FURTHER RESULTS ON DOUBLE ±1 ERROR CORRECTING CODES OVER RINGS Z_m

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***Abstract**: In this paper further results on double ±1 error correcting codes over rings are presented. In particular optimal linear codes correcting ±1* type of errors over *rings Z_7 and Z_9 are constructed. A method allowing to construct (2N, 2N-6) double ±1error correcting codes over rings Z_m from the given (N, N-4) double ±1 error correcting codes over rings Z_m is also developed.*

***Keywords**: Error correcting codes, Codes over the rings* Z_7 and Z_9, Asymmetrical errors, Optimal codes.

1. Introduction

From practical point of view the codes over rings Z_{2m} or Z_{2m+1} are interesting, because they can be used in 2^{2m} – QAM (Quadrature amplitude modulation) schemes. Codes over finite rings, particularly over integer residue rings and their applications in coding theory have been studied for a long time. Errors happening in the channel are basically asymmetrical; they also have a limited magnitude and this effect is particularly applicable to flash memories.

There have been couple of papers regarding to optimal ±*1* single error correcting codes over alphabet Z_m [Martirossian, 1996; Kostandinov, 2010]. Also there are many linear codes capable to correct up to two error of type ±*1* for different alphabets which have been found by computer search, but there are not optimal. The optimality criteria for the linear codes over fixed ring Z_m can be considered in two ways. First of all, recall that the code of the length n is optimal-1 if it has a minimum possible number of parity check symbols. Secondly, optimality-2 criteria for the code is that for a given number of parity check symbols, it has a maximum possible length. The linear code (12, 8) correcting double error over ring Z_5 of value ±*1* was presented in [Khachatrian, 2014] satisfies the optimality criteria -1:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 3 & 4 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 2 & 2 & 2 & 2 & 2 & 1 & 1 \\ 3 & 2 & 4 & 4 & 2 & 3 & 2 & 4 & 4 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 3 & 2 & 4 & 4 & 2 & 0 & 4 \end{bmatrix},$$

this code was given by the parity check matrix *H*, which has 8 information and 4 parity check symbols.

At this point we do not know any codes that satisfy the optimality criteria-2. In [Khachatrian, 2014] a method how to compare two code constructions over different size of alphabets when both satisfy the optimality – 1 criterion has been presented. Two factors are considered, namely:

1) The first factor should be the rate of the code i.e. the ratio of the number of information symbols over the length of the code.
2) Second, the ratio between the number of possible amplitude errors corrected by the code over the size of alphabet minus 1, which corresponds to the number of all possible amplitude errors.

The product of these two factors is chosen as a merit to compare optimal codes over different size of alphabets.

For the code over ring Z_5 mentioned above the product is: (8/12) * (2/4) = 0.3333.

In this paper constructions of the optimal-1 codes (16, 12) and (20, 16) over the rings Z_7 and Z_9 correcting double ± 1 errors is presented. For these codes the products will be (12/16) * (2/6) = 0.25 for code over Z_7 and (16/20) * (2/8) = 0.2 for Z_9 . These products are a little bit smaller then for the code (12, 8) over ring Z_5 , although there are much better comparing with codes over Z_{16} and Z_{128} in [Kostandinov, 2010; Han Vinck, 1998; Kostandinov, 2008].

Moreover, in this paper a constructions of codes C (N, N-6), which are 2 times longer than previous ones, and have 6 parity check symbols are presented.

