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NEURAL NETWORK APPROACH TO THE FORMATION MODELS OF MULTIATTRIBUTE UTILITY*

Stanislav Mikoni

Abstract: *The inverse problem of choice is formulated as a problem of the settings multiattribute utility model for known-rank alternatives. This problem has the analogy with neural network learning. There is the difference between multiattribute utility model and neural network learning: useful function parameters are subjects for updating instead weight inputs of neurons. It is formulated the main condition alternatives reordering. They must be not comparable in Pareto-dominance analyses. Two alternatives are changed their places due to useful function ratio under its parameters variation. The algorithm is proposed to train the choice model. Algorithm is illustrated by an example.*

Keywords*: multiattribute utility, inverse problem of choice, utility function, training model, aggregate objective functions.*

ACM Classification Keywords*: H.4.2 [Information Systems Applications]: Types of Systems decision support*

Introduction

Rational choice is to solve the problem of multi-criteria optimization based on the input data model and choice conditions model. The choice conditions model contains all the decision-makers preferences. So let's call it a model of preferences. The more preferences the model contains, the more accurate the problem should be solved choice.

The dominant analysis requires the smallest amount of preferences. Its number is limited by specifying optimization direction for each attribute of evaluated alternatives. The multiobjective optimization models occupy an intermediate position. In addition to optimization direction it is given the importance of each criterion, unified criterion scale and function for criteria aggregation. The multiattribute utility model contains maximum amount of preferences. In addition to those preferences are set preferences on the scales of the attributes in the form of non-linear utility functions.

However, accuracy of the solution of choice depends not only on the preferences included in the model choice conditions, but also on the accuracy of their target [Mikoni, Burhakov, 2013]. The transition from a qualitative assessment to quantify preferences has multiple variants. This is the main reason for differences between the choice result and intuitive decision-maker assessment. In connection with this urgent problem setting up a choice conditions model on the evaluation of alternatives, the decision maker made in rank scale. This problem is called the inverse problem of choice as opposed to the direct problem of choice when evaluating alternatives in the rank scale formed on the basis of estimates of utility, measured in a stronger scale.

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Thus, the inverse problem of choice can be seen as the task of teaching choice conditions model on preferences of the decision maker specified in rank scale, similar to training a neural network model with the teacher. Task of preferences model training formulate as follows. It is given the importance and the utility function for the each attribute. The desired order relation is defined on the set of evaluated objects. The task is to sort objects by multiattribute utility functions in accordance with a predetermined order.

Conditions of Model training

The training of any model with a constant of its structure and functions assumes such change function parameters, which causes a change in the simulation results. The constancy of the structure and function of neural network model assumes the immutability of the chosen architecture and activation functions of neurons of the network. Changeable parameters of neural network model are the weights on the inputs of all neurons of the network. The result of training a neural network model is a set of vectors of weights corresponding to all specified output states network.

Analogue of neural network structure in models of choice is valued attributes structure and formed on the attributes basis criteria reflecting the decision maker preferences. Assuming unchanged criteria importance the only choice conditions model using a utility functions has parameters suitable for training. Models used for the dominant analysis and multiobjective optimization are deprived of such opportunities.

To change the shape of the utility function is necessary to know its formula, the parameters of which are subject to change. Analytical model of the utility function can be obtained by approximating a discrete function, built on the points. Methods for creating the utility function using points were proposed in [Neumann, Morgenstern, 1953] and [Keeney, Raiffa, 1976]. However, the diversity of functions generates a variety of parameters to be changed.

To unify these parameters in [Mikoni, 2013] it was proposed to use standard utility function, reflecting both aversion and propensity for risk of the decision maker. In cases where the target value of the attribute coincides with one of the boundaries of the scale its utility function is accepted, convex upward or downward. In the more general case, when the target value c_j of j-th attribute is set between the boundaries of the scale, the domain of the utility function is divided relative to the target value for desire and forbidden parts [Mikoni 2009]. This separation suggests the possibility of constructing a composite utility function that reflects both aversion and propensity for risk decision-makers. Seems reasonable that relative to forbidden and desire values of attribute scale utility function can reflect propensity and aversion for risk correspondingly. Indeed, the growth in utility forbidden values cannot be large, while towards the target utility increases and then decreases slowly.

If the decision maker arranges the same growth of utility ranges of forbidden and allowed values of the attribute scale, then, in accordance with the above considerations, the schedule of the utility function should not reflect risk propensity in the forbidden values and risk aversion – in the range of allowed values. Such a pattern is displayed logistic utility function (1)

$$u(y) = \frac{1}{1+e^{-\beta(y-c)}}. \qquad (1)$$

The logistic function is presented In Figure 1.

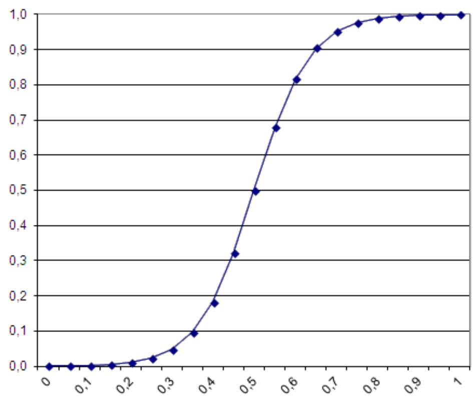

Figure 1.

Parameter *c* characterizes the target value and the variable *y* is the actual value of the attribute. Opportunity of utility growth in case of exceeding the target value is created by appointment of 50 percent utility at achieving *c*: *u*(*c*)=0.5. Measure of risk aversion / risk propensity is adjusted coefficient of β. It is useful to note the similarity of this function with the sigmoid activation function of the neuron. When β→∝, logistic function transforms at a threshold function. Such dependence is too presented by Harrington function:

$$u(y)=\exp(-\exp(-(y-c))).$$

In the case where the attribute preferences on a scale of relative values *c* vary in opposite directions with the same speed, the utility function acquires a bell shape with a maximum value at point *c*.

$$u(y) = e^{-\beta \cdot (y-c)^2} \qquad (2)$$

The bell shape function is presented In Figure 2.

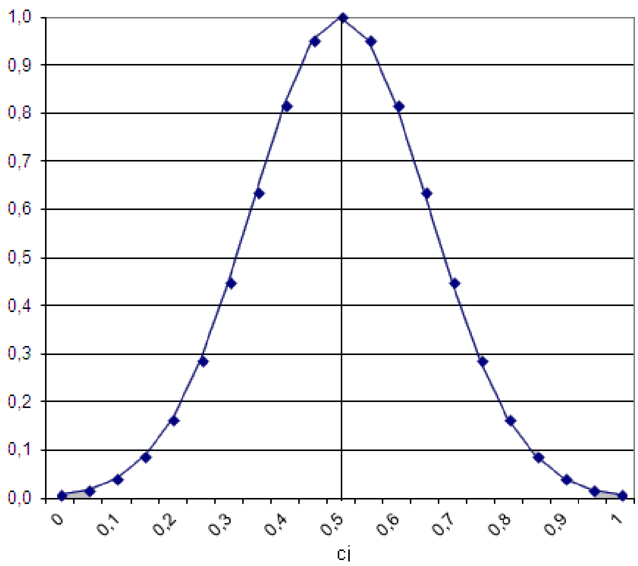

Figure 2.

Formulas (1) and (2) have the same parameter that unifies learning utility functions. List of typical utility functions are not restricted to the two above examples.

Conditions for changing ranks of alternatives on the attributes usefulness

According to useful theory expected utility of alternative assembles expected utility of values of all attributes.

Consider two objects x_i and x_k, x_i, $x_k \in X$. Evaluate their two numerical attributes f_1 and f_2 values: $\mathbf{y}(x_i)=(y_{i1}, y_{i2})$ и $\mathbf{y}(x_k)=(y_{k1}, y_{k2})$.

The inequalities $y_{i1} > y_{k1}$, $y_{i2} < y_{k2}$, or $y_{i1} < y_{k1}$, $y_{i2} > y_{k2}$ are the primer condition of alternative ranks change on vector estimates of attributes. Satisfying these conditions objects x_i and x_k are incomparable with respect to Pareto-dominance. They belong to one of the levels ranged graph of domination and therefore have the same place in the general order of objects.

Ordering of disparate objects is possible by converting vector estimates in scalar values using generalizing function. Let it be additive generalizing function. It gives the following scalar evaluation of objects x_i and x_k:

$y(x_i)= w_1 \cdot y_{i1} + w_2 \cdot y_{i2}$;

$y(x_k)= w_1 \cdot y_{k1} + w_2 \cdot y_{k2}$.

If $y(x_i)>y(x_k)$, it $\rho(x_i)<\rho(x_k)$, i.e. object x_i has lower rank (the best place) than to x_k.

When $w_1 \cdot y_{i1} + w_2 \cdot y_{i2} = w_1 \cdot y_{k1} + w_2 \cdot y_{k2}$ objects have the same scalar evaluation and as a result, occupy the same space. Transform identity in the following expression:

$w_1 \cdot y_{i1} - w_1 \cdot y_{k1} + w_2 \cdot y_{i2} - w_2 \cdot y_{k2} =0$;

$w_1 \cdot (y_{i1} - y_{k1}) + w_2 \cdot (y_{i2} - y_{k2}) =0$;

$w_1 \cdot (y_{i1} - y_{k1}) = - w_2 \cdot (y_{i2} - y_{k2})$.

Then:

$$\frac{y_{i1} - y_{k1}}{y_{i2} - y_{k2}} = -\frac{w_2}{w_1} \qquad (3)$$

In expression (3) difference values features f_1 and f_2 are constants. Consequently, their contributions to the overall estimations of objects can be influenced only through the change in the ratio of weights of attributes.

The magnitude of weight w_2, which involves alignment of contributions on both attributes, determined by the formula:

$$w_2 = -w_1 \cdot \frac{y_{i1} - y_{k1}}{y_{i2} - y_{k2}} \qquad (4)$$

The magnitude of weight w_1, which involves alignment of contributions on both attributes, determined by the formula:

$$w_1 = -w_2 \cdot \frac{y_{i2} - y_{k2}}{y_{i1} - y_{k1}} \qquad (5)$$

Unlike linear scales of attributes utility functions scales are non-linear: $u_j = \varphi(y_j)$, $j = \overline{1,n}$. This means that the scale equal intervals correspond unequal lengths thereof utility scale [0, 1], which is clearly shown in Figure 3.

If we take $y_i = 6$, and $y_k = 4$, the difference values utility increases from the left to the logistic function neighboring features:

$u_i' - u_k' =0,73 - 0,27=0,46 \rightarrow u_i'' - u_k'' =0,88 - 0,12=0,76$.

Difference of utility values is changed to 0.3.

This effect makes it possible to change the ratio of overall ratings utility objects by changing the steepness of nonlinear utility functions.

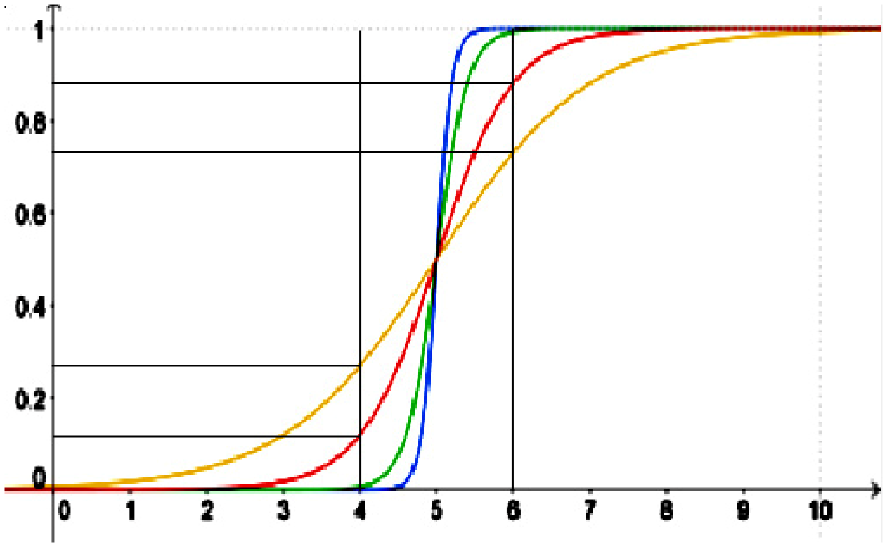

Figure 3.

Substitute in the formula (3) the utility of attribute values

$$\frac{u_{i1} - u_{k1}}{u_{i2} - u_{k2}} = -\frac{w_2}{w_1} \quad (6)$$

changing of the steepness of non-linear utility function fronts. Alignment condition overall ratings of the second attribute are the basis of the expression:

$$u_{i1} - u_{k1} = -\frac{w_2}{w_1} \cdot (u_{i2} - u_{k2}) \quad (7)$$

Alignment condition overall ratings of objects at the first attribute are the expression:

$$u_{i2} - u_{k2} = -\frac{w_1}{w_2} \cdot (u_{i1} - u_{k1}) \quad (8)$$

Violation of the identities (3) and (4) leads to a change of alternatives ranks on the opposite ones. Effect of change in utility is reached at a scale factor greater than 1 when the numerator is more than the denominator – $w_2 > w_1$ in (3) and $w_1 > w_2$ in expression (4).

Based on the examination of the conditions change ranks of objects by their usefulness, it should be an opportunity to train the model multiattribute utility by changing the parameters of the utility functions. Under a given object ordering training procedure is similar to training a neural network with the teacher. The difference is that the training of the neural network is the selection of the weighting values of neurons and training multiattribute utility model is realized by finding the values of the parameters of the utility functions.

The algorithm of objects ordering relative to the sample

The initial data for solving the problem are vector estimations of objects in the feature space, the importance of attributes and the required order of objects.

The objects are ordered by Pareto dominance relation. The initial strict order of object is determined by multiattribute optimization. The difference reveals between received and required order of objects. The pairs of objects are defined to be reordered. Each pair is analyzed with respect to the identity of Pareto dominance. If one object from the pair dominates another one reordering of the pair is impossible. In the absence of Pareto dominance an attribute is defined with maximum difference of useful function. The front steepness of the useful function is tuned. It is the iterative process. It is completed when the ratio multiattribute utility functions compared objects is reversed. The algorithm consists of the following steps.

1. While not considered all pairs of objects, selected a pair of objects x_i, $x_k \in X$.
2. If one need to reordering pair (x_i, x_k) to a pair of (x_k, x_i), the possibility of the operation is analyzed. If the objects x_i and x_k are related by Pareto dominance, when $x_i \underset{P}{\succ} x_k$ or $x_k \underset{P}{\succ} x_i$, then they cannot be reordered, go to 1.
3. The utility difference $\Delta u_{ik,j} = u_j(x_i) - u_j(x_k)$ is calculated between objects x_i and x_k in all attributes, $j = \overline{1,n}$.
4. The maximum utility difference $\Delta_{u_{j,\max}} = \max_j (\Delta_{u_{ik,j}})$ is found.
5. If $\Delta u_{j,\max} > 0$, decrease the front steepness on the ΔM, otherwise increase one on the ΔM.
6. The next objects estimates $u_j(x_i)$ and $u_j(x_k)$ are determined.
7. If $u_j(x_i) > u_j(x_k)$ and the front steepness can be changed, go to 5, otherwise go to 1.

The presented algorithm has been tested on the instrumental system choice and ranking by utility SVIR-U.

The simulation results showed the legitimacy of the proposed approach.

We illustrate the process of changing places of objects on the example of ranking five apartments on three attributes: total area (TA), price and kitchen area (KA). For these attributes expert appointed the following utility function: bell-shaped, decreasing logistics and increasing logistics respectively. The initial values of these functions for the assessed apartments listed on the left side table. 1. Weighted Average utility estimations of these attributes are shown in the column "Utility". The apartments ranking is performed by this estimations. According to a predetermined order of apartment 1 cannot be favored apartments 5. To change the order of these apartments their utility difference is analyzed for each attribute. The largest difference occurs in "Price" attribute. To decrease difference it is necessary to reduce the steepness of the decreasing logistic function by parameter M. Instead initial value of function parameter M=0.3333 was found value M=0.3350, which reduces the difference between the apartments utility by price (see right column "Price"). As a result of changes in the ratio of multiattribute utility of apartments 5 and 1 its ranks are exchanged (see right column "Rank").

Table 1. Rating apartments for utility

	TA	Price	KA	Utility	Rank	TA	Price	KA	Utility	Rank
№1	0,699	**0,881**	0,269	0,62	4	0,699	**0,795**	0,269	0,588	5
№2	0,902	0,761	0,378	0,68	2	0,902	0,692	0,378	0,657	2
№3	1,000	0,592	0,500	0,70	1	1,000	0,567	0,500	0,689	1
№4	0,902	0,408	0,622	0,64	3	0,902	0,433	0,622	0,653	3
№5	0,699	**0,239**	0,777	0,57	5	0,699	**0,308**	0,777	0,595	4

Conclusion

There are many methods for ordering of alternatives. One of them can be accept for predominant one. Then the question arises – is it possible to setting some choice model for known-rank alternatives? In the paper there was shown such possibility for multiattribute utility model of choice. That possibility exists due to nonlinear utility functions. Increase of attribute value on equal measures leads to Increase of utility function value on not equal ones. That effect permits to change ratio of values of multiattribute useful functions for reordered alternatives. So its ranks change too. Places exchange is possible for only not comparable alternatives in Pareto-dominance analyses. The algorithm is proposed to train the choice model. It is based on choosing of standard utility functions for each attribute and its parameters turning with the purpose of alternatives reordering. The algorithm is illustrated by an example. Program system choice and ranking SVIR, produced at Saint Petersburg State Transport University in Russia (www.mcd-svir.ru/refer07.html) can be applied to calculating the results.

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Major Fields of Scientific Research: System analysis, Multicriteria choice, Intelligent technologies, Simulation

SIMULATION OF WEATHER IN AGENT MODELS

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Abstract: *The article deals with the agent-based approach to creating simulation models. A classification is proposed for agent models depending on their behavior and ability to move. Depending on the ability to move and affect other objects, we have been proposed to distinguish between active and passive agents, spatial and time series agents. Particular attention in this article is paid to time series agents and their simulation using neural networks. The article also considers training and testing tool for neural networks developed within the research for the possibility of using neural networks for time series simulation.*

Keywords: *agent, agent simulation, rescue neural network, time series.*

Introduction

Agent [1] is an entity, which has an activity, autonomous behavior may interact with the environment and other agents make decisions based on some set of rules. Agent models are useful for studying systems, the behaviour of which is determined not by global laws, but by contrast, these laws are the result of individual activity of multiple agents.

The transition from the concept of the object to the concept of the agent occurs when at least one of the three types of behaviour is exist for object: reactive, collective or management behaviour. Reactive behavior occurs in the object as a response to external factors such as ambient temperature, rainfall. Agent, based on these factors can change its settings and move in space. Collective behavior occurs when there is a space of at least two objects that can change its state and move in space in relation to each other. We can say about managerial behavior when one or more agents control the behavior management group of agents.

Agent simulation is a convenient way of simulation modeling, which allows describing the behaviour of each individual agent, which in turn allows distinguishing the development and testing of each agent as a separate independent subtask.

Operation is a set of actions of different forces, coherent and related by purpose, tasks, place and time, which are conducted simultaneously and consistently in accordance with a single concept and plan to solve problems in the strategic or operational direction (in a certain zone, area) within a reasonable period of time. The operation is performed in accordance with one of several plans developed in case one of the possible situations encountered.

Types of agents in operation models

Model agents differ from each other by sets of properties, but the behavior of agents is more important for agent simulation. In terms of spatial agent simulation, behaviour is a change by the agent of the location in space, as well as interaction with other agents. Depending on the presence or absence of the ability to move in space, we shall distinguish active and passive agents. Active agents can freely change their location in space, while passive agents lack this capability. A special type of agents is spatial agents, which cannot move in space like passive agents, but actively interact with active agents in the process of their moving in space. For example, when moving the rescuer agent on the map, diverse spatial agents, such as rivers, roads, forests, can affect the velocity property, as well as agent's energy reserve. The effect of spatial agents on the velocity property will lead to a change in the rate of movement of the active agent in space depending on the spatial agent. For example, the active agent will move across the field or road much faster than across the forest, or when crossing the river. The effect of spatial agents on the energy supply, namely the reduction of the value of this parameter while moving will allow to avoid the active agent's infinite movement in space. Spatial agents allow to create more detailed models of interactions. Parameters of active agents depend on the resources, which are limited, and their number decreases depending on the actions of the active agent in the course of model time. Resources can be destroyed, exhausted or captured by agents of the counteracting plan of operation, resulting in a rapid decrease of parameters of active agents. For example, when a resource "Fuel" is exhausted, the agent "Car" stops its movement, but remains intact, and the exhaustion of the resource "Fuel" for aircrafts leads to their destruction.

Time series agents

Simulation of the behaviour of agents, depending on the factors, which can be represented as a time series, also has a very important meaning. The examples of such factors may include temperature, humidity, rainfall, solar radiation. For the simulation of the factors, which can be represented as a time series, a special type of agents has been introduced – time series agents.

One of the problems to be addressed in the simulation of the movement of active agents is the selection of the optimal route or several routes, depending on practicability and cost of passing along this route, which fits the traveling salesman problem. The traveling salesman problem is to find the most profitable route through the points given. The statement of the problem includes the profitability criterion of the route (shortest, cheapest, cumulative criterion, etc.) and corresponding distance matrices, cost matrices, etc.

Let us consider an example of how a change in external factors may affect the change in the balance of route sections and eventually the change in the route of agricultural machinery. Figure 1 show a map with two agricultural enterprises, which are in different localities (Anisov and Baklanova Muraveyka). Each of agricultural enterprises owns a certain set of agricultural machinery. The enterprises have entered into an agreement that in case of shortage of agricultural machinery at one of the enterprises, the other should help if the vehicles are available.

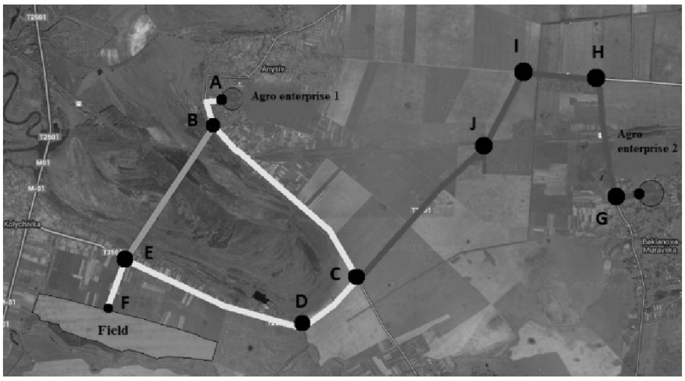

Figure 1. Model of movement of the agricultural machinery

The agricultural enterprise located near the vertex A owns the field located near the vertex F, and, throughout the year, performs a series of agricultural operations in this field, such as planting, processing and collection of crops.

The agricultural machinery from this agricultural enterprise can reach the field by two routes, one of which passes through the wetland (A-B-E-F), and the second one – along the asphalt road (A-B-C-D-E-F). The second route is considerably longer and, therefore, to transfer the agricultural machinery along this route, one need to use a significantly greater amount of fuel and time. In the summertime, the agricultural machinery can travel along the route segment B-E, but in some years, heavy rains flood the lowlands, through which the above section passes, which prevent the movement of vehicles. Thus, the effect of weather conditions can alter the routes of movement of the active agents.

Simulation of weather factors using neural networks

Since the simulation should be carried out considering weather factors, one of the subtasks to be solved is to simulate a change in weather conditions, for which neural networks are convenient to use. Weather factors are often presented in the form of a time series – an array of pairs <time>-<parameter value>. One of the methods, which can be used for the simulation of time series, is neural networks, which can be trained through the use of weather time series. The network trained based on the weather time series can be used as an element of the time series agent. For the topology, for example, the Ward network and multilayer perceptron, which are shown in Figures 2 and 3, are suitable for simulation of time series.

The multilayer perceptron is a network consisting of several layers of neurons connected in series. At the lowest level of the hierarchy, the input layer is located, consisting of sensor elements, the task of which purpose is only reception and distribution of input information through the network. Then, there are one or, more rarely, several hidden layers.

Each neuron in the hidden layer has a number of inputs connected to the outputs of the neurons of the preceding layer, or directly to the input sensors X_1, ..., X_n, and one output. Neuron has a unique weight vector w. The weights of all the neurons of a layer form a matrix, which shall be denoted by V or W.

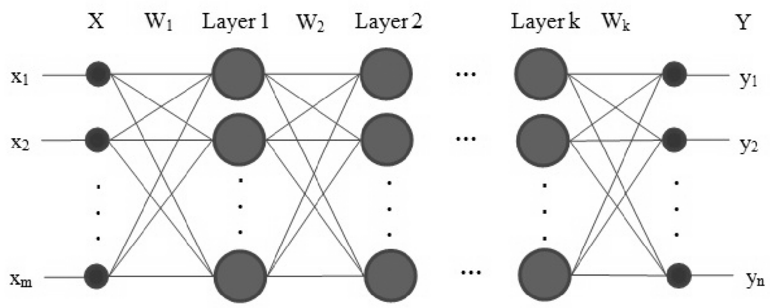

Figure 2. Multilayer perceptron

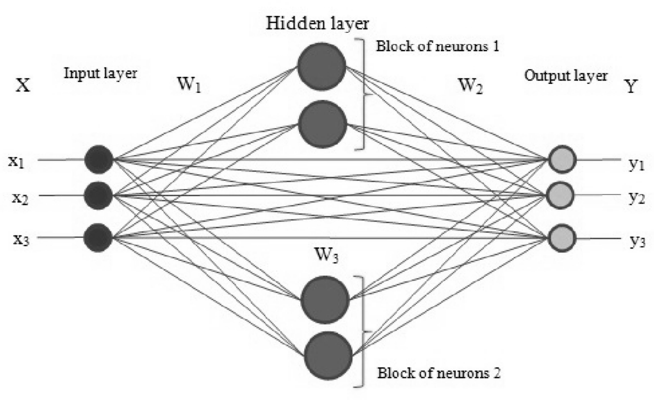

Figure 3. Ward networks

To represent the state of the neural network, the analog logic is used, in which the valid states of synaptic connections are determined by arbitrary real numbers and the degree of neuronal activity – by real numbers between 0 and 1. Sometimes, models with discrete arithmetic are studied, in which the synapse is characterized by two Boolean variables: activity (0 or 1) and polarity (-1 or +1), which corresponds to the three-valued logic. States of neurons can thus be described by a Boolean variable. This discrete approach makes the configuration space of states of the neural network finite (not to mention the benefits of the hardware implementation).

The Ward network also comprises several layers of neurons connected in series. The partition of hidden layers into blocks allows using different transfer functions for different blocks of the hidden layer. Thus, same signals received from the input layer are weighed and processed in parallel using several methods, and the result is further processed by the neurons of the output layer. For the input layer neurons, a linear activation function is usually set, and for neurons from the blocks of the hidden and output layer, it is determined experimentally. Figure 3 shows the structure of the Ward network consisting of one input, one output and one hidden layer, and the hidden layer is divided into two blocks of neurons, each of which implements its own activation function.

Before creating a neural network, one shall choose its parameters, such as the size of the input vector and the size of the training sample. The size of the input vector in the prediction of time series is the number of samples of the time series, which is simultaneously applied to the inputs of the neural network at each step of training. The size of the input vector equals to the number of inputs of the neural network. Figure 4 shows the size of the input vector of the neural network during the simulation of time series.

The size of the training sample is the number of samples of the time series, which is applied to the inputs of the neural network in the process of training the neural network. For an adequate time series model, a sample shall be applied covering as much states of the object studied as possible. The size of the training sample often equals to the period of the time series. The correct choice of the size of the input vector affects the imprecision of the model and the time of training the neural network, as well as the choice of the depth of the training sample during training the neural network (the higher the sampling depth is, the more time is required to spend on training the neural network).

Training of the neural network for modeling of weather factors

Within the study of the possibility of using neural networks for the time series simulation, a distributed training and testing tool for neural networks has been developed. The use of distributed architecture was due to the fact that the neural network training takes a lot of time and resources.

The architecture of the distributed training tool is presented in Figure 4. The RMI protocol has been selected a communication protocol between the manager and teaching and testing neural network servers. The use of this protocol allowed transmitting messages between the manager and servers as serialized objects. The neural network creation module allows creating sets of neural networks with various parameters such as the size of the input vector, the number of hidden layers, and the number of neurons in hidden layers. The neural network parameter selection module helps to select the above parameters. To select the neural network parameters, the methods of statistical analysis are used.

The tool user can create a set of untrained neural networks using the neural network creation module. Teaching and testing neural network servers establish a connection with the manager of testing and training servers, and the manager adds them to the list of available servers. Based on the list of untrained neural networks and the list of teaching and testing servers, the user can create a set of tasks for teaching and testing servers.

The user can perform a list of tasks for training neural networks; multiple neural networks can be assigned to one server. In this case, the server will perform all the tasks sequentially. All the servers perform tasks simultaneously and in a distributed manner. Upon completion of the task, the server transmits the trained neural network or network testing results through the RMI protocol to the server management dispatcher. The trained neural network is stored in a database, which allows saving the state of the trained neural network when exiting the tool.

The neural network training occurs at the teaching and testing neural network server, the interface of which is shown in Figure 5. The teaching and testing server can connect to the Server Manager, and the manager adds the server to the list of teaching and testing servers. The indicator at the bottom of the interface shows the completion percentage of the task sent to the server.

The tool allows to check the results of the neural network training using a set of tools substituted at the Experiments management tab. Figure 6 shows the result of checking the trained neural network. Training the neural network was conducted using a temperature time series. The neural network was trained using the time series, and after the neural network training, the training result has been tested using the means of the Experiments management tab.

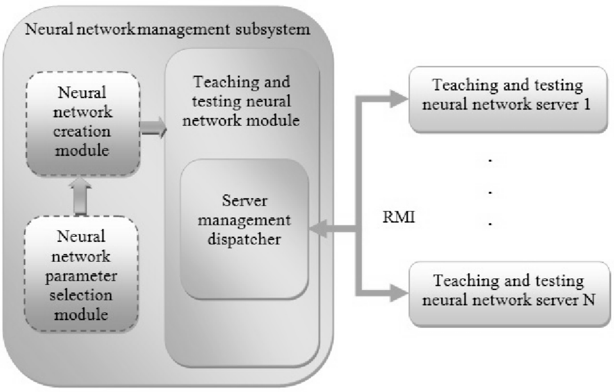

Figure 4. Architecture of the neural network training tool

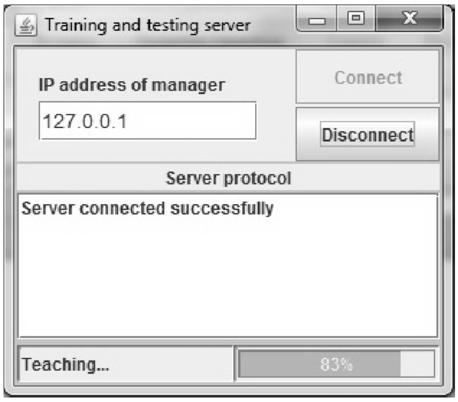

Figure 5. Neural network training window

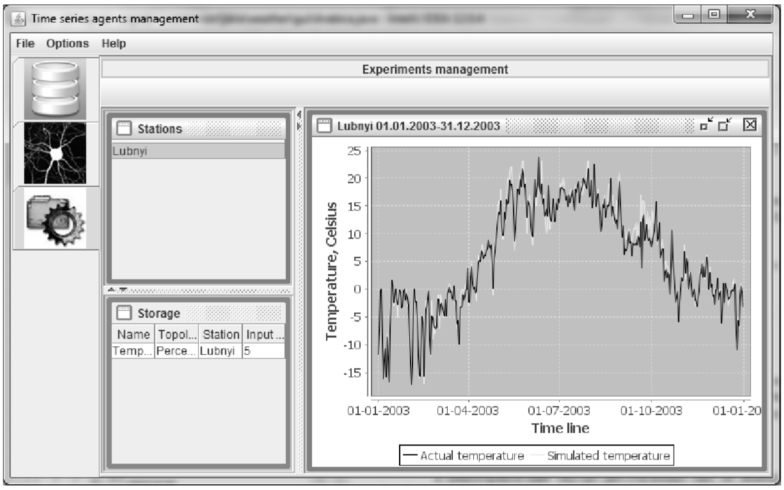

Figure 6. Neural network testing window

Model of the active agent's movement over the time series agent

Figure 6 shows the model of the active agent's movement over the time series agent. The time series agent was trained using a temperature time series. The active agent starts its movement from the point with coordinates specified during the model initialization. The active agent's movement occurs from the point A to point B. The agent's route is over the time series agent. During the time series agent's movement outside the agent time series, its parameters do not depend on the time series agent's parameters. During the active agent's movement over the time series agent, the interaction between the active agent and the time series agent occurs, and they both can affect each other. For example, the time series agent can reduce the number the active agent's movement velocity or reduce the active agent's reserve forces. In its turn, the active agent can change the time series agent's parameters. Time series agents can also affect passive agents and spatial agents if they are within its range. For example, when modeling agricultural operations, the time series agent, trained on the rainfall time series can change the parameters of the spatial field agent, which the possibility of applying a certain agricultural operation depends on.

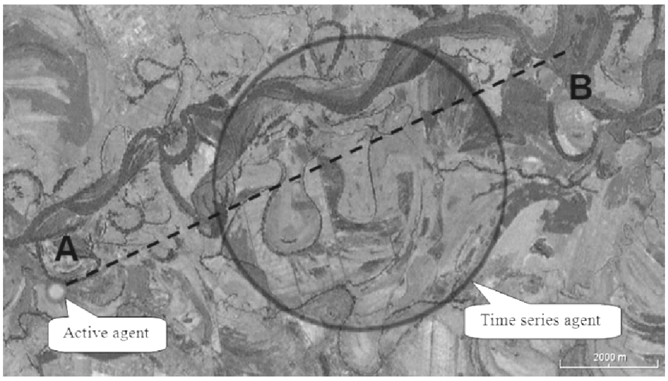

Figure 7. Model of the active agent's movement over the time series agent

Conclusion

Depending on the behavior, active and passive agents have been distinguished, as well as a special type of agents: spatial agents. For modeling the factors, which can be represented as a time series, a special type of agents has been introduced – time series agent. A neural network has been selected a simulation means. Within the study of the possibility of using neural networks for time series simulation, a distributed teaching and testing neural network tool has been developed.

