COMPONENT MODELING: ON CONNECTIONS OF DETAILED PETRI MODEL AND COMPONENT MODEL OF PARALLEL DISTRIBUTED SYSTEM

Elena Lukyanova

Abstract: Connections of detailed Petri *N* model and its component *CN*-net on the level of their structural and dynamic properties are investigated.

Keywords: *component Petri net, components-places, components-transitions, languages of component net with components-places and component net with components-transitions.*

ACM Classification Keywords: *Petri net, language of Petri net, homomorphism, epimorphism, model, model verification.*

Introduction

One of the stages of process of research of the dynamic systems of any complication is a construction of qualitative model of the investigated system. Realization of researches of model makes it possible to set a number of specific properties of model with subsequent interpretation of the results in relation to the target system. Petri nets [Kotov, 1984] are convenient means of modeling (detailed modeling) of various parallel distributed systems. In a detailed modeling of real systems and objects we can receive a larger network, which makes the analysis of detailed models impracticable. When taking component Petri net (CN -net) as a model of parallel distributed system [Lukyanova 1, 2012, Lukyanova, 2011], we get the possibility to work with the model that is much smaller than the original Petri detailed model. It is important that CN -model does not lose its conformity for the description of the initial investigated system.

This paper describes currently obtained results on the links between the detailed Petri model and component Petri model of investigated parallel distributed system. These are the results of the relationship of structural properties of CN - and detailed Petri models, of the language of detailed Petri model and the language of CN -model with only components-transitions. In the paper, we determine the language of component Petri net containing only components-places, its relations with the language of detailed model and characteristics of input languages.

Component modeling of parallel distributed systems

On the initial stage of component modeling, analysis of the original complex system for allocating its constituent simpler objects – groups of identical and single-type processes (processes that are of the same type differ only in the number of identical parallel processes). It allows forming allocated groups of identical and single-type processes as blocks of composite model components at the phase of model construction. Thus, in the modeling process, we obtain a detailed model of the original system, which has identical and single-type processes placed in the appropriate blocks – composite components (components-places Cp and components-transitions Ct) [Lukyanova 2, 2012]. Among the constituent components there may be identical and single-type composite components [Lukyanova 3, 2012]. Petri net constructed is called component Petri net.

Definition 1. Component Petri net (CN -net) is a directed graph, described by the ordering quinary $CN = (P, T, F, W, M_0)$, where P is a finite set of places consisting of subsets P_1 and P_2 (P_1 is a finite set of component-places, P_2 – a finite set of places that are left after the separation of component-places); T – final set of transitions, consisting of subsets T_1 and T_2 (respectively, the set of components-transitions and a set of transitions that are left after the separation of the component transitions); $F \subseteq P \times T \bigcup T \times P$ – the incidence relation between places and transitions; $W : F \to N \setminus \{0\}$ – the multiplicity function of arcs; M_0 – the initial marking of net.

The ratio of the incidence F and multiplicity function of arcs W determine the function of the incidence I, defining the rule $I : (P \times T \bigcup (T \times P) \to N)$. Incidence function defines that the elements of one set of arcs cannot be connected, and describes the sets of input and output elements.

Component-place Cp designs some single-type processes of the detailed Petri model of the investigated system, which begins and ends with the place (places). Component-place is triple $Cp = (N, X, Y)$, where N is Petri net, $X \subseteq P$, $Y \subseteq P$ – the sets of its initial and final places correspondingly, not having respectively the input and output arcs, and $X \bigcap Y = \varnothing$. Component Cp, as a structural element of CN -net, is a place that has input and output arcs, and as in the regular Petri nets is a condition that determines the possibility of an event – transition firing in CN -net.

Component-transition Ct is an area of net of the detailed model, designing some of the single-type process, beginning and ending with transition (transitions).

Component transition Ct is triple $Ct = (N, U, V)$ where N is Petri net, $U \subseteq T$, $V \subseteq T$ – respective sets of its initial and final transitions that do not have the respective input and output arcs, and $U \bigcap V = \varnothing$. Component Ct as part of CN -net has input and output arcs, and is an event and transition of CN -net.

Remark 1. It is natural to denote corresponding places and transitions in identical and single-type composite components with the same symbols, and corresponding places and transitions with the same symbols in identical parallel processes.