2. Construction of Optimal (16, 12) code over ring Z_7

Our purpose is to construct an optimal linear code over ring Z_7 correcting double errors of the type ± 1. It is well known, that a linear code given by the parity check matrix H , can correct up to two errors of the type ± 1, only when H has a property according to which all the syndromes resulting from adding and subtracting operations between any two columns of the matrix H are different $\left(\pm h_i \pm h_j , where \ (h_i \neq \pm h_j)\right)$. For constructing this kind of matrix H , at first we will find a difference set in Z_7. For example, a difference set for a linear code (12, 8) constructed in [Khachatrian, 2014] is the set {3, 2, 4, 4, 2}. A difference set is defined to have a property that the differences for any 2 components in the set are different in Z_5 given that difference is taken for the elements located at the same distance from each other where the distance itself can be from the set (1, 2, 3, 4). Note also that the distance between positions of elements is calculated modulo 5 in this case. For an example if the distance is chosen to be three, we have to take a difference between 4-th and 1-th positions of the set which is equal = 1, a difference between 5-th and 2-th positions of the set will be 0, a difference between 1-th(6-th) and 3-th positions of the set will be -1(4), a difference between 2-th(7-th) and 4-th positions of the set will be -2(3), and finally a difference between 3-th(8-th) and 5-th positions of the set will be 2.

For the ring Z_7 it is easy to check that the difference set is a set – {4, 3, 6, 6, 3, 4, 2} of the length 7.

For instance, for the distance equal to 1 all corresponding differences resulting from (3 – 4 = **6**, 6 – 3 = **3**, 6 – 6 = **0**, 3 – 6 = **4**, 4 – 3 = **1**, 2 – 4 = **5**, 4 – 2 = **2** (**mod7**)) are different. 4-corresponds to the position with index 0 and the last position 2 corresponds to the position with index 6 and 0 – 6 = 1 (mod 7) (all operations are in Z_7).

A linear (16, 12) code over ring Z_7 is given by the following parity check matrix *H*:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 1 & 1 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 \\ 4 & 3 & 6 & 6 & 3 & 4 & 2 & 4 & 3 & 6 & 6 & 3 & 4 & 2 & 1 & 6 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 4 & 3 & 6 & 6 & 3 & 4 & 2 & 0 & 0 \end{bmatrix}. \quad (2.1)$$

An approach how this matrix is designed is similar to one in [Khachatrian, 2014]. It consists of three parts, namely the first 7 columns, the next 7 columns and a tail of two last columns. The first two rows of the first and second parts is a code correcting one error of the type ± 1 third rows of the parts 1 and 2 as well as the fourth row of the second part is a difference set for Z_7. It can be checked that a linear code over Z_7 , given by the parity check matrix *H* in (1) can correct up to two errors of the type ± 1. This can be done in the similar manner demonstrated in [Khachatrian, 2014] and of course also by computer.

Lemma 1. *A linear code (16, 12) correcting up to two errors of the type ±1 is optimal in the sense that it has a minimal possible number of parity check symbols.*

Proof: In this case the number of combinations for each code word that can be corrected is

$$(1 + 16*2 + (16 \text{ choose } 2) * 4) = 513.$$

Thus, we have that $513 * 7^{12} \leq 7^{16}$ and the cardinality of the best possible code is $7^{16} / 513 < 7^{13}$.

3. Construction of Optimal (20, 16) code over ring Z_9

In this section we will construct an optimal (20, 16) linear code over ring Z_9. As same as in previous construction we need to find a difference set of length 9 for Z_9**.** In this case we could not find a difference set of length 9. So, to fix this problem we find a difference set of the length 8: {7,3,2,4,4,2,3,7}. Similarly in this set, for all distances (1, 2, 3, 4…) the differences of any 2 components should be different in Z_9. For instance, for the distance-1 all corresponding differences resulting from (3 - 7 = **5**, 2 - 3 = **8**, 4 - 2 = **2**, 4 - 4 = **0**, 2 – 4 = **7**, 3 – 2 = **1**,7-3=**4**(mod9)) are different (all operations are in Z_9).

In the previous construction, sequences consisting of all integers in Z_7 - {0, 1, 2, 3, 4, 5, 6} have been used in rows of the matrix. Since we have for Z_9 a difference set with only 8 components, we should take either a sequence {0,1,2,3,4,5,6,7} or {1,2,3,4,5,6,7,8}.