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INFORMATION SYSTEM OF FORECASTING BASED ON COMBINED MODELS WITH TIME SERIES CLUSTERING

Alexander Berzlev

Abstract: *The article offers information system of time series forecasting based on the combined models of hybrid and selective types for various criteria of selection with the previous time series clustering by the methods of nearest neighbor and K - nearest neighbors.*

Keywords: *time series, combined forecasting model, information system, K-nearest neighbors' method.*

ACM Classification Keywords: *G.3 Probability and statistics - Time series analysis; H.4.2 Information Systems Applications: Types of Systems: Decision Support; H.1.1.Systems and Information Theory*

Introduction

The growth rate of national markets, trends of globalization, the complication of business relationships are some of the preconditions that today the development of economic systems takes place under uncertainty, instability and risk that is characterized by a significant degree of variability. In this regard, in unstable economic markets, the use of classical econometric statistical models and appropriate methods of time series forecasting, which reflect the financial and socio-economic indicators, is rather limited. Considerable part of the time series, for which forecasting problem appears, as a rule, is usually characterized by instability and non-stationary. Many easy-to-use forecasting models: exponential [Brown, 1959; Holt, 1957], linear regressive, autoregressive of ARIMA type [Box, 1976] are not intended to non-stationary time series forecasting, and those models that are appointed for this purpose, have some disadvantages, in particular, the model ARIMAX is characterized by the complexity of large number of parameters calculating and nonlinear regression model is limited by the problem of identifying functional relationships [Vercellis, 2009]. Complexity of forecasting process is also linked with the necessity to process and analyze large amounts of data. Therefore, today the state of forecasting methods development is closely related with the development of information technology. The so-called forecasting information systems that reflect this relationship within econometrics, financial mathematics, and statistics are used in the wide range of applied areas of science as well as in manufacturing, financial planning in economy and trade. Effective forecasting information systems can be used in decision-making, used by analysts to assess the risk of financial investments, etc.

Under the forecasting information system will consider interrelated totality of programs, each of which carries out a specific function: incoming information receiving, its maintenance and processing, forming of input data based on defined algorithms for the forecast realization for the user purposes. The main components of time series forecasting are the database with information about the object of study, in this case, the retrospective time series data, which is continuously updated, forecasting models complex and methods for their quality assessing, which are grouped depending on the problem of forecasting. The operation of such a system should carry out in interactive mode with the user or person who makes a decision. This system should be characterized by the required accuracy, flexibility, time saving and betimes react on changes in the dynamics of economic processes.

Developing of effective time series forecasting information systems is an urgent task for both for theory and practice. In particular, this system can be used by analysts, investors and traders to solve problems of money management, investment, planning, and so on. The purpose of the study is to build an information system of time series forecasting based on combined forecasting models with the previous time series clustering by nearest neighbor method and K-nearest neighbors to solve forecasting problems of values and time series signs increments.

Problem Statement

Discrete time-series $\{z_i\}_{i=1}^{n} = \{z_1, z_2, \ldots, z_n\} = \{z(t_1), z(t_2), \ldots, z(t_n)\}$ will be called a finite sequence of values that are fixed at discrete time points $t_i \in S$, $i = \overline{1,n}$, S – some discrete set. We'll assume that the measurements are valid, are fixed at defined time points and are the economic or financial indexes.

There are major problems that can be solved within the framework of information systems: forecasting of time series values, increments signs forecasting, identifying of time series local extreme points. In this paper restrict ourselves to the first two. Formulate each of them separately.

Problem of time series values forecasting. Based on retrospective values z_n, z_{n-1}, z_{n-m+1}, $m \le n$ of series $\{z_i\}_{i=1}^{n}$, to estimate most accurately its behavior in future in moments $t_{n+1}, t_{n+2}, \ldots, t_{n+\theta}$ i.e. to build a sequence of forecasting values $\{\hat{z}_i\}_{i=n+1}^{n+\theta} = \{\hat{z}_{n+1}, \hat{z}_{n+2}, \ldots, \hat{z}_{n+\theta}\}$ where θ – the prediction horizon, and m – volume retrospective sample. Denote by $\hat{z}_\tau(n)$ – forecast, that is calculated at the time t_n (at the point n) for τ points forward. Forecast of time series values $\{z_i\}_{i=1}^{n}$ for one point forward can formally be written as $\hat{z}_{n+1} = \hat{z}_1(n) = f(z_{n-m+1}, z_{n-m+2}, \ldots, z_n)$, where f – forecasting model. In the case of forecasting with horizon $\theta > 1$ such approach can be used:

$$\hat{z}_{n+1} = \hat{z}_1(n) = f(z_{n-m+1}, z_{n-m+2}, \ldots, z_n),$$
$$\hat{z}_{n+2} = \hat{z}_2(n) = f(z_{n-m+2}, z_{n-m+3}, \ldots, z_n, \hat{z}_{n+1}),$$
$$\vdots$$
$$\hat{z}_{n+\theta} = \hat{z}_\theta(n) = f(z_{n-m+\theta}, \ldots, \hat{z}_{n+\theta-2}, \hat{z}_{n+\theta-1}), \ m \le n .$$

It should be noted that different forecasting models may have different mechanisms for calculating the forecasts; external factors, the various options that require separate assessment may be taken into account. To build the most accurate forecast means to build a model that meets the criteria for assessing the quality of prediction. For this problem the average absolute deviation, average square error, standard deviation, relative error and so on may be such criteria.

Problem of time series increments signs forecasting. Based on a $\{z_i\}_{i=1}^{n}$ build up the series, which consists of first differences $\{\Delta z_i\}_{i=1}^{n}$, where $\Delta z_i = z_i - z_{i-1}$, $i = \overline{2,n}$. Denote by $\{\chi_i\}_{i=2}^{n}$ signed series, where $\chi_i = \mathrm{sgn}(\Delta z_i)$. Sign increment forecast, which is calculated at the point n for τ points forward denote by $\hat{\chi}_\tau(n)$. Denote by F increment sign forecast model on one point forward. Then forecast can be formally written as: $\hat{\chi}_{n+1} = \hat{\chi}_1(n) = F(z_{n-m+1}, z_{n-m+2}, \ldots, z_n)$. Some quality criteria for the forecasting of this model, in particular that based on Heaviside function, are described in the work [Berzlev, 2013].

Consider the general formulations of values and time series signs increments forecasting problems. Problem statement of time series values forecasting is formulated as follows: let give the set of forecasting models $f_1, f_2, \ldots, f_N$, as the basis of which for a series $\{z_i\}_{i=1}^{n}$ in the point n the estimates of time series future elements can be built, $\{\hat{z}_{n+1}^p, \hat{z}_{n+2}^p, \ldots, \hat{z}_{n+\theta}^p\}$, $p = \overline{1,N}$, N – the number of forecasting models. Based on this models set and retrospective series $\{z_i\}_{i=n-m+1}^{n}$ at the point n to calculate the most accurate predictive values sequence $\hat{Z} = \{\hat{z}_i\}_{i=n+1}^{n+\theta} = \{\hat{z}_{n+1}, \hat{z}_{n+2}, \ldots, \hat{z}_{n+\theta}\}$.

Statement of increments signs forecasting problem is formulated as follows: let the set of increments signs forecasting models $F_1, F_2, \ldots, F_L$ is given, on the base of them in the point n of time series $\{z_i\}_{i=1}^{n}$ the estimates of increments signs can be calculated $\hat{\chi}_1^p(n)$, $p = \overline{1, L}$, L – the number of increments signs forecasting models. The based on a set of data models and retrospective series $\{z_i\}_{i=n-m+1}^{n}$ at a point n to calculate the most accurate predictive assessment of increment sign on one point forward.

The objective of this study is the development of such information system, which based on combined forecasting models with prior time series clustering in interactive mode with the user, defines forecast with the desired accuracy for the user's purposes. When combined we'll mean models that are based on a certain number of other models by hybridization or selection. When clustering for nearest neighbor method will understand the partitioning of time series into clusters of fixed length and location for the last specified cluster (which precedes the point where the forecast is calculated) based on proximity measure of similar to it cluster, distance to which is the minimal. The problem of such clusters finding is solved on the basis of K - nearest neighbors [Singh, 2000; Fernández-Rodríguez, 2002; Berzlev, 2013].

Development of information system forecasting

At the initial stage of information systems development problems, that should be solved, should be identified, the goals of systems building should be determined; methods for finding solutions should be defined, formalizing and structuring of knowledge occurs. The next step is the implementation of the system and its testing. Consider the key stages of the information system.

Step 1. The time series clustering by nearest neighbor or K - nearest neighbors methods. Do the length cluster m of the time series $\{z_i\}_{i=1}^{n}$ as the subsequence $\{z_{k_j}\}_{j=1}^{m}$ of m elements, $m < n$, $k_{j+1} = k_j + 1$ for $j = \overline{1, m-1}$ (the sequence order of elements in the subsequence is the same as in the time series). Clusters can be represented directly as the subsequences of the input time series elements, or by entering the distances between elements in the middle of the cluster.

In the case of increments signs forecast the clusters representations based on signed sequences are considered. If $\{\chi_i\}_{i=2}^{n}$ – signed sequence of the time series $\{z_i\}_{i=1}^{n}$, signed clusters will be:

$$\chi_{(m)}^{s} = \{\chi_{k_1^s}, \chi_{k_2^s}, \ldots, \chi_{k_m^s}\} = \{\chi_{k_j^s}\}_{j=1}^{m},\ k_{j+1}^{s} = k_j^s + 1,\ \chi_{k_1^s} = \chi_s,\ s = \overline{2, n-m},$$

where $\chi_{(m)}^{s}$ – cluster, that consists of m elements, s – initial element series index. Do the $\chi_{(m)}^{n-m+1}$ – pivotal cluster, in other words, last available cluster, the other ones $\chi_{(m)}^{s}$, $s = \overline{2, n-m}$ – non-pivotal clusters. It is evident that the number of non-pivotal cluster of m elements, that are building based on series of n elements is equal $n - m - 2$ [Berzlev, 2013].

In the case of time series values forecasting the clusters representations are considered directly on the time series elements:

$$z_{(m)}^{s} = \{z_{k_1^s}, z_{k_2^s}, \ldots, z_{k_m^s}\} = \{z_{k_j^s}\}_{j=1}^{m},\ k_{j+1}^{s} = k_j^s + 1,\ j = \overline{1, m-1},\ z_{k_1^s} = z_s .$$

The cluster $z_{(m)}^{n-m+1}$ is called the pivotal cluster, all other clusters are called non-pivotal clusters $z_{(m)}^{s}$, $s = \overline{1, n-m}$.

The set of all non-pivotal clusters denote as $\Re$. It should be noted that in [Singh, 2000] the term pattern is used for the term cluster. In paper [Fernández-Rodríguez, 2002] the term vector is used, also different authors use such terms as set, pieces, etc.

Since each cluster can be represented by a point in m - dimensional space, it is possible to calculate the proximity measure or metric distances between the pivotal cluster and all non-pivotal clusters. As proximity measures it can be used: Euclidean, Minkowsky, Mahalanobis distances, or in the case of cluster-based representation of sign sequences: Hamming similarity, Rogers-Tanimoto measure, etc. As the result of clustering algorithm application by K- nearest neighbors' method, the non-pivotal clusters, similar to the pivotal cluster (the distance of which to the pivotal cluster is minimal) will be received.

Step 2. Since the forecasting accuracy determines not only by forecasting models possibilities, but also by forecasting objects characteristics, i.e. time series on which these models are implemented, the key steps are the time series preprocessing and analysis of its structure.

Step 3. Formation of the general set of forecasting models. On the next step for the problem of time series values forecasting the general set $\Im_{PS}$ of forecasting models $f_1, f_2, \ldots, f_N$, which can consist as from different classes models and so as from identical models but with different sets of parameters, is formulated.

Polynomial models with adaptive parameters: Brown, Trigg-Leach, Holt, Winters adaptive polynomial models, etc, [Berzlev, 2011; Berzlev, 2012] are recommended to include to the set. For the increments signs forecasting problem the set of specific models $F_1, F_2, \ldots, F_L$ is formed. To this type of models moving averages models can be included. In particular, the paper [Berzlev, 2013] proposes to choose the following indicators:

$\chi_1(n) = \mathrm{sign}\left(\frac{1}{p} \sum_{i=1}^{p} z_{n-i+1} \right)$ – increment sign forecasting based on simple moving average with $p > 0$ period;

$$\chi_1(n) = \mathrm{sign}\left(\frac{1}{\nu_p} \sum_{i=1}^{p} (p-i+1) z_{n-i+1} \right), \ \nu_p = \sum_{i=1}^{p} i \text{ – weighted moving average;}$$

$$\chi_1(n) = \mathrm{sign}\left(\sqrt[p]{\prod_{i=1}^{p} z_{n-i+1}} \right) \text{ – geometric moving average;}$$

$$\chi_1(n) = \mathrm{sign}\left(\prod_{i=1}^{p} (z_{n-i+1})^{p-i+1} \right)^{\frac{1}{\nu_p}}, \ \nu_p = \sum_{i=1}^{p} i \text{ – weighted geometric moving average.}$$

Step 4. Identification and adaptation of models. Let to the general sets of models adaptive polynomial models of Brown, Trigg-Leach, Holt, Winters, each of which has a number of parameters that need to adapt in each time series point, in which the prognosis is implemented, were included. Time series is divided into two sections: experimental and predicted, on each of them the forecast by models from the general set $\Im_{PS}$ is implemented. Experimental section, in turn, is divided into two parts, the model parameters adaptation, which allows eliminating of the impact of the initial parameters choice on the forecasts results that are implemented in the second part of the experimental plot of the series, takes place on the first of them. We'll also assume that the mechanism of models adaptation from the general set of parameters $\Im_{PS}$ by the refinement of at each point of the input series according to the forecasts errors in the previous steps, were assigned. Note that the initial parameters of the models are given a priori, based on the structure of the time series and the specific characteristics of each forecasting model or are calculated on the experimental time series interval.

Step 5. The forecast implementation for each of the models, which are the components of an information system, depending on the user's purpose. Consider for example two models for each of the prediction tasks set out above.

Adaptive combined forecasting model of selective and hybrid types with previous clustering by nearest neighbor method.

Selective approach is the selection for each value τ at each point of time series from programming set $\Im_{PS}$ of a single model, that provides highly accurate forecasting for a particular selection criterion: B-criterion [Lukashin, 2003], R-criterion [Berzlev, 2012] on the current site time series. Parameters selection criteria usually have an adaptive character. Also, often, to improve the accuracy of forecasting, the selection criteria do not apply to the program, but to the so-called basic set $\Im_{BS}^{\tau}$. It is a subset of the basic set consisting of such models, which give the most accurate predictions for the current site time series. The selection of models to the basic set can be done, for example, based on the D-criterion [Lukashin, 2003]. Forecast for the hybrid approach is calculated as the weighted sum of forecasts by all models that make up the basic set $\Im_{BS}^{\tau}$. Build up a forecast based on these approaches, using the results of the time series clustering (step 1).

Selective method. Let the series clustering by nearest neighbor method was conducted i. e. based on certain proximity measure, such as Euclidean distance, the cluster $z_{(m)}^{x-m+1} \in \Re$ was defined, which is most similar to the pivotal cluster $z_{(m)}^{n-m+1}$. The last element of the cluster $z_{(m)}^{x-m+1}$ will be an element z_x, the last element of the cluster $z_{(m)}^{n-m+1}$ will be an element z_n.

For each fixed τ do a basic set of forecasting models $\Im_{BS}^{\tau}$ using D-criterion. D-criterion value at the time t_x can be calculated by the formula:

$$D_p(\tau) = \frac{1}{x-c+1}\sum_{j=0}^{x-c}\left(\hat{z}_{\tau}^{p}(x-\tau-j) - z_{x-j}\right)^2,$$

where τ – the forecast period, c – prehistory period, $\hat{z}_{\tau}^{p}(x-\tau-j)$ – the forecast, which is calculated at the moment $t_{x-\tau-j}$ (в точці $x-\tau-j$) for τ steps forward by model f_p, $p=\overline{1,N}$. Then the basic set of models for fixed τ is defined as:

$$\Im_{BS}^{\tau} = \left\{f_p \in \Im_{PS} \middle| D_p(\tau) \le \lambda D_{min}(\tau), p=\overline{1,N}\right\},\ D_{min}(\tau) = \min_{p=\overline{1,N}} D_p(\tau),$$

where $\lambda \in \mathbb{R}$ – selection parameter, in practice it is recommended to choose $\lambda \in [1.2, 1.5]$.

Denote the models that have been included into the set $\Im_{BS}^{\tau}$ by $f_1^{\tau}, f_2^{\tau}, \ldots, f_{L_\tau}^{\tau}$, L_τ – the number of models in this set, $\tau=\overline{1,\theta}$, $L_\tau \le N$.

For each model in the basic sets $\Im_{BS}^{\tau}$ we calculate the value of the B-criterion:

$$B_{x,\tau}^{q_\tau} = (1-\alpha_B)B_{x-1,\tau}^{q_\tau} + \alpha_B e_{\tau}^{q_\tau}(x-\tau), \tag{1}$$

where $0 < \alpha_B \le 1$ – the smoothing parameter, and $e_{\tau}^{q_\tau}(x-\tau) = \left|\hat{z}_{\tau}^{q_\tau}(x-\tau) - z_x\right|$ – the absolute forecast error, which is calculated at the moment $t_{x-\tau}$ for τ steps by models $f_{q_\tau}^{\tau}$, $\tau=\overline{1,\theta}$, $q_\tau=\overline{1,L_\tau}$. Then such a model f^{τ^*}, for which B-criterion minimum value is provided, would be considered as the most accurate model for fixed τ by B-criterion $\min\limits_{q_\tau=\overline{1,L_\tau}}\left\{B_{x,\tau}^{q_\tau}\right\}$. Forecast that is based on models f^{τ^*} at the point n on τ steps further denote as $\hat{z}_\tau^*(n)$. Then the forecast at a point n for combined model of selective type by the B- selection criterion (1) with time series clustering using nearest neighbor will be calculated by the formula:

$$\hat{z}_\tau(n) = \alpha\,\hat{z}_\tau^*(n) + (1-\alpha)\,z_{x+\tau}, \tag{2}$$

where $z_{x+\tau}$ – the time series value, which follows on τ points after $z_{(m)}^{x-m+1}$ cluster, similar to the pivotal cluster, $\alpha \in [0,1]$ – parameter which indicates the importance of taking into account the estimated values of selected model in forecast.

A hybrid method. Let after the clustering in point x for each τ the basic $\Im_{BS}^\tau$ sets were formed and values of B-criteria $B_{x,\tau}^{q_\tau}$ (1), $q_\tau = \overline{1,L_\tau}$ were calculated. Denote by $\hat{z}_\tau^{q_\tau}(n)$ the forecast, which is calculated at the point n on τ points forward for the $f_{q_\tau}^\tau$ models from the basic set $\Im_{BS}^\tau$, Then the forecast of combined model of hybrid type with clustering by nearest neighbor method is given as:

$$\hat{z}_\tau(n) = \alpha \sum_{q_\tau=1}^{L_\tau} \omega_\tau^{q_\tau}\,\hat{z}_\tau^{q_\tau}(n) + (1-\alpha)\,z_{x+\tau}, \tag{3}$$

where $\alpha \in [0,1]$, weights $\omega_\tau^{q_\tau}$ can be defined on the B-criterion (1) [Lukashin, 2003] taking into account the factor of proportionality, which is determined from the unity weights sum equality, $\sum\limits_{q_\tau=1}^{L_\tau} \omega_\tau^{q_\tau} = 1$, i.e.

for $L_\tau = 2$ $\quad \omega_\tau^1 = \dfrac{B_{x,\tau}^2}{B_{x,\tau}^1 + B_{x,\tau}^2}$, $\omega_\tau^2 = \dfrac{B_{x,\tau}^1}{B_{x,\tau}^1 + B_{x,\tau}^2}$,

for $L_\tau = 3$ $\quad \omega_\tau^1 = \dfrac{B_{x,\tau}^2 B_{x,\tau}^3}{B_{x,\tau}^1 B_{x,\tau}^2 + B_{x,\tau}^1 B_{x,\tau}^3 + B_{x,\tau}^2 B_{x,\tau}^3}$, $\omega_\tau^2 = \dfrac{B_{x,\tau}^1 B_{x,\tau}^3}{B_{x,\tau}^1 B_{x,\tau}^2 + B_{x,\tau}^1 B_{x,\tau}^3 + B_{x,\tau}^2 B_{x,\tau}^3}$,

$$\omega_\tau^3 = \frac{B_{x,\tau}^1 B_{x,\tau}^2}{B_{x,\tau}^1 B_{x,\tau}^2 + B_{x,\tau}^1 B_{x,\tau}^3 + B_{x,\tau}^2 B_{x,\tau}^3},$$

and so on.

After the forecast calculating predicted point $\hat{z}_1(n)$ is used to build a new pivotal cluster $z_{(m)}^{n-m+2} = \{z_{k_1^{n-m+2}}, z_{k_2^{n-m+2}}, \ldots, z_{k_{m-1}^{n-m+2}}, \hat{z}_1(n)\}$, $z_{k_1^{n-m+2}} = z_{n-m+2}$, and the old pivotal cluster $z_{(m)}^{n-m+1}$ becomes non-pivotal cluster that is included to the set $\Re$, and the calculation process starts over again, that is based on certain proximity measure to the pivotal cluster, $\Im_{BS}^\tau$ sets are formed for each τ, the B-criterion value is calculated for each forecasting model that uses a similar non-pivotal cluster sequence as retrospective information. Further the forecast is built according to the selective or hybrid principles (1-3).

Signs increment forecasting model with the prior clustering by K-nearest neighbors' method.

Let on certain proximity measure basis in point z_n K clusters $\chi_{(m)}^{y-m+1} \in \aleph$, $y \in [m+1, n-1]$, $card(\aleph) = K$, similar to the pivotal cluster $\chi_{(m)}^{n-m+1}$, were defined, $\aleph$ - clusters set, similar to the pivotal cluster. The last element of each clusters $\chi_{(m)}^{y-m+1}$ are the elements z_y. In this method we'll be limited to the selective principle of forecasting building. As the selection criterion in z_y points the assessment:

$$I = \frac{1}{m}\sum_{j=1}^{m}\gamma_j H_h\left(\hat{\chi}_1^p(y-j)\Delta z_{y-j}\right). \tag{4}$$

will be used, where γ_j - weights coefficients, $\sum_{j=1}^{m}\gamma_j = 1$, $H_h(x) = \begin{cases} 0, x < 0 \\ h, x = 0 \\ 1, x > 0 \end{cases}$, $x \in \mathbb{R}$ – Heaviside function.

Differences Δz_{y-j} , $j = \overline{1,m}$ are build from clusters elements $\chi_{(m)}^{y-m+1} \in \aleph$, $p = \overline{1,L}$, [Berzlev, 2013]. To simplify in the points z_y we'll select a single model for which criterion I (4) is maximal. Denote the increments signs projections of selected models to one point forward, which are calculated in the points z_y (for each cluster) by $\chi_{1,d}^*(y)$, $d = \overline{1,K}$,. Then increment sign forecast to one point forward, which is calculated at point z_n can be defined by formula:

$$\chi_1(n) = sign\left(\sum_{d=1}^{K} H_h(\chi_{1,d}^*(y)) - \frac{K}{2}\right) \tag{5}$$

Conclusions

Scientific novelty. The forecasting information system that is based on adaptive combined models of hybrid and selective types according to various criteria of selection, the previous time series clustering methods for nearest neighbor and K- nearest neighbors was described. Clustering options, proximity measure, and other models indicators can be defined by users in the course of the forecast. Information system solves two actual problems: time values series forecasting with determined predicted horizon and time series increments forecasting to one point forward.

The practical value. The described information system was realized in the software environment Delphi. A comparative analysis of time series forecasting of prices (petrol, silver, aluminum, data were selected for the last three years, total 700 measurements) using the conventional adaptive combine models based on adaptive polynomial models of Brown, Trigg-Leach etc. and combined models with the prior time series clustering was conducted. Results of the analysis suggest that the proposed an article models (1-3), which are the components of information system provide a higher accuracy in case of forecasting with period $\tau > 5$.

Signs increments forecasting model (4,5) also allows to improve forecasting accuracy compared with naive algorithm and the known series forecasting model with unstable nature of oscillations, as described in the [Lukashin, 2003]. Details of the model test and numerical results are shown in the paper [Berzlev, 2013].

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SEMANTIC NET FROM CONCEPTS AS A MODEL OF STUDENT'S KNOWLEDGE: HOW STABLE ARE THE RESULTS OF EXPERIMENTAL STUDY?

Evgeny A. Eremin

Abstract: *Original method to study students' conceptual knowledge was reported at previous ITHEA conferences. According to this method, to show their educational achievements students had to pass through some testing procedure, which requires combining of basic terms from a learnt course into interrelated pairs. Then students' answers, saved in text files, were analyzed by computer software that joined all interrelated concepts into groups, independent from each other. The larger these groups were the better students' success in digestion of learnt course was supposed.*

As human cognitive processes are very complex, it is important to make sure that experimental method is really valid and steady. The present paper examines stability of the experimental results; new implemented experiments were also aimed at estimation of method's inaccuracy. Several diversified tests were made to assure that method error do not deface observed picture: the difference between repeated tests was evaluated; the stability of student's pair selection and of knowledge fragmentation were analyzed to make sure that we can trust the results of experiments; regression towards the mean effect, which can skew the outcome of low-performed students, was estimated – the situation was found to be secure; another discipline was checked using the same method and there was no noticeable dependence of numeric scaled gauges from a learning content.

Detailed study shows that the generality of students demonstrate a growth of the proposed measure of learning success and its magnitude distinctly exceeds inaccuracy of method. The value of this inaccuracy was also estimated.

Keywords: *concept, relation, semantic network, knowledge representation, learning*

ACM Classification Keywords: *H.1.2 User/Machine Systems – Human information processing, I.2.4 Knowledge Representation Formalisms and Methods, Experimentation – Semantic networks, I.2.6 Learning, Human factors – Concept learning, Knowledge acquisition.*

Introduction

The accepted classification, described for instance in well known book [Anderson and Krathwahl, 2001], itemizes several different kinds of knowledge: factual, conceptual, procedural and meta-cognitive. It becomes more and more evident every year that remembering of numerous facts is useless without any system in acquisition. Continuous flow of new data puts forward conceptual organization of incoming information, so the level of the conceptual knowledge now comes before the level of the factual one. Systematization of learning facts is absolutely essential today because of rapidly growing amount of data about outward things. Hence problems, connected with building the structure of digesting concepts, become more and more urgent.

We can find demonstrative examples of knowledge organization role in many research works from different fields. For instance, the significance of structural knowledge in physics was reported in [Koponen et al, 2002]. The authors even established a special course, which "purpose is not to teach more physics but to organise what has already been learnt". The illustration from other knowledge domain was developed in [Loubser et al, 2001], where the conceptual base for environmental literacy was analyzed.

Experimental research also supports the discussed above theoretical thesis about the importance of knowledge systematization. Very decisive case can be found in detailed experiments [Lee et al, 2011]: this publication clearly showed that assessment based on knowledge integration can measure a wider range of knowledge levels than traditional multi-choice items.

Following ideas about importance of conceptual knowledge, in 2008 I started series of experiments to study how students organize basic concepts of a learning course in their minds and how to evaluate the level of their competency in systematization. A special testing procedure was developed to check ability to group the main terms of learnt discipline and to find some quantitative measures for this learning property. Developed experimental method was reported at the IV International Conference “Modern e-Learning” (MeL 2009) and published in its proceedings in Russian [Eremin, 2009]. Later on English version of these materials appeared in the associated international journal [Eremin, 2010]. Obtained during the study results were discussed at MeL 2011 [Eremin, 2011] and also at the “Multidisciplinary Academic Conference” (MAC 2012) [Eremin, 2012].

As human cognitive processes are very complex, it is important to make sure that experimental method is really valid and steady. So the aim of the present paper is **to consider possible method’s inaccuracy**. It describes the experimental study of a question how stable are the results we get. Different checks are made for this purpose; details of these control experiments will be discussed in the next sections.

In the issue all revisions proved that the magnitude of method errors can not deface a picture observed in experiments. The approximate value of this error was also calculated.

Basic experiment

This section briefly describes the main features of the basic experiment [Eremin, 2009-2012], which was implemented in order to study students’ skills in organization of a system from fundamental terms. Later on this experiment will be carefully checked up for stability and validity of results.

The first step to organize the original experiment was to form a full list of basic terms from the learning course, which teacher suppose good student must know. According to personal author’s preferences the educational course about computer architecture was under consideration.

In our case the list of concepts, used as experimental base, contained more than 120 terms. The most general concepts – like *computer*, *software* and *hardware*, *theoretic basis* etc., complemented by the terms that expand the previous ones – *operating system*, *processor*, *memory*, *DMA*, *principle of hierarchy*, *byte* and many others, were included into this base. Some related terms from connected disciplines such as microelectronics, logics and number notations were also added to the list. Contrary, the list did not include the names of concrete operating systems, external devices and their manufacturers, and other similar data, less essential from the position of learning the main course’s regularities. Using the standard terminology from object-oriented programming, we may say that classes of the concepts were under consideration but not their instances.

The next preparatory step was to build a semantic network from selected concepts. This is an optional operation, but it is very useful because gives us possibility to picture to ourselves what we want to get from a scrupulous student. The possible variant of such network, imaging the teacher’s view on the selected knowledge domain, is presented in fig. 1. Note that not all terms are visible in the picture: the numbers in the right top corner of rectangles with various concepts indicate how many related terms are hidden.

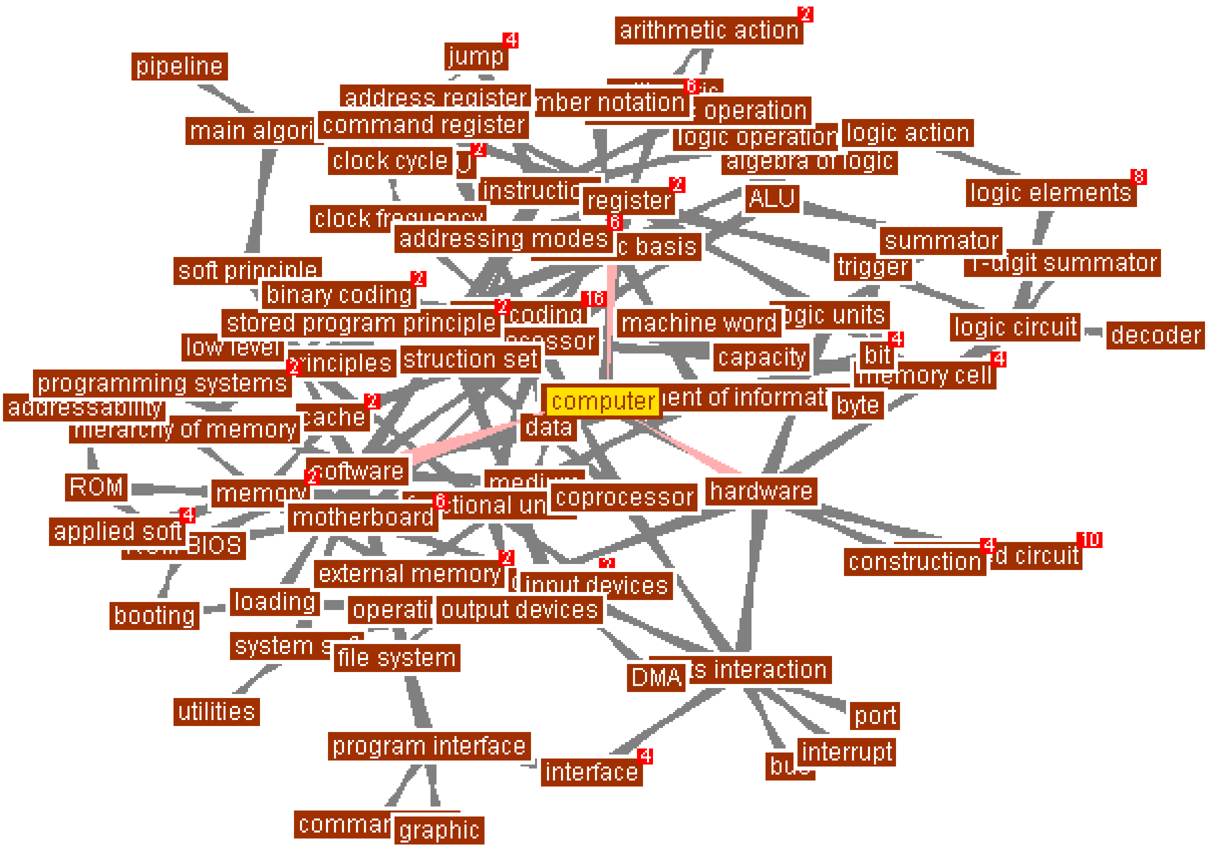

Figure 1

Semantic network in the picture has minimum two features that are important for further discussion. Firstly, this is really *network* but not a simple tree structure: many key concepts in our scheme are interlinked with large number of neighbors. Secondly, we see *connected* graph in the picture, because at least one way between two arbitrary terms always exists. From pedagogical point of view it means than all terms from the learning course are interrelated and form one single structure. As experiments showed, students could not obtain suchlike fully connected network even if they tried to do it.

When this complete list of terms had being formed, it became a base for experimental testing. The procedure of testing was the following. A student had to point out two terms and fix a link of definite kind between them. The testing software saved all constructed pairs into a text file that was analyzed after accomplishment of experiments by means of another computer program. Details of testing procedure can be found in [Eremin, 2010].