As an example of CN -model, Fig. 1 shows a small CN -net. This CN -net includes:

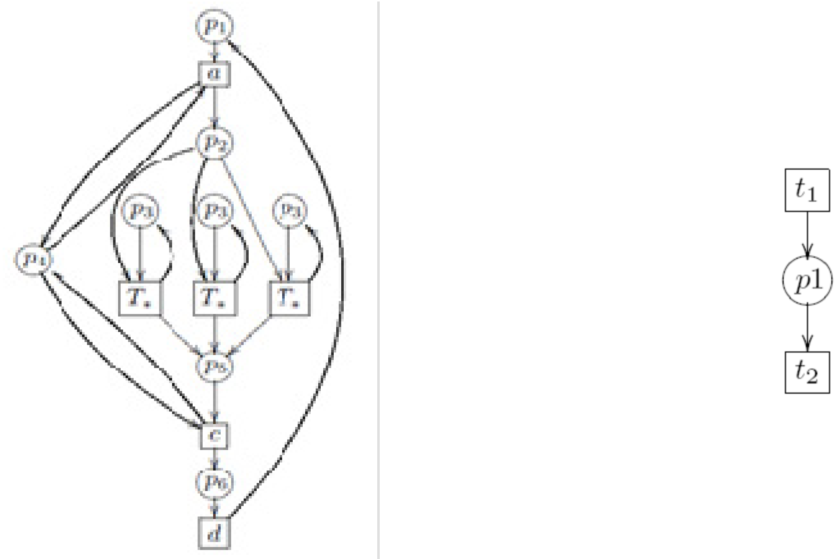
1). Three identical parallel processes. According to [Lukyanova 3, 2012, Lukyanova 4, 2012], in these processes, relevant places and transitions, according to Remark 1, are indicated by the same markings (Fig. 1, a);

2). Composite components – components-transitions *T** (Fig. 1, b).

At that, CN -net of Fig. 1 is itself one of composite components-places P_8^*, P_{10}^*, P_{14}^* P_{16}^*, P_{17}^*, P_{20}^* – component-place P_{16}^* in CN -net [Lukyanova 3, 2012], as shown at Fig. 2.

CN-model analysis to determine the structural properties

The effectiveness of analysis of CN -models to determine the structural characteristics of a detailed Petri model of parallel distributed system has been stated in [Lukyanova, 2011, Lukyanova 3, 2012]. The proposed analysis is based on the analysis of the component net (CN -net), in which the component parts are considered only as places and transitions, and on the analysis of one representative from each group of single-type components. From this group of single-type components, "minimal" representative is selected – an integral component with the least number of identical parallel processes. So here is the final theorem [Lukyanova 3, 2012].



a) **b)**

Figure 1. a) CN -net; b) component-transition *T** in CN -net.