In this case the parity check matrix for an optimal linear code (20, 16) correcting double errors of the type ± 1 has the following form:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 1 & 1 & 2 & 4 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 2 & 4 \\ 7 & 3 & 2 & 4 & 4 & 2 & 3 & 7 & 7 & 3 & 2 & 4 & 4 & 2 & 3 & 7 & 1 & 1 & 2 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 7 & 3 & 2 & 4 & 4 & 2 & 3 & 7 & 6 & 3 & 7 & 2 \end{bmatrix}. \qquad (3.1)$$

As for previously constructed codes a corresponding parity check matrix (3.1) also consists of three parts. Since a difference set in this case has only 8 elements a corresponding split between those parts will be (8, 8, 4). This is because our target is to have a code of the length 20 which will be optimal and therefore we will need for the tail part to have 4 columns which is in fact the last four columns of the matrix (3.1). A linear code over Z_9, given by the parity check matrix H (3.1), can correct up to two errors of the type ± 1. The proof of this statement can be done similarly by manner demonstrated in [Khachatrian, 2014], as well as by computer.

Lemma 2. *A linear code (20, 16) code given by* (3.1) *correcting up to two errors of the type* ± 1 *is optimal in the sense that it has a minimal possible number of parity check symbols.*

Proof: In this case the number of combinations for each code word that can be corrected is

$$(1 + 20*2 + (20 \text{ choose } 2) * 4) = 801.$$

Thus, we have that $801 * 9^{16} \leq 9^{20}$ and the cardinality of the best possible code is

$$9^{20} / 801 < 9^{17} .$$

4. Construction of C (2N, 2N-6) codes based on optimal C (N, N-4) codes

In this section we will describe a method, which allows us to construct new codes with a double length at the expense of just two parity check symbols. We will assume that we have at our disposal (N, N-4) double error correcting code like codes constructed in part 2 in this paper as well as in [Khachatrian, 2014]. In this paper we construct codes of length N with 4 parity check symbols C (N, N-4). Using method, which will be described below, we can construct codes of length 2N with 6 parity check symbols C (2N, 2N-6). Further in this paper by codes we mean double error correcting code of the type ±*1*.

Let C (N, N-4) be a code over ring Z_m . Our construction of a new code will have a parity check matrix with 6 rows and 2N columns where the first 4 rows will be just a repetition of 4 rows of the code C (N, N-4). Now we will describe how we add 2 additional rows for the parity check matrix. The first N columns and the last N columns of two additional rows will be referred as group 1 and a group 2 respectively. In the first row of group 1 we put integer 2 repeated n times then integer 1 repeated n times and then (N -2n) times any integer x from Z_n . In the second row of the group 1 we put all integers from Z_n {0, 1 ,…, n-1} repeated twice and then but the first (N-2n) elements of {0,1,…, n-1}. Consequently, in the first row of group 2 we put the second row of group 1, and in the second row of group 2 accordingly we put integers 3 and 4 repeated n times, and in the rest of positions the same integer x from group 1. Note that an integer x should differ from (1, 2, 3, 4) and must satisfy the condition (($x + x \neq 1 \pmod n$)),

Group 1	Group 2
C (N, N - 4)	C (N, N - 4)

$$\begin{bmatrix} 2 & 2 & \dots & 2 & 1 & 1 & \dots & 1 & x & x & \dots & 0 & 1 & \dots & n-1 & 0 & 1 & \dots & n-1 & 0 & 1 & \dots \\ 0 & 1 & \dots & n-1 & 0 & 1 & \dots & n-1 & 0 & 1 & \cdots & 3 & 3 & \dots & 3 & 4 & 4 & \dots & 4 & x & x & \dots \end{bmatrix}$$

Now we can prove the following theorem.