At the last step all resulting files fell under final computer processing. Its main aim was to educe linked groups of concepts for every student. For example, fig. 2 shows how computer combines concepts *functional units*, *processor*, *memory*, *input devices* and *output devices* into one group, using four associations *class/subclass* for central category *functional units*. Then *ALU* and *CU* (arithmetic and logic unit and control unit) will be joined to this group through the relations *whole/part* with term *processor*. Thus we get a set of seven interrelated concepts as a result.

As we already mentioned, in ideal case all concepts of a course must be interrelated. But experiments showed, that real students' files represented more scattered picture, which consisted of several isolated groups from concepts, and some groups were very small (only 2-3 terms). Every such small group can be interpreted as a separate fact that student did not associate with other facts from the course. You must also note that an increased rate of fragmentation indicates that student's knowledge is sparser.

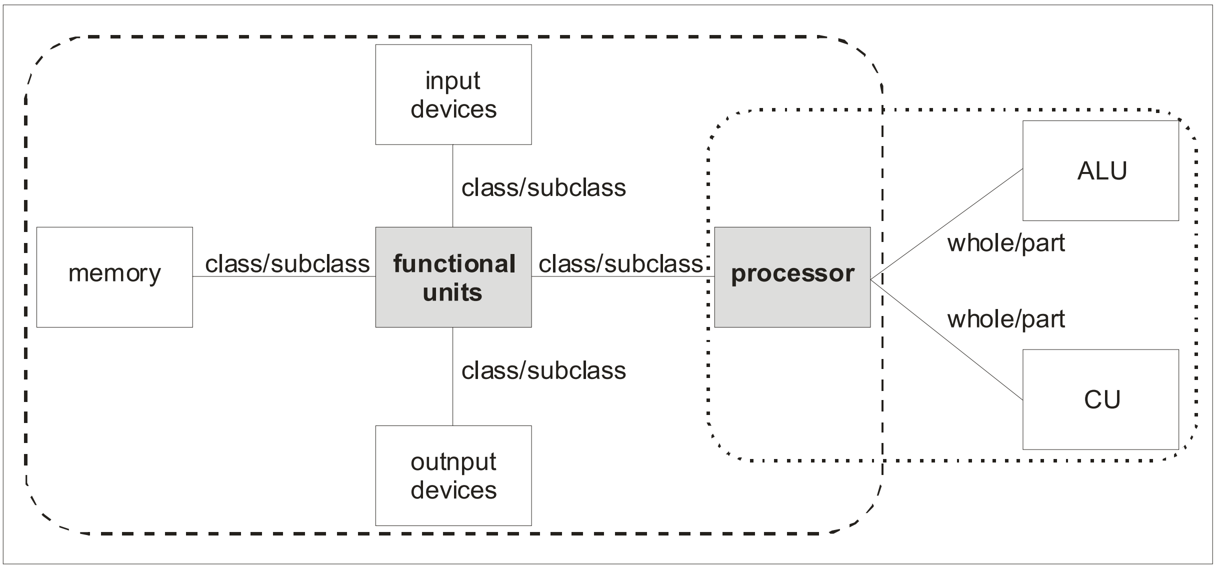

Figure 2

Typical result for ordinary student is shown in fig. 3, where 3 largest groups of concepts are depicted (there are also two groups from 3 terms and two from 2 ones, but they are not included into fig.3 as non-essential). Experiment shows that student's knowledge is fragmental and not clearly recognized as common picture (compare with the network in fig. 1). The most essential limitation is that general concepts like *hardware* and *software* seem to exist isolated from most of others and in similar way theoretical subjects like *scale of notation* are absolutely separated.

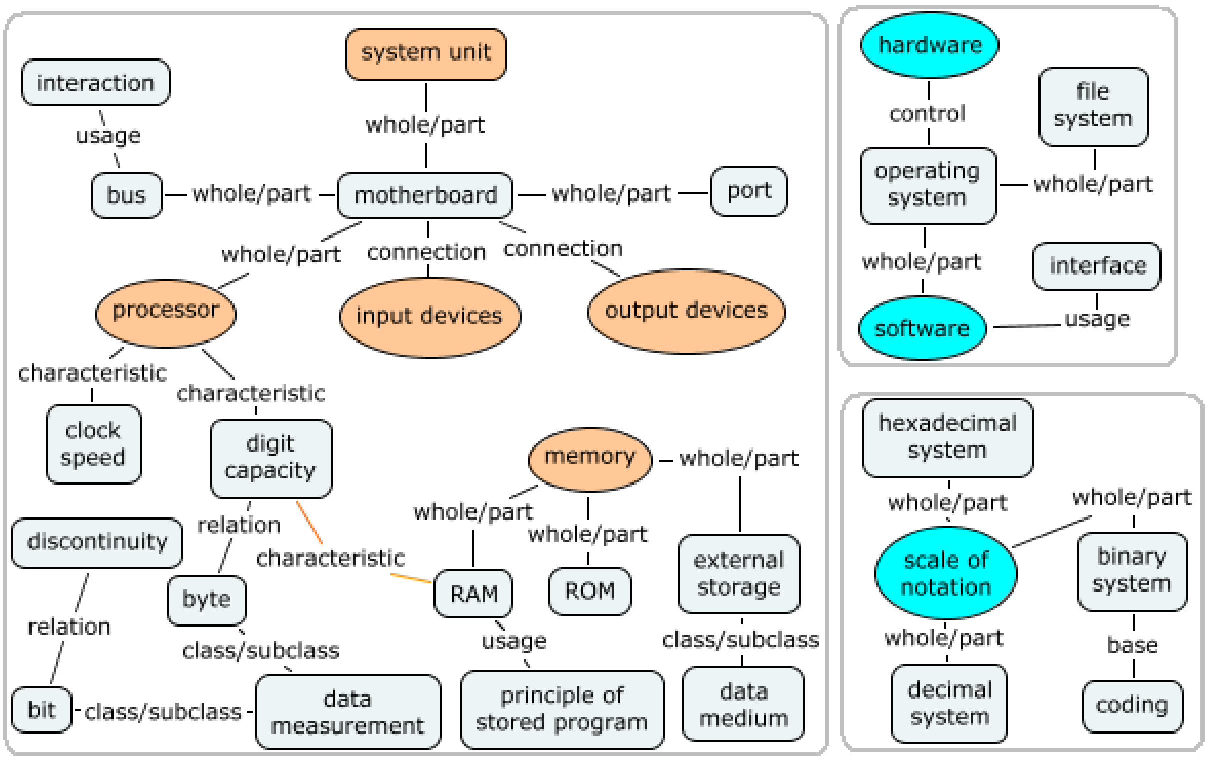

Figure 3

Fragmentation of knowledge was also evidently presented at specific “spotted” diagrams with independent groups of concepts, discussed earlier in publications [Eremin, 2009-2011].

More detailed discussion about grouping of concepts can be found in section “Test 3: stability of knowledge fragmentation”.

Analysis of numeric results in publication [Eremin, 2011] allowed selecting a simple but effective quantitative characteristic for evaluation of conceptual knowledge. It is based on resulting data about size and number of groups. If we divide *total number of concepts T* that student has named by *number of groups G*, we get *average size of a group*, which must be large when student learnt the course profoundly. As experiments showed, the results for average number of terms per group *T/G* had the most essential numeric changes among all other examined gauges. This feature prompts that such value can be considered as a reliable experimental measure [Rogosa and Willett, 1983].

Checking experimental method for errors

Now let us enumerate the main results of our basic experiment in order to check its accuracy. So the main findings we going to check are the following.

- Experiments showed that students’ knowledge is far from ideal entire picture – learnt terms are strongly fragmented.
- Average size of groups from terms that student demonstrates may be proposed to be a gauge of learning success.
- Several kinds of students can be educed according to introduced above gauge. The most distinct result is that “weak” students demonstrate very stable growth.

Conclusions formulated in the previous research will be investigated on validity in this paper. For this purpose we shall vary several conditions and examine the influence of changes; correspondent fruits are described in the following sections.

Section “**Test 1: repetition of experiments**” depicts how stable are the obtained results when we reduplicate the same test for the same students once more. Calculating the numeric difference between these two checks, we derive a value for confrontation with growth of the evaluating parameter *T/G* after completing a course: the last gain must distinctly exceed random changes between repetitions of the same test. Such experiments also give a numeric value of method’s error.

Section “**Test 2: stability of pair selection**” checks are pairs that students construct during their testing identical or not. It is important if we want to have stable experimental results. Such results also indirectly indicate are student’s knowledge deep or superficial.

Section “**Test 3: stability of knowledge fragmentation**” continues the discussion about stability of results. Its main question may be stated like following: is fragmentation real or maybe it is some sequence of inexact method.

Section “**Test 4: evaluation of RTM effect**” examines specific mechanism of statistical error and evaluates its significance for our experiments, using numeric method from [Ostermann et al, 2008]. This effect may artificially make the lowest results higher and so its estimation is very fateful to assure us about low-performed students’ success.

Section “**Test 5: different disciplines**” argues that the results of evaluation do not depend upon the contents of learning material. All previous study was based on “*Computer architecture*” course. Another educational discipline – “*Mathematical logic*”, considered in this section, showed very close results.

Before we begin the discussion of stability problems, let us introduce some brief denotation for students groups. In order to make the description more definite, let us continue the tradition from [Eremin, 2011] to use short tokens to refer to tested groups. So whereupon group G0 is the first trial of method [Eremin, 2010], results of groups G1-G4 are the subject of analysis in [Eremin, 2011], and some aspects of student's practical assessment were investigated for groups G1-G7 in [Eremin, 2012]. The present paper that continues the previous research, considers new results for groups G8-G10.

Total number of students in all groups G0-G10 is equal to 116. So on the average experimental group consists of approximately 11 students (from 5 to 17).

Test 1: repetition of experiments

The meterage theory states that the simplest way to check an accuracy of any experimental result is to repeat a measurement several times. Although our evaluations are not so easy to reduplicate as in the case of meterage with ruler or electrical device, we may try to test students twice (no one will agree for more!) So let us consider the results of two small academic groups G8 (7 students) and G9 (5 students), which were tested *twice before* learning the control course and *once after* it.

Group G8 passed two pretests (tests before learning a course) with interval more than one month; in group G9 pretests were separated by 7 months. In both cases students could not remember their answers during the first pretest, hence they had to pass the second pretest anew. Then they learnt “Computer architecture” course and final testing as usual was implemented after completion of the course.

Let us define values *T/G* for two input tests as A_1 and A_2 correspondently. Then according to foundations of meterage theory the result of measurement must be calculated as arithmetical mean $A = (A_1 + A_2) / 2$. Absolute error is computed using formula $\Delta A = |A_1 - A_2| / 2$, and relational one – as $\Delta A / A$.

Calculations for students from group G8 give the following values for relational error: minimum – 2%, maximum – 16% and average – 8%. The results for group G9 are worse: minimum – 1%, maximum – 24% and average – 16%. Total average relational error for both groups is about 11%. This value can be taken as a total estimation of average error of method.

Obtained values of relational error we can compare with learning progress *P*, computed as $P = A_3 - A$, were A_3 is ratio *T/G* for output testing. The excess of this characteristic over relational error is equal to $(P - \Delta A) / A_3$.

The last parameter for group G8 is always positive: it means that values of learning growth for all students are higher than random method's error and hence can be distinctly fixed. Minimal magnitude of this parameter is 3%, maximum – 86% and average – 28%. As before, G9 shows lower results: maximum – 42%, average – 14%. The most disagreeable is that one student has negative result -13%: this means that her measured growth is less than error of method and hence can not be evaluated correctly.

Thus we see that 11 students from 12 in our experiment have valid learning progress and it suggests us that our experimental method generally has valid results.

Fig. 4 illustrates these calculations by diagrams. There we can see total results for all students from both groups. Every student is represented by means of 3 columns: two pretest B1 and B2 (dark colors) and posttest A (light color). It is clear that students S2, S4 and S5 in G8 and S1 and S2 in G9 have evident learning progress. Student S4 in G9 has negative (undetermined) results: value for output testing A is lower than input one for B2. Other students have less obvious but positive educational progress.

Test 2: stability of pair selection

My previous publications concentrated an attention on statistics of experimental results. Now, studying the stability of obtained results, let us examine concrete pairs that students construct and evaluate their permanence.

To estimate this feature we have 9 doubled pretests for group G8 (note that two students do not finished the semester, so in the previous section you saw number 7 for G8!) and 5 test results for G9. The analysis is easy: a short program reads both pretest files for every student and calculates percentage of pairs that concur.

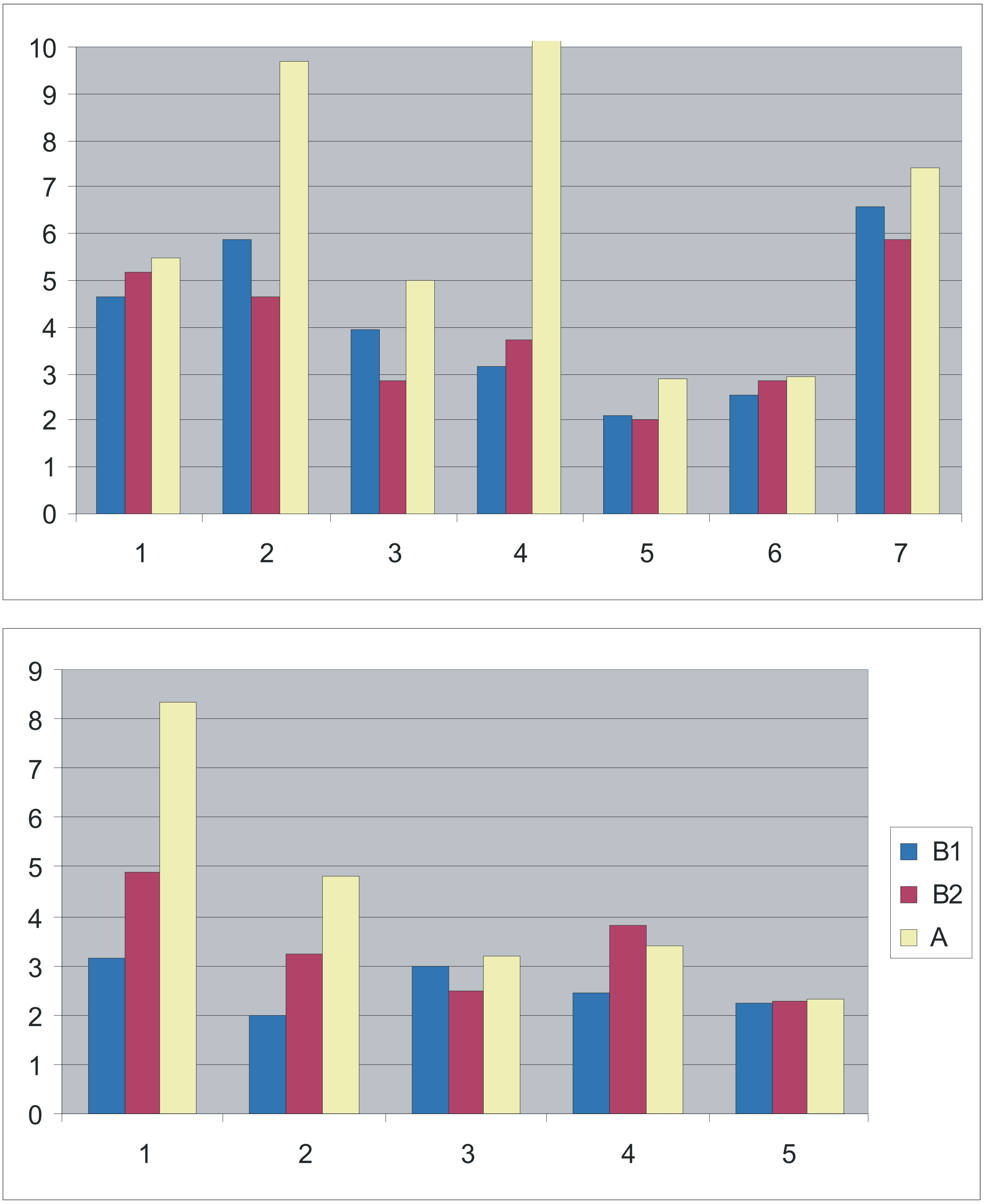

Figure 4

The results of such processing are the following. Students from group G8 demonstrate minimum 13% and maximum 48% congruence; average value is 30%. “Weaker” students from small group G9 gave maximum 24% and average value only 11%. To understand such difference we must have in view that groups has different specialties and G8 students named much more pairs: for G8 average number of pairs was 33 and they included on the average 41 topics, while G9 had only 14 links from 21 terms! On my opinion all results are too low, but this is what my students really showed.

The statistical analysis also showed a very interesting fact: a plot for number of overlap pairs looks very similar to correspondent plot for our gauge *T/G*, selected in present research as a measure of learning success. The linear correlation between these variables is very high and equals to 0.92 for G8; value 0.69 was formally calculated for 5 students of G9 and total magnitude for all 14 students is 0.88. I suppose such accordance is a certain indirect confirmation that we are really evaluating some objective ability of a student to work with conceptual knowledge. Unfortunately the number of students in G8 and G9 is too small to make a confident conclusion.

Test 3: stability of knowledge fragmentation

Results of our study indicate that students can not build (or maybe can’t show that they are able to built) an entire picture of the discipline they learnt. It is a serious blame, so we must check this situation more carefully.

The first easy to do control is to give students a direct task to build an entire picture. We must emphasize that during the basic experiment students did not know its aim [Eremin, 2010]. What if they will specially try to build one interrelated picture?

Such estimation experiment was implemented in 2009 for several students. They were asked to build one connected picture, but nevertheless the results were fragmented. Although student specially tried to construct non-fragmented system of terms and total number of groups decreased 2-5 times, nobody reached only one group of concepts. Furthermore the best result among these students was 4(!) groups.

Usually when students do not specially worry about minimization of group’s number, the result 4-5 groups is good. Only two from all 116 students managed to form 2 groups. In the first case student formed 2 groups in pretest, but her posttest contains 6 groups. In the second case in two pretests results were 12 and 7, and in posttest he showed 2 groups. So it seems to be rather chance than consistent pattern.

Hence we can see from all these facts that it is very long distance from students’ answers (remember fig. 3) to ideal teacher’s picture (fig. 1). But let us check this thesis more careful yet.

For this purpose in 2013 another method was trialed for building of groups from topics. The list of terms was absolutely the same as earlier, but the task sounds in other way. Students were proposed to link to every key concept as many related concepts as possible. To make the task clearer it was an example, demonstrating that to category *fruit* we can link several *subclasses* like *apple*, *orange* and so on. Applying this method to computer architecture, we may specify for instance that concept *functional unit* has subclasses *processor*, *memory*, *input devices* and *output devices* (remember fig. 2).

The experimental task looks quite similar, but upon a close view it stimulates grouping of concepts: it directly prompts a student that he must link to selected concept as many terms as possible – pair strategy has no such prompt. Besides new modification of method makes irrational to construct the smallest groups from two terms: it means introducing generalized concept for one term – such generalization is useless!

Let us mark the basic method as **MCP** (*Method of Constructing Pairs*) and proposed modification as **MGT** (*Method of Grouping Terms*). The results for group G10, tested both ways, are the following (see tab. 1).

Table 1

	method	1	2	3	4	5	6	7	8	9	10	11	12	mean
terms	MCP	57	49	50	54	40	36	41	58	60	61	71	50	52.3
	MGT	44	41	57	47	25	34	42	39	34	62	49	20	41.2
groups	MCP	15	14	8	15	13	9	10	12	12	16	14	14	12.7
	MGT	6	8	10	7	5	6	7	7	7	14	10	5	7.7
T/G	MCP	3.8	3.5	6.3	3.6	3.1	4.0	4.1	4.8	5.0	3.8	5.1	3.6	4.2
	MGT	7.3	5.1	5.7	6.7	5.0	5.7	6.0	5.6	4.9	4.4	4.9	4.0	5.4

The most essential changes were observed in the number of groups – it decreased approximately 1.7 times in average. At once total number of terms also decreased 1.3 times, so finally ratio T/G grew about 1.3 times.

We see that although the number of group noticeably decreased, even its minimal value 5 is far from the ideal. Hence essential change of method did not lead to disappearance of fragmentation.

Comparing data for MCP and MGT variants of method, I also determined common pairs for both experiments. Average ratio of common pairs to total number of links is approximately 34% for MCP and 40% for MGT (as it follows from tab. 1, total number of terms for MCP is greater, so fraction for it is smaller). Comparing with values 30% and 24% for experiments in groups G8 and G9, described in the previous section, we can discover some accordance of these results.

Test 4: evaluation of RTM effect

Beginning to learn the course “Computer architecture”, students already know many facts about computer. So after the final control we must compare the results before and after the course digest, expecting to seclude student’s real growth. Analysis of testing results before (pretest) and after teaching (posttest) is wide-spread practice in educational research.

So natural to general logic of education, pretest/posttest method is in a center of an active theoretical discussion. It traces back to 1956 when F.M. Lord published his work “The measurement of growth” [Lord, 1956], in which concluded that “differences between scores tend to be much more unreliable than the scores themselves”. Later Rogosa and Willett [Rogosa and Willett, 1983] investigated the problem wider and proved that limitations of difference measurement are not universal: in many important situations the difference scores are reliable. The authors formulated a very clear practical guideline: “the difference score cannot detect individual differences in change that do not exist, but it will show good reliability when individual differences in true change are appreciable”. We already used this principle of maximum pretest/posttest difference while selecting a suitable experimental gauge.

As careful investigation shows, difference scores have limitations of various natures. One of them is **regression towards the mean** (RTM; e.g. [Barnett et al, 2005]). As RTM may cause noticeable experimental error, it is very useful to explore this phenomenon.

RTM was first described in 1886 by F. Galton, who studied the dependence of adult children heights from specially calculated middle height of parents. He discovered that extremely tall parents had shorter children and, conversely, children of short parents tend to be taller than their parents. Later on such effects were found in many various areas, especially in medicine and education [Bland and Altman, 1994].

The substance of RTM may be explained by means of the following example. Suppose a normal student who felt himself bad or was strongly disconcerted with something while doing pretest and therefore demonstrated extremely low result. Posttest was long after and our imaginary student naturally got his average score (much closer to the mean than the primary one). Such situation has two abnormalities: first, our examinee will be mistakenly classified according to pretest to low level of primary knowledge and, second, his numeric growth will be undeservedly high. Conjunction of these circumstances may lead to wrong conclusion that students with low initial knowledge show more essential growth than others: unfortunately this effect is indistinguishable from really high learning outcome. Trying to avoid such fault we ought to do primary measurement more carefully, i.e., like theory usually recommends, several times; but is it real to oblige students to pass through the same pretest 3-4 times? To reduce RTM effect practically, Davis [Davis, 1976] suggested measuring pretest baseline twice, using one data set to calculate differences and the other one for baseline definition. In my experiments the idea of independent evaluation of students' growth and their baseline was put to use even more strictly: students were ranked according to rating, absolutely independent from pretest.

A special mathematical methods were developed to evaluate is this kind of experimental inaccuracy significant or not. The most suitable estimation was proposed in [Ostermann et al, 2008]. To apply it we must know only general statistical characteristics of pre- and posttest such as the mean and the standard deviation; calculating formula also contains correlation between the results of two measures.

The authors of the cited paper not only described the mathematical basics of evaluation technique, but kindly attached an Excel file to realize all calculations (http://www.biomedcentral.com/1471-2288/8/52). It remains just to substitute our values into definite cells of a spreadsheet and get immediate outcome. My experimental parameters for groups G1-G6 were the following: pretest – the mean 4.87 with deviation 2.66 and posttest – 5.58 with 2.76; correlation between scores 0.46; total number of students – 67. The main estimation result for specified values is that if the "true mean" (term "true" in publication represent value in "real nature", but not the statistical one!) of test is lower than 5.1, RTM effect is not essential with p-value < 0.025. It is reasonable to assign average value 5.23 to this "true" variable in our case. Then, as Excel diagram shows, p-value will not exceed 0.05. According to such small magnitude, the RTM effect on our experiments must not be significant.

Test 5: different disciplines

This section shows that the results of experiments do not depend upon learning discipline. All previous results were obtained for "*Computer architecture*". What will change if we take another educational discipline?

In order to answer this question some additional experiments were fulfilled during learning "*Mathematical logic*" course. Method and software for testing were identical to previous experiments, I just changed the text file with the list of terms (new list contained 88 concepts).

The only difference in processing of the experimental data was the following. During "Computer architecture" control, students were rank-ordered according to some rating: the criterion of such arrangement was time of finishing all the tasks, given by the teacher. Students with small numbers finished the course earlier; hence they are supposed to demonstrate better results in learning the course content. In opposite, columns for «the slowest» students form the right part of the picture (see fig. 4).

The course of mathematical logic is organized in slightly different manner: instead of control tasks in computer class, students must solve several tasks in a written form. The marks for this work also allowed arranging students according to their results. It is important to note, that in both cases students' rating was defined *independently* from the results of experiments: this requirement is aimed at reducing RTM statistical effect, which was discussed in the previous section.

It is evident that the results of experiments for both educational courses can not be compared directly – we previously need to normalize them, using common scale. The normalization procedure must be executed for both coordinates – arguments and results.

Let us begin from abscess – student's rating *n*. It is integer dimensionless value, but the problem is that groups have different size. Hence we must reduce values of this rating according to total number of students *N* in the group. To solve this problem we introduce the formal variable *X*, which has a meaning of improved non-dimension student's rating. We assign value X = 0 to the first student with n = 1, and X = 1 – to the last student with n = N. For any other student with rating *n* we can calculate variable X_n, using formula

$$X_n = (n-1) / (N-1), \text{ where } n = 1, 2, \ldots, N$$

We see that *X* is a real variable, possesses the value in the interval from 0 to 1.

Now we consider the ordinate – our learning progress gauge *T/G*. As semantic nets for computer architecture and mathematical logic are quite different, it is incorrect to put them together. Following [Eremin, 2011] let us introduce the ratio

$$K_{tg} = (T_2/G_2) / (T_1/G_1),$$

where index 2 relates to posttest and 1 – to pretest. Non-dimension value K_{tg} describes the growth of conceptual knowledge and we can suppose is suitable for comparison of results for different disciplines.

Using the introduced above scales, we can put dots for growth coefficient K_{tg}, preliminary translating abscissa X_n for every student. The result scatter plot is shown in fig. 5. Every round black point conforms a student from groups G1-G6; not filled dots were got for mathematical logic: round ones describe group G9 and square dots – G8. Note that mathematical logic course was in the first semester, when 7 students studied in G9.

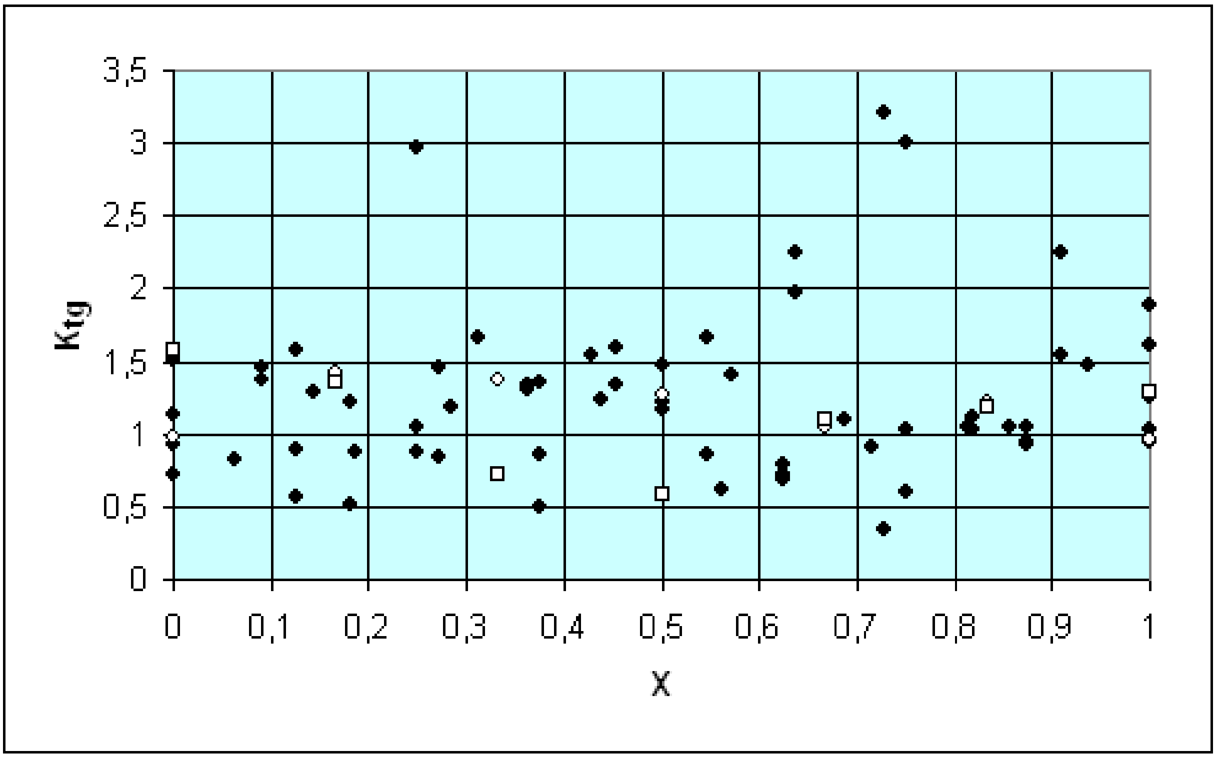

Figure 5

Diagram shows that dots for mathematical logic lie among the dots for computer architecture and variation of their positions do not exceed the dispersion of the last ones. Hence no dependence from learning material can be noticed.

Contrariwise, the values of *T/G* for mathematical logic are noticeably lower than for computer architecture. So as it was expected, these variables for different disciplines can not be compared without conversion.

And one more interesting detail. By occasional concatenation of circumstances one student made his output test twice. Two resulting values, processed as it was described in the section “Test 1: repetition of experiments”, gave for posttest relational error about 11%, i.e. very close to average pretest error.

Conclusion

Additional experiments were implemented to check the stability of results, obtained in previous research of students’ conceptual knowledge. Variation of some method’s conditions gave possibility to estimate how these changes influence on observed picture. Several different tests were organized for these purposes. First of all calculation of the difference between two pretests in the same group evaluates the magnitude of relational error of testing procedure (its average value was near 11%). Then the stability of grouping concepts process was investigated. In spite of some difference in numeric results, the main conclusions were found to be stable. Special evaluation of RTM effect, which can cause incorrect results for low-performed students, showed that we may not worry about this effect in our study. And at last after confrontation of results for two different educational disciplines we come to a decision that scaled statistical characteristics do not depend upon learning material.

So all control experiments described in the paper assured that we can rely on previous study of conceptual knowledge.

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FUZZY EXPECTED VALUE MODEL WITH INSPECTION ERRORS AND TWO LEVEL OF TRADE CREDIT IN ONE REPLENISHMENT CYCLE

Olha Yegorova

Abstract: *In this paper economic order quantity model with two level of trade credit in one replenishment cycle, time value of money, inspection errors, planned backorders, and sales returns is offered. In those problem demand rate, selling price for good quality items, selling price for defective items, interest paid rate and interest earned rate consider as fuzzy variable. Following expected value model criterion and a fuzzy expected value model are constructed. Fuzzy simulations are employed to estimate the expected value of fuzzy variable and maximum return on sale. In order to solve the fuzzy expected value model, a genetic algorithms based on the fuzzy simulation is designed.*

Keywords: *EOQ, imperfect quality, misclassification errors, deterioration items, inventory backordering, trade credit, fuzzy simulation, genetic algorithm.*

ACM Classification Keywords: *I.6.5 Model Development*

Introduction

Effective inventory management is one of the main issues for any trade company. The traditional economic order quantity model is widely used by decision maker as inventory control tool in real-world environmental. It was tacitly assumed that the buyer must pay for the items purchased as soon as the items are received. Sometimes retailer can't pay at the given period just for economic reasons and uses trade credit. Before the end of the trade credit period, the retailer can sell the goods, accumulate revenue, and earn interest.

During last several years, a great deal of the economic order quantity (EOQ) models with trade credit has been offered in the scientific literature. A lot part of these works shows the main research efforts in the stochastic environmental. For example, an EOQ model under the conditions of permissible delay in payments studied in [Goyal, 1985; Chung, 1998]. Resently, Huang [Huang, 2003] and Sharma et al. [Sharma, 2012] modified this assumption to two level of trade credit. Aggarwal and Jaggi [Aggarwal, 1995], Chung and Liao [Chung, 2004], Shah and Raykundaliya [Shah, 2009], Tripathy and Pradhan [Tripathy, 2012] deal with the problem of determining the economic order quantity for deterioration items under permissible delay in payments. Chang and Dye [Chang, 2001] published the paper for deteriorating items with partial backlogging and permissible delay in payments. Hou and Lin [Hou, 2009] suggested a cash flow oriented EOQ model with deteriorating items under permissible delay in payments.

An analytical review of the research efforts in the fuzzy environment performed on EOQ and economic production quantity models had done by Hsu [Hsu, 2012], Yegorova [Yegorova, 2012]. While the decision variables are similar to the ones in the stochastic environment, the demand and inventory costs are considered fuzzy variables. Similar approach is constrained with constraints such as budget, space, service level.

A careful observation of the given above papers reveals that there are not considered inventory model under the conditions of trade credit with human error in screening process. We notice that Schwaller [Schwaller, 1988] extended the EOQ model by adding the assumptions that defective items of known proportion were present in incoming lots, and that fixed and variable inspection cost were incurred and in finding and removing these defective items. Salameh and Jaber [Salameh, 2000] developed an economic order quantity model for the case where a random proportion of the items in a lot are defective. But human error in the screening process was ignored in their model rather than in Duffuaa and Khan [Duffuaa, 2005]. Later Khan et. al [Khan, 2011] proposed an EOQ with defective items, inspection errors, and sales returns, where shortage are not allowed, defective items are sold at a discounted price at the end of the 100% screening process. More discussions are given in notes by Hsu [Hsu, 2012]. It is clear that production processes are often imperfect. Therefore, in order to provide good service to attract customers and keep them coming back, development of such adequate model is needed with possibility of its clarification and adaption to the changing external terms. Besides, optimal inventory policies that consider both the supplier and retailer viewpoints are more reasonable than those those that consider only from one perspective.

