$$

and function $\tau(q)=(q-1)D$ is linear. If the distribution of points in the boxes isn't the same, the fractal set is heterogeneous, i.e. multifractal, and $\tau(q)$ is a nonlinear function. If $q\to+\infty$, the main contribution to the partition function is made by the boxes that contain the greatest number of particles n_i and, consequently, most likely characterized by the filling p_i. Conversely, if $q\to-\infty$, the main contribution to the partition function is made by the most sparse boxes with small values p_i. Thus, the function $\tau(q)$ shows how heterogeneous set of points is investigated.

Along with the scale exponent $\tau(q)$ for the multifractal characteristics of the set the function of multifractal spectrum (the spectrum of singularities) $f(\alpha)$ is used. The dependence of the probability from the box size $p_i(\varepsilon)$ has an exponential character

$$p_i(\varepsilon)\propto \varepsilon^{\alpha_i}\,, \tag{4}$$

where α_i is some exponent, in general various for the diverse boxes (a measure of the singularity). For the homogeneous fractal all of the exponents α_i are the same and equal to the fractal dimension D.

Function of multifractal spectrum $f(\alpha)$ characterizes a probability distribution for the diverse values α_i. If value $n(\alpha)d\alpha$ is probability of the fact that α_i is in the interval $(\alpha, \alpha + d\alpha)$, i.e. the number of the boxes i that have the same measure $p_i(\varepsilon)$ with $\alpha_i \in (\alpha, \alpha + d\alpha)$, then

$$n(\alpha) \approx \varepsilon^{-f(\alpha)}. \quad (5)$$

So function $f(\alpha)$ is fractal dimension of the some homogeneous fractal subset ξ_α from the original set ξ that is characterized by the same probabilities of the box filling $p_i(\varepsilon) \approx \varepsilon^{\alpha}$.

Taking into account the expressions (1) and (5), the generalized partition function $Z(q,\varepsilon)$ can be written by using function of multifractal spectrum $f(\alpha)$ the next way:

$$Z(q,\varepsilon) = \sum_{i=1}^{N(\varepsilon)} p_i^q(\varepsilon) \approx \int d\alpha\, n(\alpha)\varepsilon^{q\alpha} \approx \int d\alpha\, \varepsilon^{q\alpha - f(\alpha)}.$$

Formally, the transition of variables $\{q, \tau(q)\}$ to the variables $\{\alpha, f(\alpha)\}$ can be made with the help of the next Legendre transformations:

$$\begin{cases} \alpha = \dfrac{d\tau}{dq} \\ f(\alpha) = q\dfrac{d\tau}{dq} - \tau \end{cases} \quad \text{and} \quad \begin{cases} q = \dfrac{df}{d\alpha} \\ \tau(q) = \alpha\dfrac{df}{d\alpha} - f \end{cases}. \quad (6)$$

Figure 1 shows plots of multifractal characteristics $\tau(q)$ and $f(\alpha)$ for monofractal and multifractal stochastic processes. In the case of a monofractal process scaling exponent $\tau(q)$ is a straight line, and the function of multifractal spectrum $f(\alpha)$ is point.

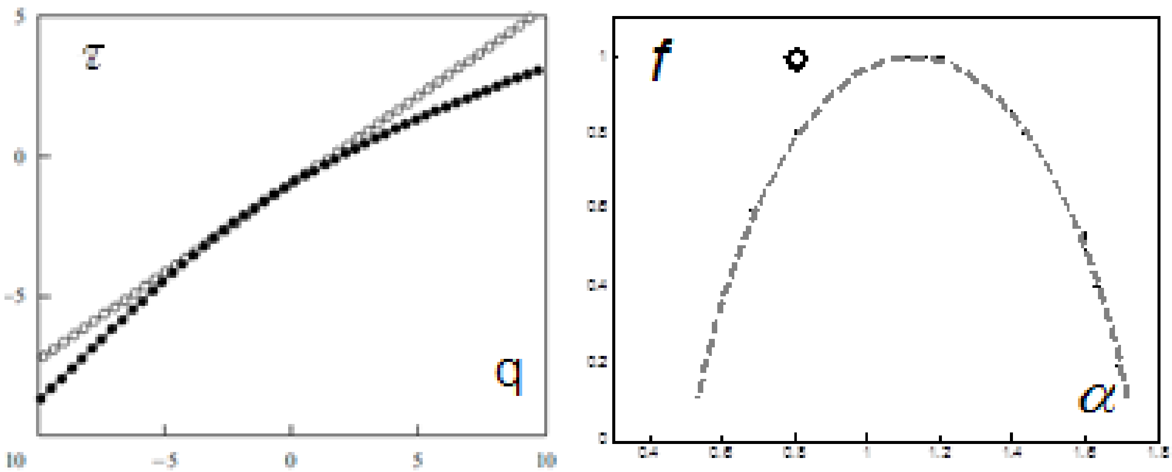


Figure 1. Functions $\tau(q)$ and $f(\alpha)$ for monofractal and multifractal stochastic processes

Now consider the basic concepts of self-similar and multifractal random processes [Calvet, 1997; Reidi, 2002; Kantelhardt, 2008; Abry, 2009].

Stochastic process $X(t)$, $t \geq 0$ with continuous real-time variable is said to be self-similar of index H, $0 < H < 1$, if for any value $a > 0$ processes $X(at)$ and $a^{-H}X(at)$ have same finite-dimensional distributions:

$$\text{Low}\{X(at)\} = \text{Low}\{a^H X(t)\}. \tag{7}$$

The notation $\text{Low}\{\cdot\}$ means finite distribution laws of the random process. Index H is called Hurst exponent. It is a measure of self-similarity or a measure of long-range dependence of process. For values $0,5 < H < 1$ time series demonstrates persistent behaviour. In other words, if the time series increases (decreases) in a prior period of time, then this trend will be continued for the same time in future. The value $H = 0,5$ indicates the independence (the absence of any memory about the past) of values of time series. The interval $0 < H < 0,5$ corresponds to antipersistent time series: if a system demonstrates growth in a prior period of time, then it is likely to fall in the next period.

One can show by choosing in (7) $a = 1/t$, that for the self-similar process, the following equality is held:

$$\text{Law}\{X(t)\} = \text{Law}\{\text{a}^{-H}X(at)\} = \text{Law}\left\{\left(\frac{1}{t}\right)^{-H} X(1)\right\} = \text{Law}\{t^H X(1)\}. \tag{8}$$

Using (8), the moments of the self-similar random process can be expressed as

$$E\left[|X(t)|^q\right] = E\left[|t^H X(1)|^q\right] = t^{qH} E\left[|X(1)|^q\right] = C(q)\cdot t^{qH}, \tag{9}$$

where the quantity $C(q) = E\left[|X(1)|^q\right]$.

In contrast to the self-similar processes (7) multifractal processes have more complex scaling behavior:

$$\text{Law}\{X(a\,t)\} = \text{Law}\{\mathcal{M}(a)\cdot X(t)\},\ a > 0, \tag{10}$$

where $\mathcal{M}(a)$ is random function that independent of $X(t)$. In case of self-similar process $\mathcal{M}(a) = a^H$.

For multifractal processes the following relation holds:

$$E\left[|X(t)|^q\right] = c(q)\cdot t^{qh(q)}, \tag{11}$$

where $c(q)$ is some deterministic function, $h(q)$ is generalized Hurst exponent, which is generally non-linear function. Value $h(q)$ at $q = 2$ is the same degree of self-similarity H . Generalized Hurst exponent of monofractal process does not depend on the parameter q : $h(q) = H$.

The generalized Hurst exponent $h(q)$ is connected with the function $\tau(q)$ by the ratio

$$\tau(q) = qh(q) - 1 . \tag{12}$$

Determination of function multifractal spectrum $f(\alpha)$ is carried out according to formulas (6).

Figure 2 shows plot of generalized Hurst exponent $h(q)$ for monofractal and multifractal stochastic processes. In the case of a monofractal process Hurst exponent is a constant..

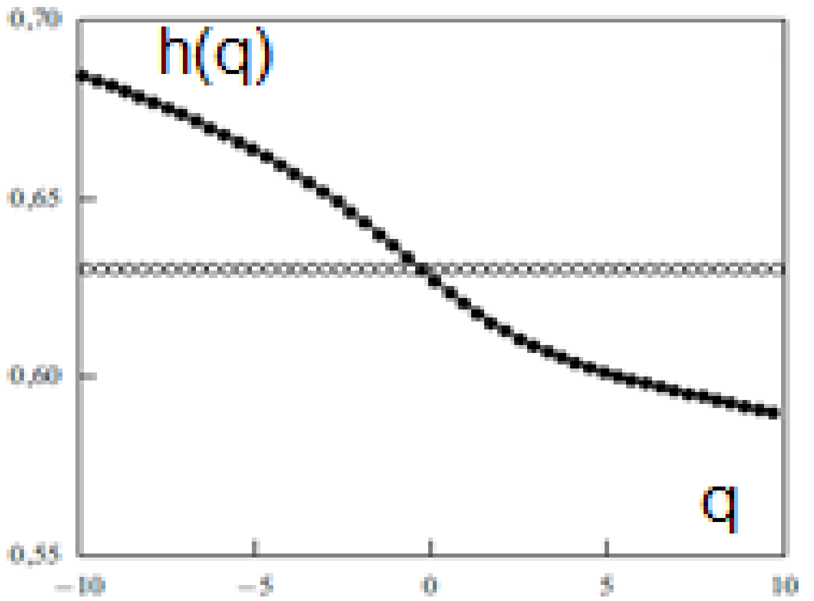


Figure 2. Functions $h(q)$ for monofractal and multifractal stochastic processes

Method of multifractal detrended fluctuation analysis

Two of the most popular research tools multifractal structure of the time series are the method of multifractal detrended fluctuation analysis (MFDFA) [Kantelhardt, 2002a, 2008] that is focused on the processing of non-stationary trended series, and the method of modulus maxima of wavelet transform (WTMM) [Muzy, 1993; Mallat, 1998; Kantelhardt, 2008]. Both methods are powerful tools for statistical processing of time-dependent processes.

The results of numerical studies show that in the investigation of a time series of unknown complex fractal structure the method MFDFA has to be used in the first place because it is easier to understand and implement [Oswiecimka, 2005, 2006; Kantelhardt, 2002a, 2008; Кириченко, 2011, 2011a]. Most investigations presented in the review were carried out by the method MFDFA.

According to the MFDFA method, for the initial time series $x(t)$ the cumulative time series $y(t)=\sum_{i=1}^{t}x(i)$ is constructed which is then divided into N segments of length τ, and for each segment $y(t)$ the following fluctuation function is calculated:

$$F^2(\tau)=\frac{1}{\tau}\sum_{t=1}^{\tau}(y(t)-Y_m(t))^2\ , \tag{13}$$

where $Y_m(t)$ is a local m-polynomial trend within the given segment. The averaged on the whole of the time series $y(t)$ function $F(\tau)$ has scaling on the segment of length τ:

$$F(\tau)\propto\tau^H\ .$$

In the study of multifractal properties the dependence of the fluctuation function $F_q(\tau)$ of a parameter q is considered:

$$F_q(\tau)=\left\{\frac{1}{N}\sum_{i=1}^{N}[F^2(\tau)]^{\frac{q}{2}}\right\}^{\frac{1}{q}}\ . \tag{14}$$

Since in the case $q=0$ expression (14) contains the ambiguity instead of it the following expression is used:

$$F_q(\tau)=\mathrm{Exp}\left\{\frac{1}{N}\sum_{i=1}^{N}\ln[F^2(\tau)]\right\}\ .$$

If the investigated series is multifractal and has a long-term dependence, the fluctuation function is represented by a power law

$$F_q(\tau)\propto\tau^{h(q)}\ , \tag{15}$$

where $h(q)$ is generalized Hurst exponent.. For monofractal time series the fluctuation function $F_q(\tau)$ is the same for all values q, and the generalized Hurst exponent does not depend on the parameter q: $h(q)=H$.

Figure 3 shows the fluctuation functions $F_q(\tau)$ for monofractal (left) and multifractal (right) processes of parameter values $q=\{-5,-2,0,2,5\}$.

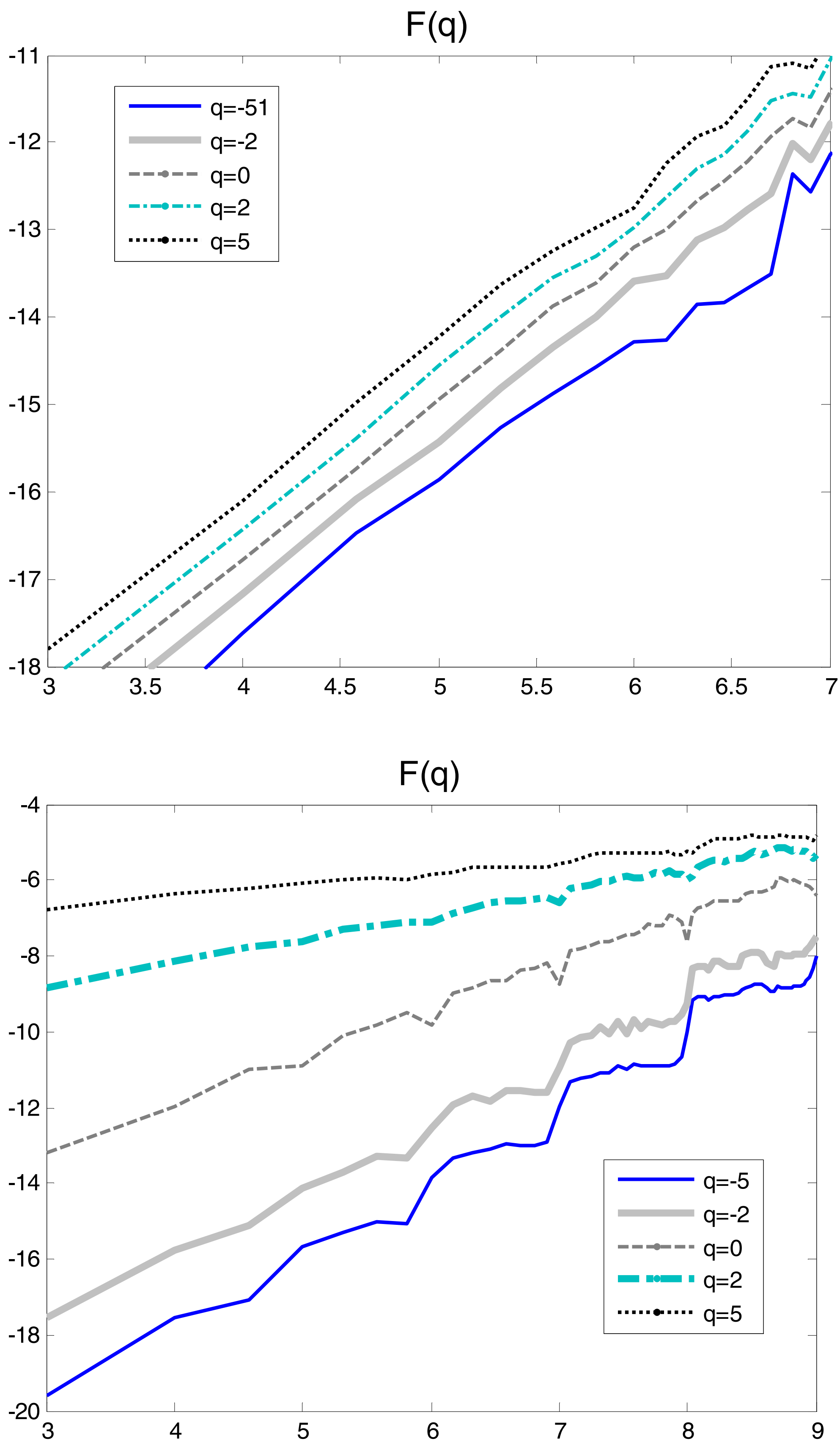


Figure 3. Functions $F_q(\tau)$ for monofractal (left) and multifractal (right) processes

At positive values q major contribution to the function $F_q(\tau)$ is given by segments which show great deviations of $F^2(\tau)$ and in case negative q the segments with small variances of $F^2(\tau)$ dominate. Thus, at negative values q of the generalized Hurst exponent $h(q)$ describes the segments, showing small fluctuations, and at positive q function $h(q)$ characterizes the segments with large fluctuations.

Visualization of inhomogeneity of time series in its multifractal characteristics

One of the most important properties of physiological signals which characterize the state of the organism is their inhomogeneity. Consider as a degree of inhomogeneity of time series which is reflected in its multifractal characteristics.

The simplest model of a multifractal process with the desired properties is a deterministic binomial multiplicative cascade [Feder, 1991; Calvet, 1997; Reidi, 2002]. In its construction, the initial unit interval is divided into two equal intervals, which are assigned weights p_1 and $p_2 = 1 - p_1$, respectively. Then the same procedure is repeated with each of the intervals. As a result, the second step has 4 intervals with weighting coefficients p_1^2, $p_1 p_2$, $p_2 p_1$ and p_2^2. If the number of steps $n \to \infty$ and $p_1 \neq p_2$, we arrive at a limit measure, which is a inhomogeneous fractal set.

Figure 4 shows the time series of values a binomial cascade for values $p_1 = 0.6$ and $p_1 = 0.8$. The number of iterations $n = 10$, i.e. the length of the realization equals 2^{10}. It is obvious that with increasing the weighting coefficient p_1 the inhomogeneity of time series increases also.

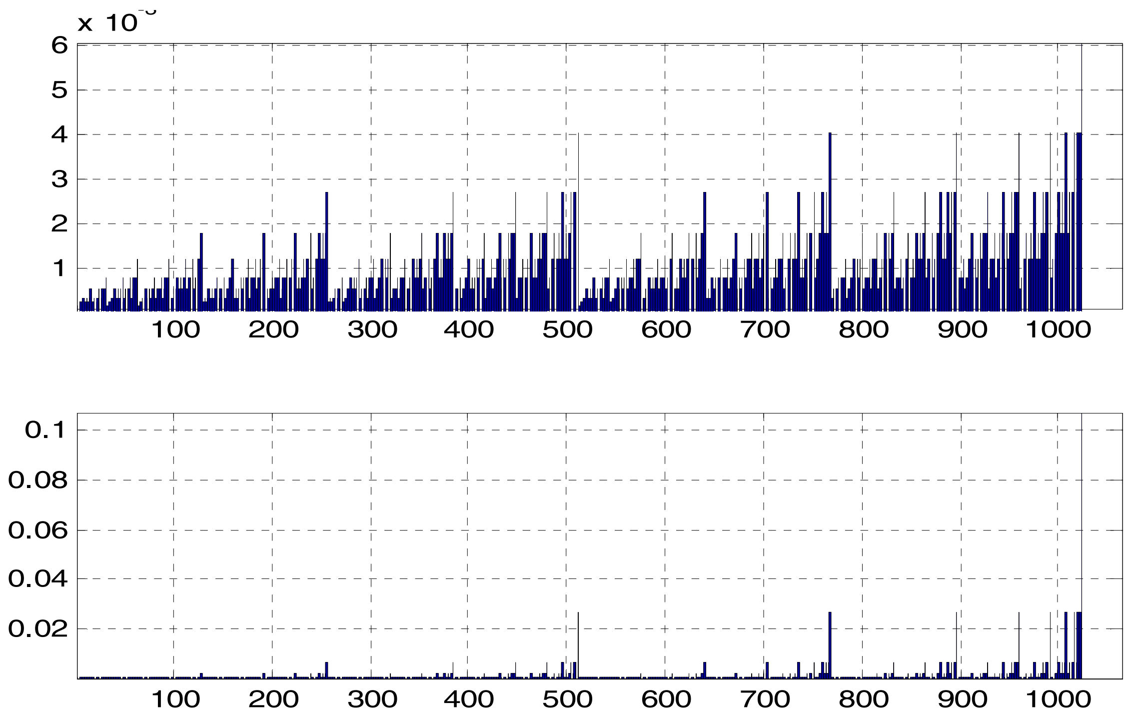


Figure 4. Time series of binomial cascade for values $p_1 = 0.6$ (top) and $p_1 = 0.8$ (bottom).