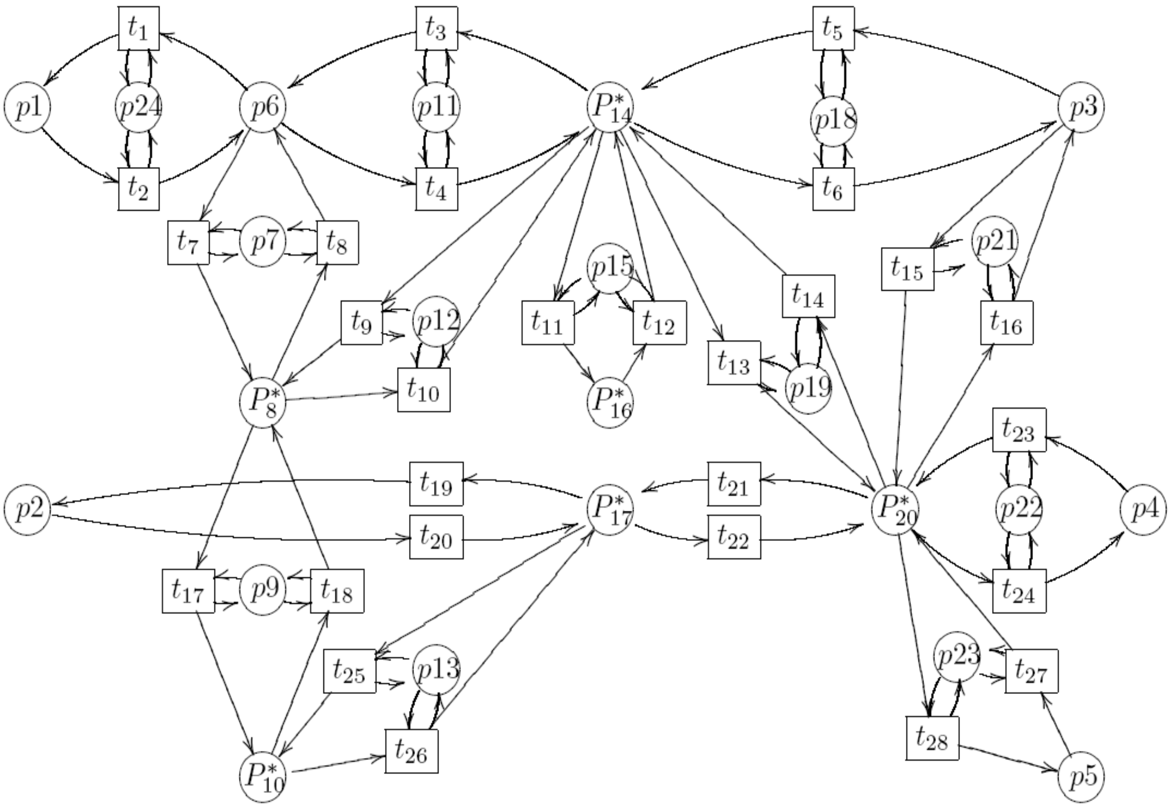


Figure 2. CN -net, in which P_8^*, P_{10}^*, P_{14}^* P_{16}^*, P_{17}^*, P_{20}^* are components-places.

Theorem 1. Detailed Petri model of the investigated system does not have structural property if corresponding CN -net does not have this property.

Detailed Petri model of the investigated system has structural property if corresponding CN -net has this property subject to the fulfillment of following properties of the structural elements of the CN -net:

a) for each group of single-type components-transitions, the component-transition with the least number of identical parallel processes is live;

b) for each group of single-type components-places, a combined system of inhomogeneous linear Diophantine equations (SILDE) corresponds to a component-location with the least number of identical parallel processes.

Languages of *CN*-model

CN -net research involves the analysis not only of structural characteristics, but also of a series of dynamic characteristics. Dynamic behavior of the modeled system is described in terms of the net functioning. Functioning of the net is formally described by the set of the sequence of firing and set of achievable net marking.

While determining language, generated by CN -net, one should consider:

- presence of two different types of composite components in the component Petri net: components-places C_p and components-transitions C_t ;
- two-aspects approach is applicable to the operation of the component Petri nets [Lukyanova, 2011, Lukyanova 2, 2012]: on the one hand, the composite components of the CN -net serve immediately; on the other hand, the study of the functioning of composite component is an integral part of the investigation of the properties of the CN -net model.

This means that we need a separate consideration of the nature of the sets of possible sequences of events or of sets of reachable markings in the composite components themselves, in component net only with components-transitions, in component net only with components-places, and in component net with both components-places and components-transitions.

In the study of CN -language net, containing only the components-transitions C_t [Lukyanova 4, 2012], operation of the net is described in terms of sequences of firing of transitions [Kapitonova, 1988, Kotov, 1984].

Given X and Y respectively finite alphabets of detailed Petri model and CN -model only with components-transitions of the investigated system, then X^* is a set of all words in alphabet X , $Y^* = (X \cup \{T_1^*, T_2^*, ..., T_n^*\})^*$ – a set of all words in the alphabet $Y = X \cup \{T_1^*, T_2^*, ..., T_n^*\}$, where T_k^* ($k = 1,2,..., n$) are names of the various components-transitions C_{t_k} $(k = 1,2,..., n)$ in the CN -net.

Definition 2. Let's call the language $L_t(N)$ of a detailed Petri model N , in which composite components – components-transitions can be separated, its free language, for which marking for the identical and single-type components-transitions C_t .is the same, according to the Remark 1.