This paper first extends the economic order quantity model for deterioration items with two level of trade credit in one replenishment cycle, inspection errors, planned backorders, and sales returns in both stochastic and fuzzy environments such that the demand, selling prices, interest paid, and interest earned assume a fuzzy nature. As a considered problem has constraints and uncertainty nature, it is necessary to foresee realization of technology which will be allow to shorten the quantity of the analyses cases and optimize the calculation process.

Preliminaries

Possibility theory was proposed by Zadeh [Zadeh, 1978], and developed by many researchers such as Dubois and Prade [Dubois, 1988] and Yager [Yager, 1993]. Recently, B. Liu and Y. K. Liu [Liu, 2002] presented a new measure named credibility measure. Moreover, Liu [Liu, 2004] proposed credibility theory.

Definition 1. Let ξ be a fuzzy variable with membership function $\mu(x)$. Then the possibility, necessity, and credibility measure of the fuzzy event $\xi \geq s$ can be represent, respectively, by [Liu, 2002]

$$Pos\{\xi \geq s\} = \sup_{x \geq s} \mu(x) \tag{1}$$

$$Nec\{\xi \geq s\} = 1 - \sup_{x < s} \mu(x) \tag{2}$$

$$Cr\{\xi \geq s\} = \frac{1}{2}\left[Pos\{\xi \geq s\} + Nec\{\xi \geq s\}\right] \tag{3}$$

Definition 2. The expected value of a fuzzy variable is defined as [Liu, 2002]

$$E[\xi] = \int_0^{\infty} Cr\{\xi \geq s\}ds - \int_{-\infty}^{0} Cr\{\xi \leq s\}ds \tag{4}$$

Definition 3. The optimistic function of α is defined as [Liu, 2004]

$$\xi_{\sup}(\alpha) = \sup\left[s \middle| Cr\{\xi \geq s\} \geq \alpha\right], \quad \alpha \in (0,1]. \tag{5}$$

Definition 4. If $\tilde{\xi} = (a,b,c)$ is a triangular fuzzy number with center *b*, left width $a > 0$, and right width $c > 0$, then its membership functions has the following form

$$\mu(s) = \begin{cases} \dfrac{s-(b-a)}{a}, & b-a \leq s \leq b, \\ \dfrac{(b+c)-s}{c}, & b \leq s \leq b+c, \\ 0, & elsewhere. \end{cases} \tag{6}$$

Definition 5. For the fuzzy variable described in Definition 4, the credibility of the event $Cr\{\xi \le s\}$ is defined based on the definition in (3) as [Liu, 2004]

$$Cr(\xi \le s) = \begin{cases} 0, & s \le b-a, \\ \dfrac{s-(b-a)}{2a}, & b-a \le s \le b, \\ \dfrac{s-(b-c)}{c}, & b \le s \le b+c, \\ 1, & elsewhere. \end{cases} \quad (7)$$

Definition 6. If $\tilde{\xi} = (a,b,c,d)$ is a trapezoidal fuzzy number with left modal level $b > 0$, right modal level $c > 0$, left width $a > 0$, and right width $d > 0$, then its membership function has the following form

$$\mu(s) = \begin{cases} \dfrac{s-(b-a)}{a}, & b-a \le s \le b, \\ 1, & b \le s \le c, \\ \dfrac{(c+d)-r}{d}, & b \le r \le b+c, \\ 0, & elsewhere. \end{cases} \quad (8)$$

Definition 7. For the fuzzy variable described in Definition 6, the credibility of the event $Cr\{\xi \le s\}$ is defined based on the definition in (3) as [Liu, 2004]

$$\mu(s) = \begin{cases} 0, & s \le b-a, \\ \dfrac{s-(b-a)}{2a}, & b-a \le s \le b, \\ \dfrac{1}{2}, & b \le s \le c, \\ \dfrac{s-c}{2d}, & c \le s \le d, \\ 1, & d+c \le s. \end{cases} \quad (9)$$

In this research, the triangular fuzzy variable is used to model the fuzzy interest paid, interest earned and trapezoidal fuzzy variable is used to model the fuzzy demand, selling price of good quality items and selling price of defective items.

Problem definition

Trade Company, which temporarily does not have working capital, plans to replenish inventory by using of trade credit. Allowable purchasing costs of inventory range from $E_{\min}$ to $E_{\max}$. the supplier offer the retailer a credit period of M -days. During this time, the retailer uses general revenue as investment resource with interest earned per monetary per unit time i_e. At the end of this period, the retailer pays off all units sold, keeping the rest for day-to-day expenses and stars paying for the interest charges on the unsold stock with the interest rate per monetary unit per unit time of i_p. Basic interest rate i_p would be lower to i_w from the date of N, if the retailer pays off all units sold during M to N.

Purchase orders are executed with constant intensity λ. Taking into account the overall dimension of unit of productions b, c, d, for holding inventory it is necessary warehouse with capacity W. Supply items lose their physical characteristics while they were kept stored in the inventory. Defective units may be found during products quality control. To ensure an adequate level of direct consumer service p_s, the maximum allowable number of defective items in the lot should not exceed u units. Inspector could incorrectly classify a non-defective item to be defective or incorrectly classify a defective item to be non-defective. Supplier is to rework (make up deficiency) or replace items with manufacturing imperfection. Production costs of defective items, which was obtained for settle a quality claims, does not include to the purchasing cost of the inventory lot and they will not be returned to the supplier. Other expenditures due to mistakes in incoming goods inspection now pay by the retailer.

Demand D is uniformly distributed on the interval between adjacent replenishment. Shortage is allowed. The items that are classification as defective and those returned by the consumers are kept in stock and sold at the end of the operation cycle. It is supposed that the inflation rate per monetary per unit time will be τ percent.

It is necessity determine the economic order quantity and length of operating cycle with the maximum revenue on sales.

Model formulation

It is known that λ is the replenishment rate per unit per unit time and D is demand rate per unit per unit time. Let us assume that the number of items that are classified as defective included those that are non-defective $(1-\beta)\lambda$, and incorrectly classified as defective (with probability a_1), and those that are defective $\beta\lambda$, and classified as defective (with probability $(1-a_2)$). Thus, we have the number of items that are classified as defective in one cycle

$$((1-\beta)a_1+\beta(1-a_2))\lambda \tag{10}$$

and the number of items that are classified as non-defective in one cycle

$$((1-\beta)(1-a_1)+\beta a_2)\lambda \tag{11}$$

where β – the probability that the item is defective, a_1 – the probability of classifying a non-defective item as defective, a_2 – the probability of classifying a defective item as non-defective.

Similarly, we estimate qualitative structure of the items obtained for settle a quality claims. The number of items that have been obtained for settle a quality claims and classified as defective are

$$((1-\beta_r)a_{r1}+\beta_r(1-a_{r2}))\lambda_r \tag{12}$$

and the number of items that have been obtained for settle a quality claims and classified as non-defective are

$$((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r \tag{13}$$

where λ_r – replenishment rate of items that have been obtained for settle a quality claims per unit per unit time, β_r – the probability that among obtained for settle a quality claims items is defective, a_{r1} – the probability of classifying a non-defective item among obtained for settle a quality claims as defective, a_{r2} – the probability of classifying a defective item among obtained for settle a quality claims as non-defective.

The proportion of the customers, who would like to accept backlogging at time t, decrease with the waiting time for the next replenishment. In this situation the backlogging rate is defined as $B(t)=1/(1+\delta(t_i-t))$, where t_i – the time at which the i^{th} replenishment is being made, δ – the backlogging parameter.

All cash flow elements will lead to the start of the investment using discounted by continuous scheme for a correct comparison the volume of money involved in the creation of stocks with following income. Determine the net discount rate of the inflation

$$k = \begin{cases} r, & \tau < 5\%, \\ (1+r)(1+\tau)-1, & \tau \geq 5\%, \end{cases} \tag{14}$$

and adjust the components of cash flow with the new rate, where r – current real discount rate in the market. If the amount of money at time t is $v(t)$, then their cost at the time $t = 0$ calculated as

$$v_0 = \int_0^H v(t)e^{-kt}dt \tag{15}$$

where v_0 – the value of money at the beginning of the planning horizon, H – planning horizon.

To determine optimal cash flow oriented inventory management strategies necessity take into account not only the amount of cash flow, but also moments of payments and revenues according to the contract provisions.

The crisp inventory model. The inventory system evolves as follows: units of items arrive at the inventory system with replenishment rate λ per unit per unit time when the shortage level becomes I_b. From Figs. 1, one can see that $[0;t_1)$ is the time taken to fill the backorders with rate $(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda - D)$. During replenishment period $[t_1;t_2)$ inventory level is declining only due to the demand and deterioration with rate $(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda - D - \gamma\cdot\theta\cdot I)$, where θ – percentage of items deteriorated per unit time, γ – percentage of deteriorated items screened out from the inventory. As soon as inventory level of serviceable items becomes I_s supplier should default the settlement of claims and maximum serviceable inventory level I_m will be accumulated. After that, inventory level due to the demand and deterioration becomes zero over time interval $[t_3;t_4)$ and backordering period $[t_4;T)$ begins.

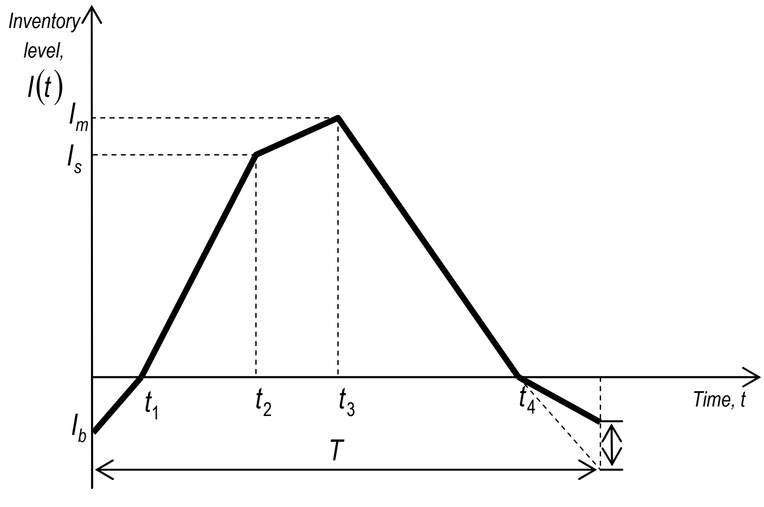

Fig. 1. Behavior of the inventory level over the time

The inventory levels of serviceable items at time t over the five periods in a cycle are determined by following differential equations:

$$\frac{I(t)}{dt}=\begin{cases}((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D, & 0\le t<t_1,\\ ((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D-\gamma\cdot\theta\cdot I, & t_1\le t<t_2,\\ ((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D-\gamma\cdot\theta\cdot I, & t_2\le t<t_3,\\ -D-\gamma\cdot\theta\cdot I, & t_3\le t<t_4,\\ -\dfrac{D}{1+\delta(t-t_4)}, & t_4\le t<T\end{cases} \tag{16}$$

with the boundary conditions $I(0)=I_b$, $I(t_1)=0$, $I(t_2)=I_s$, $I(t_3)=I_m$, $I(t_4)=0$, $I(T)=I_b$. The solutions for the above differential equations (16) are

$$I(t)=\begin{cases}(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)t+c_1, & 0\le t<t_1,\\ \dfrac{1}{\gamma\cdot\theta}(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)+c_2e^{-\gamma\cdot\theta\cdot(t-t_1)}, & t_1\le t<t_2,\\ \dfrac{1}{\gamma\cdot\theta}(((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D)+c_2e^{-\gamma\cdot\theta\cdot(t-t_2)}, & t_2\le t<t_3,\\ -\dfrac{1}{\gamma\cdot\theta}D+c_4e^{-\gamma\cdot\theta\cdot(t-t_3)}, & t_3\le t<t_4,\\ -\dfrac{D}{\delta}\ln|1+\delta(t-t_4)|+c_5, & t_4\le t<T.\end{cases} \tag{17}$$

The final solutions of above equations are given by

$$I(t)=\begin{cases}(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)t+I_b, & 0\le t<t_1,\\ \dfrac{1}{\gamma\cdot\theta}(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)\left(1-e^{-\gamma\cdot\theta\cdot(t-t_1)}\right), & t_1\le t<t_2,\\ \dfrac{1}{\gamma\cdot\theta}(((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D)\left(1-e^{-\gamma\cdot\theta\cdot(t-t_2)}\right)+I_se^{-\gamma\cdot\theta\cdot(t-t_2)}, & t_2\le t<t_3,\\ \left(I_m+\dfrac{1}{\gamma\cdot\theta}D\right)e^{-\gamma\cdot\theta\cdot(t-t_3)}-\dfrac{1}{\gamma\cdot\theta}D, & t_3\le t<t_4,\\ -\dfrac{D}{\delta}\ln|1+\delta(t-t_4)|, & t_4\le t<T.\end{cases} \tag{18}$$

Hence, it can be deduced from (18) at $I(t_1)=0$, $I(t_2)=I_s$, $I(t_3)=I_m$, $I(t_4)=0$, $I(T)=I_b$ that

$$t_1=\frac{I_b}{D-\lambda((1-\beta)(1-a_1)+\beta\cdot a_2)} \tag{19}$$

$$t_2=t_1-\frac{1}{\gamma\cdot\theta}\ln\left(1-\frac{\gamma\cdot\theta\cdot I_s}{(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)}\right) \tag{20}$$

$$t_3=t_2-\frac{1}{\gamma\cdot\theta}\ln\left(\frac{I_m-\dfrac{1}{\gamma\cdot\theta}(((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D)}{I_s-\dfrac{1}{\gamma\cdot\theta}(((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D)}\right) \tag{21}$$

$$t_4 = t_3 - \frac{1}{\gamma \cdot \theta} \ln\left(\frac{D}{\gamma \cdot \theta \cdot I_m + D} \right) \tag{22}$$

$$T = t_4 + \frac{1}{\delta}\left(e^{\frac{-I_b \cdot \delta}{D}} - 1 \right) \tag{23}$$

where T – length of the cycle.

We may also deduce that economic order quantity is

$$Q = \lambda t_2 \tag{24}$$

Our aim is to maximize return on sale per unit time, which can be expressed as

$$ROS_i(I_b, I_m, I_s) = \frac{(R_s)_i - K - (P_c)_i - C_{si} - C_{rg} - C_h - C_d - C_s - C_{un} - C_{adi} - C_{rnd} - IP_i + IE_i}{(R_s)_i} \to \max, \quad i = 1,2,\ldots,14 \tag{25}$$

subject to the constraints

- investment amount on total production cost cannot be infinity, it may have an upper and lower limits on the maximum investment

$$E_{\min} \le P_C \le E_{\max} \tag{26}$$

- warehouse space where the items are to be stored is limitation

$$b \cdot c \cdot d \cdot \lambda \cdot (t_1 + t_2) \le W \tag{27}$$

- holding cost cannot be more than total production cost

$$C_h < P_C \tag{28}$$

- providing good service of the customers

$$P\left(a_2 \le 1 - \frac{u+1}{\beta \cdot \lambda (t_1 + t_2)} \right) \le p_s \tag{29}$$

where R_s – sale revenue is given by

$$R_s = \frac{1}{T}\left(\begin{array}{l} p_v \left(\int_0^M e^{-kt} dt \left(\int_{\Xi_1} D dt \right) + \Psi_0 \int_0^N e^{-kt} dt \left(\int_{\Xi_2} D dt \right) + \Psi_1 \int_0^{t_4} e^{-kt} dt \left(\int_{\Xi_3} D dt \right) + \int_0^T e^{-kt} dt \left(\int_{t_4}^T \frac{D dt}{1 + \delta (t - t_4)} \right) \right) + \\ + p_b \int_0^T e^{-kt} dt \left(\beta \cdot a_2 \cdot \lambda \cdot t_2 + \beta_r \cdot a_{r2} \cdot \lambda_r (t_3 - t_2) + ((1 - \beta_r) a_{r1} + \beta_r (1 - a_{r2})) \lambda_r (t_3 - t_2) \right) \end{array} \right),$$

$$\Xi_1 = \begin{cases} [0; M], t_2 \le M < t_3 < T,\ t_3 \le M < t_4 < T,\ t_2 \le M < N < t_3 < T, t_3 \le M < N < t_4 < T, \\ \quad t_2 \le M < t_3 < N < t_4 < T, t_2 \le M < t_3 < t_4 < N < T, t_3 \le M < t_4 < N < T, \\ \quad t_2 \le M < t_3 < t_4 < T < N,\ t_3 \le M < t_4 < T < N, \\ [0; t_4], t_4 \le M < T, M > T, t_4 \le M < N < T, N > M > T, t_4 \le M < T < N; \end{cases}$$

$$\Psi_0 = \begin{cases} 1, t_2 \le M < N < t_3 < T,\ t_3 \le M < N < t_4 < T,\ t_2 \le M < t_3 < N < t_4 < T,\ t_2 \le M < t_3 < t_4 < N < T, \\ \quad t_3 \le M < t_4 < N < T,\ t_2 \le M < t_3 < t_4 < T < N,\ t_3 \le M < t_4 < T < N, \\ 0, \textit{otherwise}; \end{cases}$$

$$\Xi_2 = \begin{cases} [M;N], \ t_2 \le M < N < t_3 < T, \ t_3 \le M < N < t_4 < T, t_2 \le M < t_3 < N < t_4 < T, \\ [M;t_4], t_2 \le M < t_3 < t_4 < N < T, t_3 \le M < t_4 < N < T, \ t_2 \le M < t_3 < t_4 < T < N, \ \ t_3 \le M < t_4 < T < N \\ 0, \ otherwise; \end{cases}$$

$$\Psi_1 = \begin{cases} 1, t_2 \le M < t_3 < T, \ t_3 \le M < t_4 < T, \ t_2 \le M < N < t_3 < t_4 < T, \ t_3 \le M < N < t_4 < T, \\ \quad t_2 \le M < t_3 < N < t_4 < T, \\ 0, otherwise; \end{cases}$$

$$\Xi_3 = \begin{cases} [M;t_4], \ t_2 \le M < t_3 < T, \ t_3 \le M < t_4 < T, \\ [N;t_4], t_2 \le M < N < t_3 < t_4 < T, \ t_3 \le M < N < t_4 < T, \ t_2 \le M < t_3 < N < t_4 < T, \\ 0, \ otherwise; \end{cases}$$

K – the ordering cost, P_c – the purchase cost is given by

$$P_c = \frac{p_c}{T}\left(\begin{array}{l} \int_0^M e^{-kt}dt \left(\begin{array}{l} \int_{\Xi_1} Ddt + ((((1-\beta)(1-a_1)+\beta a_2)\lambda - D)(t_2 - t_1) - I_s) + \\ + ((((1-\beta_r)(1-a_{r1})+\beta_r a_{r2})\lambda_r - D)\cdot \Xi_4 - (\Phi - I_s)) + \mathrm{H} \end{array} \right) + \\ + \Psi_0 \int_0^N e^{-kt}dt \left(\int_{\Xi_2} Ddt + \mathrm{H}_1 \right) + \Psi_1 \int_0^{t_4} e^{-kt}dt \left(\int_{\Xi_3} Ddt + \mathrm{H}_2 \right) \end{array} \right),$$

$$\Xi_4 = \begin{cases} (M - t_2), t_2 \le M < t_3 < T, t_2 \le M < N < t_3 < T, t_2 \le M < t_3 < N < t_4 < T, \\ \qquad t_2 \le M < t_3 < t_4 < N < T, t_2 \le M < t_3 < t_4 < T < N, \\ (t_3 - t_2), t_3 \le M < t_4 < T, t_4 \le M < T, M > T, \ t_3 \le M < N < t_4 < T, t_4 \le M < N < T, N > M > T, \\ \qquad t_3 \le M < t_4 < N < T, \ t_3 \le M < t_4 < T < N, t_4 \le M < T < N; \end{cases}$$

$$\Phi = \begin{cases} I_m, \ t_3 \le M < t_4 < T, \ t_4 \le M < T, M > T, \ t_3 \le M < N < t_4 < T, \ t_4 \le M < N < T, \ N > M > T, \\ \qquad t_3 \le M < t_4 < N < T, t_3 \le M < t_4 < T < N, \ t_4 \le M < T < N, \\ I(M), otherwise; \end{cases}$$

$$\mathrm{H} = \begin{cases} (I_m - D(t_4 - t_3)), \ t_4 \le M < T, \ M > T, \ t_4 \le M < N < T, \ \ N > M > T, \ t_4 \le M < T < N, \\ (I_m - I(M) - D(M - t_3)), \ t_3 \le M < t_4 < T, t_3 \le M < N < t_4 < T, t_3 \le M < t_4 < N < T, t_3 \le M < t_4 < T, \\ 0, \ otherwise; \end{cases}$$

$$\mathrm{H}_1 = \begin{cases} (((1-\beta_r)(1-a_{r1})+\beta_r a_{r2})\lambda_r - D)(N - M) - (I(N) - I(M)), t_2 \le M < N < t_3 < T, \\ (I(M) - I(N) - D(N - M)), \ t_3 \le M < N < t_4 < T, \\ (((1-\beta_r)(1-a_{r1})+\beta_r a_{r2})\lambda_r - D)(t_3 - M) - (I_m - I(M)) + (I_m - I(N) - D(N - t_3)), \ t_2 \le M < t_3 < N < t_4 < T, \\ (((1-\beta_r)(1-a_{r1})+\beta_r a_{r2})\lambda_r - D)(t_3 - M) - (I_m - I(M)) + (I_m - D(t_4 - t_3)), \ t_2 \le M < t_3 < t_4 < N < T, \\ \qquad t_2 \le M < t_3 < t_4 < T < N, \\ (I(M) - D(t_4 - M)), t_3 \le M < t_4 < N < T, \ \ t_3 \le M < t_4 < T < N, \\ 0, \ otherwise; \end{cases}$$

$$
\mathrm{H}_2 = \begin{cases} \left(\left(\left(1-\beta_r\right)\left(1-a_{r1}\right)+\beta_r a_{r2}\right)\lambda_r - D\right)\left(t_3 - M\right) - \left(I_m - I(M)\right) + \left(I_m - D\left(t_4 - t_3\right)\right),\ t_2 \le M < t_3 < T, \\ \left(I(M) - D\left(t_4 - M\right)\right),\ t_3 \le M < t_4 < T, \\ \left(\left(\left(1-\beta_r\right)\left(1-a_{r1}\right)+\beta_r a_{r2}\right)\lambda_r - D\right)\left(t_3 - N\right) - \left(I_m - I(N)\right) + \left(I_m - D\left(t_4 - t_3\right)\right),\ t_2 \le M < N < t_3 < T, \\ \left(I(N) - D\left(t_4 - N\right)\right),\ t_3 \le M < N < t_4 < T,\ t_2 \le M < t_3 < N < t_4 < T, \\ 0,\ otherwise; \end{cases}
$$

C_{si} – the (inspection) screening cost is given by

$$
C_{si} = \frac{c_{sc}}{T}\left(\lambda t_2 + \lambda_r\left(t_3 - t_2\right)\right)\int_0^{t_3} e^{-kt}dt,
$$

C_{rg} – the cost for return the rejection items to supplier is given by

$$
C_{rg} = \frac{c_r}{T}\left(\left(\left(1-\beta\right)a_1 + \beta\left(1-a_2\right)\right)\left(\lambda t_1\int_0^{t_1} e^{-kt}dt + \lambda\left(t_2 - t_1\right)\int_0^{t_2} e^{-kt}dt\right)\right),
$$

C_h – the inventory holding cost per cycle is given by

$$
C_h = \frac{1}{T}\left(h_s\left(\int_{t_1}^{t_4} I(t)dt\int_0^{t_3} e^{-kt}dt\right) + h_r\left(\begin{array}{l} \frac{1}{2}\left(\left(1-\beta\right)a_1 + \beta\left(1-a_2\right)\right)\lambda t_1^2\int_0^{t_1} e^{-kt}dt + \\ +\frac{1}{2}\left(\left(1-\beta\right)a_1 + \beta\left(1-a_2\right)\right)\lambda\left(t_2 - t_1\right)t_2\int_0^{t_2} e^{-kt}dt + \\ +\frac{1}{2}\left(\left(1-\beta_r\right)a_{r1} + \beta_r\left(1-a_{r2}\right)\right)\lambda_r\left(t_3 - t_2\right)\left(T - t_2\right)\int_0^{T} e^{-kt}dt + \\ \frac{1}{2}\left(\beta a_2\lambda t_2 + \beta_r a_{r2}\lambda_r\left(t_3 - t_2\right)\right)T\int_0^{T} e^{-kt}dt \end{array}\right)\right),
$$

C_d – the deterioration cost is given by

$$
C_d = \frac{1}{T}\left(c_{dg} + \frac{c_{out}\left(1-\gamma\right)}{\gamma}\right)\left(\begin{array}{l} \left(\left(\left(\left(1-\beta\right)\left(1-a_1\right)+\beta a_2\right)\lambda - D\right)\left(t_2 - t_1\right) - I_s\right) + \\ +\left(\left(\left(\left(1-\beta_r\right)\left(1-a_{r1}\right)+\beta_r a_{r2}\right)\lambda_r - D\right)\left(t_3 - t_2\right) - \left(I_m - I_s\right)\right) + \left(I_m - D\left(t_4 - t_3\right)\right) \end{array}\right)\int_0^{t_4} e^{-kt}dt,
$$

C_s – the shortage cost per cycle due to backlog is given by

$$
C_s = \frac{c_{sh}}{T}\left(\int_0^{t_1} -I(t)e^{-kt}dt + \int_{t_4}^{T} -I(t)dt\int_0^{T} e^{-kt}dt\right),
$$

C_{un} – the opportunity cost due to lost sale per cycle is given by

$$
C_{un} = \frac{c_u D}{T}\left(\int_{t_4}^{T}\left(1 - \frac{1}{1+\delta\left(t - t_4\right)}\right)dt\int_0^{T} e^{-kt}dt\right),
$$

C_{adi} – the cost of accepting a defective items

$$
C_{adi} = \frac{c_{ad}}{T}\left(\left(1-\beta\right)a_1\lambda t_2\int_0^{t_3} e^{-kt}dt\right),
$$

C_{rnd} – the cost of rejection a non-defective items

$$C_{rnd} = \frac{c_{nd}}{T}\left(\left(\beta a_2 \lambda t_2 + \beta_r a_{r2} \lambda_r (t_3 - t_2)\right) \int_0^T e^{-kt} dt \right),$$

IP – interest paid is given by

$$IP = \frac{1}{T}\left(p_c \left(\Phi_1 \int_0^{t_4} e^{-kt} dt \int_{\Xi_3} I(t) dt + \Psi_0 \cdot i_p \int_0^N e^{-kt} dt \left(\mathrm{H}_3 \int_{\Xi_2} I(t) dt \right) \right) \right),$$

$$\Phi_1 = \begin{cases} i_p, \ t_2 \le M < t_3 < T, \ t_3 \le M < t_4 < T, \\ i_w, \ t_2 \le M < N < t_3 < t_4 < T, \ t_3 \le M < N < t_4 < T, \ t_2 \le M < t_3 < N < t_4 < T, \\ 0, \ otherwise; \end{cases}$$

$$\mathrm{H}_3 = \begin{cases} (1 + N - t_4), \ t_2 \le M < t_3 < t_4 < N < T, \ t_3 \le M < t_4 < N < T, \ t_2 \le M < t_3 < t_4 < T < N, \ t_3 \le M < t_4 < T < N, \\ 0, \ otherwise; \end{cases}$$

IE – interest earned is given by

$$IE = \frac{1}{T}\left(p_v \cdot i_e \left(\int_0^M e^{-kt} dt \left(\int_{\Xi_1} Dtdt + \Theta_1 \int_{\Xi_1} Ddt \right) + \Psi_0 \int_0^N e^{-kt} dt \left(\int_{\Xi_2} Dtdt + \Theta_2 \int_{\Xi_2} Ddt \right) \right) \right),$$

$$\Theta_1 = \begin{cases} (M - t_4), \ t_4 \le M < T, \ M > T, \ t_4 \le M < N < T, \ N > M > T, \ t_4 \le M < T < N, \\ 0, \ otherwise; \end{cases}$$

$$\Theta_2 = \begin{cases} (N - t_4), \ t_2 \le M < t_3 < t_4 < N < T, \ t_3 \le M < t_4 < N < T, \ t_2 \le M < t_3 < t_4 < T < N, \\ \qquad t_3 \le M < t_4 < T < N, \\ 0, \ otherwise; \end{cases}$$

p_v – selling price per unit of good quality items, p_b – selling price per unit for defective items, p_c – purchasing cost per unit, c_{sc} – screening (inspection) cost per unit, c_r – the cost for return the rejection item to supplier per monetary per unit, h_s – holding cost of serviceable items per monetary per unit per unit time, h_r – holding cost of imperfect quality items per monetary per unit per unit time, c_{dg} – deterioration cost per monetary per unit, c_{out} – penalty cost of selling deteriorated items to customer per monetary per unit, c_{sh} – unit shortage cost per monetary per unit per unit time, c_u – cost of lost sale per monetary per unit, c_{ad} – cost of accepting a defective item, c_{nd} – cost of rejection a non-defective items.

Hence our crisp problem is

$$\max\ ROS_i(I_b, I_m, I_s) = \begin{cases} ROS_1(I_b, I_m, I_s), & t_2 \le M < t_3 < T, \\ ROS_2(I_b, I_m, I_s), & t_3 \le M < t_4 < T, \\ ROS_3(I_b, I_m, I_s), & t_4 \le M < T, \\ ROS_4(I_b, I_m, I_s), & M > T, \\ ROS_5(I_b, I_m, I_s), & t_2 \le M < N < t_3 < T, \\ ROS_6(I_b, I_m, I_s), & t_3 \le M < N < t_4 < T, \\ ROS_7(I_b, I_m, I_s), & t_4 \le M < N < T, \\ ROS_8(I_b, I_m, I_s), & N > M > T, \\ ROS_9(I_b, I_m, I_s), & t_2 \le M < t_3 < N < t_4 < T, \\ ROS_{10}(I_b, I_m, I_s), & t_2 \le M < t_3 < t_4 < N < T, \\ ROS_{11}(I_b, I_m, I_s), & t_3 \le M < t_4 < N < T, \\ ROS_{12}(I_b, I_m, I_s), & t_2 \le M < t_3 < t_4 < T < N, \\ ROS_{13}(I_b, I_m, I_s), & t_3 \le M < t_4 < T < N, \\ ROS_{14}(I_b, I_m, I_s), & t_4 \le M < T < N. \end{cases} \tag{30}$$

subject to the constraints

$$E_{\min} \le P_C \le E_{\max} ,$$

$$b \cdot c \cdot d \cdot \lambda \cdot (t_1 + t_2) \le W ,$$

$$C_h < P_C ,$$

$$P\left(a_2 \le 1 - \frac{u+1}{\beta \cdot \lambda (t_1 + t_2)} \right) \le p_s ,$$

where $ROS_i(I_b, I_m, I_s)$, $i = 1,2,...,14$ is given by Eq. (25).

Fuzzy expected value inventory model. In this paper, we have considered the demand rate, selling prices, interest paid rate and interest earned rate as fuzzy variables to tackle the reality in more effective way. When the parameters $\tilde{D}$, $\tilde{p}_v$ $\tilde{p}_b$, $\tilde{i}_p$, $\tilde{i}_w$, $\tilde{i}_e$ (as per assumption) treated as fuzzy variables, the above inventory expressions become fuzzy and thereby the return on sales becomes fuzzy variables on the credibility space $(X, P(X), \mathrm{Cr})$. Let $\tilde{D}$, $\tilde{p}_v$, $\tilde{p}_b$, $\tilde{i}_p$, $\tilde{i}_w$ and $\tilde{i}_e$ be defined by trapezoidal and triangular fuzzy numbers such that $\tilde{D} = [D_1, D_2, D_3, D_4]$, $\tilde{p}_v = [p_{v_1}, p_{v_2}, p_{v_3}, p_{v_4}]$, $\tilde{p}_b = [p_{b_1}, p_{b_2}, p_{b_3}, p_{b_4}]$, $\tilde{i}_p = [i_{p_1}, i_{p_2}, i_{p_3}]$, $\tilde{i}_w = [i_{w_1}, i_{w_2}, i_{w_3}]$ and $\tilde{i}_e = [i_{e_1}, i_{e_2}, i_{e_3}]$, where $(D_1 < D_2 < D_3 < D_4)$, $(p_{v_1} < p_{v_2} < p_{v_3} < p_{v_4})$, $(p_{b_1} < p_{b_2} < p_{b_3} < p_{b_4})$, $(i_{p_1} < i_{p_2} < i_{p_3})$, $(i_{w_1} < i_{w_2} < i_{w_3})$ and $(i_{e_1} < i_{e_2} < i_{e_3})$ based on the subjective judgments. If the decision maker wants to determine optimal pricing and inventory police such that fuzzy expected value of the return on sale is maximal, a fuzzy EVM can be constructed as follows,

$$\max \quad E[ROS_i(I_b,I_m,I_s)] = \begin{cases} E[ROS_1(I_b,I_m,I_s)], & t_2 \le M < t_3 < T, \\ E[ROS_2(I_b,I_m,I_s)], & t_3 \le M < t_4 < T, \\ E[ROS_3(I_b,I_m,I_s)], & t_4 \le M < T, \\ E[ROS_4(I_b,I_m,I_s)], & M > T, \\ E[ROS_5(I_b,I_m,I_s)], & t_2 \le M < N < t_3 < T, \\ E[ROS_6(I_b,I_m,I_s)], & t_3 \le M < N < t_4 < T, \\ E[ROS_7(I_b,I_m,I_s)], & t_4 \le M < N < T, \\ E[ROS_8(I_b,I_m,I_s)], & N > M > T, \\ E[ROS_9(I_b,I_m,I_s)], & t_2 \le M < t_3 < N < t_4 < T, \\ E[ROS_{10}(I_b,I_m,I_s)], & t_2 \le M < t_3 < t_4 < N < T, \\ E[ROS_{11}(I_b,I_m,I_s)], & t_3 \le M < t_4 < N < T, \\ E[ROS_{12}(I_b,I_m,I_s)], & t_2 \le M < t_3 < t_4 < T < N, \\ E[ROS_{13}(I_b,I_m,I_s)], & t_3 \le M < t_4 < T < N, \\ E[ROS_{14}(I_b,I_m,I_s)], & t_4 \le M < T < N, \end{cases} \tag{31}$$

subject to the constraints

$$E_{\min} \le E\left[\tilde{P}_C\right] \le E_{\max},$$

$$b \cdot c \cdot d \cdot \lambda \cdot \left(E\left[\tilde{t}_1\right] + E\left[\tilde{t}_2\right]\right) \le W,$$

$$E\left[\tilde{C}_h\right] < E\left[\tilde{P}_C\right],$$

$$P\left(a_2 \le 1 - \frac{u+1}{\beta \cdot \lambda\left(E\left[\tilde{t}_1\right] + E\left[\tilde{t}_2\right]\right)}\right) \le p_s,$$

where

$$E[ROS_i(I_b,I_m,I_s)] = E\left[\frac{\left(\tilde{R}_s\right)_i - K - \left(\tilde{P}_c\right)_i - \tilde{C}_{si} - \tilde{C}_{rg} - \tilde{C}_h - \tilde{C}_d - \tilde{C}_s - \tilde{C}_{un} - \tilde{C}_{adi} - \tilde{C}_{rnd} - I\tilde{P}_i + I\tilde{E}_i}{\left(\tilde{R}_s\right)_i}\right],$$

$i = 1,2,\dots,14$.