In this case multifractal characteristics of the binomial process depend only on the weighting coefficient p_1 and are calculated analytically:

$$\tau(q) = \frac{-\ln(p_1^q + p_2^q)}{\ln 2}, \quad h(q) = (\frac{1}{q} - \frac{\ln(p_1^q + p_2^q)}{q \ln 2}),$$

$$\begin{cases} \alpha = -\dfrac{p_1^q \ln(p_1) + p_2^q \ln(p_2)}{\ln 2\,(p_1^q + p_2^q)} \\ \\ f(\alpha) = -\dfrac{q}{\ln 2}\dfrac{p_1^q \ln(p_1) + p_2^q \ln(p_2)}{(p_1^q + p_2^q)} + \dfrac{\ln(p_1^q + p_2^q)}{\ln 2} \end{cases}.$$

Figure 5 shows plots of the generalized Hurst exponent $h(q)$, the scaling exponent $\tau(q)$ and the function of multifractal spectrum $f(\alpha)$ for time series of binomial cascade for values of weighting coefficient $p_1 = \{0.6, 0.7, 0.8, 0.9\}$. It should be noted that with increasing inhomogeneity of series the value $\Delta h = h(q1) - h(q2)$ increases, the scaling exponent $\tau(q)$ becomes more convex and the range of multifractal spectrum $f(\alpha)$ becomes a wider.

Investigation results

1. Research of multifractal characteristics of RR-interval's sequences.

It's known that for the diagnosis and detection of diverse heart's diseases analysis of the electrocardiogram has an important place. ECG is a recording of electrical heart's activity. The slightest deviation from the norm may indicate the violation of the cardiac rhythm and the presence of diverse diseases. One of the methods of diagnosing heart diseases is analysis of the series constructed by the RR-intervals.

RR-interval is the time interval between adjacent teeth of electrocardiogram and it equals to the duration of the cardiac cycle. These intervals are very important in determining the heart rate and diagnosis of diverse types of cardiac arrhythmias. Figure 6 [PhysioNet] shows the construction of RR-interval's sequences. It's known, that these types of series have chaotic structure [Shuster, 1988; Hoyer, 1997], so it's possible to analyze them using multifractal methods.

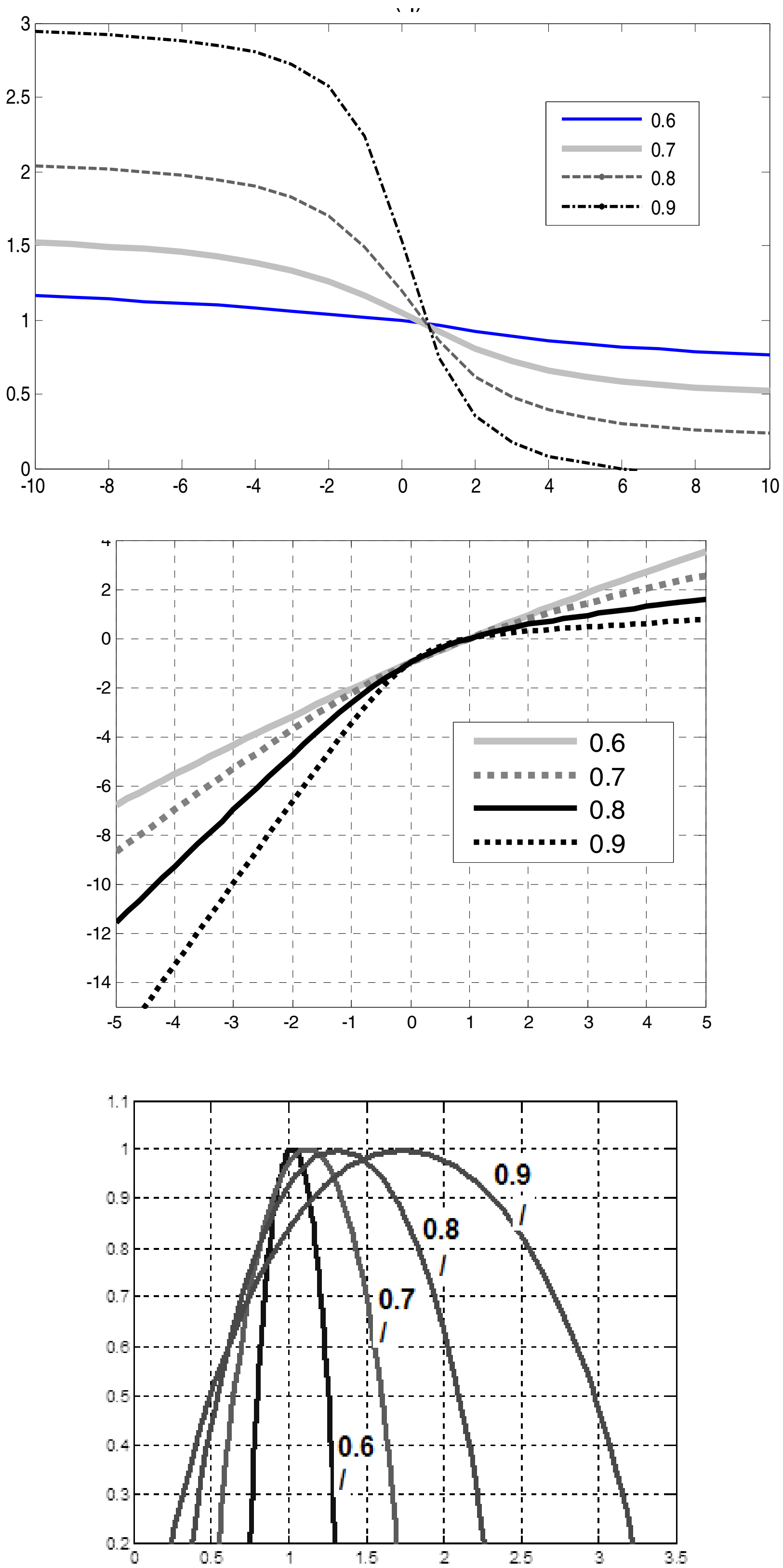


Figure 5. Functions $h(q)$, $\tau(q)$ and $f(\alpha)$ of binomial cascade for different values p_1

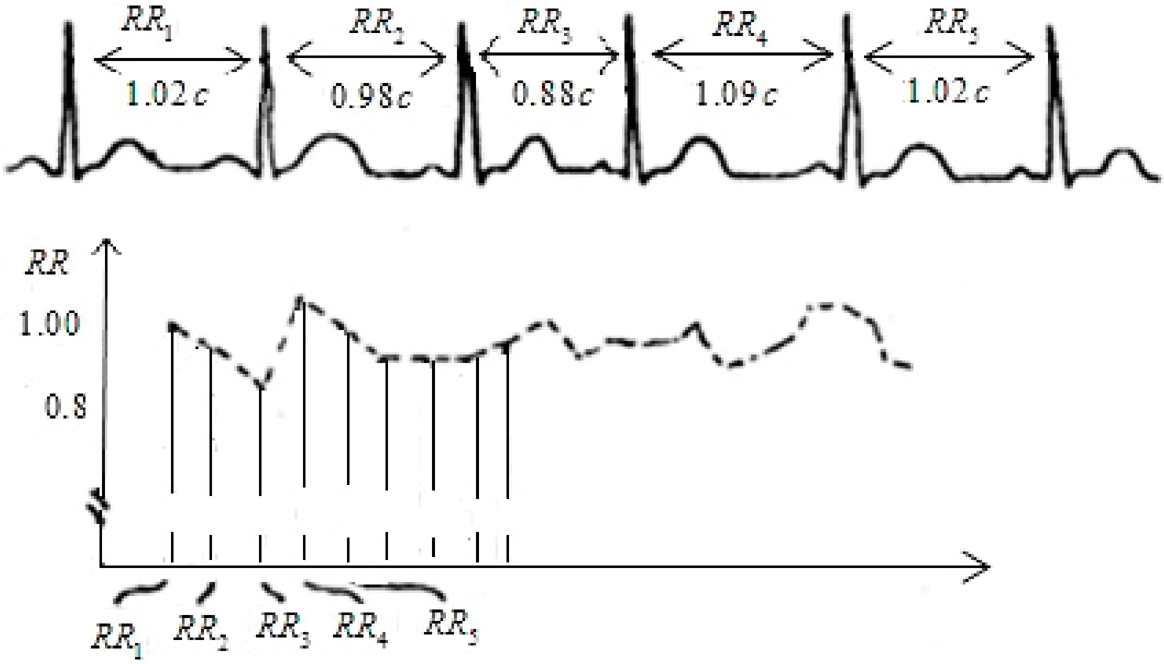


Figure 6. Image of normal ECG-signal with RR-intervals and constructing of RR-interval's sequence

Initial data for the research were obtained on the particularized website [PhysioNet] containing an extensive medical database. Figure 7 shows RR-interval's sequence, generalized Hurst exponent $h(q)$, scaling exponent $\tau(q)$ and multifractal spectrum $f(\alpha)$ which are typical for RR-intervals of the person, who has no heart diseases.

The database contains cardiogram records of the patients involved in medication trials. The medical investigation included patients belonging to the age group from 45 to 69 years who have a heart arrhythmia. The data of RR-intervals before and after taking medication were used to treat and prevent tachycardia by increasing heart rate. Figure 8 shows the RR-interval's sequences, generalized Hurst exponents $h(q)$, scaling exponents $\tau(q)$ and multifractal spectrums $f(\alpha)$ of patient that were a typical for the majority of patients before and after drug application.

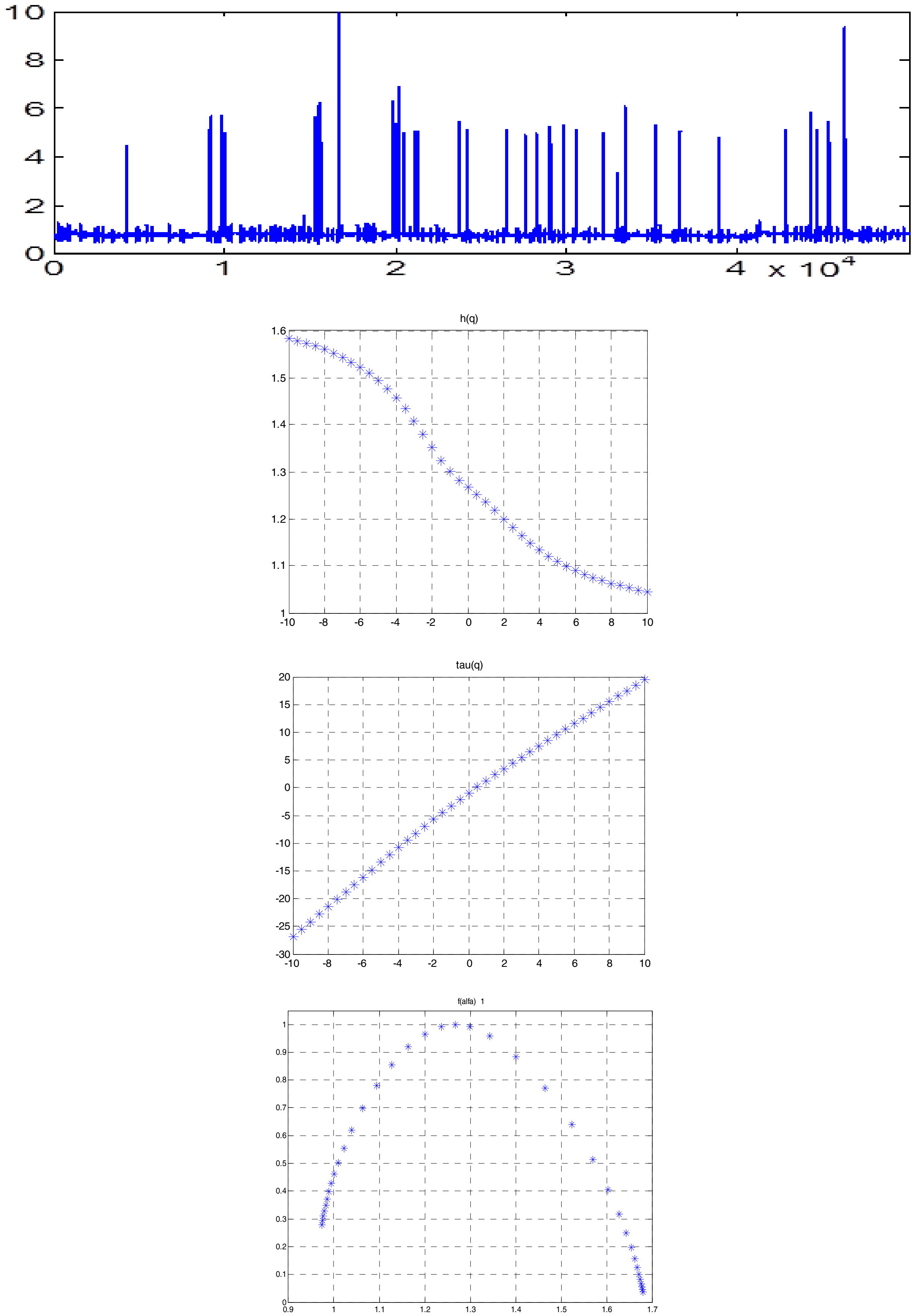


Figure 7. RR-intervals, generalized Hurst exponent, scaling exponent and multifractal spectrum of healthy person

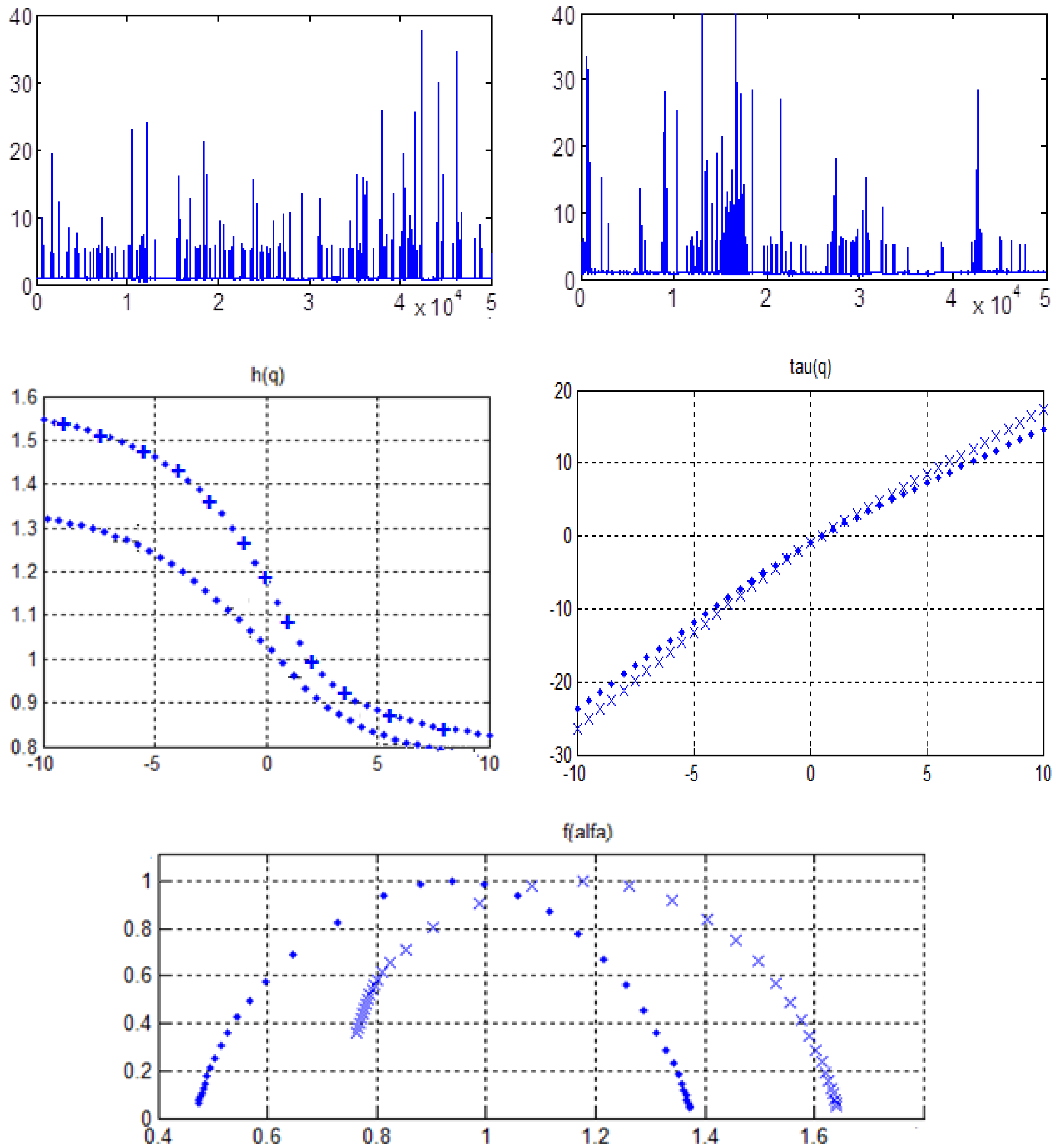


Figure 8. RR-intervals, generalized Hurst exponent, scale exponent and multifractal spectrum before drug's application (points) and after application (crosses).

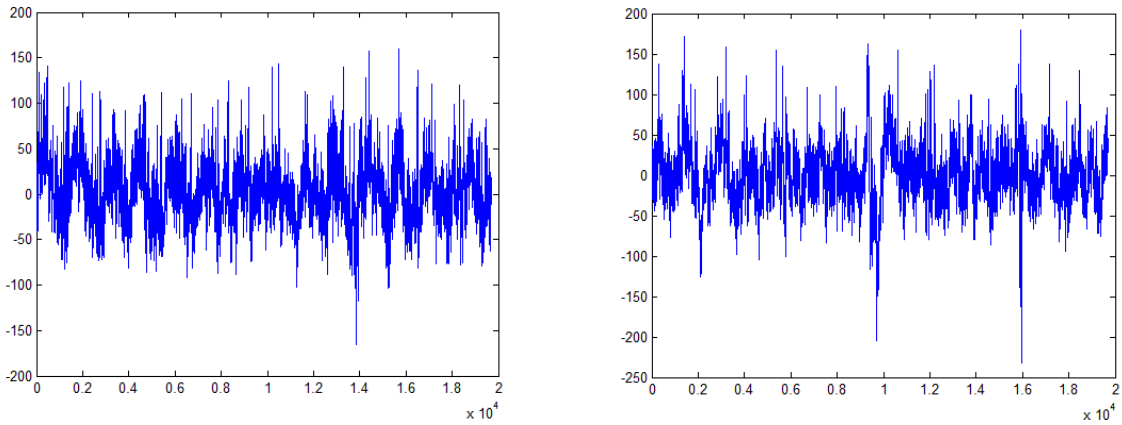
Researchers have shown that drug's application causes changes of multifractal characteristics of RR-interval's sequence. More visual characteristics that distinguishes time series before and after medication are function of multifractal spectrum $f(\alpha)$ and generalized Hurst exponent $h(q)$. Almost all patients have shift to the right of $f(\alpha)$ or shift up of $h(q)$ after medication, i.e. values of these functions has increased and became closer to the characteristics of healthy persons.