Definition 3. Language $L_t(CN)$ of CN -net, containing only components-transitions, is the set of firing sequences of CN -net, which is a subset of the set of all words in alphabet Y , resulting from expansion of the alphabet X of original detailed model N with names of components-transitions T_k^* . At that, in Y^* the words are allowed, not containing those characters from X , which are used in marking of transitions in components-transitions in the net N .

It is stated in [Lukyanova 4, 2012] that:

- languages of the composite components, differing only in number of identical parallel processes and modeling single-type processes are congruent;
- transition of words in alphabet X of detailed Petri model N of the investigated parallel distributed system in set of words of alphabet Y of its -model, containing only the components-transitions, is determined up to epimorphism preserving concatenation of words;

– the language of -net, containing only components-transitions, is surjective homomorphic mapping of the language of original detailed Petri model N of investigated parallel distributed system.

Let's examine component Petri net, which contains only components-places as an integral components. In this component net, some areas of detailed Petri model are accumulated in special places of CN -net – components-places Cp . Place in a Petri net is a condition for determining the possibility of an event – transition firing. Therefore, in the case of CN -net, containing only the components-places, we will examine a set of reachable markings in nets N and CN for the study of its language. We can present possible changes in the net marking, that result from transition firing, in the form of a graph marking – a directed graph whose set of nodes is formed by the set of reachable net marking. Such a graph, describing the dynamics of the net functioning, reflects the distribution of tokens – change of the conditions for the possibility of an event in the net. Then the operation of the net can be described in terms of reachable marking [Kapitonova, 1988, Kotov, 1984]. To do this, let's mark the nodes of the graph of reachable markings with symbols so to have valid character string (set of words) in some alphabet T , obtained by writing out symbols of nodes along the paths in the graph of reachable markings starting at the initial marking. We have set $T*$ of all words in alphabet T , made up of the symbols of the nodes of net reachable markings, a subset of which is the set of possible sequences of nodes of net reachable markings. Then each step of the composite component also generates a symbol from the set of names that corresponds to the node of the graph of reachable markings of this component.

Given net N contains m places, then marking of net N is described by m -dimensional vector which coordinates correspond to places, ordered according to the numeration of places in the net. In the net N , components-places are separated, suppose the total number of places that were allocated to the components is $Cp_i \ (i = 1,2,...,n)$, k . Then marking of CN -net will be configured by l -dimensional vector ($l = m - k$), which coordinates correspond to places, ordered according to the numeration of places in the CN -net. In this case, the numeration of places, adopted in the net N , is preserved in CN -net.

Let A and B , respectively, are finite alphabets of detailed model N and CN -model only with components-places. Then A – set of names of m -dimensional vectors corresponding to the nodes of the graph of reachable markings of net N , B – set of names of l -dimensional vectors corresponding to the nodes of the graph of reachable markings of net CN . Set $A*$ – set of all words in the alphabet A , $B* = (\phi(A) \cup \{P_1^*, P_2^*, ..., P_n^*\})^*$ – set of all words in the alphabet $B = \phi(A) \cup \{P_1^*, P_2^*, ..., P_n^*\}$, where $P_i^* \ (i = 1,2,...,n)$ are the names of the different nodes in the graph of reachable markings of CN -net, that correspond to the names of nodes of components-places $Cp_i \ (i = 1,2,...,n)$. Mapping ϕ transits names of m-dimensional vectors in the names of l -dimensional vectors.

Let a – name of m -dimensional vector $(a_1, a_2, ..., a_m)$ from the alphabet A , then $\phi(a) = a'$ – name of l -dimensional vector $(a_1', a_2', ..., a_l')$ from the alphabet B . As a result of combining of areas of the net N , which are separated in the components Cp in the CN -net, in places – each element $a_j' \ (j = 1,2,...,l)$ is a mapping at least of one element $a_s \ (s = 1,2,...,m)$. Here comes a theorem.

Theorem 2. The mapping ϕ is surjective mapping, which transits alphabet A in alphabet B .

Definition 4. The language $L_p(N)$ of detailed Petri model N, in which composite components – components-places Cp can be separated, is its free language that is defined in terms of the set of reachable marking in the net. Places and transitions in the identical and single-type components-places are marked according to Remark 1.