Next sections carried out the simulation technique to estimate the fuzzy parameters and solution methodology for fuzzy expected value model along with theoretical results to identify global optimal solution for (I_b, I_m, I_s).

Fuzzy simulation technique

In above-mention inventory system, frequently, the decision maker wants to control the fuzzy revenue on sales in each replenishment cycle such that the critical value of the fuzzy revenue on sales is maximal. Similarly, in model (31) it needs to find the appropriate vector (I_b, I_m, I_s) such that satisfies the constraints and reaches it maximal value. For the fixed value of (I_b, I_m, I_s) a fuzzy simulation technique [Taleizadeh, 2013] is employed to estimate the fuzzy parameters such as demand, selling prices, interest paid rate and interest earned rate.

Step 1. Set $E = 0$ and initialized G and O.

Step 2. We randomly generate sequences $\left(D^g, p_v^g, p_b^g, i_p^g, i_w^g, i_e^g\right)$ from the α-level sets of fuzzy variables $\tilde{D}$, $\tilde{p}_v$, $\tilde{p}_b$, $\tilde{i}_p$, $\tilde{i}_w$, $\tilde{i}_e$, $g = 1,2,\dots,G$, where α is a sufficiently small positive number.

Step 3. Calculate $ROS_i\left(l_b, l_m, l_s, D^g, p_v^g, p_b^g, i_p^g, i_w^g, i_e^g\right)$ for $g = 1,2,...,G,\ \ i = 1,2,...,14$.

Step 4. Set $a_i = ROS_i\left(l_b, l_m, l_s, D^1, p_v^1, p_b^1, i_p^1, i_w^1, i_e^1\right) \wedge ... \wedge ROS_i\left(l_b, l_m, l_s, D^g, p_v^g, p_b^g, i_p^g, i_w^g, i_e^g\right)$,

$$b_i = ROS_i\left(l_b, l_m, l_s, D^1, p_v^1, p_b^1, i_p^1, i_w^1, i_e^1\right) \vee ... \vee ROS_i\left(l_b, l_m, l_s, D^g, p_v^g, p_b^g, i_p^g, i_w^g, i_e^g\right).$$

Step 5. Randomly generate s_i from $[a_i, b_i]$.

Step 6. If $s_i \geq 0$, then $E_i \leftarrow E_i + Cr\left\{ROS_i\left(l_b, l_m, l_s, \tilde{D}, \tilde{p}_v, \tilde{p}_b, \tilde{i}_p, \tilde{i}_w, \tilde{i}_e\right) \geq s_i\right\}$,

where

$$Cr\left\{ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) \geq s_i\right\} = \frac{1}{2}\left(\begin{array}{l} \underset{j=1,2,...,O}{Max}\left\{\mu_{ij} \middle| ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) \geq s_i\right\} + 1 - \\ -\ \underset{j=1,2,...,O}{Max}\left\{\mu_{ij} \middle| ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) < s_i\right\} \end{array}\right)$$

and $\mu_i\left(D^g, p_v^g, p_b^g, i_p^g, i_e^g\right) = \mu_i\left(D^g\right) \wedge \mu_i\left(p_v^g\right) \wedge \mu_i\left(p_b^g\right) \wedge \mu_i\left(i_p^g\right) \wedge \mu_i\left(i_w^g\right) \wedge \mu_i\left(i_e^g\right)$.

Otherwise, $E_i \leftarrow E_i - Cr\left\{ROS_i\left(l_b, l_m, l_s, \tilde{D}, \tilde{p}_v, \tilde{p}_b, \tilde{i}_p, \tilde{i}_w, \tilde{i}_e\right) \leq s_i\right\}$,

where

$$Cr\left\{ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) \leq s_i\right\} = \frac{1}{2}\left(\begin{array}{l} \underset{j=1,2,...,O}{Max}\left\{\mu_{ij} \middle| ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) \leq s_i\right\} + 1 - \\ -\ \underset{j=1,2,...,O}{Max}\left\{\mu_{ij} \middle| ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) > s_i\right\} \end{array}\right).$$

Step 7. Repeat 5 and 6 for O times.

Step 8. Calculate

$$E\left[ROS_i\left(l_b, l_m, l_s, \tilde{D}, \tilde{p}_v, \tilde{p}_b, \tilde{i}_p, \tilde{i}_w, \tilde{i}_e\right)\right] = \int_0^\infty Cr\left\{ROS_i\left(l_b, l_m, l_s, \tilde{D}, \tilde{p}_v, \tilde{p}_b, \tilde{i}_p, \tilde{i}_w, \tilde{i}_e\right) \geq s_i\right\} ds - \\ - \int_{-\infty}^0 Cr\left\{ROS_i\left(l_b, l_m, l_s, \tilde{D}, \tilde{p}_v, \tilde{p}_b, \tilde{i}_p, \tilde{i}_w, \tilde{i}_e\right) \leq s_i\right\} ds$$

as $E\left[ROS_i\left(l_b, l_m, l_s, \tilde{D}, \tilde{p}_v, \tilde{p}_b, \tilde{i}_p, \tilde{i}_w, \tilde{i}_e\right)\right] = a_i \vee 0 + b_i \wedge 0 + E_i \dfrac{b_i - a_i}{O}$.

So far, we have constructed EOQ in fuzzy sense, and known that the EVM can be solved with generic approaches. However, the model needs to be solved with the heuristics algorithm owing the complexity of problem. There are several heuristic algorithms inspired from evolution of nature, such as Genetic Algorithms [Goldberg, 1989], Evolutionary Strategies [Rechenberg, 1994] and Ant Colony Algorithm [Dorigo, 1996]. Here in order to solve the EVM, we choose Genetic Algorithm as the foundation to design an algorithm which integrates fuzzy simulation and Genetic Algorithm, where the fuzzy simulation is employed to estimate the maximal revenue on sales, and Genetic Algorithm is used to find the optimal solution.

Genetic algorithms based on the fuzzy simulation

Genetic algorithm is a member of the wide category of evolutionary algorithms, which was originally proposed by Goldberg [Goldberg, 1989]. It is a heuristic optimization technology based on such natural evolution mechanisms as crossover, mutation and natural selection. Till now, GA has been successfully applied to a wide range of applications. The comprehensive survey of GA and its application to solve inventory control problems can be found in [Taleizadeh, 2013].

In what follows the main characteristics of the genetic algorithm employed in this research are described. In this paper, the chromosomes are the strings of the unfilled order backlog I_b, inventory level of serviceable items I_s and maximum inventory level of serviceable items I_m. Therefore, a binary vector as a chromosome to represent real values of the variables I_b, I_s and I_m. The length of the vector depends on the required precision. Moreover, infeasible chromosomes, the ones that do not satisfy the constraints of the model in (31) are not considered.

The crossover operator creates two new chromosomes from a pair of selected chromosomes of the parents' generation. There are a number of special crossover operations for binary-coded data: one-point crossover, two-point crossover, n-point crossover, uniform crossover. In this research, a single point crossover with different probabilities P_c of 0.5, 0.6, 0.7 is utilized.

The mutation operator changes the value of a random selected element of the chromosome. To do this, a random number RN between $(0,1)$ is generated for each gene. If RN is less than a predetermined mutation probability P_m, than the mutation occurs in the gene. Otherwise, it does not.

In a maximization problem, the more adequate the solution, the greater the objective function (fitness value) will be. Therefore, the fittest chromosomes will take part in offspring generation with a large probability. The fuzzy simulation is used to evaluate the objective function of this research.

The selection operator ensures the selection of well-performing chromosomes for the reproduction process. The most popular techniques to duplicates of good individuals are tournament and elitism. The practical design witnessed advantage of exactly elitism as at him optimum vectors-decisions are not lost. From all types of selection only for elitism it is proved [Harti, 1990] in a theory that the iterative search process of optimum decision meets. Also there are other selection methods such as panmictic, inbreeding, outbreeding and proportional selection.

Stopping condition for the model optimization can be defined in a few different ways:

- By entering the total number of the generations;
- Achievement of necessary value of fitness-function;
- Selective population consists of identical elements;
- By introducing a positive number ε such that the optimization is stopped, if the condition $f_{\max} - f_{\min} < \varepsilon$ is fulfilled, $f_{\max}$ and $f_{\min}$ being maximum and minimum objective function value, respectively.

Before starting the optimization algorithm, it is necessary select the value ε in the stopping condition (or the number of generation), the population size L, and using the penalty function method converted constrained problem to an unconstrained.

In short, the steps involved in the GA algorithm used in this research are

1. Set the parameters P_c, P_m and randomly create the initial population of size L (the individuals should satisfy the constraints).

2. Calculate the fitness value for each chromosome.

3. Select an individual for mating pool by tournament selection method using elitism.

4. Apply a crossover operator to each selected pair of chromosome with probability P_c.

5. Apply a mutation operator to a randomly selected chromosome with probability P_m.

6. Replace the current population by the resulting new feasible population.
7. Calculate the objective function.
8. If the stopping conditions are met, stop. Otherwise, go to step 4.

Conclusion

In this paper, a stochastic replenishment inventory model was development. Nonlinear programming model for deterioration items with two level of trade credit in one replenishment cycle, time value of money, inspection errors, planned backorders, and sales returns have been proposed. We have considered the EOQ inventory problem by characterizing the demand rate, the selling price for good quality items, the selling price for defective items, interest paid rate and interest earned rate the as fuzzy variable and constructed the fuzzy expected value model. Applying the fuzzy simulation technique, we have estimated the expected value of fuzzy cost parameters and interest rates. Whereas the fuzzy expected value model is hard to solve with analytic methods, and we have designed a genetic algorithm based on the fuzzy simulation to solve it.

Some avenues for future works follow:

- the holding cost or other parameters of the problem may take uncertain forms (stochastic or rough) as well;
- some other deterioration function rather than exponential may be considered for the deterioration rate;
- some other meta-heuristic algorithms such as particle swarm may be employed to solve the problem;
- Evolutionary Technologies of Directed Optimization [Snytyuk, 2004] can be considered as an effective technique to solve the problem.

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SHAPING THE CITATION-PAPER RANK DISTRIBUTIONS: BEYOND HIRSCH'S MODEL

Vladimir Atanassov, Ekaterina Detcheva

***Abstract**: It is well known that the h-index has been originally introduced by analyzing a simple deterministic model of individual's scientific activity. The basic features of Hirsch's model are listed as follows: i) constant publication rate in time; ii) constant number of citations gained by each paper per unit time. One of its predictions is the formation of (approximately) linear negative slope citation-paper rank distribution. However, the actual citation-paper rank distributions obtained by analyzing real life scientometric data are, apart from some rare exceptions, convex and far from linear.*

In this paper we consider two possible reasons that might explain the deviation from linear negative-slope (Hirsch) citation-paper rank distribution. The first one is an increasing in time publication rate, reflecting the fact of growing productivity in the course of a scientific career, as well as the increase of publication rates on global scale. The second one is an increasing in time number of citations gained by individual papers per unit time, associated, e.g. with improving publication quality or growing popularity. Both factors lead to a shaping of convex citation-paper rank distribution from the linear one, in a way that generalizes Hirsch's results.

***Keywords**: citation-paper rank distributions, Hirsch's model, linear negative-slope distribution, convex distributions, time dependence, empirical relationships, scientometric data analysis*

***ACM Classification Keywords**: H. Information Systems, H.2. Database Management, H.2.8. Database applications, subject: Scientific databases; I. Computing methodologies, I.6 Simulation and Modeling, I.6.4. Model Validation and Analysis*

Introduction

Since the beginning of scientometric studies citation dynamics has been established as one of their most fascinating parts. This might be due to the fact that it gives more answers to questions 'why?' rather than 'who?' the latter being of main interest for the application-oriented citation analysis. Both areas complement each other by exchanging data and theoretical models. Citation dynamics itself has miscellaneous aspects, *e.g.* the psycho-socio-economical issues of the publication-citation process ([Asknes, 2005; Bar-Ilan, 2008; Markov et al, 2013]), including dynamics of visibility and quality, as well as citation motivation and tactics. The main efforts, however, have been concentrated on studying the citation distributions and the (stochastic) processes behind them. A bouquet of references, not necessarily going on the same line, follows further on: [de Solla Price, 1976], [Bookstein, 1990 a, b], [Sen, 1996], [van Raan, 2001 a, b], [Egghe, 2005], [Burrell 2007 a, b]. Studies on a global scale (time spans sometimes exceeding a century, worldwide range, various scientific fields, huge amount of citation data) have been performed ([Redner, 2005], [Wallace et al, 2008]) to find out what kind of distribution function fits best a given data set, to explain deviations, to refine details as distribution tails *etc*. The variety of distributions starts with the one parameter beta function resulting from a Cumulative Advantage (or Success Breeds Success, Preferential Attachment) process ([de Solla Price, 1976]). The latter reproduces in a limiting case Bradford and Lotka laws, as well as Pareto and Zipf distributions (comments on all this could be found in

[Bookstein, 1990 a, b] and [Watts and Gilbert., 2011]). It is worth to note the two-step (first to publish and second, to get cited) competition process [van Raan, 2001 a, b], where a paper-citation distribution represented by a modified Bessel function (K_0) has been obtained rather than the expected power-law one. A rather comprehensive model [Burrell 2007 a, b] assumes Poisson publication process of rate equal to the publication rate and gamma-distributed citation rate for an individual paper, which eventually leads to beta-distributed citations to individual publications. Further on, [Wallace et al, 2008] prefer the stretched exponential function to fit their data. The problems of Zipf's law to explain empirical citation distributions motivated [Gupta et al, 2005] and [Peterson et al, 2010] to suggest a 'truncated' power-law distribution, specified by cumulative advantage processes or random choice (exponential decay), for large and small citation counts, respectively. Discrete generalized beta distribution has been implemented by [Petersen et al, 2011] to analyze rank citation data of 300 distinguished physicists. In a recent study on modeling citation dynamics ([Eom and Fortunato, 2011]) it has been found that citation distributions are best described by a shifted power-law, the citation dynamics is determined by citation bursts appearing few years after publication and the microscopic mechanism, responsible for the evolution of the citation networks is the preferential attachment with time-decaying initial attractiveness (see also [Redner, 2005]). Preferential attachment together with aging (loss of novelty in the course of time) and fitness (overall community response) are the basic mechanisms responsible for the long-term citation behavior of individual papers for [Wang et al, 2013]. It has been also shown [Golosovsky and Solomon, 2012] that in addition to the preferential attachment, papers exceeding a certain threshold of overall citation count (50-70, meets about 10 percent of all papers) continue to be cited, due to an epidemic-like self-excited cascade process.

One may conclude from this somewhat hectic development that citation dynamics will remain on the top of scientometric research (probably) for a long time. All these theoretical and empirical studies have additional motivation in practical applications, like improvement of scientific assessment – from establishing the milestones of excellence in science up to introducing more and more scientometric indices that better represent citation paper rank distributions.

The scope of this paper is studying some possible reasons for deviation of citation-paper rank distributions from the linear negative slope one, obtained in [Hirsch, 2005] by assuming time-independent publication and citation rates. Hirsch's model, sometimes commented as elementary, base and averaged one [Liu and Rousseau, 2008], inspired us to treat the problem of publication-citation processes from pure dynamical point of view, rather than entering into much detail by following the modern trends in statistical physics. For the purpose of our analysis we consider (as usual in similar studies) continuous real valued distributions of real argument, without further discussion on integer-real and discrete-continuous aspects of citation-paper rank distributions ([Bookstein, 1990b], [Atanassov and Detcheva, 2013]).

The paper is organized as follows. In the first section we briefly consider Hirsch's model and its assumptions and consequences. A set of equations that govern the publication-citation process for time-dependent sources (publication rate and citation rate of each paper – not to be mistaken with the 'items being produced by sources' approach in [Egghe, 2005]) is derived in the second section and the conditions for existence of rank convex, linear and concave distributions are obtained. The third section contains analysis of the basic equations for the case of stationary time-dependent papers' citation rate. Next three sections illustrate the theory with examples of citation-paper rank distributions (obtained in explicit form or numerically) for several combinations of more or less realistic time-dependent sources.

Hirsch's model outline

In his well known paper [Hirsch, 2005] where the h-index has been introduced, Hirsch considered a simple deterministic publication-citation process, briefly described as follows. An author publishes p papers per unit time (*e.g.* per year) and each published paper gains c citations per unit time (year). Assuming constant p and c we arrive at the following equations for the paper P published at time τ ($P+1$ represents the total number of author's papers at this time) and for the citations C gained by this paper within a time interval $t-\tau \geq 0$:

$$P = p\tau \,,\; C = c(t-\tau)\,. \tag{1}$$

At time moment t this model produces a *linear negative slope* citation-paper rank distribution (or *Hirsch distribution*)

$$C = C_{\max} - sP\,, \tag{2}$$

where the slope $s = c/p = C_{\max}/P_{\max}$, $C_{\max} = ct$, $P_{\max} = pt$. Since s is time-independent, the distribution does not change its shape in the course of the publication-citation process.

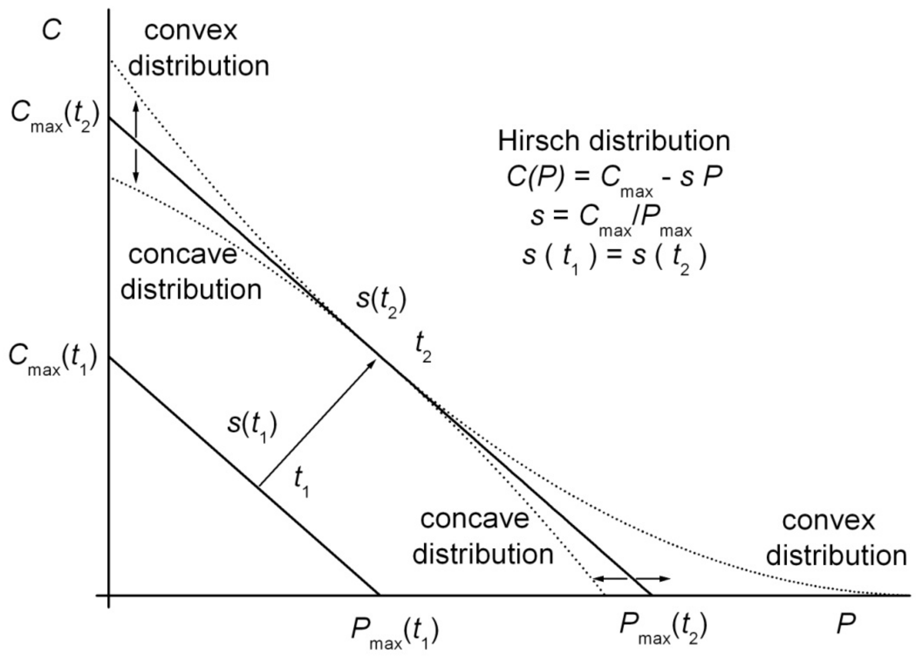

Figure 1. Time evolution of a linear negative slope (Hirsch) citation-paper rank distribution with corrections for surplus or deficit of citations C and/or papers P, respectively

Two questions appear in connection with the distribution (2): i) why the real life citation-paper rank distributions are usually convex and seldom linear or concave; ii) what happens if the publication and citation rates p and c change in time and cannot be considered as constants. Some intuition leads us to a conclusion (illustrated by Fig. 1), that *increasing* in time publication rate p would result in a surplus of papers in the distribution tail containing least cited papers; in the same way, *increasing* in time citation rate c would result in a surplus of citations in the core of most cited papers. Both factors could lead to formation of a *convex-shaped* citation-paper rank distribution. Analogously, *decreasing* in time publication rate p and/or citation rate c could produce a *concave* citation-paper rank distribution. A combination of increasing and decreasing in time p and c could even form distributions with at least one *inflection* point.

One may find some support of these considerations in the results of a case study on time evolution of scientific activity (see [Atanassov, 2012] for more detail). On Fig. 2a we observe a sharp increase in publication rate (productivity). Fig. 2b demonstrates, apart from some statistical variability, a slight positive trend in *average* citation rate per paper (impact). The corresponding extremely convex citation-paper rank histogram is shown on Fig. 2c. Similar increase in *average* citation rate per paper could be deduced also from the results of a global study on citation distributions [Wallace et al, 2008]. This makes clear the need of generalizing Hirsch's model for time-dependent publication and citation rates. This could be done by deriving equations that govern the publication-citation process and eventually form the citation-paper rank distribution.

Equations governing the publication-citation process

Let us assume that a scientist publishes $p > 0$ papers per unit time. At a moment τ the paper count $P(\tau)$ is

$$P(\tau) = \int_0^\tau dt' p(t') . \tag{3}$$

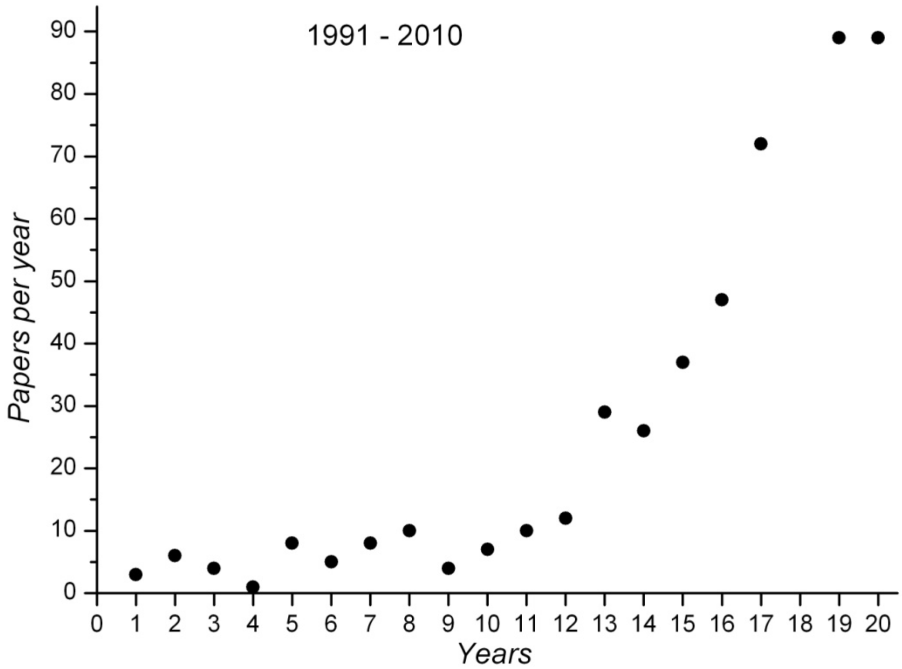

Figure 2a. Example for time evolution of *average* publication rate

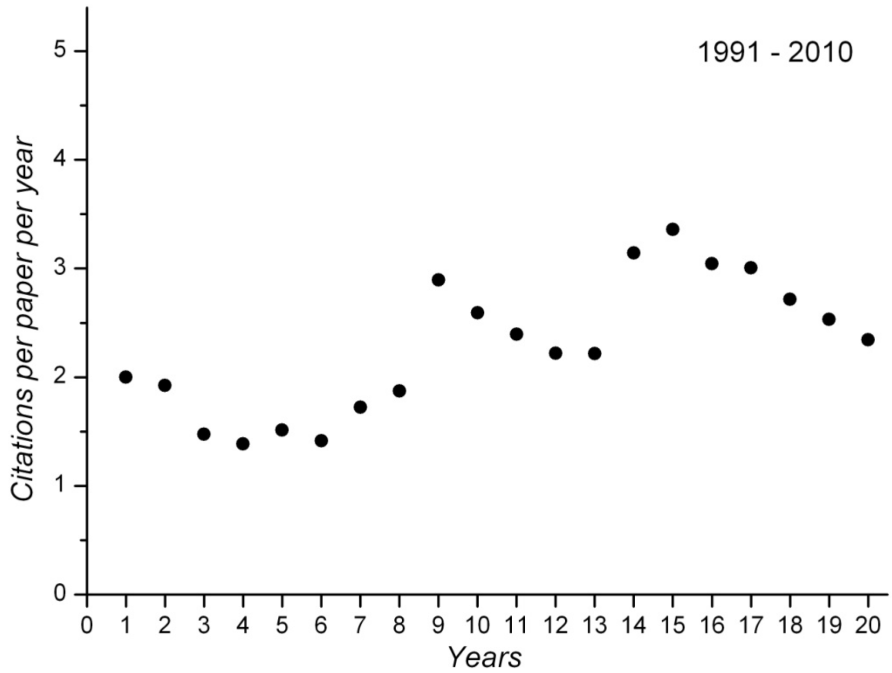

Figure 2b. Example for time evolution of *average* citation rate per paper

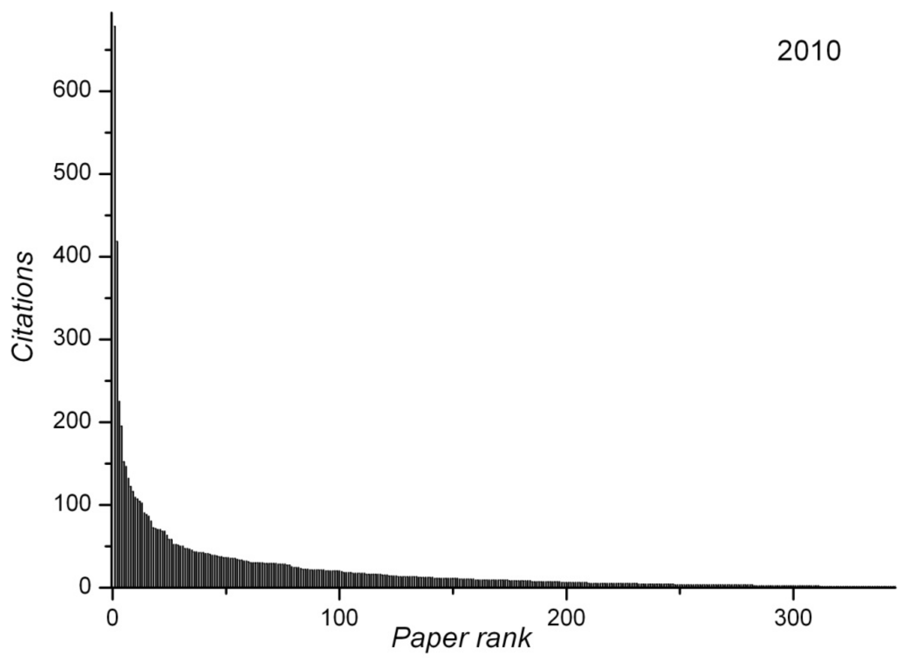

Figure 2c. Citation-paper rank histogram for the case study presented on Figure 2a, b

Equation (3) establishes one-to-one correspondence between a paper P and the time of its publication τ and

$$\frac{d\tau(P)}{dP} = \frac{1}{p\left[\tau(P)\right]}.$$

Each paper $P(\tau)$ starts earning $c(\tau,t) \geq 0$ citations per unit time, provided that $t \geq \tau$ (causality). Hence, for the number of citations gained by a paper $P(\tau)$ within a time interval $[\tau,t]$ we have

$$\underset{\sim}{C}(\tau,t) = \int_{\tau}^{t} dt'\, c(\tau,t'),\ \tau \leq t. \tag{4}$$

The set of equations (3)-(4) govern the time evolution of the publication-citation events. The citation-paper distribution at time t is

$$C(P,t) \equiv \underset{\sim}{C}\left[\tau(P),t\right] = \int_{\tau(P)}^{t} dt'\, c\left[\tau(P),t'\right]. \tag{5}$$

It is worth to emphasize the difference between τ (time of publication of paper P) and t (time of 'measurement', when all citations for all published papers have been taken into account to construct the histogram/distribution).

It should be noted that Eq. (5) alone does not necessarily specify a citation-paper *rank* distribution. The latter is obtained (when processing real life scientometric data) by rearranging the citation-paper sequence in descending order of the number of citations gained, *i.e.* most cited paper placed first. The model considered here cannot account for such procedure; it can only rely on the specifics of the publication-citation process that automatically produce rank distribution. Therefore, an additional condition must be imposed on the publication and citation sources p and c. By definition, a rank distribution $C(P,t)$ satisfies $\partial C(P,t)/\partial P \leq 0$. We have

$$\frac{\partial C(P,t)}{\partial P} = \frac{1}{p\left[\tau(P)\right]}\left\{\int_{\tau(P)}^{t} dt' \left[\frac{\partial c(\tau,t')}{\partial \tau}\right]_{[\tau(P),t']} - c\left[\tau(P),\tau(P)\right]\right\}. \tag{6}$$

Hence the condition for existence of rank distribution is the inequality

$$\int_{\tau(P)}^{t} dt' \left[\frac{\partial c(\tau,t')}{\partial \tau}\right]_{[\tau(P),t']} \leq c\left[\tau(P),\tau(P)\right] \tag{7}$$

that must be fulfilled throughout the publication-citation process.

If Eq. (7) takes place it makes sense to define a maximum citation count C_{max}, representing the number of citations to the first, most cited paper:

$$C_{max}(t) \equiv C(0,t) = \int_{0}^{t} dt'\, c(0,t'). \tag{8}$$

Further on, bearing in mind that τ cannot exceed t one may introduce a maximum paper number (equal to the maximum number of papers) of *nonzero* citation count:

$$P_{max}(t) \equiv P(t) = \int_{0}^{t} dt' p(t'), \tag{9}$$

where, obviously, $C\left[P_{max}(t),t\right] = 0$. One should bear in mind that, although t, C_{max} and P_{max} may tend to infinity, in order to have a *distribution* the total number of citations

$$C_{total}(t) \equiv \int_0^{P_{\max}} C(P,t)dP = \int_0^t d\tau p(\tau) \underset{\sim}{C}(\tau,t) = \int_0^t d\tau p(\tau) \int_\tau^t dt' c(\tau,t') \qquad (10)$$

must remain finite.

The citation-paper rank distribution shape is determined by the its second derivative

$$\frac{\partial^2 C(P,t)}{\partial P^2} = \left\{ \frac{1}{p^2(\tau)} \left[\int_\tau^t \frac{\partial^2 c(\tau,t')}{\partial \tau^2} dt' - 2\frac{\partial c(\tau,t'')}{\partial \tau} - \frac{\partial c(\tau,t'')}{\partial t''} - \frac{dp(\tau)}{d\tau} \frac{\partial C(P,t)}{\partial P} \right] \right\}_{\tau=\tau(P),t''=\tau(P)} . \qquad (11)$$

Thus we have a convex, (negative-slope) linear or a concave distribution depending on whether $\partial^2 C(P,t) / \partial P^2$ is positive, zero or negative, respectively.

Analysis for a stationary citation rate

We start our analysis of the basic set of equations derived in the previous chapter by considering publication-citation processes with a *stationary* paper citation rate

$$c(\tau,t) \equiv c_s(t-\tau) \geq 0 , \ \tau \leq t . \qquad (12)$$

The meaning of this approach is that all published papers have the same citation behavior, however shifted in time due to the different moments of publication τ. The dependence of individual paper's citation rate on its age $t-\tau$ could be quite complicated (see, *e.g.* [Wang et al, 2013]). The empiric results ([Asknes, 2003]) demonstrate that a typical behavior (of rather universal character) consists of an approximately linear growth, reaching a maximum followed by an exponential-like decay. Another type of citation behavior can be observed for the so called *sleeping beauties* ([van Raan, 2004]) or *revived classics* [Redner, 2005]. Obviously, Eq. (12) cannot hold simultaneously for both types. On the other side, the *average* citation rate per paper on a global scale could be considered as slowly varying (increasing) within the hundred years time interval under investigation [Wallace et al, 2008]. Stationarity approach might require reducing this time interval in order to make such variations small enough.

Now the number of citations gained by a paper $P(\tau)$ within a time interval $[\tau,t]$ is

$$\underset{\sim}{C}(\tau,t) \equiv \underset{\sim}{C}_s(t-\tau) = \int_0^{t-\tau} dt' c_s(t') , \ \tau \leq t . \qquad (13)$$

The citation-paper distribution at time t is

$$C(P,t) \equiv \underset{\sim}{C}_s \left[t - \tau(P) \right] = \int_0^{t-\tau(P)} dt' c_s(t') . \qquad (14)$$

Further on, from Eq. (6) we obtain

$$\frac{\partial C(P,t)}{\partial P} = -\frac{c_s\left[t-\tau(P)\right]}{p\left[\tau(P)\right]} \leq 0 . \qquad (15)$$

Hence *stationary citation rates always produce rank distributions*.