Theorem 1: *For a given C (N, N-4) code over ring* Z_n *correcting double ±1 errors, it is possible to construct a code with 6 parity check symbols of a length C(2N, 2N-6) correcting ±1 double errors.*

Proof. In order to prove this theorem it must be shown that all corresponding syndromes resulting from operations ±*1* between all columns of both groups should be all different. Let us split all columns of the group 1 into 3 subgroups, namely the first subgroup (subgroup 1.1) contains the first columns of group 1, which first component is 2, starting from the left, the second subgroup (subgroup 1.2) contains next n columns, which first component is 1, and the third subgroup (subgroup 1.3) contains the last columns with x. Accordingly, we can do the same with columns of group 2 and split them into three subgroups

(2.1, 2.2 and 2.3) in accordance with (3.1). We need to consider only those cases, when the first four components have the same syndromes. We will demonstrate the proof for the case, when the first error has upward direction (+1), and second downward (-1). Let us perform the proof by 3 cases:

1) Let's suppose that two errors occurred in the first group. Because the both parts of parity check matrix H consist of the same code in first 4 rows, we do not know in which part the errors occur: whether in the first part of matrix H or in the second one. We can check it using next 2 rows. There can be only 3 possible subcases:

1.1) If errors are in subgroup 1.1 the first position will always be 0, otherwise in group 2 it cannot be 0 (due to the property of the set {0,1,…, n-1}), and the syndromes will be different.

1.2) If one error occur in subgroup 1.1 and the second in subgroup 1.2 in group 1 the first position will be 1, but in group 2 the second position will be -1, and the syndromes will always be different, because we have the same components in two other positions (the second row of group 1 and the first row of group 2 are the same).

1.3) If one of the errors occur in subgroup 1.3, then if next is in subgroup 1.1 resulting syndromes will always be different, because in subgroup 1.1 the first position is 2 and in subgroup 2.1 the second position is 3, but we have the same components as in subgroups 1.3 and 2.3. Like the case b) the other two components of the syndrome always will be the same (the second row of group 1 and the first row of group 2 are the same). If second error occur in subgroup 1.2 (2.2) the way of the proof is the same.

Thus, the first case of the proof is complete.

2) Let one error occur in group 1 and second in group 2. We need to check whether both errors are in the same group or not. Again there can be 3 possible subcases:

2.1) Let an error occur in subgroup 1.1 and second error in subgroup 2.1. As the first four components of matrix H are in these subgroups we have the same columns the errors might be in the same subgroup. How we can distinguish these cases? If both occur in subgroup 1.1, then the first component will be 0, otherwise, in our case 0 can be only with column 3 of subgroup 2.1. In this case, in the third column of subgroup 1.1 the second position is 2, and in the same column of subgroup 2.1 it is 3, consequently the syndromes will be different (if both occur in subgroup 2.1 the proof is the same (the second component will be 0)).

2.2) Let one error occur in subgroup 1.1 and the second in subgroup 2.2. If both errors occur in group 1, then the first component will be 1, in our case 1 can be only with column 2 of subgroup 2.2, here the second component is 4, but in subgroup 1.2 it is 1, consequently the syndromes will be different. If both occur in group 2, then the second component will be -1, in our case -1 can be only with column 4 of subgroup 1.1, here the first component is 2, but in subgroup 2.1 it is 3, and syndromes will be different. (For the case when errors occur in subgroup 2.1 and subgroup 1.2 the way of proof is the same).

2.3) Let one error occur in subgroup 1.1 or in subgroup 1.2 and the second in subgroup 2.3. If both errors occur in group 1 (subgroup 1.3) the resulting syndromes will be different, because in subgroups 1.3 and 2.3 the corresponding rows are swapped like $\begin{pmatrix} x & x & \dots \\ 0 & 1 & \dots \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 & \dots \\ x & x & \dots \end{pmatrix}$. Consequently, when we subtract them from the same subgroup 1.1 or 1.2 the resulting syndromes will be different. (For the case when errors occur in subgroup 1.3 and in subgroups 2.1 or 2.2 the way of proof is the same).

Thus, the second case of the proof is complete.