2. Research of multifractal characteristics of EEG records for real and imagined actions.

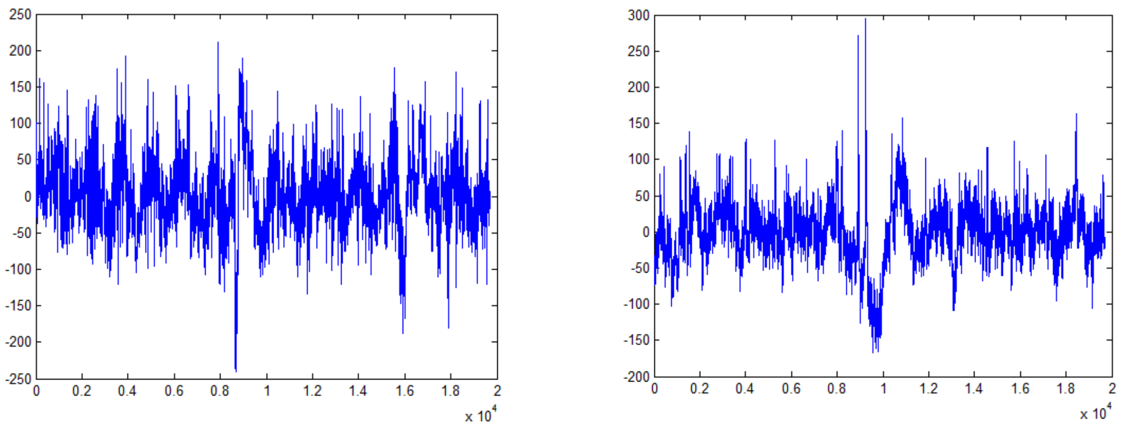
In the work investigation is conducted as multifractal characteristics of EEG of person change when he performs any physical action, and when he just imagines that he does. As experimental data the EEG signals of subjects performing certain actions were taken [PhysioNet]. Each subject performed the following complex of actions:

- In the right (left) corner of the computer screen a circle appeared and the subject clenched respectively his right (left) hand into a fist;
- In the right (left) corner of the computer screen a circle appeared and the subject just imagined that he clenched his hand into a fist, although in fact hand remained motionless.

In Figure 9 the plots of initial EEG time series for the two subjects who performed described above tasks are given. To the left EEG records when the subject clenches his hand into a fist are presented. To the right we can see EEG records when the subject just imagines that he does.



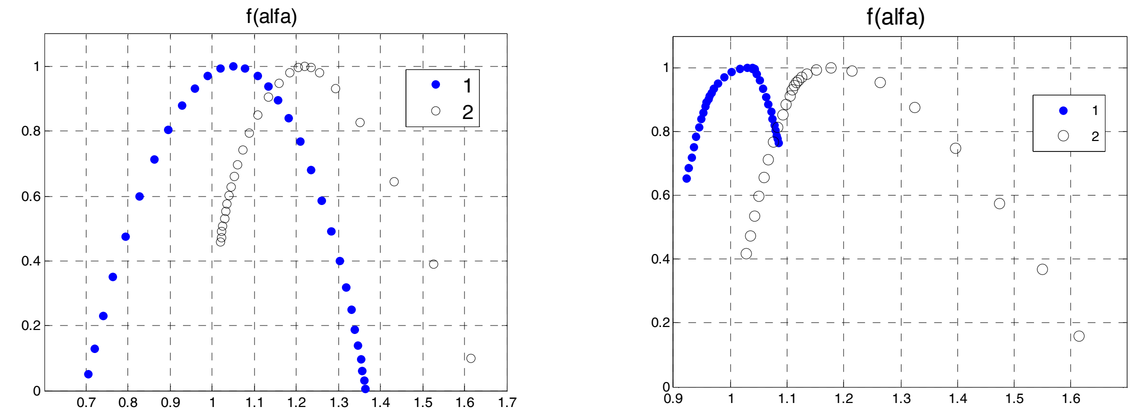
Subject 1



Subject 2

Figure 9. EEG records: subject clenches his hand into a fist (left) and subject just imagines that he does (right)

The multifractal characteristics of the corresponding time series were studied. The function of multifractal spectrum $f(\alpha)$ demonstrates the differences between the two states most graphically. Figure 10 presents functions of multifractal spectrum corresponding to the EEG records shown in Figure 9. Line 1 corresponds to the state when subject clenches his fists, and Line 2 corresponds when subject just imagines that he does.



Subject 1 Subject 2

Figure 10. Functions of multifractal spectrum of EEG

Line 1: subject clenches his fists; **Line 2:** subject just imagines that he does.

Thus, for this experiment function under the imagined action is significantly shifted to the right that makes it possible to distinguish between two states of the subject. During the study EEG records obtained with different electrodes were examined. It was found that EEG records of a number of electrodes are more sensitive to variations in the physical activity of the person and multifractal characteristics obtained from these records for the real and imagined actions differ significantly. In the investigation of EEG records of other electrodes the explicit relationship between the multifractal characteristics has not been identified.

They also the EEG records when subjects performed (imagined) other actions were considered. In the case of the movements of one hand the function of multifractal spectrum was shifted to the right as to whether when a subject was just imagined motion data. For the task with the movement of both hands the function of multifractal spectrum was contrary shifted to the right. Moreover, the shift of multifractal spectrum depends on the selected electrode.

3. Research of multifractal characteristics of EEG records for the different phases of wakefulness and sleep.

In the work we investigated EEG records of laboratory animals, which were divided into phases of wakefulness (AWAKE), slow-wave sleep (SWS) and rapid eye movement sleep (REM). Figure 11 shows typical realizations of the EEG for the different phases of wakefulness and sleep.

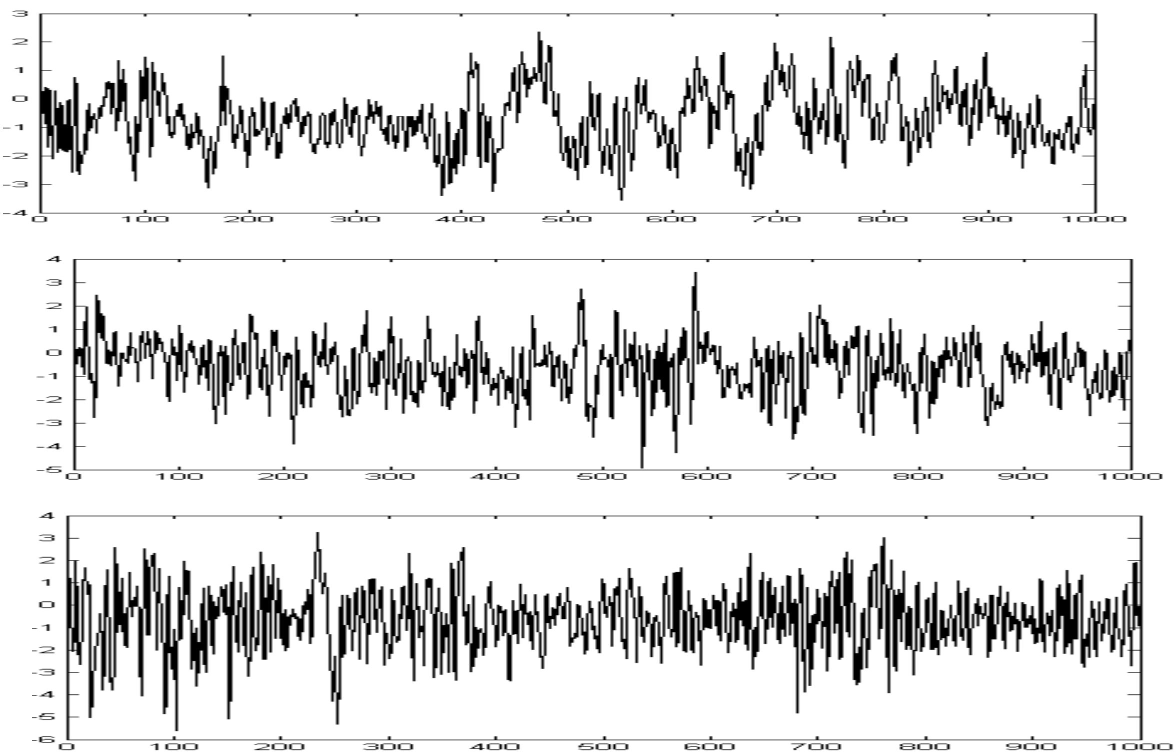


Figure 11. EEG records: AWAKE (top), SWS (middle) and REM (down)

The conducted multifractal analysis showed significant differences in the characteristics of the EEG records of phases of wakefulness and sleep. Figure 12 shows generalized Hurst exponent $h(q)$, scaling exponent $\tau(q)$ and multifractal spectrum $f(\alpha)$ which corresponding to the EEG records shown in Figure 11.

The analysis also shows that there is undoubted long-term dependence for EEG of wakefulness: in this case Hurst exponent $H = h(2)$ appreciably more than 0.5. The phase of slow-wave sleep is characterized by antipersistence, in this case the Hurst exponent H takes values in the range of less than 0.5. For REM sleep estimates of the Hurst exponent are close to the value 0.5, they have the meanings and larger and less than 0.5. It characterizes a very weak dependence EEG autocorrelation in this case.

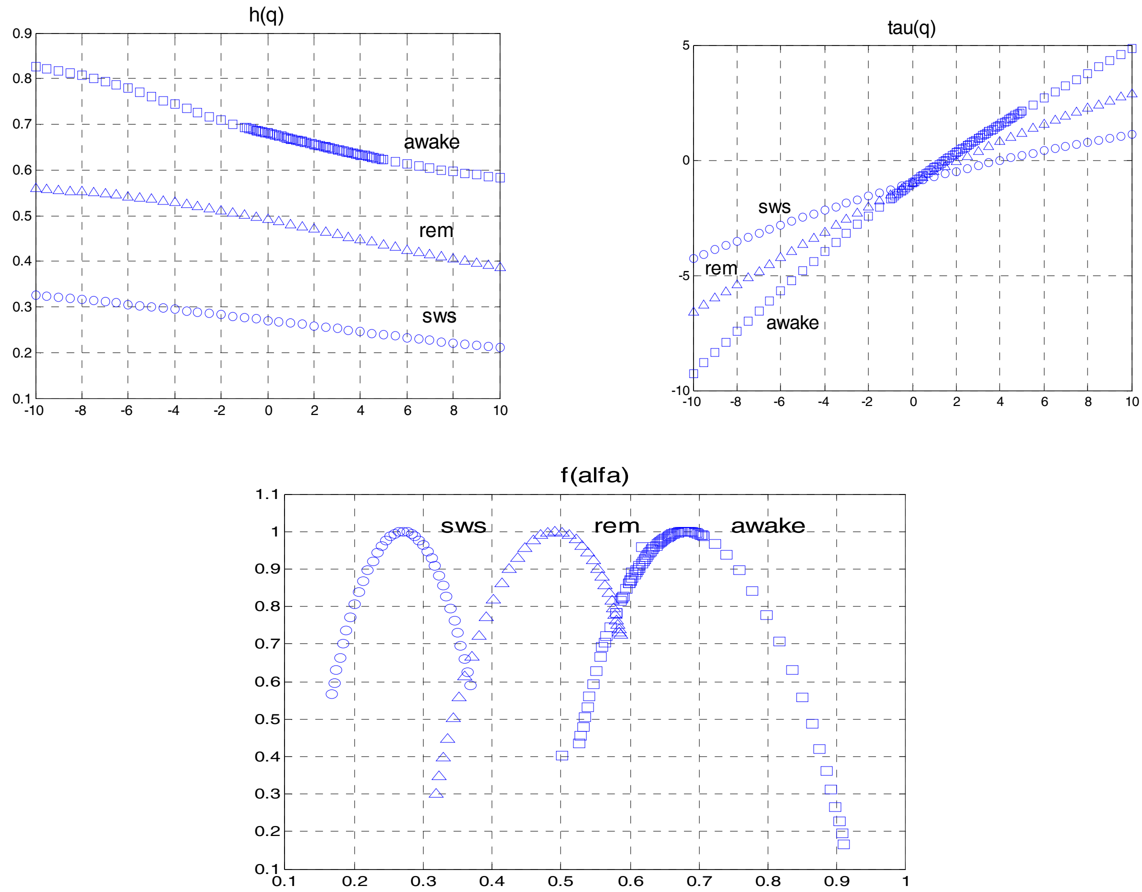


Figure 12. Generalized Hurst exponent, scale exponent and multifractal spectrum of the EEG records: AWAKE (boxes), SWS (bubbles) and REM (triangles)

Quantitative characteristic of multifractality degree

In practice, we are dealing with estimates of multifractal characteristics obtained by the realizations of finite length. In addition to variations in characteristics associated with features and uniqueness of organism, the variations related to inhomogeneity and lengths of the physiological signal have essential value. The accuracy of sample characteristics strongly depends on the length of signal realization.

Consider which of the multifractal characteristics: generalized Hurst exponent $h(q)$, scaling exponent $\tau(q)$ and multifractal spectrum $f(\alpha)$ is the most appropriate to apply for quantitative estimating. Since the functions $h(q)$, $\tau(q)$ and $f(\alpha)$ have one-to-one correspondence between themselves it is sufficient to use only one function as the basic characteristic. It is suitable use the generalized Hurst exponent $h(q)$ for this because the scaling exponent $\tau(q)$ in many cases is visual poorly informative (see for example Fig. 8) and the values of function of multifractal spectrum $f(\alpha)$ essentially depend on length of the of realization [Oswiecimka, 2006; Кириченко, 2011a] and need of more complex description.

For multifractal processes, the question of the distribution law of generalized Hurst exponent estimates was considered in a number of works, where it was shown numerically and analytically that the estimates are normal random variables. In the [Kantelhardt, 2002a; Павлов, 2007] it is shown that the large values parameter q lead to large errors. In [Kantelhardt, 2002a; Oswiecimka, 2006; Кириченко, 2011] the laws of distribution estimates $h(q)$ at different values of the parameter q were investigated. The analysis of the sample distribution laws $h(q)$ has shown that at $q > 0$ estimates has normal distribution the parameters of which depend on the value q . At $q < 0$ the sample values of the generalized Hurst exponent $h(q)$, in general, are not normally distributed. Figure 13 shows the values of the function $h(q)$ ($0 \le q \le 5$) and histograms of the estimates $h(q)$ received from realizations of the stochastic binomial cascade of length 1024 at $q = 1$ and $q = 5$.

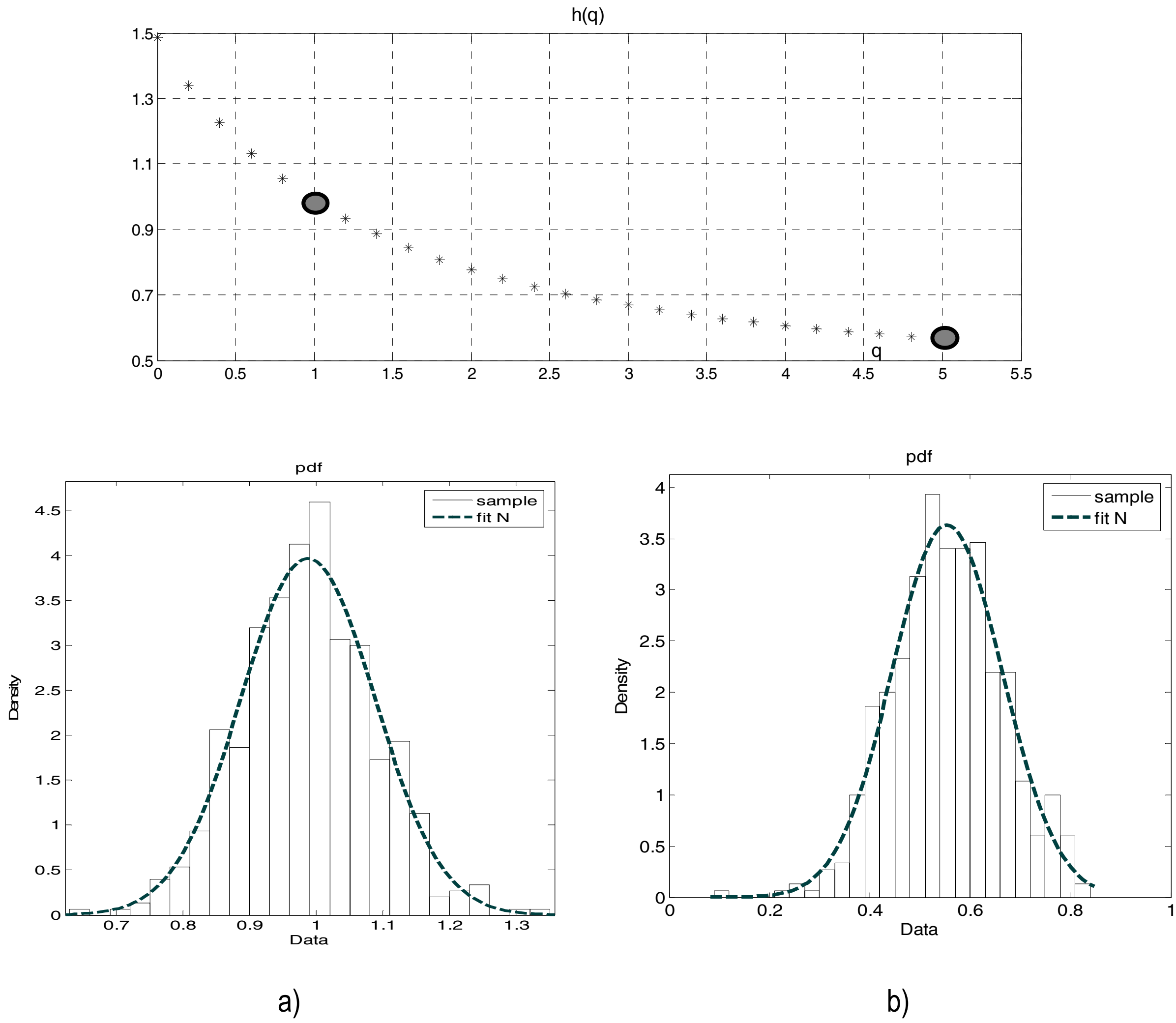


Figure 13. Generalized Hurst exponent $h(q)$ ($0 \le q \le 5$) and histograms of the estimates $h(q)$ at $q = 1$ (a) and $q = 5$ (b)

Numerical analysis showed that a random variable $\Delta h = h(q1) - h(q2)$ at $q > 0$ has normal distribution $N(m_h, s_h)$ the parameters of which depend on the realization length and values q. As the basic values $h(q)$ on which it is possible to carry out numerical comparison of multifractal properties we can use values $\hat{h}(0.1)$, $\hat{h}(5)$, $\Delta\hat{h} = \hat{h}(0.1) - \hat{h}(5)$, $\hat{H} = \hat{h}(2)$ (see Figure 14).