The expression for $\partial^2 C(P,t) / \partial P^2$ (Eq. (11)) is reduced to

$$\frac{\partial^2 C(P,t)}{\partial P^2} = \frac{1}{p^3\left[\tau(P)\right]} \left\{ c_s\left[t-\tau(P)\right] \left[\frac{dp(\tau)}{d\tau}\right]_{\tau=\tau(P)} + p\left[\tau(P)\right] \frac{dc_s\left[t-\tau(P)\right]}{dt} \right\} . \qquad (16)$$

It follows from Eq. (16) that:

- At least one of the sources p, c_s must *increase/decrease* in time in order to have a *convex/concave* citation-paper rank distribution;
- If *both* p and c_S are *increasing/decreasing* functions of time, the sources produce *convex/concave* distributions;
- For a constant citation rate c_s the distribution is *convex, linear or concave*, depending on whether the publication 'acceleration' $dp / d\tau$ is *positive, zero or negative*;
- For a constant publication rate p the distribution is *convex, linear or concave*, depending on whether the citation 'acceleration' of each paper dc_S / dt is *positive, zero or negative*;
- A citation-paper rank distribution is (negative slope) linear one if and only if the ratio c_s / p does not depend on the time of publication τ (or, which is equivalent; on the paper count P);
- A citation-paper rank distribution has no inflexion points if and only if the ratio c_s / p is a monotonously increasing/decreasing function of the time of publication τ (or, equivalently; on the paper count P);

These conclusions are drawn for quite general case of sources: strictly positive publication rate p, a nonnegative stationary papers' citation rate c and finite total number of citations (Eq. (10)).

Examples I (constant paper citation rate)

The simplest example of a publication-citation process is the one considered by Hirsch [Hirsch, 2005]. It is characterized with a constant publication rate $p > 0$ and a constant citation rate for each paper $c \geq 0$. The citation-paper rank distribution (Eq. (2)) as well as the relations for $P_{\max}$ and $C_{\max}$ are easily obtained from Eq. (5) (or Eq. (14)) and Eqs. (8), (9), respectively.

Further on in this section we consider the effect of *time-dependent* publication rate $p(\tau)$ on the shape of citation-paper rank distributions by assuming constant papers' citation rate $c \equiv c_0 \geq 0$. This represents the simplest case of a stationary citation rate. Therefore, all distributions discussed later on are *rank* distributions. They are convex, linear or concave for increasing, constant or decreasing in time publication rates $p(\tau)$, respectively.

- **linear dependence** $p(\tau) = p_0 + p_1\tau$,

where $p_0 > 0$ and p_1 are constants and $p_0 > -p_1 t$ must be fulfilled to keep p positive. By solving the integral in Eq. (3) one arrives at

$$P(\tau) = p_0\tau + \frac{1}{2}p_1\tau^2 \,,\; P_{\max}(t) \equiv P(t) = p_0 t + \frac{1}{2}p_1 t^2 \tag{17}$$

and

$$\tau(P) = \left(2P / p_0\right) / \left[1 + \sqrt{1 + \left(2p_1 P / p_0^2\right)}\right], \tag{18}$$

provided that $-2p_1 P_{\max} < p_0^2$. The citation-paper rank distribution

$$C(P,t) = C_{\max}(t) - c_0\tau(P) \,,\; C_{\max}(t) = c_0 t \tag{19}$$

is a convex, linear or concave one for p_1 greater than, equal to or less than zero, respectively (Fig. 3a).

- **power law dependence** $p(\tau) = \alpha\tau^{\beta}$,

where $\alpha > 0$, $\beta > -1$, and $\tau > 0$. Now we have

$$P(\tau) = \alpha\tau^{1+\beta} / (1+\beta),\ P_{\max}(t) \equiv P(t) = \alpha t^{1+\beta} / (1+\beta) \tag{20}$$

and

$$\tau(P) = \left[(1+\beta)P / \alpha\right]^{\frac{1}{1+\beta}}. \tag{21}$$

The citation-paper rank distribution

$$C(P,t) = C_{\max}(t)\left[1 - (P / P_{\max})^{\frac{1}{1+\beta}}\right],\ C_{\max}(t) = c_0 t, \tag{22}$$

appears in [Atanassov and Detcheva, 2013] as the *three-parameter positive exponent power-law distribution*. It is convex, linear or concave for $\beta > 0$, $\beta = 0$ or $-1 < \beta < 0$, respectively.

- **exponential dependence** $p(\tau) = \alpha \exp(\beta\tau)$,

where $\alpha > 0$. It is easy to obtain

$$P(\tau) = (\alpha / \beta)\left[\exp(\beta\tau) - 1\right],\ P_{\max}(t) \equiv P(t) = (\alpha / \beta)\left[\exp(\beta t) - 1\right] \tag{23}$$

and

$$\tau(P) = (1/\beta)\ln\left[1 + (\beta / \alpha)P\right]. \tag{24}$$

The citation-paper rank distribution

$$C(P,t) = C_{\max}(t) - (c_0 / \beta)\ln\left[1 + (\beta / \alpha)P\right],\ C_{\max}(t) = c_0 t, \tag{25}$$

is convex, linear or concave for positive, zero or negative values of β, respectively.

Examples II (constant publication rate)

Further on in this section we consider the effect of *time-dependent* (however, *stationary*) paper citation rate $c(\tau,t) = c_s(t-\tau) \geq 0$ on the shape of citation-paper rank distributions by assuming constant publication rate $p \equiv p_0 > 0$. It follows then that $P(\tau) = p_0\tau$, $P_{\max}(t) \equiv P(t) = p_0 t$. As in the previous chapter, all distributions discussed later on are *rank* ones. They are convex, linear or concave for increasing, constant or decreasing in time (t) paper citation rates $c_s(t-\tau)$, respectively.

- **linear dependence** $c_s(t-\tau) = c_0 + c_1(t-\tau)$,

where $c_0 \geq 0$ and c_1 are constants and $c_0 \geq -c_1 t$ must be fulfilled to keep c non-negative. Eq. (14) yields

$$\frac{C(P,t)}{C_{\max}(t)} = 1 - \left(1 + \frac{1}{2}\rho\right)\frac{P}{P_{\max}} + \frac{1}{2}\rho\left(\frac{P}{P_{\max}}\right)^2, \tag{26}$$

where

$$C_{\max}(t) = c_0 t + \frac{1}{2}c_1 t^2,\ \rho = c_1 t^2 / C_{\max}(t).$$

This citation-paper rank distribution appears in [Atanassov and Detcheva, 2013] as the *three-parameter polynomial distribution*. It is convex, linear or concave for $c_1 > 0$, $c_1 = 0$, or $-c_0 / t \le c_1 < 0$, respectively (Fig.3b).

- **power law dependence** $c_s(t-\tau) = \alpha(t-\tau)^{\beta}$,

where $\alpha > 0$, $\beta > -1$, and $t - \tau > 0$. Now we have

$$C(P,t)/C_{\max}(t) = \left[1 - \left(P/P_{\max}\right)\right]^{1+\beta}, \; C_{\max}(t) = \alpha t^{1+\beta}/(1+\beta). \tag{27}$$

As it could be expected, this citation-paper rank distribution is convex, linear or concave for $\beta > 0$, $\beta = 0$ or $-1 < \beta < 0$, respectively.

- **exponential dependence** $c_s(t-\tau) = \alpha \exp\left[\beta\left(t-\tau\right)\right]$,

where $\alpha > 0$. We obtain

$$\frac{C(P,t)}{C_{\max}(t)} = \frac{\exp\left[-(\beta/p_0)P\right] - \exp\left[-(\beta/p_0)P_{\max}\right]}{1 - \exp\left[-(\beta/p_0)P_{\max}\right]}, \; C_{\max}(t) = \frac{\alpha}{\beta}\left[\exp\left(\beta t\right) - 1\right]. \tag{28}$$

The rank distribution is convex, linear or concave for positive, zero or negative β, respectively.

Examples III (time dependent publication and citation sources)

Let us consider situations, where both publication rate $p(\tau)$ and (stationary) paper citation rate $c_s(t-\tau)$ are *time-dependent*.

- **linear dependence** $p(\tau) = p_0 + p_1\tau$, $c_s(t-\tau) = c_0 + c_1(t-\tau)$,

where $p_0 > 0$, p_1, $c_0 \ge 0$ and c_1 are constants and $p_0 > -p_1 t$, $c_0 \ge -c_1 t$ must hold to keep p positive and c non-negative. The citation-paper rank distribution is given by

$$C(P,t) = c_0\left[t - \tau(P)\right] + \frac{1}{2}c_1\left[t - \tau(P)\right]^2, \tag{29}$$

with $\tau(P)$ specified by Eq. (18). The shape of this distribution depends on the sign of

$$\partial^2 C(P,t)/\partial P^2 = (p_1 c_0 + p_0 c_1 + p_1 c_1 t)/p^3\left[\tau\left(P\right)\right]. \tag{30}$$

Although $\partial^2 C(P,t)/\partial P^2$ varies with P, it obviously cannot change its sign. Therefore, *rank distributions shaped by sources that depend linearly on time, have no inflection points*.

In order to illustrate the general idea (Fig. 1) and the analytics, we have solved numerically Eqs. (3) - (5) for sources linearly varying in time. The results are plotted on Fig. 3, as follows: (a) – for a constant papers' citation rate; (b) – for a constant publication rate; (c) – for varying publication and citation rates. In particular, we have demonstrated (Fig. 3c, dot curve) that *a linear citation-paper rank distribution may exist even for non-constant publication and citation rates*. This has been achieved by appropriately choosing p_1 and c_1 (obviously of opposite signs) to anneal the right part of Eq. (30).

- **power-law dependence** $p(\tau) = \alpha_P \tau^{\beta_P}$, $c_s(t-\tau) = \alpha_C (t-\tau)^{\beta_C}$,

where $\alpha_P > 0$, $\alpha_C > 0$, $\beta_P > -1$, $\beta_C > -1$, and $0 < \tau < t$. The citation-paper rank distribution is

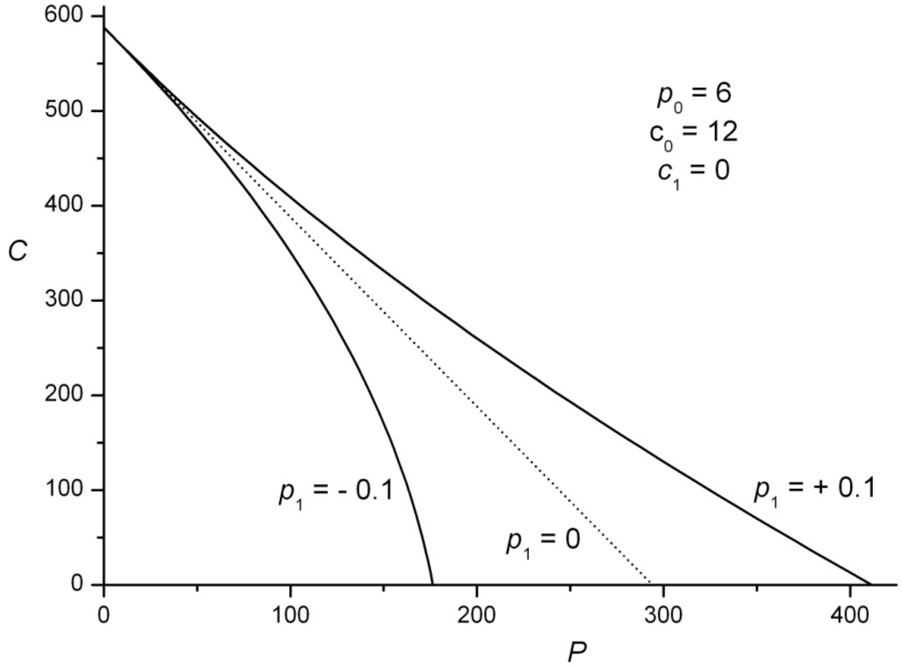

Figure 3a. Citation-paper rank distribution for linearly varying publication rate and constant citation rate

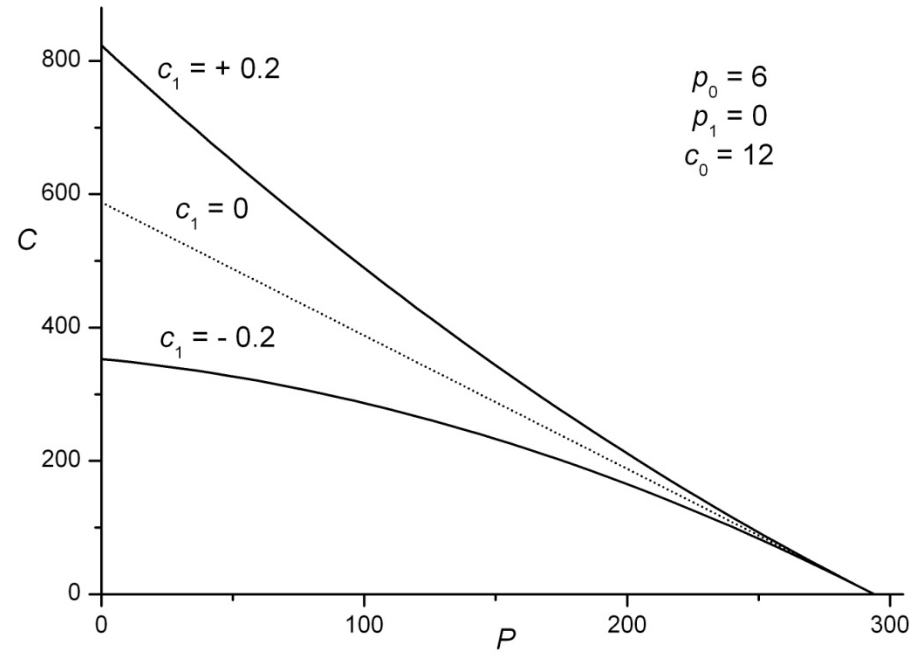

Figure 3b. Citation-paper rank distribution for constant publication rate and linearly varying citation rate

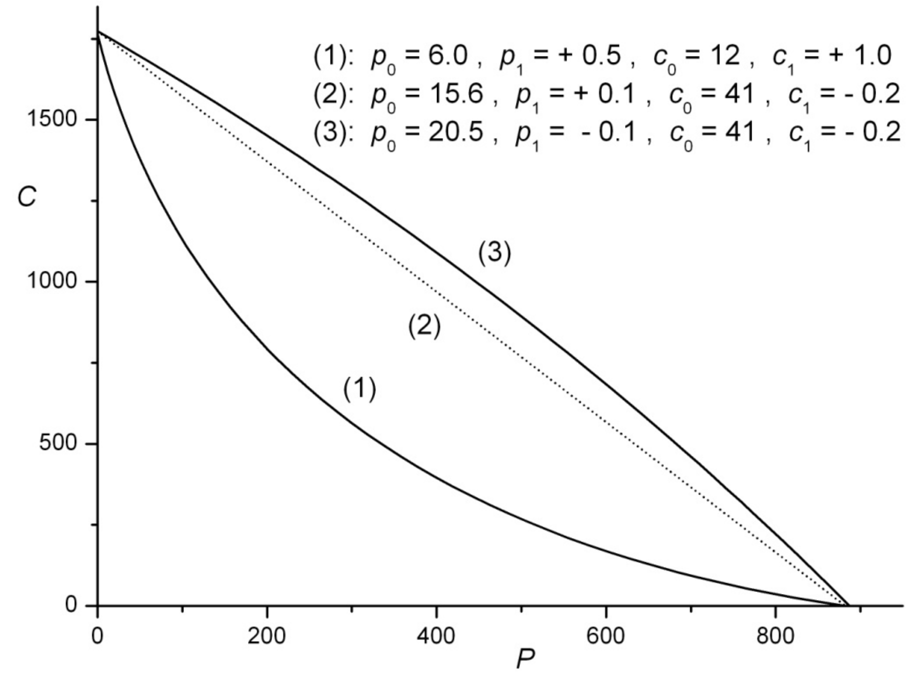

Figure 3c. Citation-paper rank distribution at linearly varying publication rate and citation rate

$$\frac{C(P,t)}{C_{\max}} = \left[1-\left(P / P_{\max}\right)^{\frac{1}{1+\beta_P}}\right]^{1+\beta_C}, \tag{31}$$

and

$$P_{\max} = \alpha_P t^{1+\beta_P} / (1+\beta_P),\ C_{\max} = \alpha_C t^{1+\beta_C} / (1+\beta_C).$$

This distribution has one inflection point

$$P_{\inf} = P_{\max}\left[1-\left(\beta_C / \beta_P\right)\right]^{-1/(1+\beta_P)}, \tag{32}$$

provided that $\beta_C / \beta_P < 0$. More precisely, it is convex-concave for $\beta_P < 0$, $\beta_C > 0$ and concave-convex one for $\beta_P > 0$, $\beta_C < 0$. If both β_P and β_C are *negative/positive*, the distribution is *convex/concave,* respectively.

- **exponential dependence** $p(\tau) = \alpha_P \exp(\beta_P \tau)$, $c_s(t-\tau) = \alpha_C \exp\left[\beta_C\left(t-\tau\right)\right]$,

where $\alpha_P > 0$, $\alpha_C > 0$. The citation-paper rank distribution is

$$\frac{C(P,t)}{C_{\max}} = \frac{\left[1+\left(\beta_P P / \alpha_P\right)\right]^{-(\beta_C/\beta_P)} - \left[1+\left(\beta_P P_{\max} / \alpha_P\right)\right]^{-(\beta_C/\beta_P)}}{1-\left[1+\left(\beta_P P_{\max} / \alpha_P\right)\right]^{-(\beta_C/\beta_P)}} \tag{33}$$

with

$$P_{\max} = \alpha_P\left[\exp\left(\beta_P t\right)-1\right] / \beta_P,\ C_{\max} = \alpha_C\left[\exp\left(\beta_C t\right)-1\right] / \beta_C.$$

In the limit $\beta_P \to 0$ (constant publication rate) Eq. (33) reproduces Eq. (28) (with $\alpha_P = p_0$). The distribution is convex, linear or concave depending on whether the exponents sum $\beta_P + \beta_C$ is positive, zero or negative. In particular, for $\beta_P \equiv \beta_C = \beta$ it follows that

$$C / C_{\max} = \left[1-\left(P / P_{\max}\right)\right] / \left[1+\left(\beta P / \alpha_P\right)\right]. \tag{34}$$

We note that for small citation count ($C / C_{\max} << 1$), where P approaches $P_{\max}$ the citation-paper rank distribution is dropping faster then expected from Zipf's law ([Bookstein, 1990a]). In the limit $P_{\max} \to \infty$ (this implies $\beta_P > 0$, $\beta_C > 0$), Eq. (33) could be written as

$$C / C_{\max} = 1 / \left[1+\left(\beta_P P / \alpha_P\right)\right]^{\frac{\beta_C}{\beta_P}}. \tag{35}$$

Eq. (35) represents a Mandelbrot-Zipf distribution with power exponent equal to the ratio of citation and publication growth rates, respectively. This result may be compared with the one derived in [Egghe, 2005] within the Lotkaian informetrics approach. In particular, if both growth rates coincide, the Lotka exponent equals 2, which corresponds to the Zipf exponent of 1, obtained here.

Summary and conclusions

This paper considers some possible reasons for the deviation of citation-paper rank distribution from the linear negative-slope (Hirsch) one. We have derived equations that govern the publication-citation process for time-dependent sources (publication rate and paper citation rate) and obtained conditions for existence of rank convex, concave and linear distributions. The basic equations are analyzed for a stationary temporal dependence of the paper citation rate. A summary of our main conclusions for this particular case follows further on:

- *Stationary* citation rates always lead to *rank* distributions;
- At least one of the sources must *increase/decrease* in time in order to have a *convex/concave* citation-paper rank distribution; publication rates and paper citation rates that simultaneously *increase/decrease* in time produce *convex/concave* distributions;
- For a constant paper citation rate the distribution is *convex, linear or concave*, depending on whether the publication rate is *increasing, constant or decreasing* function of time; for a constant publication rate the distribution is *convex, linear or concave*, for an *increasing, constant or decreasing* paper citation rate;
- The citation-paper rank distribution is a linear one if and only if the ratio of (stationary) paper citation rate and publication rate does not depend on the time of publication (or equivalently, on the paper count); linear negative-slope citation-paper rank distributions may exist even for time-dependent publication and citation rates.

Thus the widely distributed convex shape of citation-paper rank distributions could be associated with increasing in time publication rate and/or paper citation rate. The factors for such *inflation* in productivity and impact might be growing productivity and popularity in the course of a scientific career as well as increase of publication and/or citation rates on global scale.

The equations that govern the publication-citation process are derived on the base of rather common (universal) principles, as *continuity* and c*ausality*. Therefore, we hope it would be possible to use them (perhaps, with appropriate modification) for treatment of more realistic (but difficult to handle) situations, *e.g.* the non-stationary citation rates, rearranging papers or stochastic sources. Moreover, since our analysis contains no presumptions for the active part of the publication-citation process, its results and conclusions could be extended to cover the scientific activity of arbitrary set of authors – a scientist, a research group *etc*.

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SELF-CITATIONS EFFECT ON SCIENTOMETRIC INDEXES

Vladimir Atanassov, Ekaterina Detcheva

Abstract: *Scientometric studies on self-citations reveal variety of aspects, like attempted fraud approaching a crime, a self-advertising tool, a standard scientific publication practice that saves space and time by reducing repetitions, and so on up to self-citations being an important element of scientific networking. It seems, however, that self-citation analysis will remain a hot topic for a long time mainly due to its effect on assessment of scientific impact. Therefore, an obvious question that arises is, to what extent self-citations could modify the basic scientometric indicators and indexes, used to perform such assessment.*

This paper represents an attempt to quantitatively estimate the effect of self-citations on several widely used scientometric indexes (Hirsch's h, Egghe's g and Zhang's e). This has been achieved by incorporating self-citations into various explicitly given citation-paper rank distributions under more or less realistic assumptions. The latter are deduced making use of well known empirical relationships, based on analyzing a considerable amount of scientometric data. The results obtained contain self-citation corrections for the scientometric indexes, as well as some indications for a 'normal' and 'extraordinary' self-citation behavior.

Keywords*: self-citations, citation-paper rank distributions, scientometric indexes, h-index, g-index, e-index, empirical relationships, scientometric data analysis*

ACM Classification Keywords*: H. Information Systems, H.2. Database Management, H.2.8. Database applications, subject: Scientific databases; I. Computing methodologies, I.6 Simulation and Modeling, I.6.4. Model Validation and Analysis*

Introduction

Studies on self-citations are an important part of the scientific activity oriented citation analysis. Pros and cons have been reported since many years ([Asknes, 2003], [Glänzel et al, 2004], [Glänzel, 2008], [Costas et al, 2010]). In particular, self-citation considered as a part of referencing to some knowledge that has been already established, thus *reducing repetitions*, is an essential paradigm for the whole science and for a huge part of the human activity as well. Self-citations are also considered as a part of knowledge distribution, and a powerful self-advertising tool, in particular for the Knowledge Markets [Markov et al, 2013]. Self-citations appear to be a substantial part of establishing *scientific networks* [Ausloos et al, 2008]. It seems, however, that self-citation analysis will remain a hot topic for a long time mainly due to the effect of self-citations on scientific activity assessment. This reveals mainly the backside of self-citations, starting with the somewhat naïve idea to pull oneself out of the swamp holding one's own hair just like the well known baron did, by inflating the citation count, up to intentionally manipulating the scientometric indexes ([Asknes, 2003], [Markov et al, 2013], [Bartneck and Kokkelmans, 2011]).

One cannot leave aside also the psychological aspects of self-citing. The almost absolute lack of self-citations over a longer period is just as pathological as an always-overwhelming share [[Glänzel et al, 2004]. A small number of self-citations could be associated with quick change of research fields, jumping from one theme to another and perhaps frivolous attitude towards the scientific problems in general. Large number of self-citations accompanied by a relatively small number of external citations could indicate severe communication problems of an introverted scientist, closed in its shell and living outside the scientific community, producing his/her own *parallel* and *self-sufficient* science. However, it could be also due to a habit of numerous coauthors that form a self-citing circle. Whatever the reason could be, it is worth noting that the real problem for a scientist is the *lack* of external citations rather than the *excess* of self-citations.

The aim of this paper is to obtain quantitative estimates for the effect of self-citations on the three most widely used scientometric indicators, namely Hirsch's *h*-index [Hirsch, 2005], Egghe's *g*-index [Egghe, 2006] and Zhang's *e*-index [Zhang, 2009]. There are already several case studies involved in this topic, that provide a good empirical base for the theory, *e.g.* on the self-citation corrections to Hirsch's ([Schreiber, 2007], [Engqvist and Frommen, 2008], [Ferrara and Romero, 2013]) and Egghe's [Schreiber, 2008] indexes. The paper is organized as follows: in the first section we consider a (single author) discrete self-citation model and suggest a notion for 'normal' *vs.* 'abnormal' (throughout this paper we call it 'extraordinary') self-citation behavior. This concept has been used to construct appropriate models for self-citation corrections to the citation-paper rank distributions. The next two sections are devoted to estimating the effect of the (almost) additive and the multiplicative corrections on the three scientometric indexes. The analytics has been illustrated in a separate section with the results of two case studies on self-citations effect on Hirsch's index, where some comments on possible applications have been made, too.

The discrete model for normal and 'extraordinary' self-citation behavior

Let us consider a sequence of papers $\{P_1, P_2, \ldots, P_N\}$ arranged in order of their appearance (the earliest first). For a *single-authored* papers P_2 may contain a reference to P_1, P_3 to P_2 and P_1 *etc*. The number of self-citations a single author produces in N papers, provided he/she cites *all* preceding papers, is

$$N_{sc}(N) = \frac{1}{2}N(N-1). \quad (1)$$

These self-citations are *linearly* distributed to the paper count I:

$$C_{sc}(I) = N - I,\ I = 1, 2, \ldots, N. \quad (2)$$

Eq. (2) specifies an absolute limit for the self-citations of a *single author*. If, however, the author cites (no more than) K of his/her last published papers, we have a *linear* dependence of self-citations number on the total number of papers N:

$$N_{sc}(K) = K\left[N - \frac{1}{2}(K+1)\right], \quad (3)$$

and (almost) *uniformly* distributed self-citations:

$$C_{sc}(K;I) = K \text{ for } 1 \le I \le N-K,\ C_{sc}(K;I) = N - I \text{ for } N-K+1 \le I \le N. \quad (4)$$

These relations allow us to distinguish between two types of self-citation behavior (of a single author) provided that K is much smaller than N. It is quite normal that scientists support their publications by citing *some* of their newest own papers ($N_{sc} \sim N$, $C_{sc}(I) \approx K = const$), while citing *almost all* own previous papers ($N_{sc} \sim N^2$, $C_{sc}(I) \approx N - I$) demonstrates a rather extraordinary self-citation practice (Fig. 1). It is worth noting that each type of self-citation behavior smoothly evolves to the other when varying K from much smaller values than (for 'normal') to values close to (for 'extraordinary') the total number of papers N, and *vice versa*.

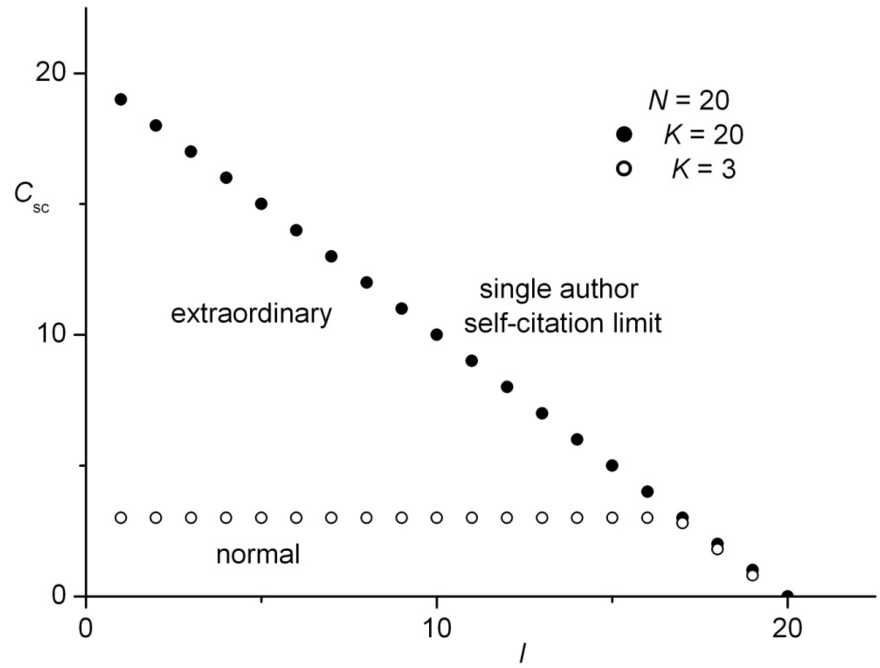

Figure 1. Self-citation distributions for ‘normal’ and ‘extraordinary’ self-citation behavior

In order to illustrate these considerations we present results of two case studies (Fig.2a, b and Fig. 3a, b) based on Thomson-Reuters Web of Science scientometric data for two scientists. Citation-paper rank distributions for the *overall* and the *external* citations (*i.e.* with and *without* self citations) are plotted on Fig. 2a and Fig. 3a, while Fig. 2b and Fig. 3b demonstrate the corresponding self-citation distributions. It should be pointed out that papers are ranked separately for each data set; in particular, the self-citation histograms are obtained by computing number of self-citations for each paper, then papers are rearranged to form a rank distribution.

There are several conclusions that could be drawn from this analysis. The ‘normal’ self-citation behavior (Fig. 2a, b) is characterized with: i) close citation-paper rank distributions and (almost) equal Hirsch indexes for overall and external citations; ii) self-citations data that is approximately uniformly distributed and lies far away from the (single author) self-citation limit. This is in contrast with the ‘extraordinary’ self-citation practice (Fig. 3a, b), where citations-paper rank distributions and Hirsch indexes with and without counting self-citations are drastically different and the self-citation distribution approaches the linear one of Fig.1. An intuitive support for the (close to) uniform distribution of self-citations over papers and their (relatively) small number could be associated with the observation that self-citation lifetime is *usually* much smaller than the one of the external citations ([Glänzel et al, 2004], [Glänzel, 2008]). There are several explanations for this phenomenon: i) paper’s aging gets faster for their

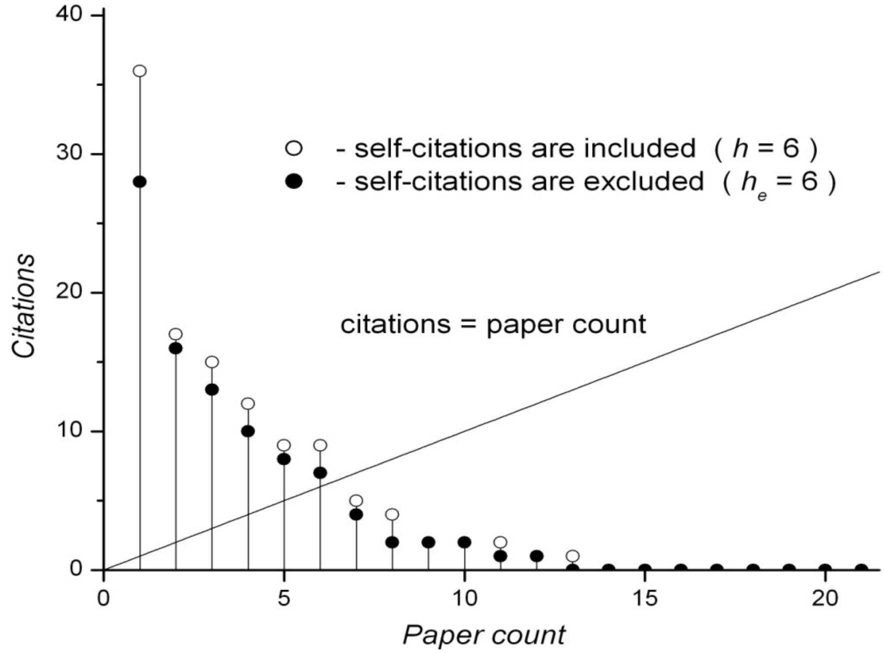

Figure 2a. Citation-paper rank distributions for a case study illustrating 'normal' self-citation behavior

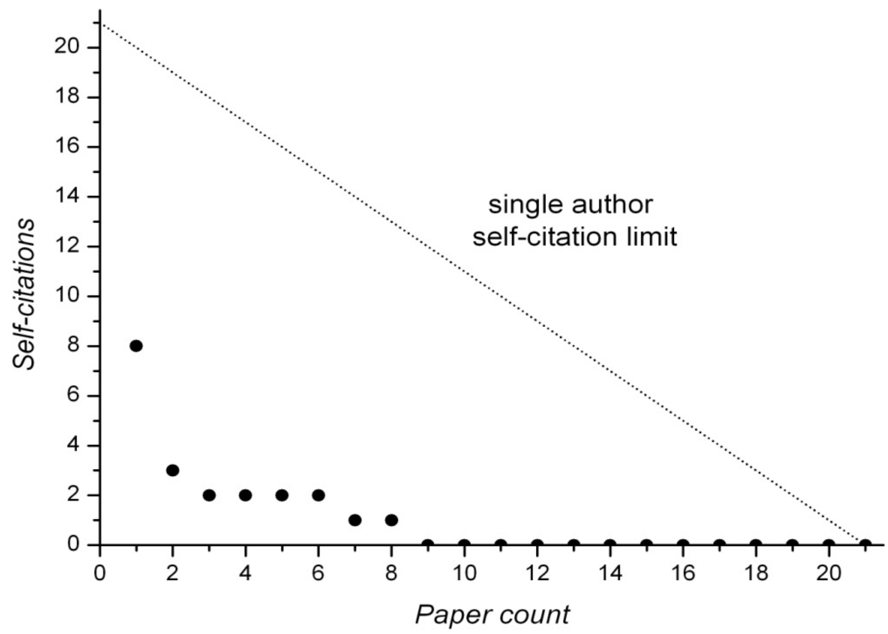

Figure 2b. Self-citation distribution for the case study illustrating 'normal' self-citation behavior

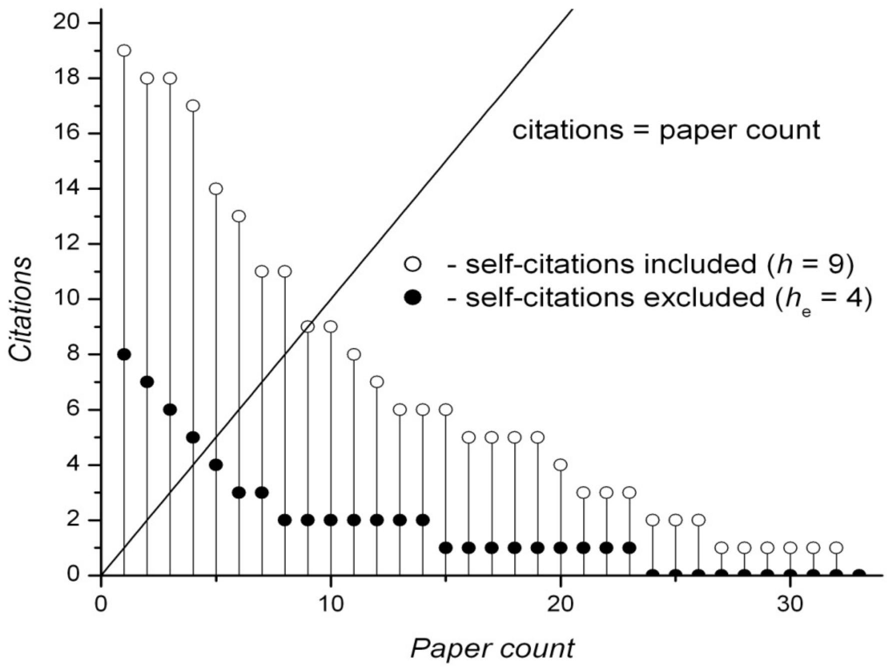

Figure 3a. Citation-paper rank distributions for a case study illustrating 'extraordinary' self-citation behavior

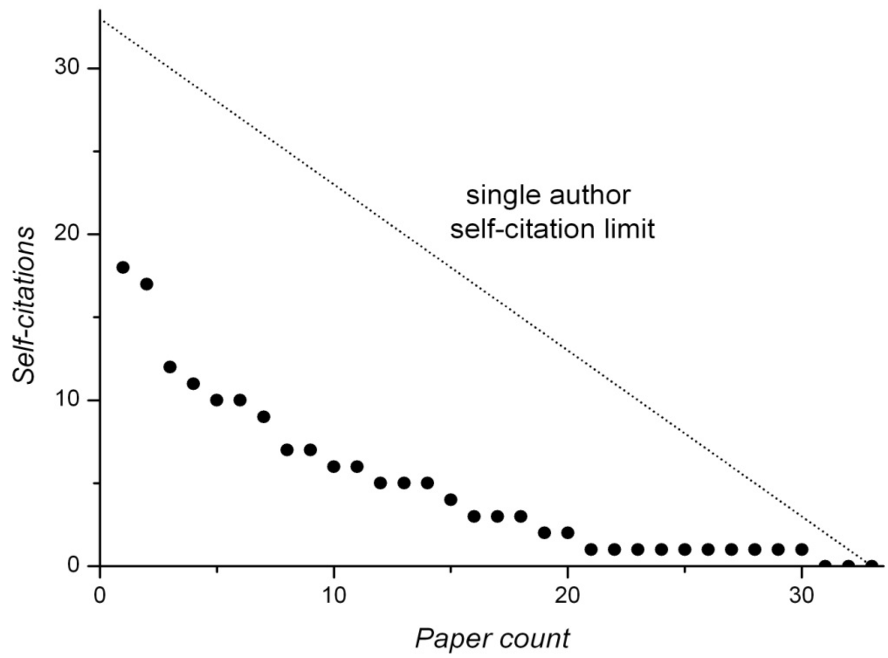

Figure 3b. Self-citation distribution for the case study illustrating 'extraordinary' self-citation behavior

authors than for the other scientists (*e.g.* newcomers in the field); ii) due to the fast and sometimes unexpected developments in science changing research themes and even research fields is rather common for individual scientists and research groups nowadays.