3) In this case both of the errors occur in the same columns of different groups. Accordingly, the first four components of the syndromes will be (0 0 0 0). In this case the number of all possible syndromes will be 2N . Due to the selection of last two rows of matrix (group 1 and group 2), it can be shown that all 2N syndromes will be different. In this case the difference between the same columns for the first two subgroups of groups 1 and 2 will be if the second element of the last column is the same first elements will be different by two, while the difference between the same columns in corresponding third subgroups will be $\begin{pmatrix} x - i \\ i - x \end{pmatrix}$ and will be different for all i's unless 2x ≠1 (mod **n**) - which is the condition for x. This analysis completes the proof of the theorem.

Here we will show some results, which we have gotten using the method described by the theorem 1:

The code (24, 18) correcting double ±*1* errors, given by parity check matrix H_5 was resulted from the optimal code (12, 8) over ring Z_5 by adding 2 parity check symbols:

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 & 1 & 2 \\ 0 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 & 1 & 2 & 3 & 3 & 3 & 3 & 3 & 4 & 4 & 4 & 4 & 4 & 0 & 0 \end{bmatrix}$$

$$H_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 3 & 4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 3 & 4 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 0 & 1 & 2 & 3 & 4 & 2 & 2 & 2 & 2 & 2 & 1 & 1 \\ 3 & 2 & 4 & 4 & 2 & 3 & 2 & 4 & 4 & 2 & 1 & 1 & 3 & 2 & 4 & 4 & 2 & 3 & 2 & 4 & 4 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 3 & 2 & 4 & 4 & 2 & 0 & 4 & 1 & 1 & 1 & 1 & 1 & 3 & 2 & 4 & 4 & 2 & 0 & 4 \\ 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 & 1 & 2 \\ 0 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 & 1 & 2 & 3 & 3 & 3 & 3 & 3 & 4 & 4 & 4 & 4 & 4 & 0 & 0 \end{bmatrix}.$$

The code (32, 26) correcting double ±*1* errors, given by parity check matrix H_7 was resulted from the optimal code (16, 12) over ring Z_7 by adding 2 parity check symbols:

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 5 & 5 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 \end{bmatrix}$$

$$H_7 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 1 & 1 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 \\ 4 & 3 & 6 & 6 & 3 & 4 & 2 & 4 & 3 & 6 & 6 & 3 & 4 & 2 & 1 & 6 & 4 & 3 & 6 & 6 & 3 & 4 & 2 & 4 & 3 & 6 & 6 & 3 & 4 & 2 & 1 & 6 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 4 & 3 & 6 & 6 & 3 & 4 & 2 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 4 & 3 & 6 & 6 & 3 & 4 & 2 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 5 & 5 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 \end{bmatrix}.$$

And lastly, the code (40, 34) correcting double ±*1* errors, given by parity check matrix H_9 was resulted from the optimal code (20, 16) over ring Z_9 by adding 2 parity check symbols:

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 6 & 6 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 6 & 6 \end{bmatrix}$$

$$H_9 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 1 & 1 & 2 & 4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 1 & 1 & 2 & 4 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 2 & 4 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 2 & 4 \\ 7 & 3 & 2 & 4 & 4 & 2 & 3 & 7 & 7 & 3 & 2 & 4 & 4 & 2 & 3 & 7 & 1 & 1 & 2 & 4 & 7 & 3 & 2 & 4 & 4 & 2 & 3 & 7 & 7 & 3 & 2 & 4 & 4 & 2 & 3 & 7 & 1 & 1 & 2 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 7 & 3 & 2 & 4 & 4 & 2 & 3 & 7 & 6 & 3 & 7 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 7 & 3 & 2 & 4 & 4 & 2 & 3 & 7 & 6 & 3 & 7 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 6 & 6 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 6 & 6 \end{bmatrix}$$

5. Conclusion

In this paper further results on construction of linear double ±*1 error correcting codes are presented. In particular a technique is developed which allows to double the length of the codes at the cost of just two parity check symbols. That technique will allow to construct codes with higher rates.*

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