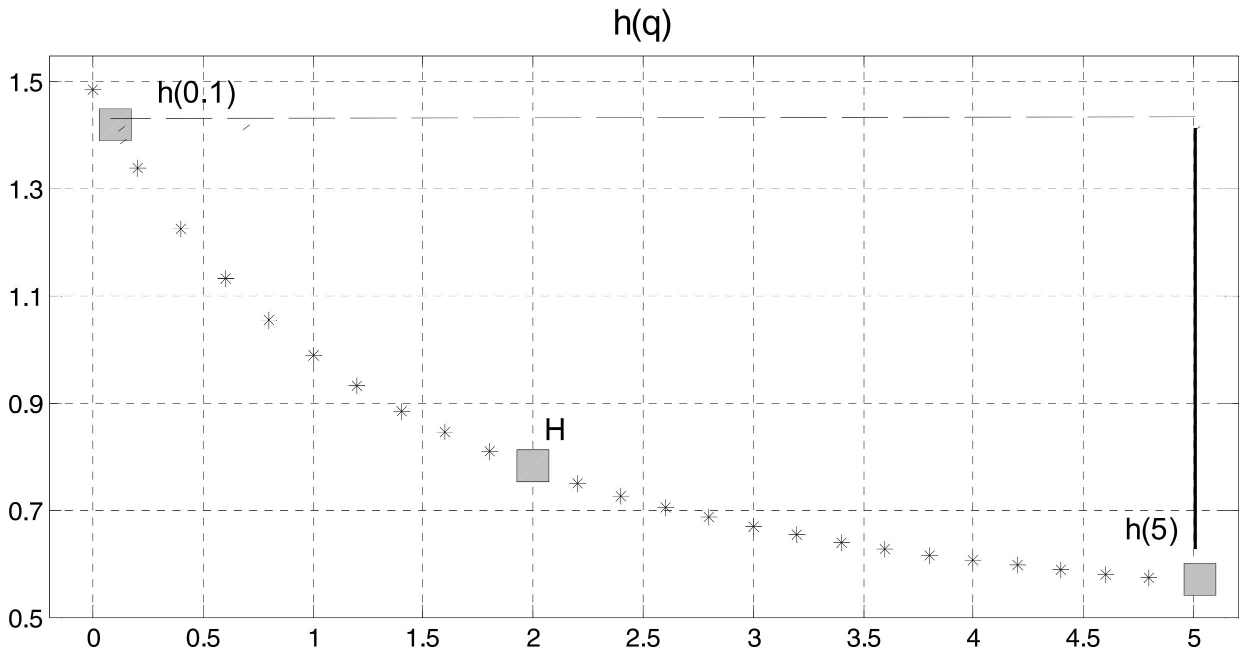


Figure 14. The basic values $h(q)$ for comparison

Then, when constructing confidence intervals it is need to use the following inequalities:

$$\bar{h}(q) - t_\alpha s_h(N) \le h(q) \le \bar{h}(q) + t_\alpha s_h(N), \quad q = 0.1,\ 2,\ 5.$$

where N is the length of investigated series, $\bar{h}(q)$ and $s_h(N)$ are mean and standard deviation of the estimate of generalized Hurst exponent $h(q)$ that was obtained from set of time series length N, α is the significance level, t_α is quantile of the standard normal distribution.

Conclusion

In the work the results of multifractal analysis for three different cases electrobiological signals are represented: RR-interval's sequences of electrocardiograms obtained from patients before drug's application and after one; electroencephalograms of subjects when they perform any physical action, and when they just imagine this and electroencephalograms of laboratory animals for the different phases of wakefulness and sleep. Research was carried out by method of multifractal detrended fluctuation analysis. In all cases, there are significant differences of multifractal characteristics.

In the work such characteristics of multifractal stochastic processes as a generalized Hurst exponent, scaling exponent, function of multifractal spectrum are discussed. It is shown that it is suitable use the generalized Hurst exponent as a quantitative measure. The results of numerical analysis showed that the estimates of the generalized Hurst exponent have normal distribution for positive values of parameter that allows to jumping to quantitative interval values. The possible specific values of the generalized Hurst exponent, which should be used in the knowledge bases of decision support systems, were proposed.

The research results presented in the review and carried out in this work show that distinct changes of fractal characteristics of physiological signals become apparent at various diseases, at change physical and mental loadings on an organism, during functional changes in the brain, etc. This would suggest that multifractal techniques can be successfully used in the analysis of various physiological signals, in particular electrobiological ones to determine the changes in functional activity of an organism. Multifractal analysis of electrocardiogram and electroencephalogram records can be basis for the statistical studies that will enable to form diagnostic methods that are relevant to clinical practice.

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SYSTEM OF AUTOMATED CHECKING OF TEXTUAL DOCUMENT DESIGN RULES

Maria Zhigalova, Alexander Sukhov

***Abstract**: The majority of text documents are made out according to predetermined rules, which regulate requirements to design of the given class of documents. However, the process of paper checking is quite time-consuming and requires from performer of high concentration and appropriate qualification. The paper presents the main methods that minimize the effort required to control the text on the absence of design errors; the analysis of software solutions is given, illustrating the principle of operation of these methods. To automate the verification of formal correctness of the text document the system designed for the compliance assessment of documents in accordance with the parameters set by the user was developed. The domain analysis is carried out; the domain conceptual model is given. The functionality of the system was described and the tools used in its development were presented. However, the verification of a text document is not reduced only to analysis of rules of its design. Also it is required to fulfill verification of document structure: presence of the required sections. The visual domain-specific language, which allows to describe structure of documents, and also connections between different documents, is designed for this purpose. The language has a simple graphical notation; therefore it can be used by as IT-specialists and clients who are not professional programmers. In practice, the developed system can be used to verify compliance with the formal requirements of projects and dissertations, scientific publications, technical documents, etc.*

***Keywords**: formatting rules, text document, compliance assessment, design documentation, domain-specific languages.*

***ACM Classification Keywords**: I.7 Document and Text Processing: I.7.2 Document Preparation – Format and notation; D.2 Software Engineering: D.2.2 Design Tools and Techniques – Computer-aided software engineering (CASE).*

Introduction

Conformity of the text to the formatting rules is one of the priority requirements for documents reflecting the results of scientific, technical and engineering activities. Uniformity of appearance and structure of text documents is mandatory for any field of knowledge since such standardization allows easier understanding of information provided. In addition to that, strict adherence to standards simplifies the process of storing and processing documents in databases.

Due to the fact that the key role in manual check of formatting rules is played by a specialist for whom increased attentiveness and special qualifications are compulsory, this process can be extremely time-consuming and inefficient in terms of time expenses. For this reason, automation of the text check seems to be a highly relevant issue. A software product that automates the compliance assessment of documents can be used by a wide range of users, including students, teachers, technical writers in organizations, etc.

In a more general case the problem considered involves determining the quality of publications, identification of potential duplication, plagiarism, partial borrowings, classification and clustering of documents, formation of databases and extensive collections of texts. Despite the fact that document check in accordance with formatting rules does not imply detailed text processing, it should be noted that this problem in one way or another is related to the general text analysis and has its own specific features.

The purpose of this work is the development of a system of text document check according to specified formatting rules. Such system will significantly reduce the amount of time and effort required for document analysis in comparison with the manual check.

Related Works

To date there are two basic methods of control of the text on the absence of formatting errors and verification of a document in accordance with certain standards including the use of ready-made formatting templates and software solutions in the process document check.

The existence of a large number of word processors (Apache OpenOffice Writer, Microsoft Word, Pages, etc.) with extensive capabilities of text editing allowing creation of complex documents for various purposes does not negate the fact that the formatting of these documents is still a laborious process. The problem is partially solved by using styles with specified parameters of fonts and paragraphs, but still, there is a high probability of violation of styling when copying text from other documents. That is why it makes sense to apply formatting templates.

One of the means of creating such templates is a markup language DocBook, an application of XML/SGML, which provides a user with a unified set of tags for setting formatting of a text document [Berdachuk, 2015]. An example of a document in a DocBook format is shown in Fig. 1.

In this approach the content of a document is isolated from its style representation. The apparent advantage of DocBook is that a predefined set of tags eliminates the possibility of errors in formatting and allows a large number of users to work with the same text simultaneously.

```
<?xml version="1.0" encoding="UTF-8"?>
<!DOCTYPE chapter PUBLIC "-//OASIS//DTD DocBook XML V4.4//EN"
"http://www.oasis-open.org/docbook/xml/4.4/docbookx.dtd">
<chapter lang="ru">
  <title>Анализ требований</title>
  <sect1>
    <title>Постановка задачи. Анализ</title>
    <listitem>
      <para>Возможность работы ...</para>
    </listitem>
  </sect1>
</chapter>
```

Figure 1. Document Structure in DocBook

Formatting templates are also proposed by the LaTeX publishing system which provides the capability for automation of the process of text entering and formatting. The content of a LaTeX document, as in the case of DocBook, is represented by structural and semantic markup; the appearance of a document is established by adding a special style file [Lvovsky, 2006] which defines formatting rules, specific to each document type. A sample document in a LaTeX format is shown in Fig. 2.

Despite a vast variety of functional characteristics, it should be mentioned that LaTeX has a number of disadvantages: firstly, working with LaTeX documents requires a special development environment installed on a personal computer, and secondly, the process of creating a document may cause difficulties for users whose primary occupation is not connected with information technologies.

Automation of text checking is implemented in a number of software products, one of which is an intelligent web-based system for spell checking "Orogrammka". Text document check is performed in terms of the norms of spelling, grammar, punctuation and stylistics [Orfogrammka, 2015]. Moreover, it is possible to conduct the compliance assessment of research papers and dissertations in accordance with requirements that are set in standards. The service has an intuitive and simple interface, however, it should be noted that text check is limited to a strictly predefined set of formatting rules (margin sizes, parameters of a title sheet, applications, reference list, etc.) without the possibility of expanding the functionality by the user, for example, checking numbering formats or tables design.

```
\usepackage[koi8-r]{inputenc}
\usepackage[english, russian] {babel}
\inputencoding{koi8-r}
\usepackage[T2A] {fontenc}
\usepackage{letterspace}
\usepackage{a4}
\usepackage{epsfig, graphicx, euscript}
\title{Пример включения файла в документ}
\author{И.И.Иванов}
\date{}
\begin{document}
Документ должен быть отформатирован правильно.
\newline
  \begin{eqnarray}
    s &=& \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}
  \end{eqnarray}
\end{document}
```

Figure 2. Document Structure in LaTeX

Another tool for automated formatting rules check was developed on the basis of Volgograd State Technical University [Sokolov, 2013]. This software solution is a Microsoft Word 2007 add-in which allows checking documents and fixing detected errors. In spite of convenience and ease of use, the service has a significant drawback: users whose personal computers are not running Microsoft Office Word are deprived of the opportunity to perform the compliance assessment of text documents.

Requirements for Formatting of a Text Document

As it was previously stated, the necessity of automation of the compliance assessment of documents is caused primarily by a substantial amount of effort required and significant time costs in case of manual text check. The developed system is aimed at automation of text document check according to formatting rules pre-established by the user.

Formatting rules are the settings applied to the content of a document in order to determine its structure and appearance. Requirements for document formatting are specified in normative documents, standards, etc.

Generally, a majority of documents are formatted in conformity with specific requirements that might change over a period of time. Therefore, it was decided to equip the system with the functionality allowing to manually add and modify formatting rules of documents of a particular class.

The analysis of documents demanding certain formatting resulted in identification of a number of essential parameters for assessing the accuracy of text document formatting including:

1) page size (name, width, length);
2) page orientation;
3) margin sizes;
4) maximum and minimum amount of work (in pages);
5) size of headers and footers;
6) page numbering (number position, number formatting);
7) description of style:
    a) style name;
    b) font:
        – name;
        – size;
        – color;
        – italic;
        – bold;
        – underlined;
    c) paragraph:
        – indent sizes (left and right);
        – first line indent size;
        – paragraph spacing (before and after the paragraph);
        – line spacing;
        – alignment.

A conceptual domain model made in the notation of Entity Relationship Diagram is shown in Fig. 3.

Once formatting rules have been entered by a user and a text document has been inputted, the system provides a report containing the results of the document check.

In this regard, main functional requirements for the system involve adding data on formatting rules of documents; editing an information entered; loading document for the check; performing the document check in accordance with the selected rules; generating a report containing the results of the document check.

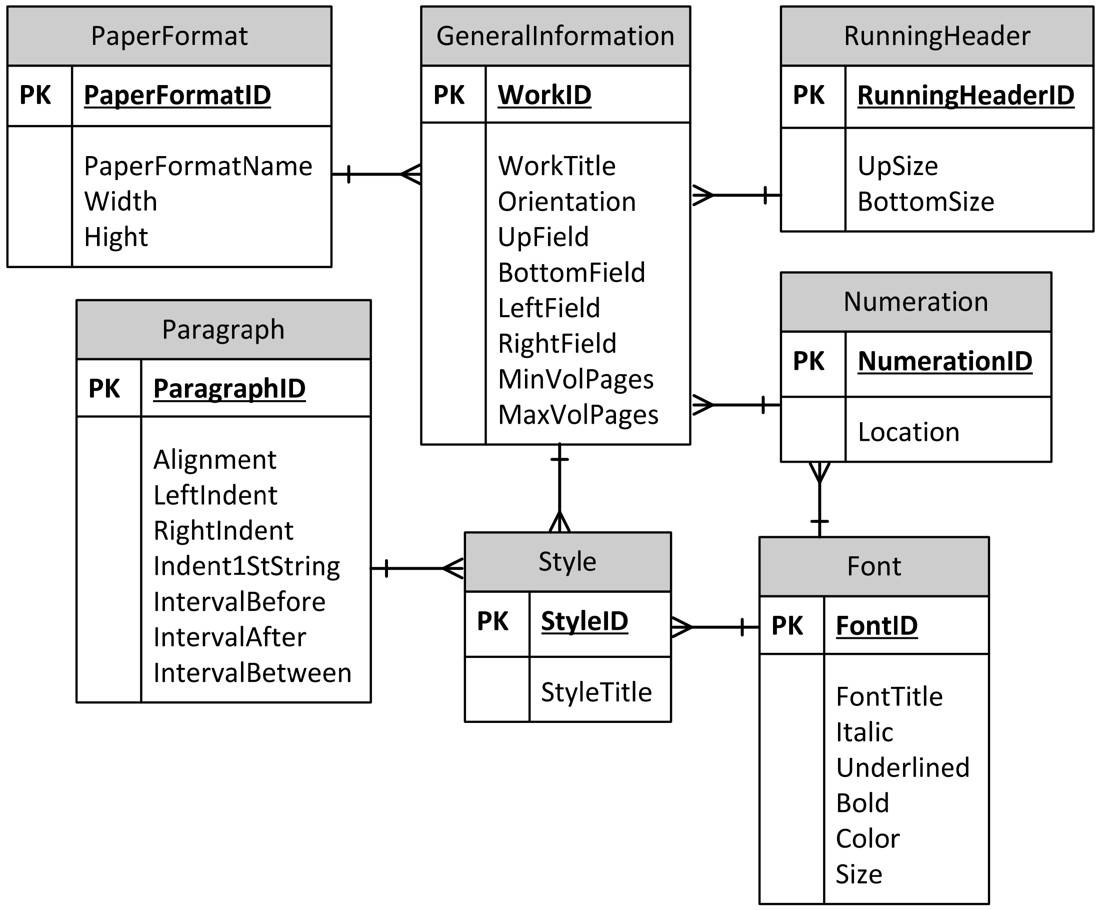

Figure 3. Conceptual Domain Model «Text Document Formatting Rules»

Office Open XML Document Format

Office Open XML (also known as OOXML or OpenXML) is a default file format of Microsoft Office electronic documents appeared for the first time in Microsoft Office 2007. The format was initially standardised by Ecma [Standard ECMA-376, 2015] and then redefined in ISO/IEC 29500 standard [ISO/IEC 29500, 2015]. OOXML is a structured archived file in a ZIP format that contains markup text of a document in an XML format, graphical information and other data included in this text document.

The advantages of an Open XML format [OpenXMLDeveloper.org, 2015] include:

1. Interoperability. The capacity of the format to interact and function with a large set of both custom and commercial applications provides a high degree of compatibility of documents for different tasks.

2. Backward compatibility. The ability of transformation of MS-DOC files into Open XML format with high accuracy allows end users to convert these documents to the Open XML format, and then programmatically access the converted documents.

3. Programmability. Minimum requirements for working with Open XML include a tool that can open and save ZIP files and an XML parser/processor. ZIP and XML libraries allow to create documents in Open XML format on a software level.

4. Integration of business data. Office applications support custom XML schemas that can extend the capabilities of the existing Office document types. Thus, users can export data from existing systems to the documents in the Office file formats.

5. Compact file format. For storing documents Open XML format uses the technology of ZIP compression, providing the possibility of reducing the storage space. Opening the file causes the automatic unpacking of the archive, and saving the file results in its compressing.

Compliance assessment in accordance with formatting rules is implemented for Microsoft Office Word text documents with the DOCX extension. A markup language for processing text files in an Open XML format is called WordprocessingML. The structure of WordprocessingML consists of a set of basic elements including main document, comments, settings, endnotes, header/footer, styles, fonts table, document glossary. Figure 4 illustrates parts of a document TestFile.docx opened with a tool Open XML Package Editor PowerTool for Visual Studio that allows to view the file hierarchy of the document archive and the relationships between them, and also to modify their markup.

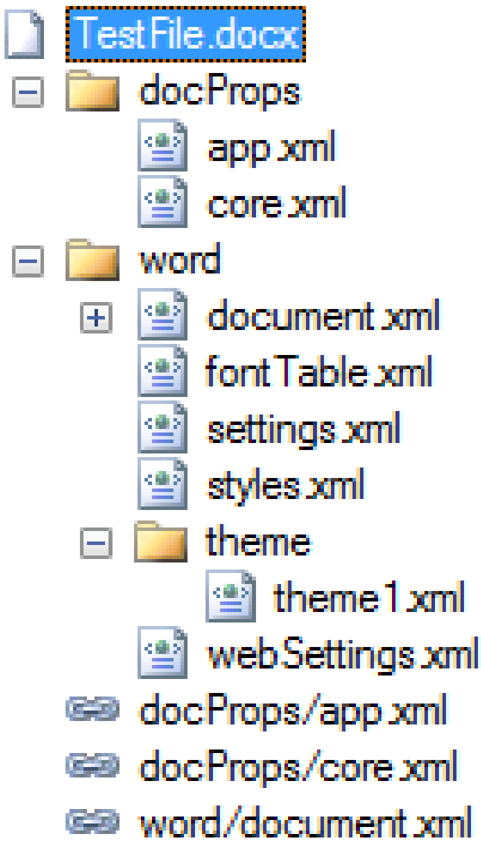

Figure 4. Text Document Structure in DOCX

In the main document part paragraphs (w:p) and tables (w:tbl) can be child elements for document body (w:body), table cell (w:tc) or text box (w:txbxContent). Paragraphs, in their turn, are a run-level content container for text runs (w:r), or images – a VML document (w:pict) or a DrawingML object (w:drawing). Finally, sub-run-level content incorporates multiple text elements (w:t).

System Development

Formatting of a text document with the use of Microsoft Word implies implementation of various styles with parametres included in styles.xml file of a document archive [Vugt, 2015]. This file contains data on styles of paragraphs, characters and tables, latent styles and standard settings of styles for an entire document (document defaults). Styles of paragraphs, characters and tables comprise information about current formatting of a document, whereas hidden styles are not used directly and serve primarily as a cache repository for style settings, for example, the ones copied from a template. Standard styles store default values for formatting of an entire document. However, it should be noted that styles.xml file does not involve data on formatting numbered and bulleted lists that is included in a special numbering.xml file.

The fact that content of a document can be formatted on multiple levels leads to a problem of determining a comprehensive set of formatting parameters used for a particular paragraph or a run of text. These levels of fomatting are schematically represented in Fig. 5.