As it can be seen on Figs. 1, 2b and 3b the border between 'normal' and 'extraordinary' self-citation behavior is rather fuzzy. The problem becomes even more complicated, when multiple co-authors are taken into account. Obviously, if a co-authors group acts as one virtual 'author', Eqs. (1) and (2) remain unchanged for each individual member of this group. This situation, however, is far from realistic and the number of self-citations for a given paper could vary from zero to the maximum number of subsequent publications authored by at least one member of the group under consideration. What is known from empirical studies is that multiple authorship boosts the external citations stronger than the self-citations [Asknes, 2003] and the ratio of self-citations to overall citation count for single-authored papers is notably lower than the one for multi-authored papers (no matter how many the co-authors are) [Glänzel and Thijs, 2004b].

Another point that should be addressed is the interdependence between self-citations and external citations. At a first glance, it seems that no clear relation between them exists at all. Glänzel and co-authors have demonstrated, however, that (from statistical point of view) there is nothing arbitrary in this relation and the conditional expectation of self-citation number a paper receives for a given number of external citations depends on the square root of the latter [Glänzel et al, 2004]. It appears that 'the more one cites oneself the more one is cited by the other scholars' [Fowler and Asknes, 2007]. Thus, going a bit ahead, we find some support for a basic assumption used in another part of our analysis, namely considering self-citation distribution as a reduced copy of the overall citation-paper rank one.

Continuous distribution analysis: additive self-citations corrections

Continuous citation-paper rank distributions are considered as an approach to the real life discrete integer-valued ones. Their advantages include the opportunity to *analytically* compute scientometric indexes, keeping at the same time most of the properties and peculiarities of the discrete ones. In what follows we extend a previous model study on citation-paper rank distributions and associated scientometric indexes [Atanassov and Detcheva, 2013] to account for self-citations. Our approach consists in computing scientometric indexes for various *continuous* citation-paper rank distributions with and without prescribed self-citation corrections. The latter are specified bearing in mind the considerations concerning the 'normal' and 'extraordinary' self-citation behavior in the previous section. Further on we distinguish between the *overall* citation count C and the *external* (*i.e.* with self-citations excluded) one C_e distributed to (the same) paper ranks P, in descending order to the citations gained (most cited placed first). The paper ranks P are considered within domains $0 \le P \le P_m$ and $0 \le P \le P_{em} \le P_m$ for the overall and external citation distributions, respectively.

By assuming (almost) *uniformly* distributed self-citations one can write down

$$C_e(P) = C(P) - a \ , \ 0 \le P \le P_{em} < P_m , \qquad (5)$$

The self-citation *additive* correction level a must obey the inequalities $0 < a < C_m$, where $C_m = C(0)$ is the maximal *overall* citation count (Fig. 4). Obviously, the maximal *external* citation count is $C_{em} = C_m - a$, and P_{em} is obtained as the least of the solutions to $C(P) = a$. Note that the support of the external citation-paper rank distribution remains finite even for $P_m \to \infty$. It should be also emphasized that, in general, the external citation-paper rank distribution $C_e(P)$ does not follow the shape and type of $C(P)$.

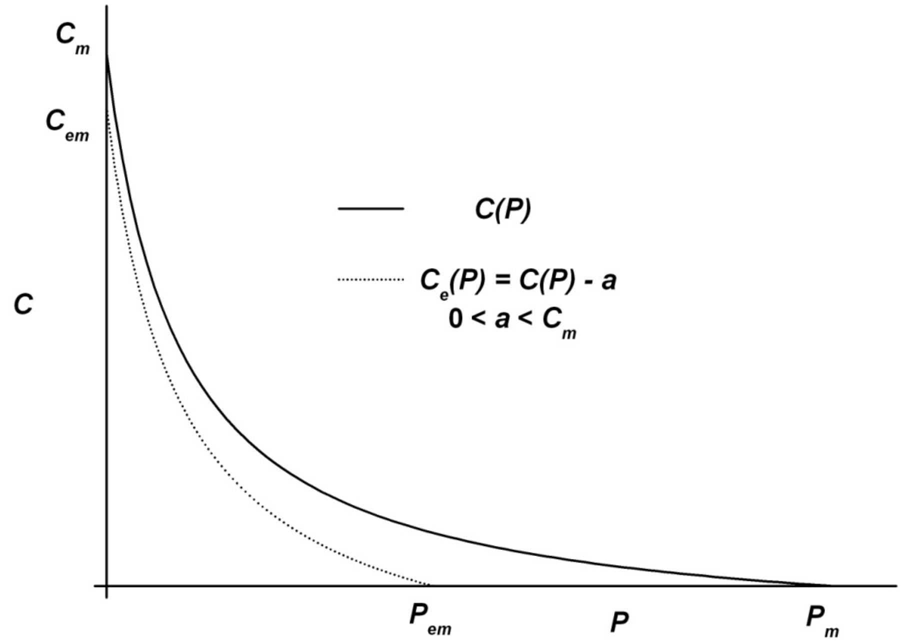

Figure 4. Additive self-citation corrections to citation-paper rank distributions

Let us now consider what happens with Hirsch's index when additive self-citations are taken into account. Denoting *h*-indexes for all citations and external citations with h and h_e, respectively, we have (by definition)

$$C(h) = h \, , \; C_e(h_e) = h_e \, . \tag{6}$$

Further on, by introducing $h_s = h - h_e$ and keeping the first three terms in Taylor series expansion of $C_e(h - h_s)$ one obtains the following quadratic equation for h_s:

$$\frac{1}{2} C''(h) h_s^2 + \left[1 - C'(h)\right] h_s - a = 0 \, , \tag{7}$$

where C' and C'' denote first and second derivatives of C, respectively. The only acceptable solution to Eq. (7) is

$$h_s = 2a \, / \left\{ [1 - C'(h)] + \sqrt{[1 - C'(h)]^2 + 2aC''(h)} \right\} \, , \tag{8}$$

provided that $2aC''(h) \geq -[1 - C'(h)]^2$ (this inequality might fail for extremely *concave* distributions).

By definition, for Zhang's *e*-indexes we have

$$e^2 = \int_0^h C(P) dP - h^2 \; , \; e_e^2 = \int_0^{h_e} C_e(P) dP - h_e^2 \, . \tag{9}$$

Following the same schema one arrives at

$$e_s^2 = (h-h_s)(a-h_s) - \frac{1}{2}C'(h)h_s^2 + \frac{1}{6}C''(h)h_s^3 . \quad (10)$$

We treat Egghe's *g*-indexes in the same way, bearing in mind that $g^2 = \int_0^g C(P)dP$ for $g^2 \le \int_0^{P_m} C(P)dP$, otherwise $g = P_m$. Now one obtains a cubic equation for $g_s = g - g_e$:

$$\frac{1}{6}C''(g)g_s^3 + \left[1 - \frac{1}{2}C'(g)\right]g_s^2 + \left[C(g) - 2g - a\right]g_s + ag = 0 , \quad (11)$$

provided that $g^2 \le \int_0^{P_m} C(P)dP$, $g_e^2 \le \int_0^{P_{em}} C_e(P)dP$. If any of these inequalities is not fulfilled, the corresponding *g*-index must be put equal to the corresponding maximal paper count P_m or P_{em}.

We would like to emphasize that the (approximate) relationships (7-8) and (10-11) are *exact* if all C derivatives of order higher than 2 are zero. Such model citation-paper rank distribution appears in [Atanassov and Detcheva, 2013] as the *three-parameter polynomial distribution*.

Further on we consider the self-citation effect on the three scientometric indexes (Hirsch's *h,* Egghe's *g* and Zhang's *e*) for *additive* self-citation corrections to several model citation-paper rank distributions (uniform, linear and Pareto).

- **Uniform distribution:** $C(P) = C_m$, $C_e(P) = C_m - a$, $0 \le P \le P_m$, $0 \le a \le C_m$.

Now $C(P)$, $C_e(P)$ and the self-citation citation-paper rank distribution have the same form. A simple, but somewhat tedious logics yields the following relationships for the self-citation corrections to Hirsch's *h*-index $h_s = h - h_e$, Egghe's *g*-index $g_s = g - g_e$ and Zhang's *e*-index $e_s^2 = e^2 - e_e^2$:

$$h_s = g_s = 0 ,\ e_s^2 = aP_m \text{ for } P_m \le C_m - a ,$$

$$h_s = g_s = a + P_m - C_m ,\ e_s^2 = P_m(C_m - P_m) \text{ for } C_m - a \le P_m \le C_m , \quad (12)$$

$$h_s = g_s = a ,\ e_s^2 = 0 \text{ for } P_m \ge C_m .$$

- **Linear negative slope (Hirsch) distribution:** $C(P) = C_m - sP$, supported for $0 \le P \le P_m$, where the distribution slope is $s = C_m / P_m$.

The external citation-paper rank distribution is written as $C_e(P) = C_{em} - sP$ for $0 \le P \le P_{em}$, where $C_{em} = C_m - a$ for $0 \le a \le C_m$ and $P_{em} = C_{em} / s$. The following relationships take place:

$$h_s \equiv h - h_e = a / (1+s) ,\ e_s^2 = e^2 - e_e^2 = sa(C_m - \frac{1}{2}a) / (1+s)^2 . \quad (13a)$$

Hence for a linear negative slope citation-paper rank distribution the additive self-citation correction to the Hirsch index is constant, depending on the self-citation level a and the slope s only; in particular, it decreases when the distribution gets steeper. This result confirms the intuitively clear notion that a set of a small number of highly cited papers is less sensitive to self-citations than numerous poorly cited ones.

The Egghe's *g*-index correction $g_s = g - g_e$ is

$$g_s = 2a / (2+s) \text{ for } 0 \le s \le 2 \text{ and } g_s = a / s \text{ for } s > 2 . \quad (13b)$$

Two different representations of g_s appear in connection with the *g*-saturation phenomenon (see [Zhang, 2009], [Atanassov and Detcheva, 2013]). Note that the self-citation correction to Egghe's index does not depend on the index itself and decreases for steeper distributions, too.

- **Pareto distribution:** $C(P) = C_m P^{-\beta}$, supported for $1 \le P < \infty$ at $\beta > 1$.

For an additive self-citation correction the external citation-paper rank distribution is no more a Pareto one:

$$C_e(P) = C_m P^{-\beta} - a\,,\ 1 \le P \le P_{em} = (C_m / a)^{\frac{1}{\beta}}\,,\ 0 < a < C_m\,. \tag{14}$$

Since both $C(P)$ and $C_e(P)$ are supported for $P \ge 1$, the conditions for existence of solutions to Hirsch's index definition equations $C(h) = h$, $C_e(h_e) = h_e$ impose the following inequalities for h, h_e, C_m and a:

$$h \ge 1\,,\ h_e \ge 1\,,\ C_m \ge 1\,,\ a \le C_m - 1\,.$$

Now Hirsch's index correction $h_s = h - h_e$ satisfies the equation

$$\left(1 - \frac{h_s}{h}\right)^{-\beta} - \left(1 - \frac{h_s}{h}\right) = \frac{a}{h}\,. \tag{15}$$

An exact solution to this equation at the limit $\beta \to 1 + 0$ is given by:

$$h_s = a \Big/ \left[1 + \frac{1}{2}\left(\frac{a}{h}\right) + \sqrt{1 + \frac{1}{4}\left(\frac{a}{h}\right)^2}\,\right]. \tag{16}$$

Eq. (16) describes a smooth transition of h_s from $\frac{1}{2}a$ (at $a / h << 1$) to $h - 1$ (for $a \to C_m - 1 = h^2 - 1$). This transition, akin to the discrete one described by Eq. (4) in the previous section, corresponds to an evolution from 'normal' self-citation behavior (almost uniform self-citation distribution, small compared to and independent on h, corrections to the Hirsch index) to 'extraordinary' one (almost linear self-citation distribution, external Hirsch's index close to 1, h_s linearly increases with h). In particular, for arbitrary $\beta > 1$, assuming that the additive self-citation level is small compared with the Hirsch index, $a / h << 1$, Eq. (16) yields

$$h_s = \frac{a}{1+\beta}\left[1 - \frac{\beta}{2(1+\beta)}\frac{a}{h} + O\left(\frac{a^2}{h^2}\right)\right]. \tag{17}$$

The (exact) self-citation correction to Zhang's *e*-index $e_s^2 = e^2 - e_e^2$ is

$$e_s^2 = \frac{\beta}{\beta - 1}\left[h(a - 2h_s) - h_s(a - h_s)\right] - a\,, \tag{18}$$

which, for $a / h << 1$, is reduced to

$$e_s^2 = a\left(\frac{\beta}{1+\beta}h - 1\right). \tag{19}$$

The *g*-index correction $g_s = g - g_e$ is estimated for $a / g << 1$ as follows:

$$g_s = \frac{(g-1)}{g}\frac{1 - g^{1-\beta}}{\left[2 - (1+\beta)g^{1-\beta}\right]}a\left[1 + O\left(\frac{a}{g}\right)\right]. \tag{20}$$

Thus, for a large enough g the self-citation correction g_s approaches $\frac{1}{2}a$. Note that $g_s = g - \left(C_m / a\right)^{1/\beta}$ must be used instead of Eq. (20) if g_e^2 exceeds the total number of external citations (i.e. a saturation of g_e occurs).

Continuous distribution analysis: multiplicative self-citations corrections

For modeling of self-citation effect on scientometric indexes in the grey zone between 'normal' and 'extraordinary' self-citation behavior it seems appropriate to introduce *multiplicative* self-citation corrections (Fig. 5):

$$C_e(P) = bC(P),\ 0 \le P \le P_{em} = P_m, \text{ where } 0 < b < 1. \tag{21}$$

Eq. (21) simply states that each paper P has $100b$ percent external citations. Moreover, all three citation-paper

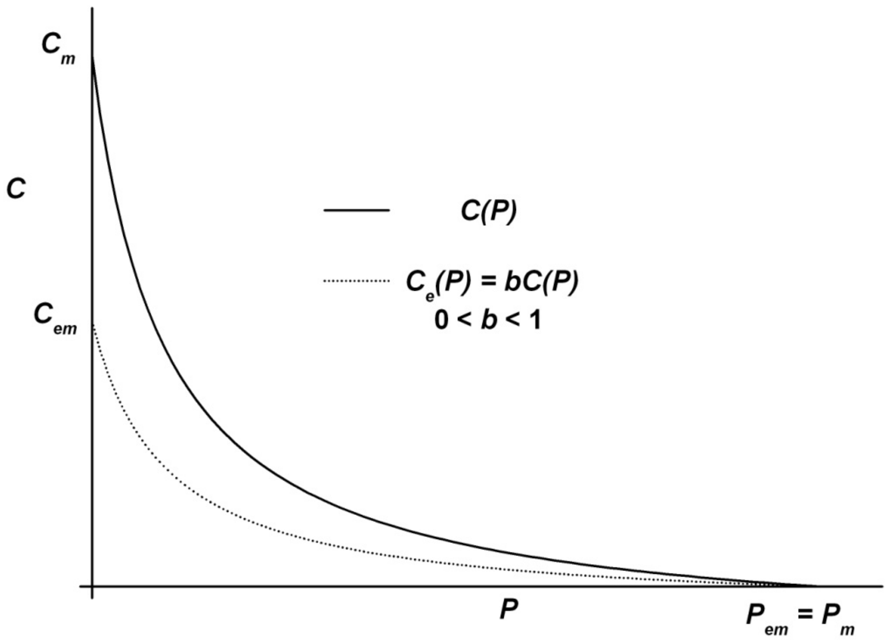

Figure 5. Multiplicative self-citation corrections to citation-paper rank distributions.

rank distributions (overall, external and self-citation ones) have the same form. Following the Taylor expansion procedure, previously used in this paper, we derive a quadratic equation for $h_s = h - h_e$:

$$\frac{1}{2}bC''(h)h_s^2 + \left[1 - bC'(h)\right]h_s - (1-b)h = 0. \tag{22}$$

The only reasonable solution to Eq. (22) is

$$h_s = 2(1-b)h / \left\{[1 - bC'(h)] + \sqrt{[1 - bC'(h)]^2 + 2b(1-b)C''(h)}\right\}. \tag{23}$$

The self-citation correction to Zhang's *e*-index is obtained from

$$e_s^2 = e^2 - e_e^2 = (1-b)\left(e^2 + h^2\right) - (2-b)hh_s + [1 + \frac{1}{2}bC'(h)]h_s^2 + \frac{1}{6}bC''(h)h_s^3 . \qquad (24)$$

The cubic equation for Egghe's *g*-index self-citation correction $g_s = g - g_e$ now looks as follows:

$$\frac{1}{6}bC''(g)g_s^3 + \left[1 - \frac{1}{2}bC'(g)\right]g_s^2 + \left[bC(g) - 2g\right]g_s + (1-b)g^2 = 0 , \qquad (25a)$$

provided that $g^2 \le \int_0^{P_m} C(P)dP$, $g_e^2 \le b\int_0^{P_m} C(P)dP$ (*i.e.* no saturation of *g*-indexes occurs),

$$\frac{1}{6}bC''(P_m)g_s^3 + \left[1 - \frac{1}{2}bC'(P_m)\right]g_s^2 - 2P_m g_s + (1-b)P_m^2 = 0 , \qquad (25b)$$

for $g > \int_0^{P_m} C(P)dP$, $g_e^2 \le b\int_0^{P_m} C(P)dP$ and

$$g_s = 0 \qquad (25c)$$

for $g > \int_0^{P_m} C(P)dP$, $g_e^2 > b\int_0^{P_m} C(P)dP$ (*i.e.* both *g*-indexes are saturated).

Once again we note that the (generally approximate) relationships (22-25) are *exact* if all C derivatives of order higher than 2 are zero, *i.e.* for the *three-parameter polynomial distribution* studied in [Atanassov and Detcheva, 2013].

In order to better illustrate the self-citation effect on scientometric indexes we consider multiplicative self-citation corrections to several model citation-paper rank distributions (uniform, Hirsch, Pareto).

- **Uniform distribution:** $C(P) = C_m$, $C_e = bC_m$, $0 \le P \le P_m$, $0 \le b \le 1$.

The multiplicative self-citation corrections are obtained by simply replacing a in Eq. (12) with $(1-b)C_m$:

$$h_s = g_s = 0 , \; e_s^2 = (1-b)P_m C_m \text{ for } P_m \le bC_m ,$$

$$h_s = g_s = P_m - bC_m , \; e_s^2 = P_m(C_m - P_m) \text{ for } bC_m \le P_m \le C_m , \qquad (26)$$

$$h_s = g_s = (1-b)C_m , \; e_s^2 = 0 \text{ for } P_m \ge C_m .$$

- **Linear negative slope (Hirsch) distribution:** $C(P) = C_m - sP$, $C_e(P) = b(C - sP)$, both supported for $0 \le P \le P_m$, where $s = C_m / P_m$ and $0 \le b \le 1$.

The relations for the Hirsch's and Zhang's indexes' corrections are as follows

$$h_s \equiv h - h_e = \left[(1-b)/(1+bs)\right]h , \; e_s^2 = e^2 - e_e^2 = e^2\left\{1 - b^3\left[(1+s)/(1+bs)\right]^2\right\} , \qquad (27a)$$

and for Egghe's *g*-index correction $g_s = g - g_e$ one obtains

$$g_s = \frac{2(1-b)}{(2+bs)}g \text{ for } 0 \le s < 2 , \; g_s = \frac{2-bs}{2+bs}g \text{ for } 2 \le s \le \frac{2}{b} \text{ and } g_s = 0 \text{ for } s > \frac{2}{b} . \qquad (27b)$$

- **Pareto distribution:** $C(P) = C_m P^{-\beta}$, supported for $1 \le P < \infty$ at $\beta > 1$.

For a multiplicative self-citation correction the external citation-paper rank distribution remains a Pareto one:

$$C_e(P) = bC_m P^{-\beta}\ ,\ 1 \le P \le P_m\ ,\ 0 < b < 1\ . \tag{28}$$

Now we have

$$h_s = h\left(1 - b^{1/(1+\beta)}\right), \tag{29}$$

$$e_s^2 = e^2 - e_e^2 = h^2\left[\left(1-b\right)h^{\beta-1} - \beta\left(1 - b^{2/(1+\beta)}\right)\right] / \left(\beta - 1\right) \tag{30}$$

and (for $0 < 1 - b << 1$):

$$g_s = \left(1-b\right)g \,/\, \left\{2 - \left[\left(\beta-1\right) / \left(g^{\beta-1} - 1\right)\right]\right\}\ . \tag{31}$$

It follows from Eqs. (27), (29) and (31) that the multiplicative self-citation corrections to Hirsch's and Egghe's indexes depend *linearly* on h and g, respectively. For Pareto distributed citations the linear dependence of g_s on g is revealed asymptotically, for a sufficiently large Egghe's index. This situation might appear if a scientist cites more those of his/her papers that gain more external citations. As far as the necessity of such self-citations is rather questionable from a scientific viewpoint, this could be considered as 'extraordinary' self-citation behavior, too.

Examples and applications

In order to illustrate the possible applications of self-citation corrections estimated in the previous two sections we briefly consider the results of two case studies. The first example (Fig. 6) is based on a data set borrowed from M. Schreiber's paper ([Schreiber, 2007], Table 1). The data has been fitted (without much success) to a power-low function. We could, however, clearly distinguish between data points lying close to the line $h_s = h$ (this line corresponds to the self-citation limit on Fig. 1) and other ones that remain constant with increasing h. In this way we could find data suspicious for 'extraordinary' self-citation behavior.

The second example (Fig. 7) represents the self-citation correction $h_s = h - h_e$ to Hirsch's index h for several research groups specialized in the field of high-power gas lasers [Krasteva et al, 2011]. Since co-authorship plays a significant role here, 164 individual scientists have been separated in 9 groups. An approximately linear dependence $h_s(h)$ (hence, an 'extraordinary' citation behavior) could be attributed to groups 2, 4, 7 and 8, while groups 6 and 9 keep the self-citation correction relatively constant and at a low level. An attempt has been also made to fit all data to a power-law function. Although not quite reliable, the result obtained is rather similar to the one presented on Fig. 6. Possible explanation for this coincidence might be that mixing data resulting from several kinds of self-citation behavior causes huge data scatter. In these conditions the fit procedure cannot distinguish between square root and a function varying between linear (at low *h*-values) and a constant (at large *h*-values), as it follows from Eq. (16).

A possible application of the bundle of formulae obtained in the previous two sections is the option to promptly estimate self-citation corrections to scientometric indexes. For this purpose one may assume that the additive self-citation level a is simply the mean self-citations number per paper (i.e. total self-citations number divided by total number of papers). For the multiplicative self-citation approach the self-citation correction factor b represents exactly the ratio of external citation number and the number of all citations. Thus, for $a = 5$ (five self-citations per paper in average) and $\beta = 1.5$, Eq. (17) yields $h_s = 3$, provided that the Hirsch's index obtained by taking into account all citations is much greater than 3. For the same β Eq. (29) states that a scientist that cites 40 percent of his/her own papers (the self-citation world average for natural sciences, [Glänzel and Thijs, 2004a])

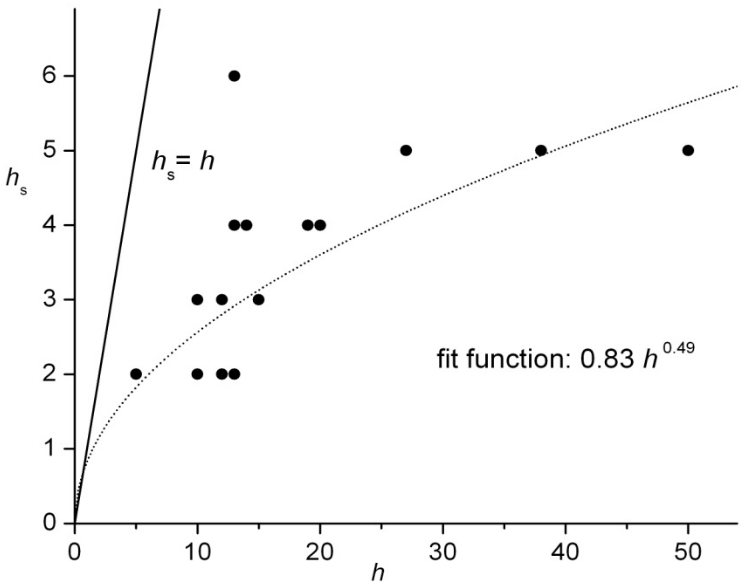

Figure 6. Self-citation correction $h_s = h - h_e$ *vs.* Hirsch's index h (after M. Schreiber's data [Schreiber, 2007])

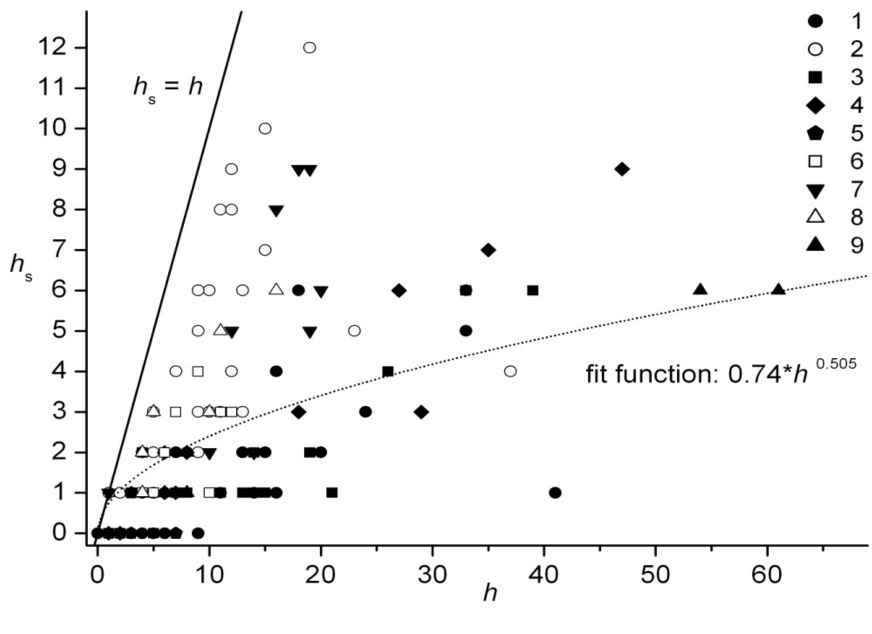

Figure 7. Self-citation correction $h_s = h - h_e$ *vs.* Hirsch's index h (a case study, [Krasteva et al, 2011])

would have (only) about 18 percent of his/her Hirsh index due to self-citations. We also apply Eq. (17) with $a = 3$ and β between 1 and 2 to confirm the results of [Engqvist and Frommen, 2008], where it has been empirically found that, on average, three self-citations per paper yield an increase of the *h* index by one.

The last point we would like to address concerns the applicability of a recently suggested self-citation correction approach for the Hirsch index. Ferrara and Romero [Ferrara and Romero, 2013] have introduced a 'discounting *h*-index' $dh = h\sqrt{R}$, where R is the ratio of external to overall (*i.e.* external plus self) citation numbers (note that $R = b$ for our case of multiplicative self-citation corrections). The authors claim that *no* prior knowledge of the self-citation distribution is required. Let us consider, for simplicity, a situation, where both overall and external citations follow linear (Hirsch) distributions with slopes s and s_e , respectively. The ratio of external and overall citations numbers is [Atanassov and Detcheva, 2013]

$$R = \frac{(C_{em}P_{em} / 2)}{(C_m P_m / 2)} = \frac{s}{s_e} \frac{(1+s_e)^2}{(1+s)^2} \frac{h_e^2}{h^2}, \qquad (32)$$

where s and s_e are the corresponding distribution slopes. Hence $h_d = h - dh = \left(1 - \sqrt{R}\right)h$ is equal to the *exact* self-citation correction $h_s = h - h_e$ if and only if $s = s_e$ or $s = 1 / s_e$, *i.e.* if the triangles associated with the corresponding linear distributions are geometrically *similar*. In particular, for additive self-citation correction one obtains $h_d = a / (1+s) = h_s$, in agreement with Eq. (13a). In case of multiplicative correction to the linear citation paper rank distribution (Eq. (27a)) we have $h_d = h_s$ if and only if $b = 0$ or $b = 1 / s^2 \le 1$ (both relations imply geometrically similar triangles, too). Further on, by comparing h_d with the multiplicative Hirsch's index self-citation correction for a Pareto distribution, given by the *exact* relationship (Eq. (29)), one concludes that $h_d = h_s$ if and only if $\beta = 1$. Hence, although rather convenient, the discounting *h*-index dh has problems with the *shape* of citation-paper rank distributions and should be used with some caution when estimating self-citation corrections to scientific impact.

Summary and conclusions

We have obtained quantitative estimates of self-citations effect on Hirsch's *h*-, Egghe's *g*- and Zhang's *e*-indexes for continuous citation paper rank distributions in general form as well as for especially chosen ones (uniform, linear and Pareto). Prior to this two types of a (single author) self-citation behavior have been considered, in order to provide support for the basic assumptions of additive and multiplicative self-citation corrections.

Our main conclusions are summarized as follows:

- Two types of self-citation behavior can be distinguished. The 'normal' one is characterized with almost uniform self-citation distribution and self-citations number that linearly depends on the number of papers. The 'extraordinary' citation practice is associated with linearly decreasing self-citation distribution and self-citations number depending on the squared number of papers. A smooth transition exists between both self-citation policies, governed by the number of self-cited papers;
- For a linear negative slope citation-paper rank distribution the additive self-citation corrections to the Hirsch's and Egghe's indexes are constants, depending on the self-citation level and the slope only; in

particular, they decrease when the distribution gets steeper. The latter means that a set of small number of highly cited papers is less sensitive to self-citations than a large number of poorly cited ones;

- For a Pareto distribution the additive self-citation correction to Hirsch's index varies from constant, depending on self-citation level and power exponent only, to linear function of the index, depending on whether the additive self-citation level is much smaller than the index, or it approaches its maximal value, indicating an 'extraordinary' self-citation behavior;
- The multiplicative self-citation corrections to Hirsh's and Egghe's indexes depend linearly on the corresponding indexes, which could also be suspicious for 'extraordinary' self-citation behavior;
- Both types of self-citation behavior have been qualitatively detected by analyzing scientometric data. The approximate formulae have been successfully compared to empirically and theoretically obtained self-citations corrections to Hirsch's index.