Definition 5. The language $L_p(CN)$ of CN -net, containing only components-places, is the set of sequences received by singling out symbols of nodes along the paths in the graph of reachable markings of CN -net, starting in initial marking and leading to each reachable marking in the net. Language $L_p(CN)$ is a subset of the set of all words in alphabet B. Alphabet B consists of the mapping of the alphabet A and the names of the nodes corresponding to component-places Cp_i.

Consider the transition ς of symbol sequences of the graph of reachable markings of net N into symbol sequences of the graph of reachable markings of net CN, containing only components-places. Thus, we consider the transition, transforming the words in the language $L_p(N)$ in the words in the language $L_p(CN)$ of CN -net, containing only components-places.

Suppose that a word $q \in A^*$ has a form

$$q = (a_1, a_2, ..., a_m)(b_1, b_2, ..., b_m)(p'_1, p'_2, ..., p'_m)(p''_1, p''_2, ..., p''_m)(c_1, c_2, ..., c_m)(d_1, d_2, ..., d_m) = \\ = abp'p''cd\ .$$

Symbols a, b, c, d denote the names of the nodes of the graph of reachable markings of detailed model N, which are not nodes of the graph of markings of any component-place Cp_i. Symbols p', p'' are the names of the nodes of graph of marking of the component-place Cp_i. So in the word under consideration $abp'p''cd$, the names of nodes of the graph of marking of one component Cp_i take part.

Let's make following notation in the word q : $ab = q_1$, $p'p'' = \overline{q}$, $cd = q_2$. Then the record of the original word *q* we get as $q = q_1\overline{q}q_2$.

At transition ς, the mapping of word $q \in A^*$ is a word $\varsigma(q) = h \in B^*$: $h = \phi(q_1)P^*\phi(q_2)$, where $\phi(q_1) = \phi(a)\phi(b)$ and $\phi(q_2) = \phi(c)\phi(d)$. Thus the effect of the transition ς on the word *q* is determined by words mapping involved in the concatenation of the word q :

$\varsigma(q) = \varsigma(q_1\overline{q}q_2) = \varsigma(q_1)\varsigma(\overline{q})\varsigma(q_2) = \phi(a)\phi(b)P^*\phi(c)\phi(d) = a'b'P^*c'd'$.

Finally, for any word $q = abp'p''cd$ from A^*, we get its mapping $h = \varsigma(q) = a'b'P^*c'd'$ – a word from B^*.

Transition ς is completely determined by the values on the letters in the alphabet A so, that each symbol $y \in B$ is a mapping at least of one symbol $x \in A$. The conclusions on transition ς are as follows:

1. $\varsigma(xy) = \varsigma(x)\varsigma(y)$ holds for all x and y over A;
2. $\varsigma(e) = e$,where e is empty word;

3. $\varsigma(x) = \phi(x)$ for words x of any length from the names of the nodes of reachable markings of network N, that are not the names of the nodes of graph of reachable markings of any constituent component Cp_i;

4. $\varsigma(x) = P_i^*$ for all words of any length x from the names of the nodes of graph of reachable markings of composite component Cp_i.

Then for $L_p(CN)$ – language of CN -net, containing only the components-places, we receive:

$$L_p(CN) = \varsigma(L(N)) = \{y / y = \varsigma(x),\ \text{где}\ x \in L_p(N)\}.$$

Resulting from what was said above we have theorems.

Theorem 3. Language of CN -net, containing only the components-places, is surjective homomorphic mapping of the language of the original detailed Petri model N of the investigated parallel distributed system.

Theorem 4. Epimorphism ς generates epimorphism ϕ.

Properties of the languages of *CN*-model

Important problems of Petri nets: a problem of membership, a problem of emptiness, finiteness problem – are solved on the language level. Let's examine the corresponding problems for the above languages. The problem of membership is connected with a check of membership of any word p to language L, in the problem of emptiness we need to find out whether the set L is empty, in the problem of finiteness one need to find out whether L is a finite set.

Theorem 5. The problem of membership is solvable for languages $L_t(N)$, $L_p(N)$, $L_t(CN)$, $L_p(CN)$.

The proof is based on the fact that the procedure for checking if p is an element of one of the mentioned languages, will end in a finite number of steps. In the case of languages $L_t(N)$ and $L_t(CN)$, for any word p of corresponding net we must determine, that p is a sequence of firing of transitions of this net. It's enough:

1) to verify that at the initial marking M_0, transition will fire, which symbol is the first in the word p;

2) to change the marking M_0 into immediately following after M_0 marking M_1;

3) to check the possibility of transition firing at M_1, transition symbol is standing second in the word p, etc.