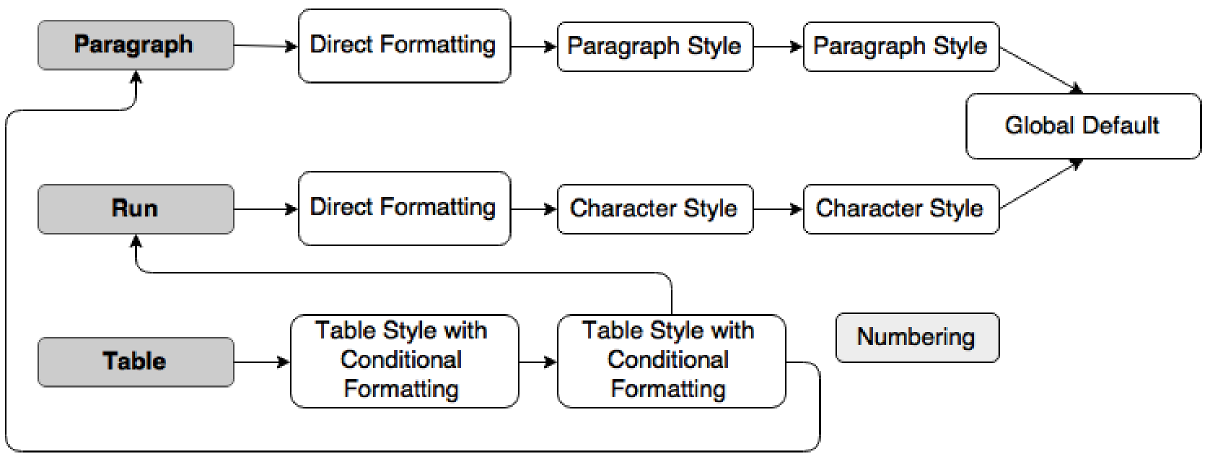

Figure 5. Levels of Microsoft Word Text Document Formatting

Thus, if it is needed to retrieve information about paragraph (e.g. line spacing or indentation), the first aspect that has to be checked is direct formatting which is specified in a file called document.xml. Yet, paragraph parametres might not be indicated in this file, and, in this case, it is necessary to inspect style which is referred to in paragraph properties. It might appear that this style does not contain data on the paragraph formatting, then it is required to check the styles from which it inherits. If this action did not bring any results, then the only option left is to process the contents of the node Global default, i.e. default settings of all styles in a document.

Similar approach is credible for check of formatting of runs of text (defining such font settings as size, name, etc.); the only difference is that character styles are considered (as opposed to paragraph styles) that can also form an inheritance hierarchy.

Data on tables formatting is defined in styles with conditional formatting that specify properties of rows and columns. Table styles are also inherited. Text inside table cells is checked according to algorithms of determining formatting of paragraphs and runs. In case of numbering, each list item may include formatting from a paragraph, a numbering format in numbering.xml or a style that is indicated by this format.

Overall, the major difficulty of text document formatting check lies in determining precise formatting parameters for paragraphs, tables, numbering and runs of text for the purpose of conducting as extensive an analysis of conformity of a document to specified rules as possible.

In order to work with WordprocessingML markup, it was decided to use Open XML SDK 2.5 for Microsoft Office. Retrieval of information on document content formatting was performed by using the FormattingAssembler module which is a part of PowerTools for Open XML.

Open XML SDK built on the System.IO.Packaging API allows manipulating documents that adhere to the Office Open XML File Formats Specification, e.g. documents created with Microsoft Office applications. This package provides a set of strongly-typed classes to obtain data about the formatting of a document and makes it possible to modify an original document (for example, to add comments).

Despite the fact that .NET offers standard interop assemblies for working with Microsoft Office, the preference was given to Open XML SDK. COM Interop (Component Object Model) provides access to Word objects (sections, paragraphs, characters, etc.) and has functionality for creating and editing documents, however, it does not support server-side automation and processes documents markedly slower than SDK.

For the reason that for all numerical characteristics (margin sizes, font sizes, line spacing, etc.) in Open XML SDK are defined in such unit of measure as points, it was needed to transform them into millimeters and centimeters which are more familiar to a user setting formatting parameters for checking. Therefore, it was decided to use the iTextSharp class library that allows converting all of the values by implementing the only method – PointsToMillimeters().

In addition, in order to view a Word document from an application, WinWordControl developed by Matthias Haenel was used. WinWordControl modified in accordance with the latest version of

Microsoft.Office.Interop.Word DLL works with a set of basic functions of Win32 API. The appearance of this control with loaded document is represented in Fig. 6.

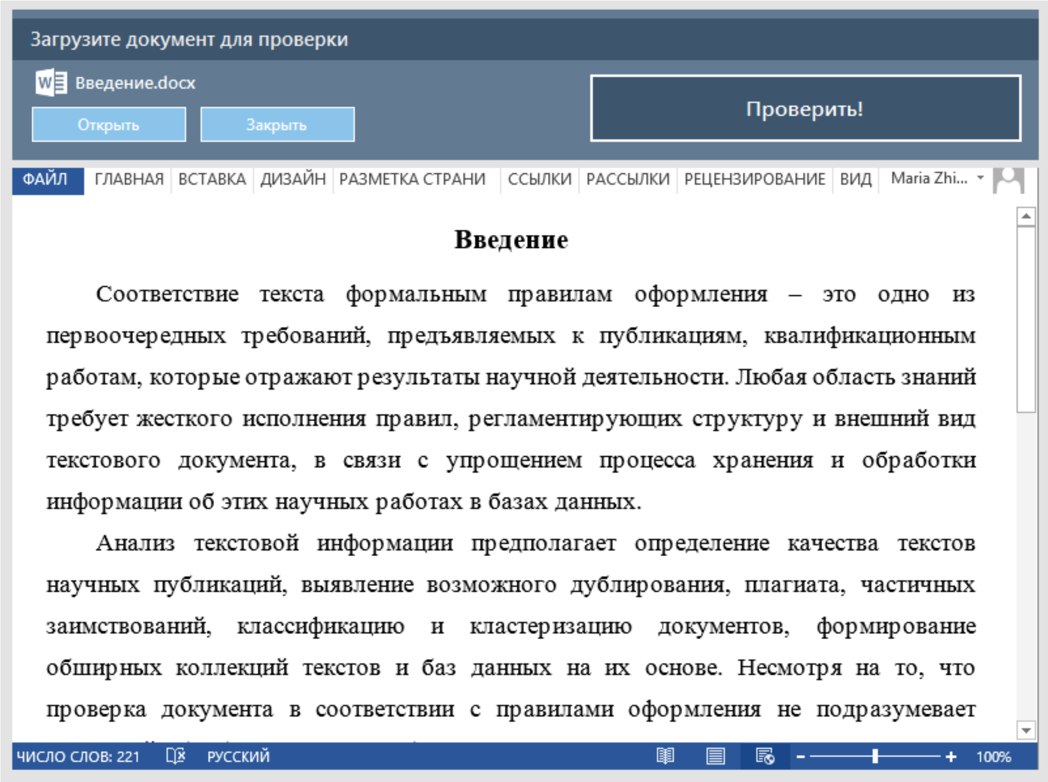

Figure 6. WinWordControl for viewing Microsoft Word documents

The necessity of using third-party control can be explained by the fact that Windows Forms does not provides standard tools to view Microsoft Office documents. The principle of operation is that an instance of Microsoft Word with an open user's document is started inside an application window, while the version of Microsoft Word corresponds to the one installed on the computer. Thus, this control can be used only if a user's computer runs Microsoft Word.

The solution "FRC System", implementing the system, contains two projects: "FormattingRulesLibrary" (class library for working with formatting rules of documents) and "FRC System" in which the application user interface was created and methods for text document check were developed.

The main window of the application that opens when the system is launched (see Fig. 7) is logically divided into two parts. On the right, there are the controls for displaying the formatting rules of text documents, downloaded from a database. The navigation through records of the database is performed by means of special navigation controls. Left side of the window (where the checked document is displayed) contains WinWordControl. Buttons "Open", "Close", "Check" provide functionality for opening, closing and document check, respectively. When the document is loaded into the system, the title of the checked Word document replaces the label "Document is not selected".

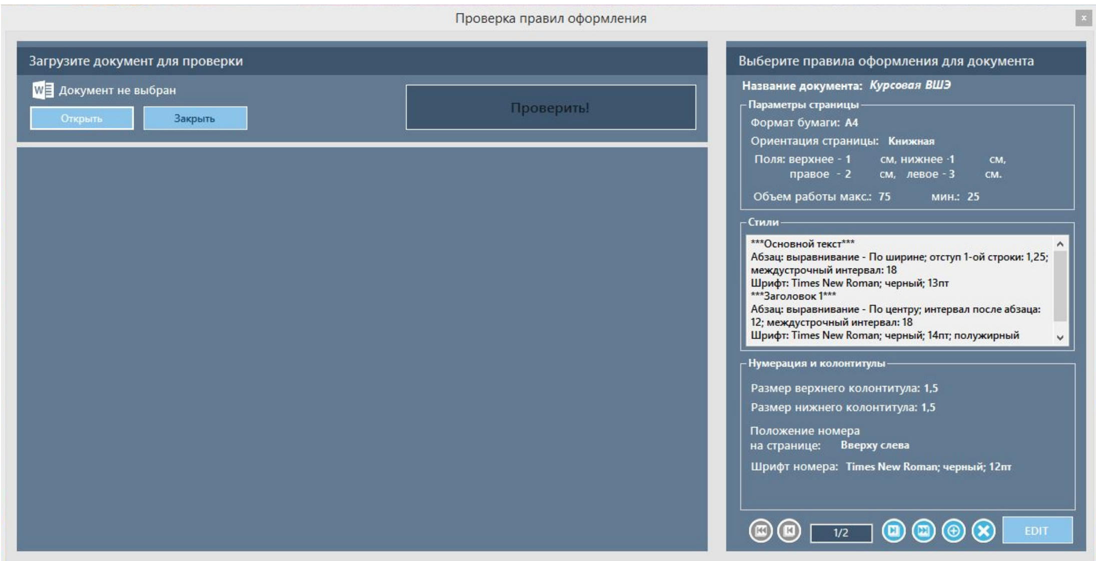

Figure 7. Main Window of the System for Document Check

The window for adding and editing new formatting rules of text documents (see Fig. 8) contains tabs, multiple input fields, comboboxes and other controls, allowing selecting necessary formatting parameters of a document.

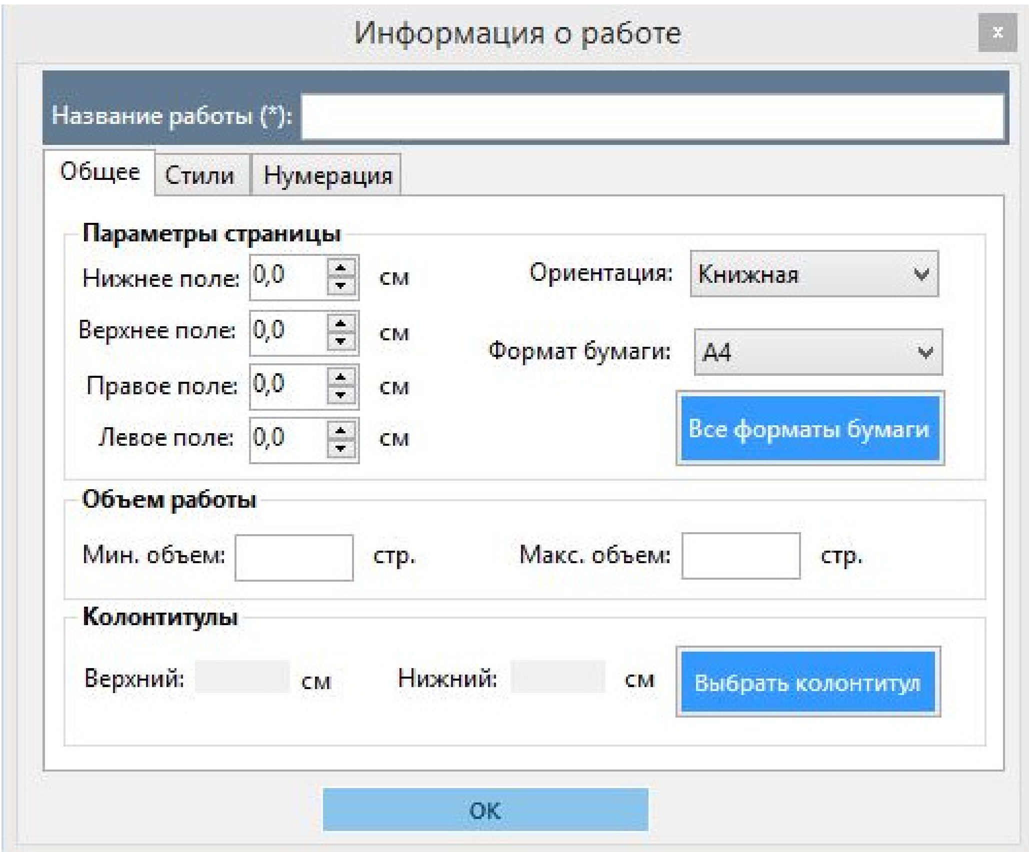

Figure 8. Window for Editing Formatting Rules

A mandatory field to fill in is the "Title of work". Various tabs allow specifying different requirements for formatting of a document. The tab "General" comprises information about page settings, number of pages, header and footer. On the tab "Styles", the user can examine styles of text formatting used in the document, add new styles or delete existing ones. The tab "Numbering" includes means for viewing and editing data on the numbering. Saving added/edited information about formatting rules occurs when the "OK" button is pressed.

The window for viewing and editing information about styles that are used for document formatting is demonstrated on Fig. 9.

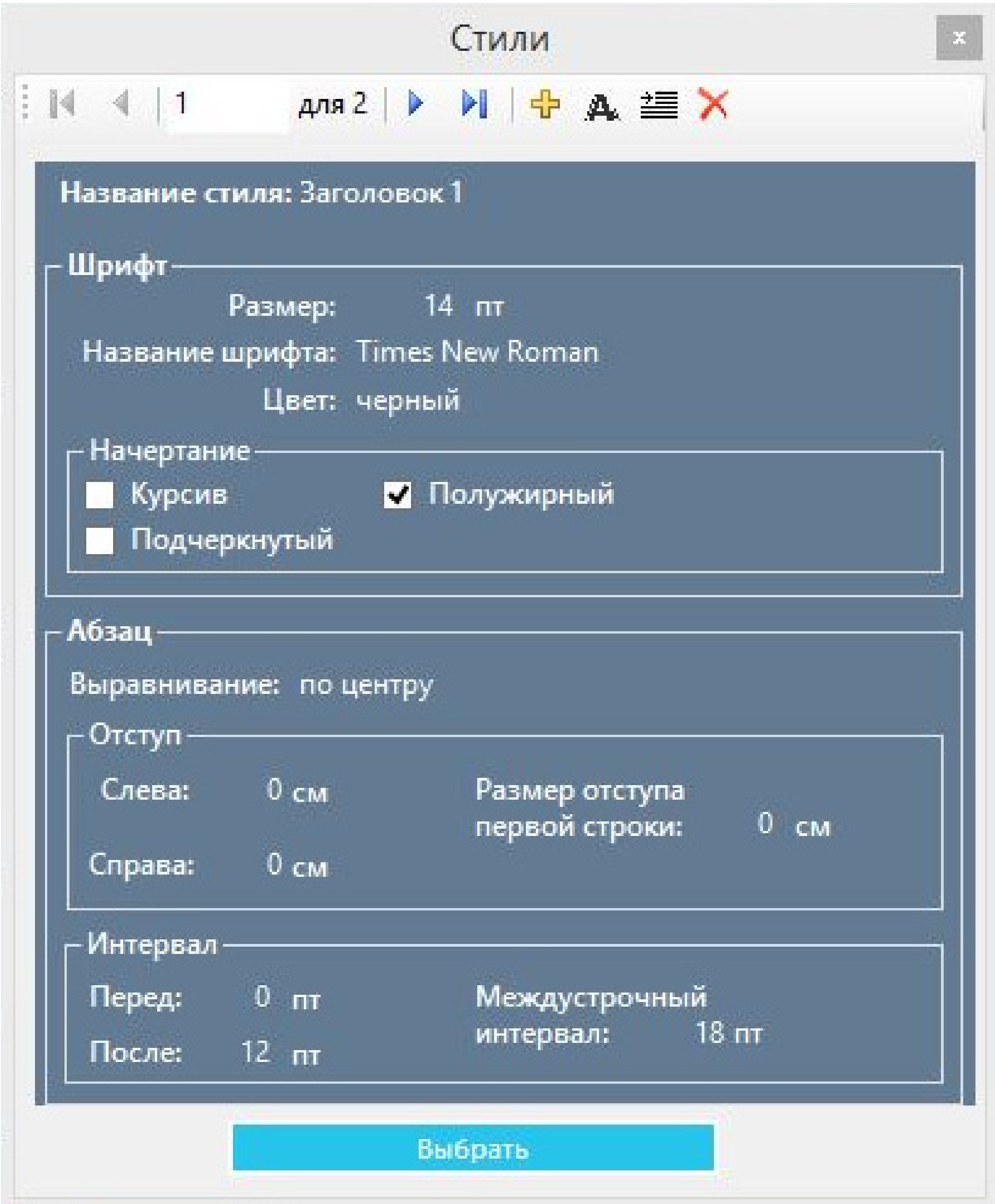

Figure 9. Window for Working with Styles

To modify font parameters used in the style, it is needed to press the button with the letter "A" on the navigation panel; editing paragraph properties requires selecting the control nearby (image depicting lines of text). Choosing style for a document occurs when the button "Choose" is pressed.

Loading a document into the system after pressing the "Load" button in the main application window is performed as follows: first, a user selects a needed file by the use of OpenDialog, then the document is opened in WinWordControl which allows displaying text of the document in the main window.

Document check is executed by the PerformChecking() method. First, a loaded Microsoft Word document is opened as an instance of the WordprocessingDocument class for working with Open XML SDK, and then, it is divided into sections and checked with the use of such methods as HeaderFooterCheck(), NumPagesCheck(), MarginCheck(document, sectPr), etc.

The interaction between a user and the system can be performed according to different scenarios. The first scenario (see Fig. 10) assumes that a user enters formatting rules manually, and then loads an original document for check. In this case, the system provides a user with either the resulting document containing the notes or the one with formatting corrected in accordance with rules specified by a user.

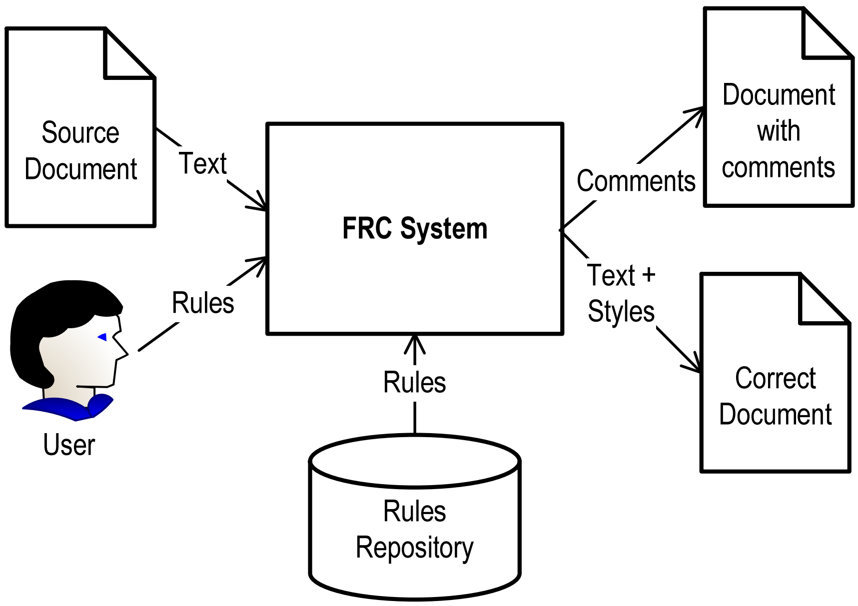

Figure 10. Correct Document Generation in FRC System

According to the second scenario (see Fig. 11) a user loads a properly formatted original document, the system performs its analysis and downloads its formatting rules into the rules repository. This significantly simplifies the entry of formatting rules of a document.

The third scenario of the interaction (see Fig. 12) suggests that a user manually enters formatting rules of a document, the system saves them in the repository, and then generates a document template with an automatically created styles which a user can use for further work with a document.

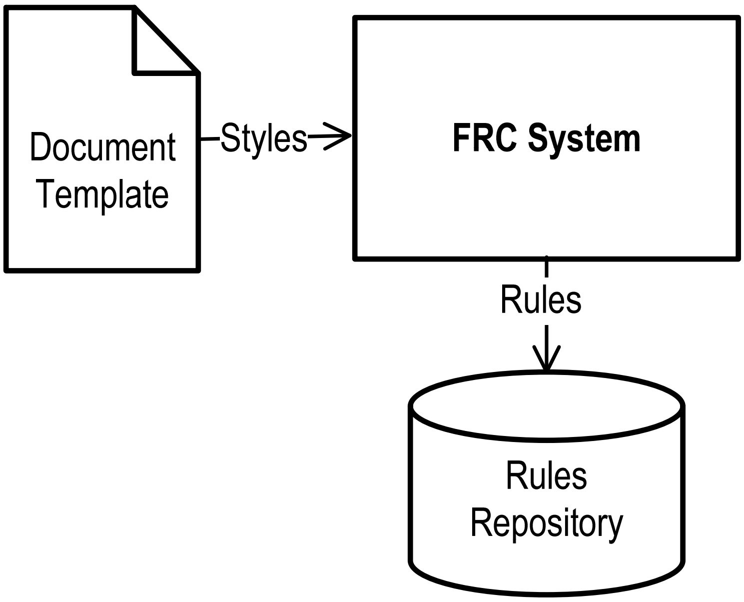

Figure 11. Creation of Rules based on Document Template in FRC System

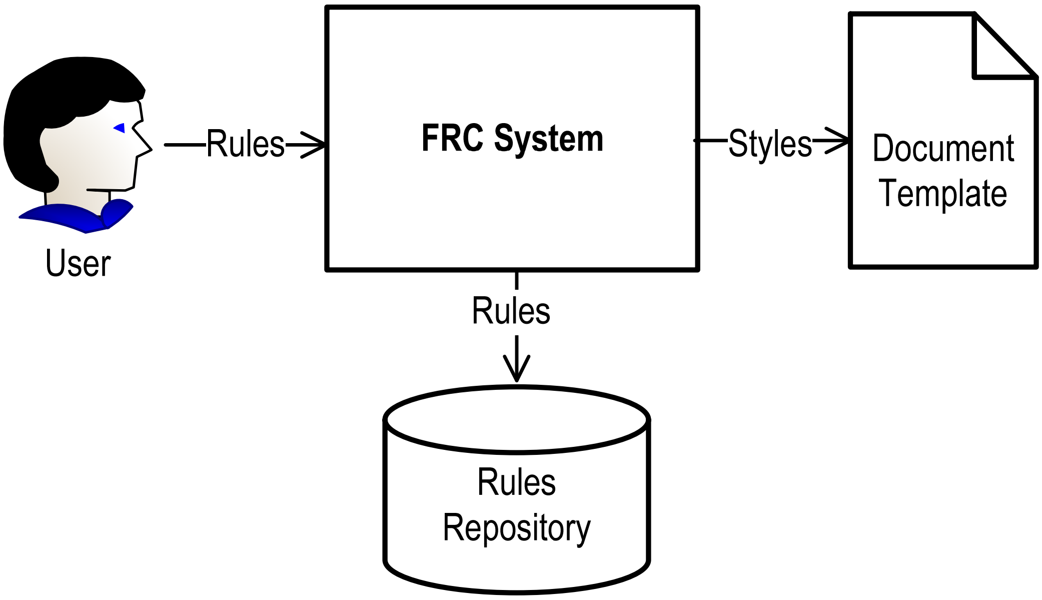

Figure 12. Creation of Document Template based on Rules in FRC System

Text Document Structure Check

As noted earlier, the task of a text document analysis is not reduced to formatting rules check. In the more general case, it is necessary to analyze document structure, i.e. verify that all required sections are included. This problem often arises in preparation of design documentation, for example, in the process of developing information systems. Design documentation has a normative function, i.e. it contains mutual obligations of participants of a project that helps to avoid misunderstandings and abuses at the stage of handover-acceptance [Zaboleeva-Zotova, 2007; Orlova, 2011].

The types and completeness of project documents are standardized. However, due to the fact that all technical documents are structurally very similar (they all consist of sections and subsections, may include additional documents, diagrams, tables, etc.), a special language for defining document structure and links between different documents can be developed. It will allow automating the process of analysis of an original set of project documents and generation of the new ones [Zhigalova, 2014]. In the same way, it is reasonable to develop tools for extracting system requirements from the project documentation, and then control their compliance in the process of implementing the system. However, the process of creating design documents is quite a laborious task that requires precise knowledge of a document structure. This process can also be automated. Means of automating the generation of project documentation will allow generating a document template on the basis of descriptions of different sections of a document specified in a convenient visual user interface. This template can later be modified manually.

In order to describe documentation used in the process of information systems design, visual domain-specific language can be developed. Domain-Specific Language (DSL) is a modeling language designed for solving problems of a certain class in a particular domain. Unlike general-purpose modeling languages, DSL is more expressive, easy to use and intelligible to various categories of professionals, since it operates with familiar terminology of the domain. Therefore, a large number of DSMLs is designed nowadays in order to describe systems in different domains: artificial intelligence systems, distributed systems, mobile applications, real-time and embedded systems, simulation systems, etc.

Since description of project documents implies not only determining their structure, but also specifying the relations between them, the developed domain-specific language describing project documentation has two levels [Zhigalova, 2015].

The first level of the language makes it possible to describe a set of documents and relations between them, the second level – the structure of a particular document. Due to a simple graphical notation of the language, the system can be used by IT-specialists, as well as clients who are not professional programmers.

Conclusion

The main result of the work done is the developed system that automates the check of a text document in accordance with formatting rules specified by a user. As it was tested, the system substantially reduces the complexity of operations performed and makes the process less time-consuming.

Moreover, the visual DSL for describing the structure of a document was created. This language can be integrated into the support system of work of an analyst when information systems are designed. On the one hand, this provides means to perform analysis and parsing of a set of design documents loaded into system, presenting the sections of a document as individual elements of a model. On the other hand, with the use of the developed language an analyst can describe each section of a design document separately, and then generate a single text description on their basis.

Despite the fact that the system performs all the main functions, there is still space for improvement. The system can be upgraded by developing web-interface for more convinient use and expanding the set of criteria for document check in order to perform more comprehensive compliance assessment.

Acknowledgements

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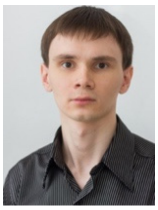

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ПОСТРОЕНИЕ ТАКСОНОМИИ ДОКУМЕНТОВ ДЛЯ ФОРМИРОВАНИЯ ИЕРАРХИЧЕСКИХ СЛОЕВ В ГЕОИНФОРМАЦИОННЫХ СИСТЕМАХ

Виталий Величко, Виталий Приходнюк, Александр Стрижак, Крассимир Марков, Крассимира Иванова, Стефан Карастанев

Аннотация: *Рассмотрены проблемы, связанные с использованием функциональности онтологических систем для обработки документов, представленных в глобальной сетевой среде. Описаны механизмы аппликативной типизации выражений, составляющих тексты документов на основе использования безтиповых инструментов теории ламбда-исчисления. Изложено использование отношения частичной упорядоченности для формирования таксономий документов на основе формирования безтиповых выражений из термов, задаваемых концептами, определяющих понятийную структуру документов. Описывается процедура лексического анализа текстов и построения таксономической структуры на основе применения процедуры анализа. Предложена технология формирование иерархий тематических слоев в геоинформационных системах (ГИС) на основе таксономий сетевых документов. Рассматривается возможность применения изложенного подхода к обработке пространственных данных в соответствии с директивой ЕС INSPIRE.*

Ключевые слова: *сетевые информационные ресурсы, онтология, таксономия, упорядоченность, безтиповое выражение, аппликация, лексема, лексический анализ, функция.*

ACM класификация: *I.2 ARTIFICIAL INTELLIGENCE - I.2.4 Knowledge Representation Formalisms and Methods, H. Information Systems – H.2 DATABASE MANAGEMENT – H.2.4 Systems*

Введение

Неструктурированная и слабоструктурированная информация, хранящаяся в глобальной сетевой среде, постоянно накапливается и архивируется [Report, 2012]. Сетевые информационные ресурсы (СИР) отображают достаточно большую совокупность тематических знаний, технологических решений, опосредованных данных, отражающих развитие современной науки и техники, состояние информационных процессов в сети и тем самым определяют контекстное содержание информационного пространства (Information networks space). Тематическое многообразие и многоаспектность сетевой информации ставит достаточно существенную проблему ее оперативного и достаточно корректного практического использования и интерпретации. При этом практически постоянно возникает необходимость в реализации и обеспечении взаимодействия сетевых информационных систем, которые характеризуются междисциплинарностью и реализованы на основе различных технологий и стандартов [Стрижак, 2015].

Эффективность процесса использования информационных ресурсов глобальной сети при организации взаимодействия между сетевыми информационными системами, существенно зависит от операциональных характеристик инструментов (сетевых программных средств), которые обеспечивают получение, обработку и хранение соответствующего контента. И здесь приходится учитывать достаточно существенное технологическое ограничение на организацию межсистемного взаимодействия, связанную с интероперабельностью, как сетевых информационных ресурсов, определяющихся разными форматами представления, так собственно и информационных систем сети, которые, как уже было отмечено, реализуются на основе использования разных технологических стандартов [Глушков, 1982], [Белов, 2005]. Контент глобального информационного пространства определяется контекстами объектов, которые определяют тематический характер сетевой среды. На этом основании можно сформулировать гипотезу, что эффективность интегрированного использования информационных систем в глобальной сети существенно зависит от технологических возможностей операционального связывания контекстов объектов, которые участвуют в сетевом взаимодействии и определяют информационный контент собственно сетевой среды.

Для обеспечения тематической связности объектов и процессов в глобальной сети, необходимо, на наш взгляд, обеспечить формирование концептуального отображения контекстных тематик используемых информационных ресурсов и систем. Одним из технологических подходов для решения указанной проблемы контекстной связности может быть использование трансдисциплинарных онтологий, обеспечивающих моделирование пассивных и активных сетевых информационных процессов накопления, обработки, отображения и взаимодействия [Стрижак, 2015], [Палагин, 2012], [Guarino, 1994], Гладун, 1994].

Целью статьи является определение механизмов формирования таксономий сетевых документов как технологической основы интеграции информационных ресурсов и систем глобальной сети.

Аппликативные механизмы таксономизации

Использование трансдисциплинарных онтологических систем для отображения семантики СИР [Глушков, 1982], [Палагин, 2012] в виде таксономических (иерархических) структур [Гладун, 1994], над которыми задается определенная расширяемая аксиоматика и между которыми определяются множества отношений, позволяет частично решить проблему корректной интерпретации использования онтологии при решении сложных прикладных задач. Одной из системных компонент онтологической системы является таксономия [Глушков, 1982], что отражает определенную иерархию взаимодействия концептов. При этом собственно иерархия задается с помощью бинарных отношений, определяющих характер взаимодействия между концептами онтологии. На основании указанных отношений, определяющих взаимодействие между концептами, реализуется процедура разбиения множества концептов онтологии на классы [Буч, 1992]. Для построения онтологических классификаторов конкретных предметных областей (ПрО) на основе сложившихся классов используются таксономические категории $\breve{T}$. Список указанных категорий определяется в процессе решения прикладной задачи на основе свойств, которые объединяют концепты по тематическим признакам [Шаталкин, 2012].

Упоминаемая категория сетевых информационных ресурсов обычно представляется в виде текстов. Множества терминов-концептов X из вышеупомянутых текстов связаны между собой разными множествами семантических отношений R_{sem} . Над отношениями из каждого такого множества R_{sem} может быть задано отношение частичного порядка $\tilde{p}$ [Малишевский, 1998]. Это отношение позволяет формулировать множество правил Rul , на основании применения которых мы можем формулировать истинные утверждения из концептов ПрО. При этом будем считать, что концепт является свободным, если он не связан с другими концептами из ПрО никакими типами отношениями. Если же между концептами установлено какое-либо отношение из множества R_{sem} , то тогда такие концепты будем определять как связанные. Тогда утверждения образуются связанными концептами, а корректность определения над ними множественного отношения частичного порядка $\tilde{p}$ позволяет считать их выполнимыми или истинными [Клини, 1957].

Построение утверждений из концептов реализуется на основании конечных наборов правил из множества Rul , которые определяют порядок применения множественного отношения частичного порядка $\tilde{p}$, как над концептами из множества X , так и над семантическими отношениями из множества R_{sem}. Также отношение $\tilde{p}$ позволяет нам формировать из концептов терминополя [Коршунова, 2009] в виде иерархических структур, где между концептами задается множественное бинарное отношение частичного порядка, которое, по сути, полностью эквивалентно отношению $\tilde{p}$.

Такие правила построения утверждений носят аппликативную форму и могут быть представлены в виде безтипового выражения вида [Барендрегт, 1985], [Стрижак, 2014]:

$$f_a = \left(\lambda x.t\left(x\right)\right)a = t\left(a\right) \qquad (1)$$

где: λ -теория – лямбда исчисление; запись λx говорит, что это λ -терм;

x – переменная, принимающая значения из множества концептов X;

t – выражение, которое может содержать переменную x;

a – аргумент функции, определяющий возможные значения переменной x;

f_a – функция, применимая к аргументу a .

Как можем видеть из (1), переменные позволяют определить свойство экстенсиональности множества концептов, которое также является финитным [Загорулько, 2008], т. к. фактически экстенсиональность концепта как определенного терма - теории определяется его принадлежностью к определенному классу. Тогда мы всегда можем ограничить истинность утверждения, введя аксиому исключающую истинность утверждения для концептов, не входящих в один класс. Тогда правила вида (1), включающие в себя в качестве переменных и аргументов отношения из множества R_{sem} и множественное отношение $\tilde{p}$, обладают свойством интенсиональности. Т.е. утверждения, обладающие свойством истинности, обладают еще набором других свойств, определяющих применимость данного утверждения.

Достоинством данного аппликативного выражения является то, что в качестве переменной и аргумента кроме концептов, могут быть отношения, т.е. мы можем строить правила, которые связывают между собой и концепты, и отношения.

Безтиповость позволяет нам поднять уровень абстракции только до рассмотрения истинности утверждений, сформулированных на основе применения правил онтологии ПрО.

Таким образом, каждая онтология представляет собой сложный объект со своей структурой и функциональностью. Любая каноническая форма онтологии [Гаврилова, 2000] отображает множество концептов ПрО, множество отношений между ними и множество интерпретирующих функций, применение которых обеспечивает взаимодействие различных состояний онтологии. Множества концептов определяют терминополе [Коршунова, 2009] ПрО, которое имеет семантическое представление в виде определенных утверждений. Все утверждения, построенные из концептов терминополя, могут обладать свойством истинности, при условии, что они связаны отношениями, которые корректно связывают между собой контексты каждого понятия.

Над всеми концептами, из которых могут быть построены утверждения онтологии ПрО, всегда определяется множественное отношение частичного порядка – $\tilde{p}$ [Малишевский, 1998]. Причем это отношение также участвует в построении истинных утверждений из концептов. Если сами концепты могут конструироваться из конечного множества термов, то все утверждения, образованные при помощи λx -термов, имеют конечную длину, максимальный путь которого не будет превышать длины последовательности 2^x . Утверждения, которые обладают свойством истинности в рамках λ -исчисления, будем называть разрешимыми [Барендрегт, 1985]. Выражение (1), при условии разрешимости, также обладает свойствами монотонности, обратимости, наследственности, аддитивности и конвертируемости.

Монотонность λx -термов может быть представлена в виде $\lambda x_1 \ldots x_n \ldots x_i L_1 \ldots L_m$, (L – λ -терм) где каждый дополнительный терм приписывается на основе правила применения множественного отношения бинарного частичной упорядоченности. Это позволяет строить цепочки утверждений на основе операции приписывания справа нового терма. Из сказанного можно сделать еще один вывод – если концепты онтологии обладают хотя-бы одним общим свойством, или хотя-бы бинарным отношением частичного порядка, то из них можно построить разрешимое утверждение в терминах λ -исчисления.

Обратимость определяет возможность строить разрешимые утверждения из отрицания существующих термов на основе изменения множественного бинарного отношения частичной упорядоченности. Т. е. справедливо отображение вида:

$$\lambda x_1 \ldots x_n \ldots x_i L_1 \ldots L_m \xrightarrow{\tilde{p}} \lambda x_i \ldots x_n \ldots x_1 L_m \ldots L_1 \quad (2)$$

Более компактно в терминах λ -исчисления это выражение можно представить следующим образом. Пусть M – λ -терм и K – λ -терм, тогда их композиция «$\circ$» представима в виде:

$$M \circ K = K \circ M = \lambda x.x \quad (3)$$

Наследственность может быть определена и впоследствии выражена как слабая и/или как сильная. Условие слабой наследственности заключается в сохранении свойства экстенсиональности λ -термом, составленного из всех концептов, обладающих этим свойством и которые образуют разрешимое утверждение. Условие сильной наследственности требует сохранения свойств экстенсиональности и интенсиональности, как для λ -терма, полученного в результате композиции определяющих разрешимых утверждений, так и для обратимого λ -терма.

Свойство аддитивности выводимо из самого определения λ -терма (1). Каждый концепт и отношение, составляющие разрешимое утверждение, задаются в λ -терме соответствующей переменной или аргументом, и так как каждый из них определяет конкретную часть выражения, то в результате формулируется разрешимое утверждение.

Конвертируемость определяется возможностью формирования утверждений из концептов иерархически связанных классов. Тогда в структуре сложного λ -терма вида (2), сформированного из соответствующих термов, возможна подстановка одного выражения вместо другого. Основным условием такой подстановки является существование между концептами указанных выражений, множественного отношения частичного порядка $\tilde{p}$.

Эффективная сетевая обработка документов, содержащих большое количество информации, реализуется на основе таксономизации их содержания, с последующим представлением в виде онтологий [Палагин, 2012],[Гладун, 1994]. Таксономизация достигается за счет применения правил из множества Rul к концептам множества X, определяющих содержание конкретного документа. Для этого из множества концептов X выделяются терминополя [Коршунова, 2009], концепты которых связываются между собой отношением частичного порядка $\tilde{p}$. Далее реализуется процедура построения цепочек утверждений из концептов терминополей на основе операции приписывания справа нового термина из множества X в виде безтипового терма. Это достигается за счет наличия у них общих семантических отношений из множества R_{sem} . Т.е. все концепты из множества X могут быть представлены в виде выражений вида (2)–(3) – $M \circ K = K \circ M = \lambda x.x$. При этом все выражения вида (2)–(3) содержат в себе термы, связанные между собой множественным бинарным отношением частичного порядка $\tilde{p}$ [Шаталкин, 2012].

Обобщенная процедура формирования таксономии документов

Практически любой текстовый документ может быть представлен в виде высказываний и утверждений, истинность которых определяется применимостью концептов их составляющих, на основе определяющих каждый концепт контекстов. Т.е. фактически все концепты, составляющие СИР, могут быть выделены в определенное множество терминополей [Коршунова, 2009] и, исходя из синтаксиса самих высказываний и утверждений, представлены в виде $\langle X, R_{synt} \rangle$, где X – множество концептов терминополя, а R_{synt} – множество синтаксических отношений между концептами, определяющих набор синтаксических правил из множества Rul построения истинных утверждений из выбранных концептов терминополей глобальной среды.