The word p has a finite number of symbols, therefore, the process of successive inspections will be completed in a finite number of steps. In the case of languages $L_p(N)$ and $L_p(CN)$, we play similarly, stating that the word p is a sequence of changing of net markings along the paths in the graph of reachable markings, starting at the initial marking and occurring as a result of firing of its transitions.

Theorem 6. Reachability problem in the given Petri net marking is reducible to the problem of language membership to corresponding component net.

The proof is based on finding a membership of some marking of net N to the set $R(N)$ of its reachable markings. For this, it is enough for its corresponding component net to figure out whether mapping of the word, corresponding to this marking, is an element of language CN.

Theorem 7. Emptiness problem is soluble for languages $L_t(N)$, $L_p(N)$, $L_t(CN)$ and $L_p(CN)$.

The proof is based on the verification of the following fact, whether at least one transition of this net fires at the initial marking.

Corollary 1. Language $L_t(N)$ ($L_p(N)$) is empty if and only if the language $L_t(CN)$ ($L_p(CN)$) is empty.

Conclusion

The results obtained in the study of connection between structural and dynamic properties of detailed Petri model and component model of parallel distributed system allow analyzing of detailed Petri model with the help of its CN-model. It is effective because CN-models meet modern requirements for models of large systems and complex real tasks to be manageable and easy to analyze. In this paper, we determine the language of component Petri net, containing only components-places, and it is stated that the language of CN-net, containing only the components-places, is a surjective homomorphic mapping of the language of the original detailed Petri model of investigated parallel distributed system; at the same time, the mapping of the alphabet A of net N in the alphabet B of CN-net (in terms of the set of reachable markings in the net) is an epimorphism, which is generated by transition of language $L_p(N)$ to the language $L_p(CN)$. In the languages under consideration $L_t(N)$, $L_p(N)$, $L_t(CN)$ and $L_p(CN)$, the problems of membership, emptiness, and finiteness are examined.

Bibliography

[Kotov, 1984] V. E. Kotov. Petri nets / V. E. Kotov – Moscow: Nauka, 1984. – 160 p. (in Russian).

[Lukyanova 1, 2012] Lukyanova E. A. On component modeling of systems with concurrency / E. A. Lukyanova // Naukovi Zapysky of NaUKMA. Computer Science. – 2012. – T. 138. – P. 47–52. (in Ukrainian).

[Lukyanova, 2011] Lukyanova E. A. On component analysis of parallel distributed systems / E. A. Lukyanova // TVIM – 2011. – № 2. – P. 71–81. (in Russian).

[Lukyanova 2, 2012] Lukyanova E. A. The structural elements of a Petri net component / E. A. Lukyanova // Problemy programuvannya – 2012. – № 2-3. – P. 25–32. (in Russian).

[Lukyanova 3, 2012] Lukyanova E. A. The study of the structural elements of the single-type CN-network during the component modelling and analysis of complex systems with concurrency / E. A. Lukyanova, A. V. Dereza // Cybernetics and Systems Analysis, 2012, – № 6. – P. 20–29. (in Russian).

[Lukyanova 4, 2012] . Lukyanova E. A. On the Relationship between the Language of CN-Model with Component Junctions, and the Language of Detailed Petri Model of Parallel Distributed System/ E. A. Lukyanova // Visnyk of Kyiv Univ. im. Tarasa Shevchenka – 2012. – in press (in Ukrainian).

[Kapitonova, 1988] Kapitonova Y. V. The mathematical theory of design of computer systems / Y. Kapitonova, A. A. Litichevsky – Moscow: Nauka, 1988. – 294 p. (in Russian).

Authors' Information

***Elena Lukyanova**– PhD, doctoral candidate of Kyiv Univ. im. Tarasa Shevchenka, Faculty of Cybernetics, Kyiv, Ukraine; Vorovsky St., Bl. 60, Flat 239, Simferopol, Crimea, 95053, Ukraine;*
e-mail: lukyanovaea@mail.ru
Major Fields of Scientific Research: Modelling, Simulation, Formal verification.