Сетевые инструменты выделения соответствующих концептов на основе механизмов аппликативного представления текстов документов в виде утверждений реализуются на основе применения IT-технологии ТОДОС [Величко, 2009], [Величко, 2014]. Первоначальную основу такого представления составляют механизмы индуктивного формирования пирамидальной сети из концептов, определяющих все контексты документа [Гладун, 1994], [Величко, 2009], [Величко, 2014]. Словоформы и синтаксические конструкции текста из аппликативной формы вида $L \circ R_{synt} \equiv \lambda x_1 \ldots x_n$ преобразуется в аппликативную форму $X \circ R_{sem} \equiv \lambda x_1 \ldots x_n$. При этом отображении всевозможные пары (x_i, x_j), определяющие отношение r_k из множества R_{synt}, заменяются парами (x_i', x_j'), которые определяют семантические отношения, такие как: «входит в», «состоит из», «класс-подкласс» или «класс-объект» из множества отношений R_{sem}. Полученное аппликативное выражение $X \circ R_{sem} \equiv \lambda x_1 \ldots x_n$ позволяет сформировать термы, которые состоят из концептов множества $\tilde{X} \subset X$ и связаны между собой множественным отношением частичного порядка $\tilde{p}$. В результате получается множество концептов, связанное отношениями следования $\langle \tilde{X}, R_{ord} \rangle$, которое и определяет первичное таксономическое отображение концептов из отобранных терминополей.

Данный процесс выполняется с помощью следующих функций:

$$c_{a,b} = (\lambda x, y.t(x, y))a, b \equiv a \text{ имеет свойство } b \quad (4)$$

$$r_{a,b,c} = (\lambda x, y, z.t(x, y, z))a, b, c \equiv \text{между } a \text{ и } b \text{ есть отношение } c \quad (5)$$

На этапе выделения множеств терминополей сетевыми инструментами IT-ТОДОС обеспечивается выделение из текста документа терминов-концептов, а также определение иерархических семантических отношений между ними. Таким образом, аппликативная форма $X \circ R_{sem} \equiv \lambda x_1 \ldots x_n$ представляет таксономию документа, на основании обработки которого было выделено терминополе $\tilde{X}$, из концептов которого формируются термы аппликативных выражений вида $\lambda x_1, x_2, \ldots, x_n . c_{x_1 p_1} \, \& \, c_{x_2 p_2} \, \& \ldots c_{x_n p_n}$. Приписывание к нему справа аппликативного выражения вида $\lambda r_{x_1 x_2 k_{12}} \, \& \, r_{x_1 x_3 k_{13}} \, \& \ldots r_{x_{n-1} x_n k_{n-1n}}$ преобразует оба выражения в логическую функцию, являющейся тавтологией при всех значениях их термов.

Идентификация указанных отношений R_{sem} между концептами из множества $\tilde{X} \subset X$, выполняется на основании словарей ключевых слов. Для этого используется дополнительная функция:

$$\overline{c}_{a,b} = \left(\lambda x, y.t\left(x, y\right)\right) a, b \equiv a = b \tag{6}$$

Функция для выделения конкретных концептов имеет следующий вид:

$$\lambda x_1, y, x_2 . c_{x_1 p_1} \, \& \, \overline{c}_{y\overline{p}} \, \& \, c_{x_2 p_2} \tag{7}$$

Определение иерархических связей R_h между концептами множества $\tilde{X}$ реализуется на основе применения аппликативной функции вида (6), у которой входные термы представимы в форме вида: $x_1, x_2, \ldots, x_n, \overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n$. На этапе формирования соответствующего класса из указанной последовательности термов уточняется тип отношения между концептами-терминами, определяющими логическую функцию вида (4)–(7), являющейся тавтологией при всех значениях составляющих ее термов.

Обобщенная процедура поддержки процессов формирования таксономий контекстов на основе обработки сетевых документов в среде IT-ТОДОС приводится на рис.1.

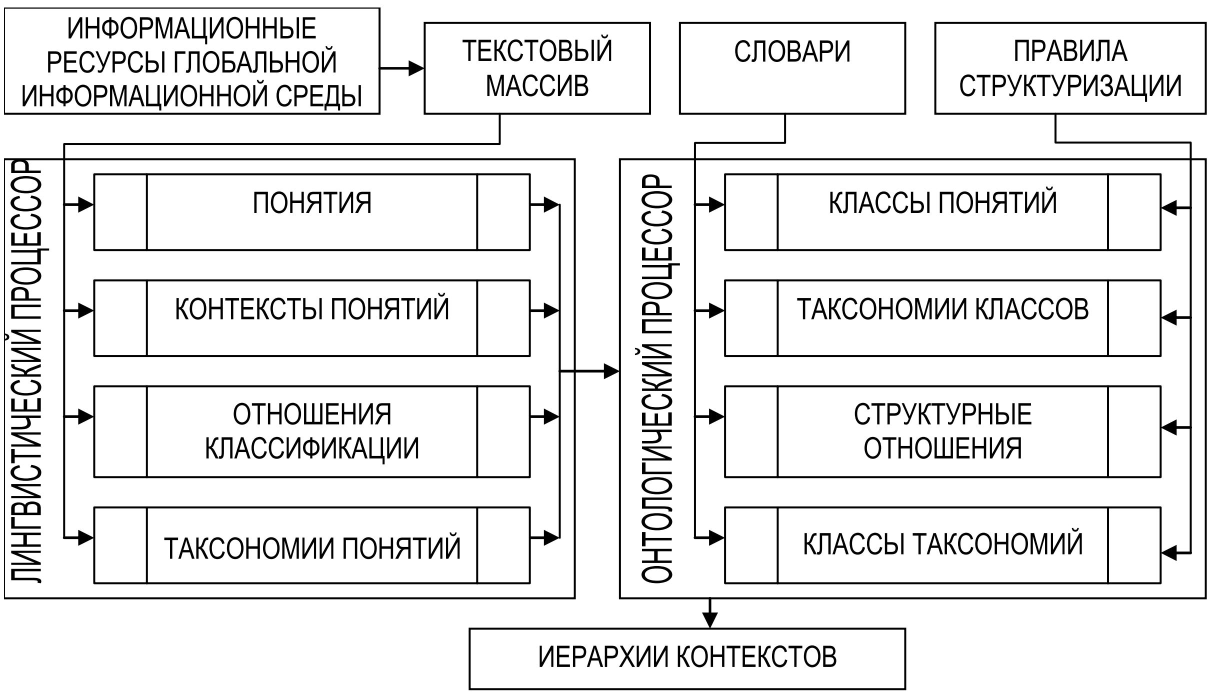

Рис. 1. Процесс формирования таксономий документов в среде IT-ТОДОС

Обеспечение взаимодействия в сетевой среде на основе формирования таксономий

Формирование таксономических структур, отображающих семантические свойства СИР и документов, составляющих их содержание, позволяет отобразить информационные процессы, активированные в глобальной среде, в виде безтиповых аппликативных выражений, каждое из которых представимо в виде онтологического графа [Гладун, 1994], [Палагин, 2012], [Величко, 2009]. Возникающая здесь проблема интероперабельности взаимодействия между компонентами СИР может быть сведена к определению множества отношений между термами, составляющих множество логических функций вида (4)–(7). Все функции указанного вида являются тавтологиями. Множество термов, определяющих синтаксическую конструкцию данных функций изоморфно множеству концептов, значения которых обрабатываются сетевыми информационными системами. Таким образом, отношения между термами, представленных в виде безтипового выражения вида (1)–(8), определяют выполнимость операций в сетевой среде над связанными компонентами СИР.

Каждый сетевой документ и каждая информационная система глобальной сети могут быть представлены в виде таксономии, поэтому процесс взаимодействия между ними сводится к установление отношений между их состояниями при смене активных концептов или отношений между концептами. Тогда проблема интероперабельности может быть разрешима на основе применения рекурсивного редуктора вида [Барендрегт, 1985], [Стрижак, 2015]:

$$\left(r_{order} \to \tilde{p}\right) \Rightarrow \left(\left(X, R, Rul\right) \to L_i \middle| i = \overline{1,n}\right) \tag{8}$$

Рекурсивный редуктор может быть сконструирован на основе следующих выражений:

$$r_{order} = \left\{\left(\lambda x.L_1\right)L_2, L_3\left[x := L_2\right]\right\}, \tag{9}$$

$$Pr\left(x_1,\dots,x_n\right) = \begin{cases} 1, \neg Pr\left(Y\tilde{p}x_i\right) \wedge Pr\left(x_1,\dots,x_n\right) \\ 0, Pr\left(Y\tilde{p}x_i\right) \end{cases} \tag{10}$$

где $x_i \in X; 1 \le i \le n$, X, Y – множества концептов,

$Pr\left(x_1,\dots,x_n\right)$ – рекурсивный предикат [Клини, 1957].

Каждый терм-концепт таксономии может быть представлен в виде предикативного выражения

$$x = P\left(r_1, r_2, \dots, r_j\right) \tag{11}$$

При этом следует учесть, что рекурсивный редуктор вида (8)–(10) сохраняет отношение эквивалентности между таксономиями всех компонентов СИР, как документов, так и информационных систем, за счет чего и обеспечивается разрешимость проблемы интероперабельности. Таксономия может быть определена для любого сложного терма-концепта, то есть по умолчанию она может быть образована бинарным отношением линейного порядка «группа объектов - объект». Отношение «группа объектов - объект» может быть расширено до множественного отношения «часть - целое» и далее до «быть элементом класса» и/или «быть элементом категории». Различать понятия «класс» и «категория» будем на основе полноты отображения предметной области. Определим понятие «категория» как достаточно полно отражающее семантические свойства предметной области, одновременно понятие «класс» определяется просто как отобранное множество концептов с общими семантическими свойствами.

Фактически применение предиката вида (10) совместно с выражением (11) позволяет определить, какие концепты таксономии сетевого компонента обладают данной свойством. Сформировав множество классов концептов таксономии с помощью выражений вида (8) - (10), мы получаем таксономии, над концептами которой задано множественное бинарное отношение «группа объектов - объект». Такая таксономия может иметь сложную структуру нисходящих иерархий. Каждый составляющая представляет собой класс термов-концептов, имеющих как минимум одно общее семантическое свойство.

$$\Pr(x_1,...,x_n) = 0 \Rightarrow \exists T \subseteq \breve{T} : \forall x \in X \exists Y \subseteq X : T = Y\tilde{p}x \qquad (12)$$

Множественное отношение частичного порядка $\tilde{p}$, на основании применимости выражений (1)–(10) ко всем компонентам СИР, определяет взаимодействие между концептами каждой таксономии, выделенной из каждого документа или информационной системы, составляющих СИР. Соответственно применение рекурсивного редуктора (8)–(10) обеспечивает формирование новых видов таксономий, из операционально используемых на всех этапах взаимодействия компонентов, составляющих СИР.

Тогда выражения вида (1)–(12), включающие в себя в качестве переменных и аргументов отношения из множества R_{sem} и множественное отношение $\tilde{p}$, обладают свойством интенсиональности, т. е. утверждения, формируемые из связанных общими отношениями термов-концептов, всегда обладают свойством истинности и еще набором других свойств, определяющих применимость данного утверждения.

Достоинством описанных аппликативных и предикатных выражений является то, что в качестве переменной и аргумента кроме концептов, могут быть отношения. Т. е. мы можем строить правила, которые связывают между собой и концепты, и отношения.

Безтиповость позволяет поднять уровень абстракции только до рассмотрения истинности утверждений, сформулированных на основе применения правил онтологии ПрО к таксономиям всех компонентов составляющих операциональное поле СИР.

Формирование иерархий тематических слоев в ГИС-системах на основе таксономий сетевых документов.

Одним из примеров обеспечения взаимодействия между документами и информационными системами в глобальной сети может служить формирование тематических слоев и их иерархической взаимосвязи в среде геоинформационных систем. На основе выделенных из сетевых документов таксономий, в среде ГИС могут быть сформированы соответствующие слои тематических карт. Каждый слой строится на основе класса, представляющего объекты таксономии отобранных документов и/или сетевой информационной системы. Сами объекты, которые входят в соответствующий класс, становятся объектами соответствующего тематического слоя.

Как уже было описано, таксономия формируется на основе установления отношений между концептами и классами вида «группа объектов - объект». При этом контексты концептов, составляющих таксономию, определяют легенду карты, определяющей тематические применения ГИС. Описание объекта на карте ограничено полями атрибутивной информации, а сервис информационных приложений позволяет определить и обеспечить доступ только до такого информационного массива, который физически присутствует у пользователя. Благодаря объединению различных типов баз данных в таксономии, атрибуты объектов могут быть представлены не только в табличном виде, но и в текстовом, а также в виде гиперссылок на соответствующие документы и информационные системы глобальной сети.

Рассмотрим процесс формирования тематических слоев минералов на основании их принятой классификации, которая описывается в определенном документе. Фрагмент такого документа, в котором описываются классы минералов и составляющие их объекты, представляется в виде таблицы 1. В соответствии с таксономическим представлением, как классы объектов, так и сами объекты, а также их свойства являются термами, которые могут быть представлены в виде выражений (11)–(12). При этом выражения (8)–(12) определяют процедуру формирования собственно таксономии.

Весь документ, включая его фрагмент в виде таблицы 1, представим в виде выражения (3) – $M \circ K = K \circ M = \lambda x.x$, где каждый сложный терм вида K, M может быть представлен в виде λ -последовательности вида (2). Выражение (2) также задает процедуру формирования всех классов минералов, как концептов редуцируемой таксономии. Каждая строка таблицы документа является классом определенных концептов минералогии и может быть представлена в виде сложного λ -терма L_m . Концепт-объект класса является простым λ -термом вида λx . Принадлежность объектов к классам определяется научной классификацией минералов и позволяет выделить соответствующие свойства каждого класса.

Таким образом, получаем из документа таблицу 1, которая сформирована на основе правил (8)–(12) множества Rul .

Применение к термам выражения (3) правил (8)–(12) позволяет из термов таблицы 1 построить таксономию, представленную на рис. 2. Отметим, что одним значением некоторых термов - выражений вида (2)–(3) - могут быть географические координаты, определяющие положение каждого концепта -минерала на географической карте.

Таблица 1. Классы и концепты-объекты минералов

	Class	Relation	Object	Object	Object	Object	Object	Object	Object	Object	Object
1	Halogenides	Belongs to class	Villiaumite	Cryolite	Halite	Sylvite	Fluorite	Weberite			
2	Organic Compounds	Belongs to class	Kerrite	Shungite	Succinite	Amber	Jet (lignite)	Coral			
3	Gemstone	Belongs to class	Turquoise	Malachite	Ruby	Opal	Obsidian	Citrine	Chrysoprase	Rauhtopaz	Amethyst
4	Ores	Belongs to class	Scheelite	Zircon	Rare earth element	Copper ore	Iron ore	Complex ore	Sphalerite	Covellite	Argentite
5	Single crystal	Belongs to class	Heliodor	Rhodonite	Topaz	Labradorite	Almandine	Phlogopite	Muscovite	Natrolite	Quartz
6	Native Elements	Belongs to class	Native cobalt	Native nickel	Native copper	Native silver	Native tin	Native gold	Native platinum	Diamond	Native sulphur
7	Oxides	Belongs to class	Smokey quartz	Hawk's eye	Tiger's-cat's eye	Cat's eye	Petrified wood	Flint	Chloropal	Spinel	Corundum
8	IMA CNMNC mineral classes	Classes	Native Elements	Halogenides	Oxides	Carbonates and Nitrates	Borates	Sulfates. Selenates. Tellurates	Phosphates. Arsenates. Vanadates	Silicates and Germanates	Organic Compounds
9	Classification by common attributes	Classes	Gemstone	Single crystal	Ores						

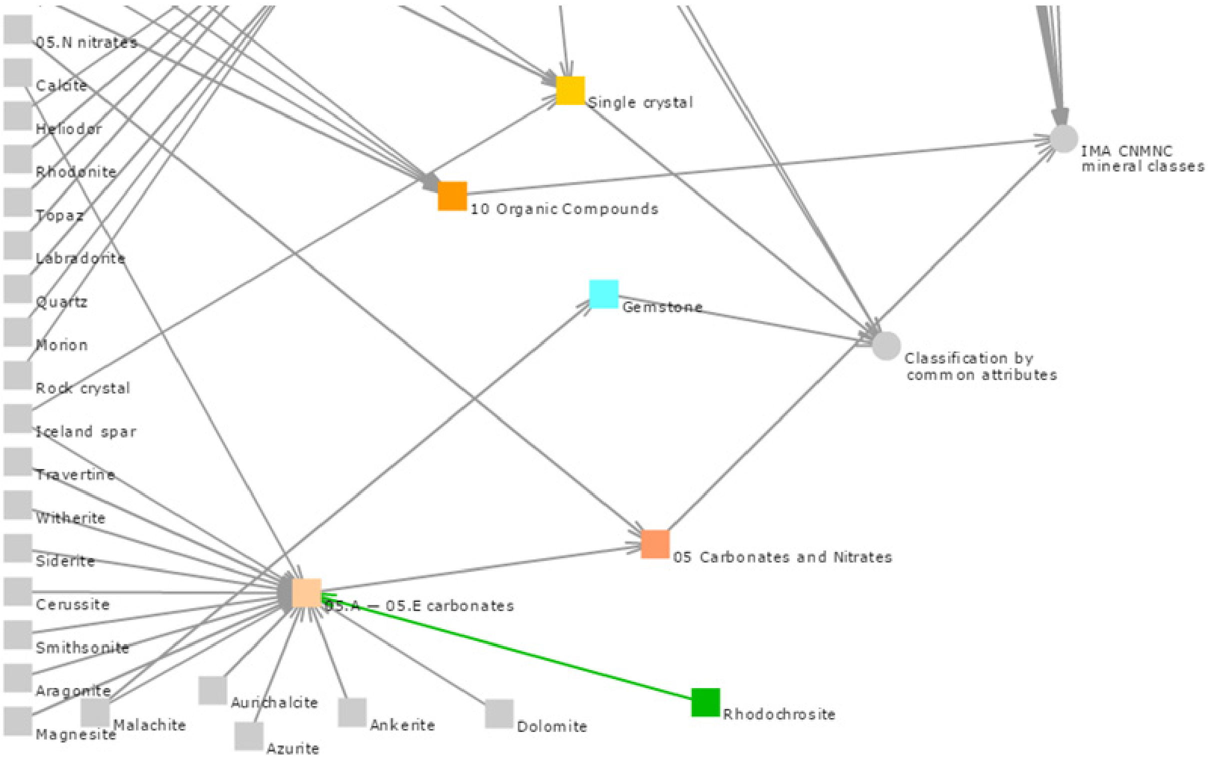

Рис.2. Фрагмент таксономии минералов

Согласно выражений (4)–(7), таксономия, представленная в виде графа (рис.2), может быть наложена на географическую карту, т.к. объекты таксономии графа имеют общее свойство с координатной сеткой карты. Т.е. описание концептов такого документа как географическая карта, а также сервисы соответствующей ГИС, могут быть включены в выражения (2)–(3). Тогда к ним применимы все правила вида (8)–(12). Это позволяет представить все концепты документа, описывающего минералы (Таблица 1) в среде ГИС (Рис.3)–(Рис 4).

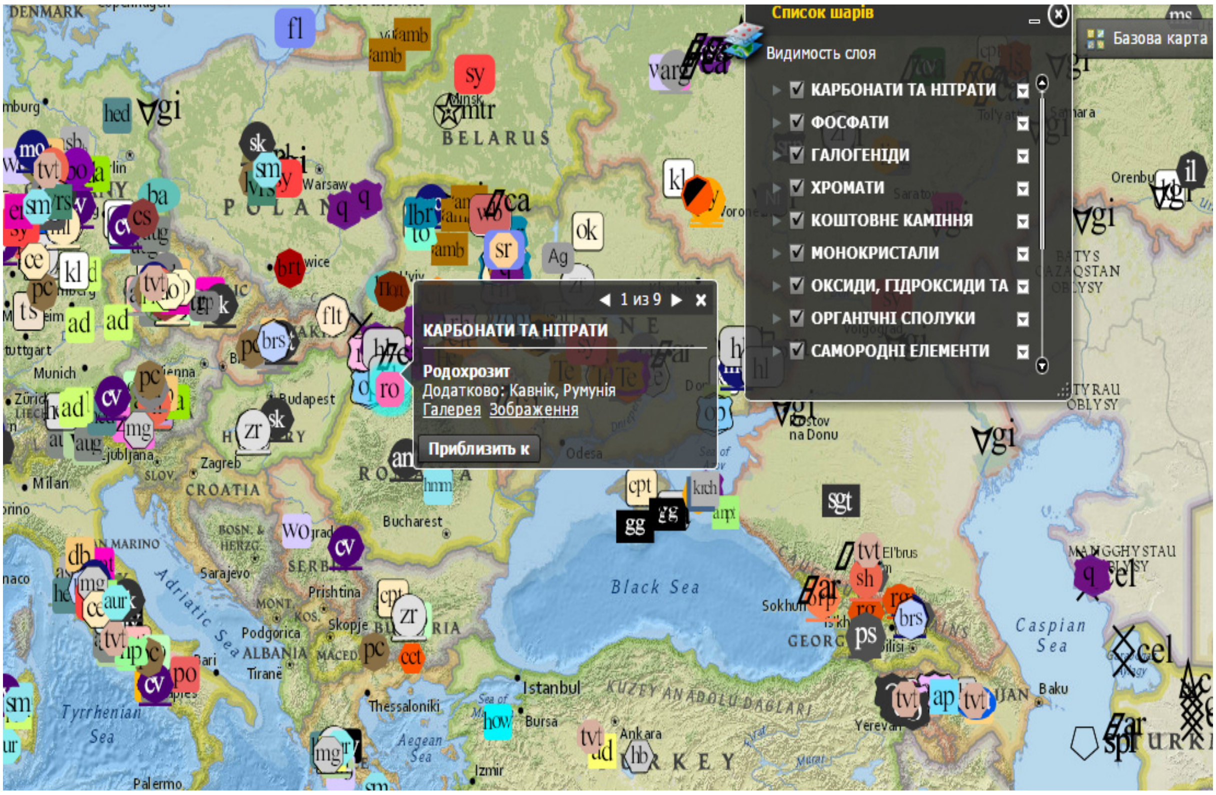

Рис.3. ГИС минералы

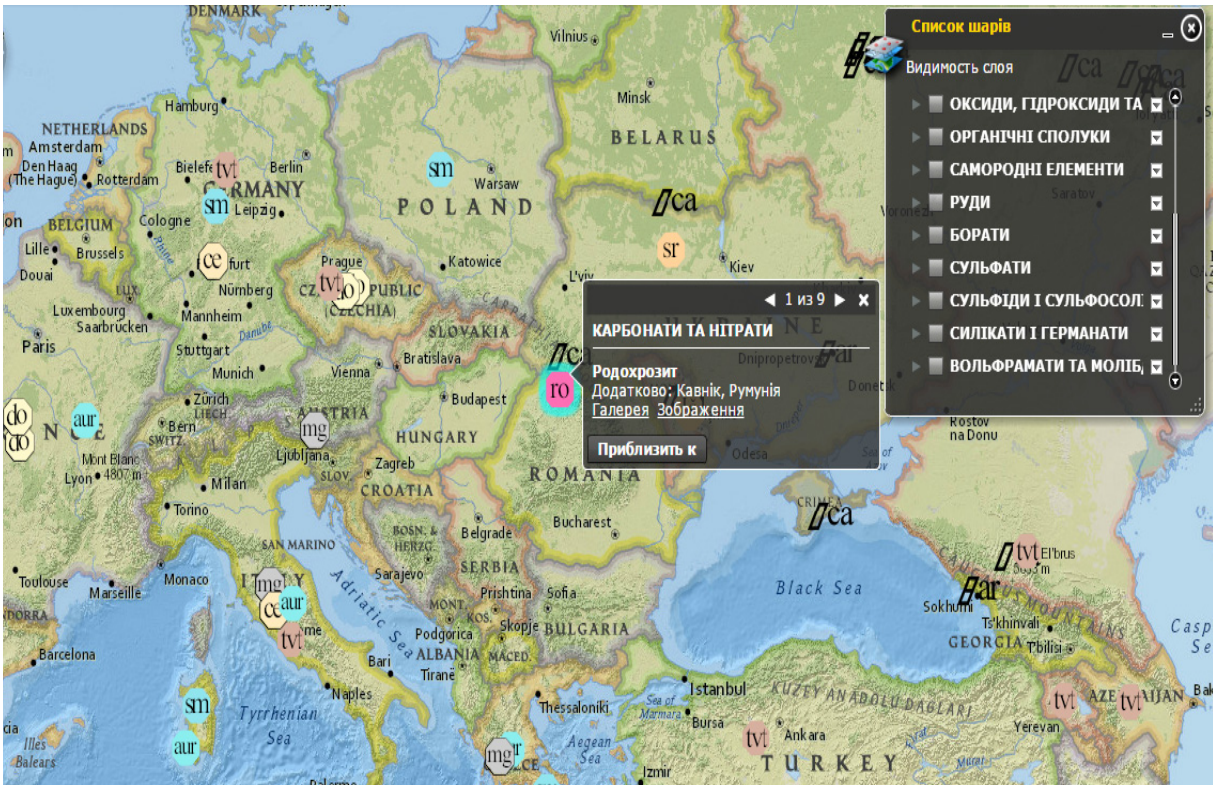

Рис.4 Фильтрация слоев карты для класса КАРБОНАТЫ и НИТРАТЫ

На основании декларирования принадлежности концепта к классу таксономии устанавливается его принадлежность к тематическому слою карты. Тогда множество всех допустимых тавтологий вида (4)–(6) состоит из утверждений о принадлежности концептов-понятий к определенной таксономии. Тавтологии формируются на основе связывания концептов классов, имеющих имена приведенных выше таксономий (Таблица 1 и Рис. 3). Как мы видим на Рис.3–4, легенда карты включает в себя тематические слои, аналогичные по имени, как классам понятий таксономии документа, так и объекты слоев, аналогичные концептам-понятиям таксономии документа.

Пространственные данные

„Пространственные данные" представляют собой данные с прямой или косвенной ссылкой на определенную точку на земной поверхности или в географической области [INSPIRE Directive, 2007]. Рассмотренные выше данные о минералах привязаны к географическим координатам месторождений. Иными словами, данные о минералах, привязанные к точке на земной поверхности, являются „пространственными данными", из чего следует, что изложенный выше подход возможно применить ко всем видам пространственных данных.

Директива 2007/2/ЕС Европейского Парламента и Совета от 14 марта 2007 года о создании инфраструктуры для пространственной информации в Европейском сообществе (INSPIRE) была опубликована в Официальном журнале Европейского Союза 25 апреля 2007 года и вступила в силу 15 мая 2007 года [INSPIRE Directive, 2007]. Основная цель директивы состоит в том, чтобы создать новый единый подход для обработки пространственных данных во всех странах-членах ЕС.

Упрощенный вид обычной обработки пространственнiх данных показан на рисунке 5. В большинстве случаев, каждая страна-член ЕС использует входные данные в соответствии с различными, часто не документированными или плохо документированными спецификациями данных. Для обработки входных данных также используются различные методы с целью получения более или менее похожей информации, имеющей отношение к деятельности в рамках ЕС [INSPIRE-DSM, 2007].

Методология, описанная в директиве INSPIRE, направлена на лучшее понимание общих требований пользователей к данным. Исходя из этих требований, методология сосредоточена на разработке согласованных спецификаций входных данных, построенных таким образом, таких чтобы все входные данные из разных стран-членов имели одинаковые характеристики и, в принципе, могли бы использовать одни и те же этапы обработки для получения необходимой информации. Обновленная схема обработки данных, разработаная с использованием предлагаемой методики, показана на рисунке 6 [INSPIRE-DSM , 2007].

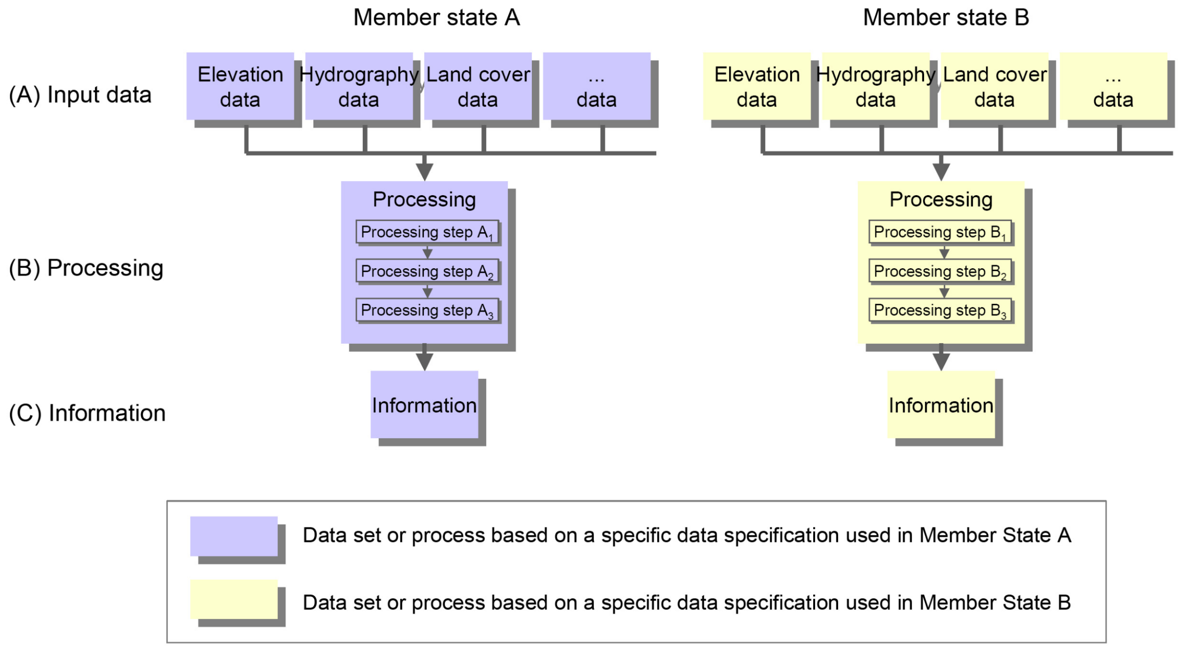

Рис. 5. Обычная ситуация: локальная (взаимонезависимая) обработка данных [INSPIRE-DSM, 2007]

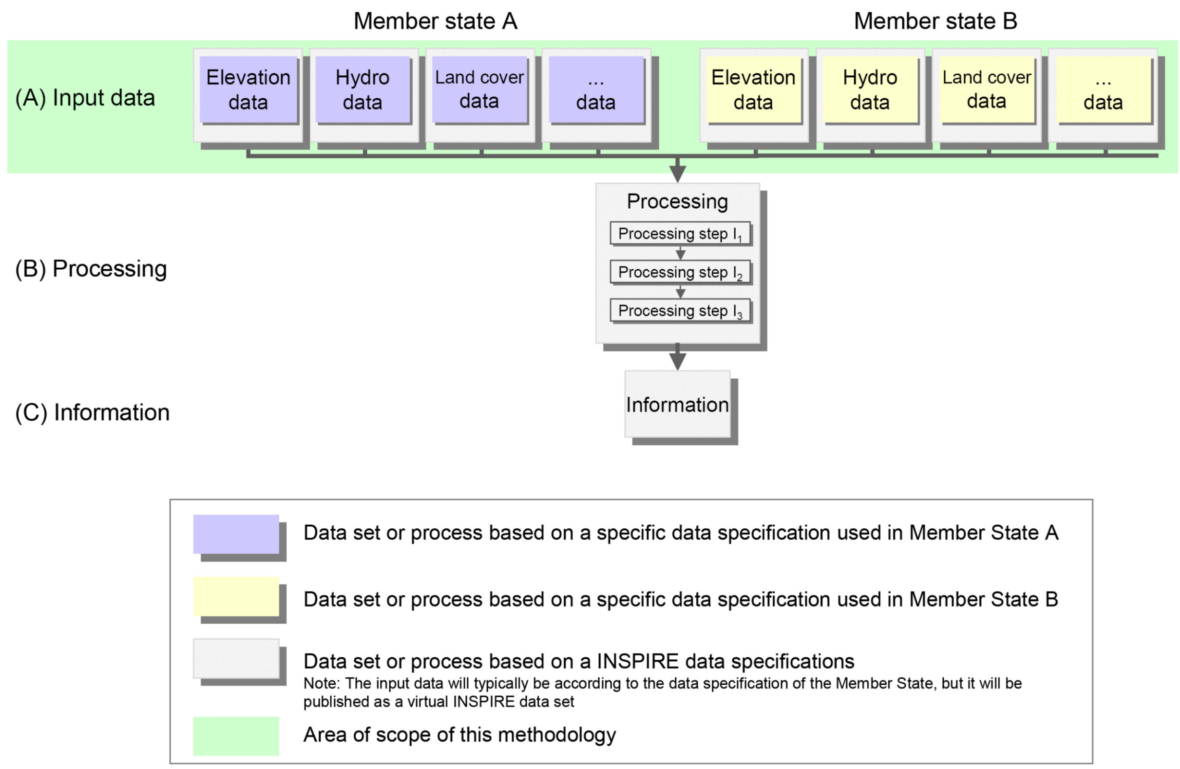

Рис. 6. Унифицированное представление данных, устраняющее взаимонезависимую обработку данных [INSPIRE-DSM, 2007]

INSPIRE - это инфраструктура для пространственной информации, которая создается государствами-членами, при которой:

– пространственные данные сохраняются, доступны и поддерживаются на наиболее приемлемом уровне;

– можно объединить пространственные данные из различных источников по всему сообществу последовательным образом и обмениваться ими между различными пользователями и приложениями;

– можно распределить пространственные данные, собранные одним органом государственной власти, между другими организациями;

– пространственные данные доступны для их широкого применения без дополнительных ограничений;

– легко обнаружить доступные пространственные данные, чтобы оценить их пригодность и узнать условия их использования.

Исходя из этих соображений, Директива фокусируется, в частности, на пяти ключевых областях:

– метаданные;

– возможность взаимодействия и согласования пространственных данных и услуг по отдельным темам (как описано в приложениях I, II, III части [INSPIRE Directive, 2007]);

– сетевые услуги и технологии;

– меры по обмену пространственными данными и услугами;

– меры по координации и мониторингу.

INSPIRE устанавливает правовую основу для создания и функционирования инфраструктуры для пространственной информации в Европе. Цель такой инфраструктуры – содействовать разработке политики по отношению к действиям, которые могут прямо или косвенно влиять на окружающую среду. Инфраструктура для пространственной информации включает:

- наборы метаданных о пространственных данных и услугах;

- сетевые услуги и технологий;

- соглашения о совместном использовании, доступе и использовании;

- координацию и мониторинг механизмов, процессов и процедур, устанавливаемых, эксплуатируемых или предоставляемых в соответствии с Директивой.

Напомним, что в Директиве [INSPIRE Directive, 2007] приняты следующие определения:

- "Метаданные" – информация, описывающая наборы пространственных данных и услуг, связанных с пространственными данными, и через которую возможно их обнаружение, систематизация и использование;

- "Пространственный набор данных" – идентифицируемое множество пространственных данных;

- "Пространственные данные" – любые данные, для которых имеется прямая или косвенная ссылка на конкретную точку земли или географического района;

- "Пространственный объект" – абстрактное представление некоторой сущности реального мира, связанной с конкретной точкой земли или географического района;
- "Сервисы для пространственных данных" – операции, которые могут быть выполнены над пространственными данными, содержащихся в наборах пространственных данных, или над соответствующими метаданными, путем вызова компьютерных приложений.

В принципе, каждый пространственный объект в пространственном наборе данных должен быть описан в спецификации, описывающей семантики данных и характеристики типов пространственных объектов в наборе данных [INSPIRE-TAO, 2007]. Типы обеспечивают классификацию пространственных объектов и определение среди прочей информации свойств, которые любой пространственный объект может иметь (тематические, пространственные, временные, функция покрытия и т.д.), а также известные ограничения (например, координатная реферативная система, которая может быть использована в наборах пространственных данных). Эта информация, в принципе, включена в схему приложения, используя язык концептуальной схемы, которая является частью спецификации данных.

В результате, спецификация данных обеспечивает необходимую информацию для интерпретации пространственных данных приложением.

Пространственный набор данных описывается набором метаданных, обеспечивающим поддержку обнаружения (а в определенной степени также оценку целесообразности использования) пространственных данных для конкретных целей.

Метаданные об сервисах предоставляют основную информацию о программах, предоставляющих услуги. Описание услуг включает в себя тип сервиса, описание операций и их параметров, а также информацию о географической информации, доступной через предлагаемые сервисы.

Формирование тематических слоев и их иерархической взаимосвязи в среде пространственных данных обеспечит взаимодействие между документами, содержащими пространственные данные, и архитектурами, ориентированными на услуги в глобальной сети. На основе выделенных из сетевых документов таксономий в среде пространственных данных могут быть сформированы соответствующие слои тематических карт. Сами пространственные данные, которые входят в соответствующий класс, становятся объектами соответствующего тематического слоя.

Напомним, что таксономия формируется на основе установления отношений между концептами и классами вида «группа объектов - объект». При этом контексты концептов, составляющих таксономию, определяют легенду карты, определяющей тематические применения пространственных данных. Благодаря объединению различных типов баз пространственных данных в таксономии, атрибуты объектов могут содержать гиперссылки на соответствующие документы и информационные системы глобальной сети.

Выводы

Трансдисциплинарная интеграция таксономии и тематической карты позволяет расширять и дополнять информационные описания понятий-объектов на базе сетевых информационных ресурсов, распределенных в интернет, а встроенная в таксономию поисковая машина - значительно расширить информацию об объектах. Такое сочетание позволяет создать единую информационно-аналитическую среду эксперта, которая перманентно пополняется территориально распределенными пользователями по различным направлениям исследований.

Предложенный подход особено полезен при работе с пространственными данными. „Пространственные данные“ представляют собой данные с прямой или косвенной ссылкой на конкретную точку земной поверхности или географической области. Директива INSPIRE ЕС установила новый вид взаимодействия между странами-членами ЕС в области пространственной информации. Основной целью Директивы является интероперабельность ("Interoperability"), что означает возможность объединения пространственных наборов данных и возможность взаимодействия сервисов в автоматическом режиме. Таким образом увеличивается добавленная стоимость наборов данных и сервисов [INSPIRE Directive, 2007].

Использование трансдисциплинарного подхода к классификации, систематизации, использованию информационных ресурсов и интеграции распределенных информационных моделей и систем на основе семантических свойств пространственных данных дает возможность каждому пользователю обнаруживать принципиально новые, ранее неизвестные взаимосвязи, способствует смещению акцентов с пассивных методов поиска, ориентированных на передачу информации, к более широкому применению активных методов анализа проблем и поиска решений, сотрудничеству пользователей и разработчиков и экспертов.

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