In conclusion, we believe that the model suggested could prove useful in analyzing self-citation effect on assessment of scientific activity. A list of appropriate topics for future studies include an extension of the discrete model for self-citation behavior by considering two and more co-authors, as well as discrete (Zeta, Zipf) citation-paper rank distributions.

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THE CREATION OF STRATEGY FOR INNOVATION DEVELOPMENT OF SOCIO-ECONOMIC SYSTEMS

Vladimir Pankratov

Abstract: *The creation the strategy of development of large socio-economic systems is based on the synthesis of the foresight methodology with the methodology of cognitive modeling, which opens up a unique opportunity in the framework of a single program-analytical complex to solve problems of strategic planning and rapid response.*

The model problem of innovative development of the recreational sector of the Southern coast of the Crimea is considered.

Keywords: *foresight methodology, cognitive modeling, sensitivity of the system, structural stability, sustainability of the perturbation, stability by value.*

ACM Classification Keywords: *H.4.2. INFORMATION SYSTEM APPLICATION: type of system strategy*

Introduction

A modeling of searching for the new ways of development of large socio-economic systems at the company, mega polis, large enterprise, and region level is a study of complex systems. A complex system is a holistic environment of a system research, which from the positions of achieving the goals is chosen, formed or created by a person and that allows to build the methodology of research and decision-making with regard to the development of the process of the object cognition in the mind of the researcher. Technological or organizational structure of such systems is hierarchical, multi-level system of interrelated same type or of different type's functional elements, which could be focused in some space or are separated for significant distances.

When creating the formalization of mathematical model of complex system it is necessary to take into account the complexity, nonformalizability, uncertainty, diversity of conceptual interconnections of organizational, technical, technological, economic and other objectives of the different phases of the life cycle of complex systems and the highest price of possible error. It follows the practical necessity of search of new concepts, principles, models and approaches to the creation of strategy of innovative development of large socio-economic systems. The most expedient is to solve the problem on the basis of the synthesis of different methodologies with modern information technologies.

Here for creation the strategy of development of large socio-economic systems is proposed the synthesis of the foresight methodology with the methodology of cognitive modeling, which opens up a unique opportunity in the framework of a single program-analytical complex to solve problems of strategic planning and rapid response.

1. Problem Formation

Methodology of foresight with high status has proven itself in many countries of the world. Application of the methodology allows answering the question "what will be, if...?" and create alternatives of scientifically-based scenarios. In accordance with the methodology of foresight [Zgurovsky M.Z., Pankratova N.D., 2005] at the last stage of the decision-making process, decision-makers are offered 3-4 scenarios that, in general, are complex semi-structured system. Currently in the framework of the foresight methodology a formalization of a number of methods of qualitative analysis (SWOT-analysis, the method of the hierarchy analysis and its modifications, methods of Delphi, cross-analysis, morphological analysis, and others), which became the basis of tools set of alternatives scenarios creation, is made [Zgurovsky M.Z., Pankratova N.D., 2011].

For a substantiated implementation of one kind or another scenario it is advisable to involve cognitive modeling, which allows on the basis of knowledge and experience to build cause-and-effect relationships, understand and analyze the behavior of a complex system and to offer science-based strategy for the implementation of the priority scenario.

Under cognitive modeling it is understood the solution of interrelated problems: the construction of cognitive models (maps'); analysis of the ways and cycles of cognitive models; substantiation of each step of stability modeling by the value, by the perturbation and structural stability; scenario analysis; solution of the inverse problems; decision-making including multifactor risks and uncertainties of different nature.

2. Technology of cognitive modeling

Technology of cognitive modeling is that on the basis of cognitive models to identify possible and rational way to manage the situation with the purpose of transition from the initial state to the desired one. The advantage of cognitive model is that it allows seeing the whole picture and details, to integrate logic and imagination, knowledge and experience.

In the modeling process at each stage of building a model the one of the important issues is substantiation of the reliability of its construction, which is achieved by mathematical justification stability of the model in the process of its creation [Zgurovsky M., Pankratov V.A., 2011]. When assessing the stability of cognitive models it is required that the system when reacting on the environmental changing had about the same equilibrium behavior for some period of time. For the assessment of stability of a complex system the following criteria is accepted:

1st criterion: the state of the system integrity is not output of trajectory of development of the system at the forecast interval of time from some set of safe operation conditions;

2nd criterion: almost monotonous growth of indicators - indicators of development of the object at the certain time interval, and then storing them in the preset interval of admissible values;

3rd criterion: putting of trajectory of development for a certain time in target set of the states;

4-th criterion: resistance to disturbance, including asymptotic stability of programmed and structural stability of the system.

Evaluation of the stability of the object development is carried out on the basis of the first two criteria. These criteria dictate the selection of specific indicators of economic stability of the object of a research, which describe and characterize the evolution of the studied object, the level of its quantitative and qualitative parameters in the system of statistics. The important means are not themselves indicators but their threshold values, i.e. limit values, the neglect values of which interfere with the normal process of various elements of reproduction, lead to the formation of negative, destructive tendencies of economic security.

The indicators used to define the threshold values are the system of indicators of economic stability. Ideally, stability is achieved under the condition that all indicators are set within the allowable limits of their threshold values, threshold values of a parameter is not achieved at the expense of others. All dependencies between sustainability indicators and their thresholds should be considered in the dynamics. In the case of massive "surge" inherent in the market, appear consistent patterns that need to be studied carefully.

The processes of propagation of the disturbances in the system are directly connected with the analysis of sensitivity of the system, its stability, adaptability; study the possibility of emergency situations in the system. That is the main issue in such trials is the question: whether the system behavior is changed significantly as a result of changes (desired, undesirable, unknown, unpredictable) in the mode of natural evolutionary development, as well as in the mode of management?

To develop recommendations for a stable development strategy the third and fourth criteria are used. The application of such criteria requires the involvement and knowledge of the theory of stability, well-developed technical and cybernetic systems and finds its application in studies of nonlinear economic systems from the second half of the 20th century. It is quite obvious that when you create a cognitive model the one of the possible approaches to the solution of the issue of determination of its «objective» is connected with the justification of model “stability”.

In this work, when creation a cognitive model the following types of sustainability as structural stability, sustainability of the perturbation and stability by value (numerical stability) are considered.

To study structural stability the cognitive map in the form of a graph is considered. Cycles of the graph conform to the contours of feedback: cycles that characterize the growing tendency to diverge from this condition, meet the contours of positive feedback, and cycles that characterize the suppression of this trend, meet the contours of negative feedback. The cycle is a contour of a positive feedback (even cycle), if it contains an even number of arcs with a minus sign. Otherwise it is a contour of a negative feedback (odd cycle). The presence of the even cycle, having positive production of signs for all of the included arcs, testifies about the structural instability of the system, since it leads to unlimited growth of the values at the vertices of a graph. Changing of the values in any of the top of negative feedback has a negative production of signs for all of the included arcs and only leads to oscillation of tops parameters [Gorelova G.V., Zakharova E.N., Radchenko S.A., 2006], which points to the structural stability of the system.

Research on structural stability, i.e. to find all of the cycles of the graph, is carried out by a recursive search, all the vertices and all possible ways are searched, after which found cycles are investigated on parity. In the program window all the even cycles of the graph, as well as their number are displayed.

The stability of the graph by the perturbation and value is based on the concept of the process of propagation of perturbations of a graph. Determine the value at the top u_i at the moment of time t through $v_i(t), i \in [1, n], t = 0,1,\dots$. Suppose that the value $v_i(t+1)$ depends on $v_i(t)$ and on the vertices adjacent to u_i. Thus, if a vertex u_j adjacent to u_i and if $p_j(t)$ represents the change in u_j at the moment of time t, it should be considered that the impact of this change on u_i at the moment of time $t+1$ will be described by the function $f(u_j, u_i)p_j(t)$, where through $f(u_j, u_i)$ the weight function of connection between the vertices u_j and u_i is denoted [4]. Thus, we have the following rule of perturbation propagation:

$$v_i(t+1) = v_i(t) + \sum_{j=1}^{N} f(u_j, u_i) \cdot p_j(t) \; \forall i = \overline{1,n} \; ,$$

$$p_j(t+1) = v_j(t+1) - v_j(t) .$$

The vertex is called stable by perturbation, if the sequence $\{|p_j(t)|\}_{t=1}^{\infty}$ is limited. The vertex is called stable by value if the sequence $\{|v_j(t)|\}_{t=1}^{\infty}$ is limited. The graph is stable by perturbation (value), if all its vertices are stable.

Such a result: from the stability by value should be the stability by the perturbation.

Thus, the stability by value is reduced to a limited matrix series and the stability by the perturbation - to limited matrix sequence [Gantmakher F.R., 1967].

Thus, the stability by value is reduced to a limited matrix series $\sum_{t=0}^{\infty} A^t$, and the stability by the perturbation - to limited matrix sequence $M_t = \{A^t\}_{t=1}^{\infty}$ [5].

Take the following stability criteria by the perturbation and value.

Criterion 1. The system in the form of signed weighted directed graph G with the adjacency matrix A is stable by perturbation IFF the spectral radius of the adjacency matrix is $\rho(A) = \max_i |\lambda_i| \leq 1$, where $\{\lambda_i\}_{i=1}^{M}$ — the eigenvalues A, and is the basis of eigenvectors.

Criterion 2. The system in the form of signed weighted directed graph G with the adjacency matrix A is stable by value IFF spectral radius of the adjacency matrix is $\rho(A) = \max_i |\lambda_i| < 1$, where $\{\lambda_i\}_{i=1}^{M}$ — characteristic numbers A, or $\rho(A) = 1$, but the Jordan form of the matrix is diagonal and there is no eigenvalue equal to 1.

3. Creation of the scenario of innovative development of the recreational sector of the Southern coast of the Crimea

Here the model problem of innovative development of the recreational sector of the Southern coast of the Crimea (SCC) is considered as an example. At the first stage of foresight methodology a SWOT analysis is involved to identify the following critical technologies of the recreation sector of the Southern coast:

Objects of research

V1 - atmosphere; V 2 - drinking water; V 3 - forests; V 4- soils.

Characteristic parameters (criteria)

V 5 - domestic waste; V 6 - sewage water; V 7 - industrial waste; V 8 - CO2 emissions; V9-transport; V10-industry; V 11 – trade; V 12–tourism; V 13 – restoration of resources; V 14 – waste recycling plants; V 15 – reorganization; V 16 – legislation.

At the next stage of the implementation of the process of foresight to identify quantitative characteristics, which are later used as the initial when using cognitive modeling, the Delphi method is involved [Pankratova N. D. and Malafeeva L. Y., 2012]. To assess different actions on objects of eco environment of Southern coast (atmosphere, drinking water, forests and soils) in accordance with the Delphi method a survey of experts was conducted, so that they express their views on selected objects of eco environment with the purpose of further ranking by selected criteria. Objects of eco environment, characteristic parameters (criteria) and their weights are given in table 1.

Table 1. Initial data

Objects of eco environment	Characteristic parameters (criteria)	Weights of characteristic parameters
$O = \{O_n \mid n = 1,4\}$	$I_n = \{I_{np} \mid p = 1,3\}$	$W = \{W_{np} \mid p = 1,3\}$
O_1 = «Atmosphere»	I_{11} = «Domestic waste»	W_{11} =0,35
	I_{12} = «Transport»	W_{12} =0,10
	I_{13} = «Recovered resources»	W_{13} =0,27
O_2 = «Drinking water»	I_{21} = «sewage water»	W_{21} =0,10
	I_{22} = «industry»	W_{22} =0,25
	I_{23} = «Waste processing plants»	W_{23} =0,35
O_3 = «Forests»	I_{31} = «Industry waste»	W_{31} =0,25
	I_{32} = «Trade»	W_{32} =0,30
	I_{33} = «Reorganization (structural change)»	W_{33} =0,30
O_4 = «Soils»	I_{41} = «Emissions of CO_2»	W_{41} =0,30
	I_{42} = «Tourism»	W_{42} =0,35
	I_{43} = «Legislation»	W_{43} =0,25

A priori defined coefficients:	
K = 0,9	Coefficient of accounting the weight of indicators of experts confidence
S^* = 0,65	The ultimate level of consistency
R^{T_1} = 0,3	Radius of the confidence interval

To ensure the reliability of expert assessment at the level of not less than 0.8, it is necessary to form a group with no less than 16 experts, each of which the indicator of competence in the subject area is reasonably obtained (table 2).

Table 2. Indicators of experts' competence

Expert	№ 1	№ 2	№ 3	№ 4	№ 5	№ 6	№ 7	№ 8	№ 9	№ 10	№ 11	№ 12	№ 13	№ 14	№ 15	№ 16
χ_k	0,61	0,61	0,94	0,63	0,99	0,94	0,99	0,61	0,99	0,61	0,94	0,61	0,83	0,99	0,99	0,99

On the basis of statistical researches, the available sources of information on the selected topic and reviews, proposed by each expert $E_k \in E$, in every cell of the questionnaire form expert puts his vision within the interval [0; 1]:

μ_{npk} – Expert evaluation of that criteria will take the specified value from the corresponding level $s = \overline{1,7}$;

ν_{npk} – Degree of expert confidence in each of the answer.

Experts fill out each section of the questionnaire form independently, without information on the evaluation of other experts.

The results of the expert assessment for the object «Atmosphere» on the criterion of «Resource Restoration» and the corresponding weights of indices (see Table. 1) are presented in table 3. Factor, allowing to take into account the weight of the indicators of experts confidence is taken as the following: K=0,8.

Table 3. The expert assessment for the object «Atmosphere» on the criterion of «Resource Restoration»

№	Expert	**Resources restoration**													
		1		2		3		4		5		6		7	
		Too low level		Very low level		Low level		Medium level		High level		Very high level		Too high level	
		μ_{k1}	ν_{k1}	μ_{k2}	ν_{k2}	μ_{k3}	ν_{k3}	μ_{k4}	ν_{k4}	μ_{k5}	ν_{k5}	μ_{k6}	ν_{k6}	μ_{k7}	ν_{k7}
1	Expert 1	0,05	0,99	0,16	0,83	0,38	0,94	0,67	0,63	0,62	0,94	0,97	0,83	0,46	0,94
2	Expert 2	0,83	0,94	0,91	0,83	0,74	0,83	0,45	0,83	0,20	0,94	0,09	0,99	0,04	0,61
3	Expert 3	0,40	0,99	0,54	0,61	0,53	0,83	0,58	0,63	0,55	0,63	0,38	0,63	0,20	0,83
4	Expert 4	0,97	0,99	0,86	0,94	0,55	0,99	0,26	0,83	0,09	0,61	0,03	0,99	0,01	0,63
5	Expert 5	0,85	0,61	0,98	0,63	0,85	0,61	0,55	0,94	0,27	0,61	0,10	0,63	0,03	0,94
6	Expert 6	0,47	0,63	0,59	0,99	0,55	0,83	0,56	0,99	0,50	0,61	0,32	0,61	0,16	0,83
7	Expert 7	0,37	0,94	0,62	0,94	0,75	0,83	0,67	0,83	0,45	0,94	0,31	0,83	0,19	0,99
8	Expert 8	0,35	0,83	0,55	0,61	0,64	0,63	0,55	0,61	0,49	0,63	0,39	0,83	0,22	0,63
9	Expert 9	0,19	0,94	0,39	0,63	0,58	0,61	0,64	0,83	0,53	0,99	0,45	0,63	0,33	0,94

10	Expert 10	0,24	0,61	0,55	0,94	0,83	0,63	0,95	0,61	0,83	0,99	0,55	0,61	0,29	0,83
11	Expert 11	0,10	0,83	0,23	0,99	0,51	0,61	0,79	0,99	0,91	0,99	0,80	0,63	0,54	0,94
12	Expert 12	0,68	0,94	0,78	0,83	0,63	0,99	0,37	0,99	0,17	0,94	0,06	0,94	0,02	0,83
13	Expert 13	0,43	0,99	0,63	0,63	0,67	0,61	0,53	0,83	0,44	0,63	0,32	0,63	0,17	0,63
14	Expert 14	0,99	0,63	0,93	0,94	0,68	0,61	0,37	0,94	0,16	0,61	0,05	0,63	0,01	0,94
15	Expert 15	0,90	0,94	0,99	0,99	0,83	0,83	0,53	0,83	0,25	0,63	0,09	0,94	0,03	0,83
16	Expert 16	0,45	0,61	0,49	0,61	0,61	0,83	0,67	0,83	0,54	0,63	0,32	0,83	0,14	0,63

According to the suggested formalizing of the Delphi method [Pankratova N. D.and Malafeeva L. Y., 2012] the process of formation of the coordinated expert assessments is carried out: interval estimates are built, the quality functional and cluster median are calculated, confidence interval is formed, the consistency of assessments in the cluster is analyzed.

Some expert results agreed by the number of experts' replies were held in trust set given in table 4.

Table 4. Agreed assessments of the expert estimation

Characteristic (criteria)	parameters	Objects of eco envionment			
		V1 Atmosphere	V2 Drinking water	V3 Forests	V4 Soils
V5	Domestic waste	0,36	0,93	0,93	0,21
V6	Sewage water	0,50	0,36	0,50	0,64
V7	Industry waste	0,50	0,36	0,50	0,36
V8	Emissions of CO_2	0,64	0,79	0,36	0,79
V9	Transport	0,83	0,71	0,8	0,61
V10	Industry	0,91	0,92	1,0	0,78
V11	Trade	0,84	0,70	0,78	0,82
V12	Tourism	0,81	0,70	0,98	0,84
V13	Resources restoration	0,62	0,73	0,76	0,79
V14	Waste processing plants	0,73	0,38	0,56	0,54
V15	Reorganization	0,55	0,77	0,64	0,72
V16	Legislation	0,74	0,63	0,82	0,80

Using the results of expert evaluation allowed building the initial adjacency matrix A for cognitive modeling, which is shown in table 5

Table 5. The initial adjacency matrix for cognitive modeling

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16
V1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.90	0.00	0.00	0.00	0.00
V4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.00	0.00	0.00	0.00
V5	0.36	-0.93	-0.93	0.21	0.00	0.40	0.00	0.00	0.00	0.00	0.00	-0.50	0.00	0.30	0.00	0.00
V6	0.50	-0.36	-0.50	0.64	0.00	0.00	0.70	0.00	0.00	0.00	0.00	-0.60	0.00	0.00	0.00	0.00
V7	0.50	-0.36	-0.50	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.60	0.00	0.00	0.00	0.00
V8	-0.64	0.79	0.36	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.40	0.00	0.00	0.00	0.00
V9	-0.83	-0.71	-0.80	0.61	0.00	0.00	0.00	0.80	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.00
V10	-0.91	-0.92	-1.00	-0.78	0.80	0.60	0.60	0.70	0.00	0.00	0.50	0.20	0.00	0.00	0.00	0.00
V11	0.84	0.70	-0.78	0.82	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.40	0.00	0.00	0.00	0.00
V12	0.81	0.70	0.98	0.84	0.30	0.00	0.00	0.00	0.80	0.00	0.50	0.00	0.30	0.00	0.00	0.00
V13	0.62	0.73	0.76	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V14	0.73	0.38	0.56	0.54	-0.50	-0.50	-0.50	-0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V15	0.55	0.77	0.64	0.72	-0.50	-0.50	-0.50	-0.50	0.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00
V16	0.74	0.63	0.82	0.80	0.40	0.40	0.40	0.40	0.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00

In table 5 elements of the matrix are: $a_{ij} = f(v_i, v_j)$ – the weight function that takes the value of the interval [-1;1]. It is equal to 0, if there are no relations between vertices V_i and V_j. The resulting cognitive graph is shown in figure 1.

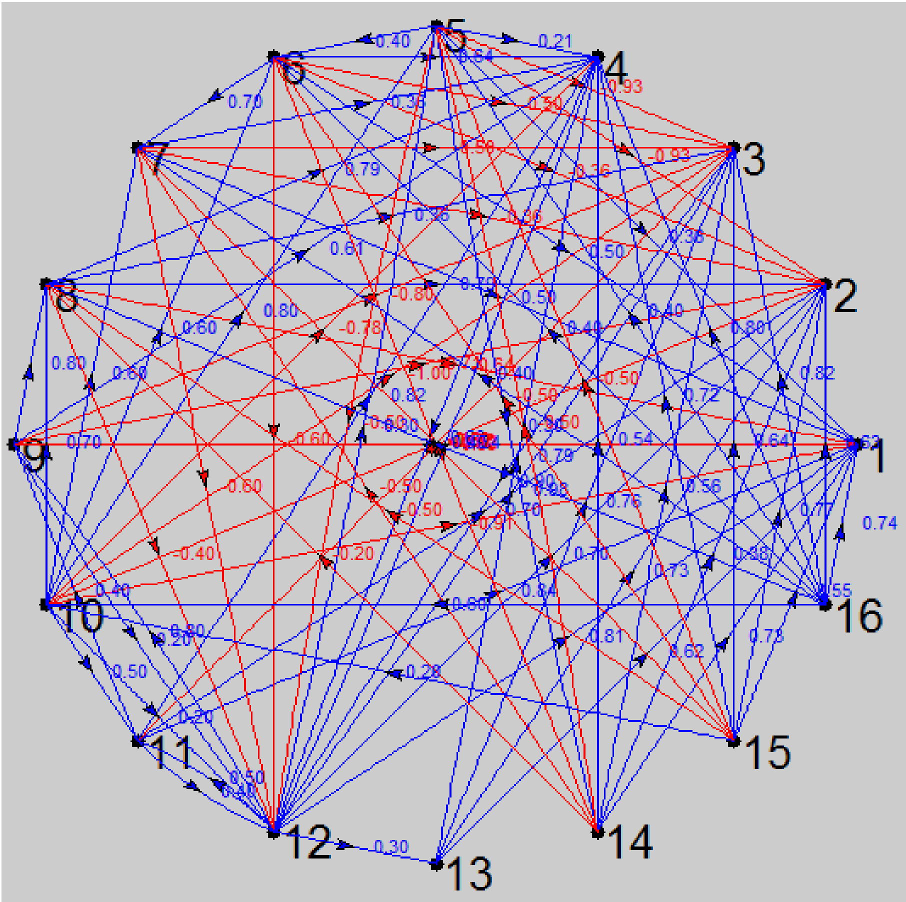

Fig. 1 The cognitive map is presented as the adjacency matrix according table 5

Substantiation of the reliability of building of cognitive maps is achieved by mathematical justification of the model stability in the process of its creation. This model is not stable by any of the considered criteria. For this model the spectral radius $\rho(A)$ **= 1.46118**, that requires a revision of the adjacency matrix to achieve the numerical stability. Considering structural stability **-** we have 26 paired cycles. To achieve structural stability is possible, if you add another factor:

Step 1. Note that we have cycles:

3-12-3 : Forests-Tourism-Forests

4-12-4 : Soils-Tourism-Soils

Improvement of forests leads to improvement of tourism. But improvement of tourism does not lead to the improvement (or deterioration) of forests. A similar situation is with soils. So here, we can remove the link. Obtain stability by the value, perturbation, numerical stability. The number of paired cycles is decreased to 7. The adjacency matrix and the cognitive map are shown in the table 6 and figure 2.

Table 6. The adjacency matrix after the 1st step of modeling

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16
V1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V5	0.36	-0.93	-0.93	0.21	0.00	0.40	0.00	0.00	0.00	0.00	0.00	-0.50	0.00	0.30	0.00	0.00
V6	0.50	-0.36	-0.50	0.64	0.00	0.00	0.70	0.00	0.00	0.00	0.00	-0.60	0.00	0.00	0.00	0.00
V7	0.50	-0.36	-0.50	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.60	0.00	0.00	0.00	0.00
V8	-0.64	0.79	0.36	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.40	0.00	0.00	0.00	0.00
V9	-0.83	-0.71	-0.80	0.61	0.00	0.00	0.00	0.80	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.00
V10	-0.91	-0.92	-1.00	-0.78	0.80	0.60	0.60	0.70	0.00	0.00	0.50	0.20	0.00	0.00	0.00	0.00
V11	0.84	0.70	-0.78	0.82	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.40	0.00	0.00	0.00	0.00
V12	0.81	0.70	0.98	0.84	0.30	0.00	0.00	0.00	0.80	0.00	0.50	0.00	0.30	0.00	0.00	0.00
V13	0.62	0.73	0.76	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V14	0.73	0.38	0.56	0.54	-0.50	-0.50	-0.50	-0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V15	0.55	0.77	0.64	0.72	-0.50	-0.50	-0.50	-0.50	0.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00
V16	0.74	0.63	0.82	0.80	0.40	0.40	0.40	0.40	0.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00

Step2. Note that we have cycles:

9-12-9 : Transport-Tourism-Transport

11-12-11 : Trade-Tourism-Trade

Now consider the transport, trade and tourism: improvement of transport leads to increase of tourists, which in turn increases the number of transport. The situation is similar with the trade. It is therefore proposed to introduce 17th factor that would connect these 3 factors: Tourism-Trade-Transport : Global development of IT technologies». Number of paired cycles is decreased to 5, and circle cycles are disappeared. The adjacency matrix and the cognitive map are shown in table 7 and figure 3.

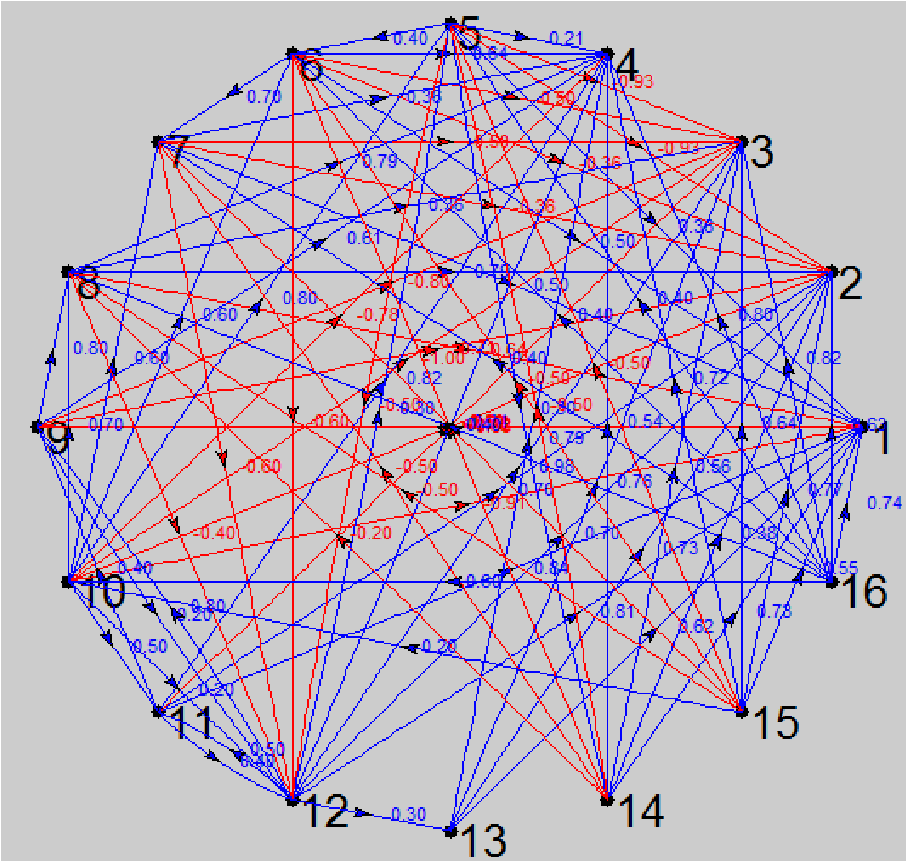

Fig. 2 The cognitive map after the 1st step of modeling

Table 7. The adjacency matrix after the 2nd step of modeling

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17
V1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V3	0	0	0	0	0	0	0	0	0	0	0	0.9	0	0	0	0	0
V4	0	0	0	0	0	0	0	0	0	0	0	0.6	0	0	0	0	0
V5	0.36	-0.9	-0.9	0.21	0	0.4	0	0	0	0	0	-0.5	0	0.3	0	0	0
V6	0.5	-0.4	-0.5	0.64	0	0	0.7	0	0	0	0	-0.6	0	0	0	0	0
V7	0.5	-0.4	-0.5	0.36	0	0	0	0	0	0	0	-0.6	0	0	0	0	0
V8	-0.6	0.79	0.36	0.79	0	0	0	0	0	0	0	-0.4	0	0	0	0	0
V9	-0.8	-0.7	-0.8	0.61	0	0	0	0.8	0	0	0	0.2	0	0	0	0	-0.4
V10	-0.9	-0.9	-1	-0.8	0.8	0.6	0.6	0.7	0	0	0.5	0.2	0	0	0	0	0
V11	0.84	0.7	-0.8	0.82	0	0	0	0	0.4	0	0	0.4	0	0	0	0	0
V12	0.81	0.7	0.98	0.84	0.3	0	0	0	0.8	0	0.5	0	0.3	0	0	0	0
V13	0.62	0.73	0.76	0.79	0	0	0	0	0	0	0	0	0	0	0	0	0
V14	0.73	0.38	0.56	0.54	-0.5	-0.5	-0.5	-0.2	0	0	0	0	0	0	0	0	0
V15	0.55	0.77	0.64	0.72	-0.5	-0.5	-0.5	-0.5	0	0.2	0	0	0	0	0	0	0
V16	0.74	0.63	0.82	0.8	0.4	0.4	0.4	0.4	0	0.6	0	0	0	0	0	0	0
V17	0	0	0	0.1	0	0	0	0	0	0.6	0	0.5	0	0	0	0	0

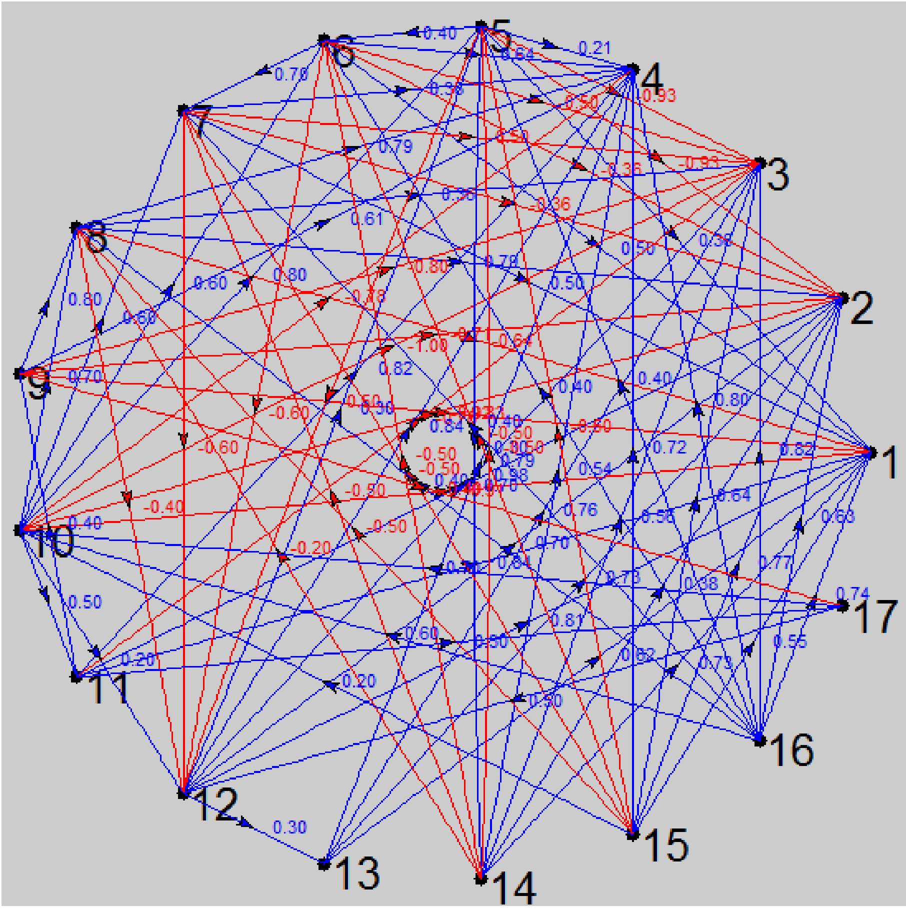

Fig. 3 The cognitive map after the 2nd step of modeling

Step 3. It can be noticed the dependency between «Waste processing plants» and «domestic waste». Domestic waste is treated at these plants. I.e. with the growing number of plants the amount of domestic waste will be decreased. On the other hand, the increasing of the amount of waste does not affect the development of factories, but simply consumes them. Therefore you can remove this link. In this case, the number of paired cycles is decreased to

1. The adjacency matrix and the cognitive map are shown in table 8 and figure 4.

Table 8. The adjacency matrix after the 3rd step of modeling

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17
V1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V5	0.36	-0.93	-0.93	0.21	0.00	0.40	0.00	0.00	0.00	0.00	0.00	-0.50	0.00	0.00	0.00	0.00	0.00

V6	0.50	-0.36	-0.50	0.64	0.00	0.00	0.70	0.00	0.00	0.00	0.00	-0.60	0.00	0.00	0.00	0.00	0.00
V7	0.50	-0.36	-0.50	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.60	0.00	0.00	0.00	0.00	0.00
V8	-0.64	0.79	0.36	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.40	0.00	0.00	0.00	0.00	0.00
V9	-0.83	-0.71	-0.80	0.61	0.00	0.00	0.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.40
V10	-0.91	-0.92	-1.00	-0.78	0.80	0.60	0.60	0.70	0.00	0.00	0.50	0.20	0.00	0.00	0.00	0.00	0.00
V11	0.84	0.70	-0.78	0.82	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30
V12	0.81	0.70	0.98	0.84	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00
V13	0.62	0.73	0.76	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V14	0.73	0.38	0.56	0.54	-0.50	-0.50	-0.50	-0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V15	0.55	0.77	0.64	0.72	-0.50	-0.50	-0.50	-0.50	0.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V16	0.74	0.63	0.82	0.80	0.40	0.40	0.40	0.40	0.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.00	0.50	0.00	0.00	0.00	0.00	0.00

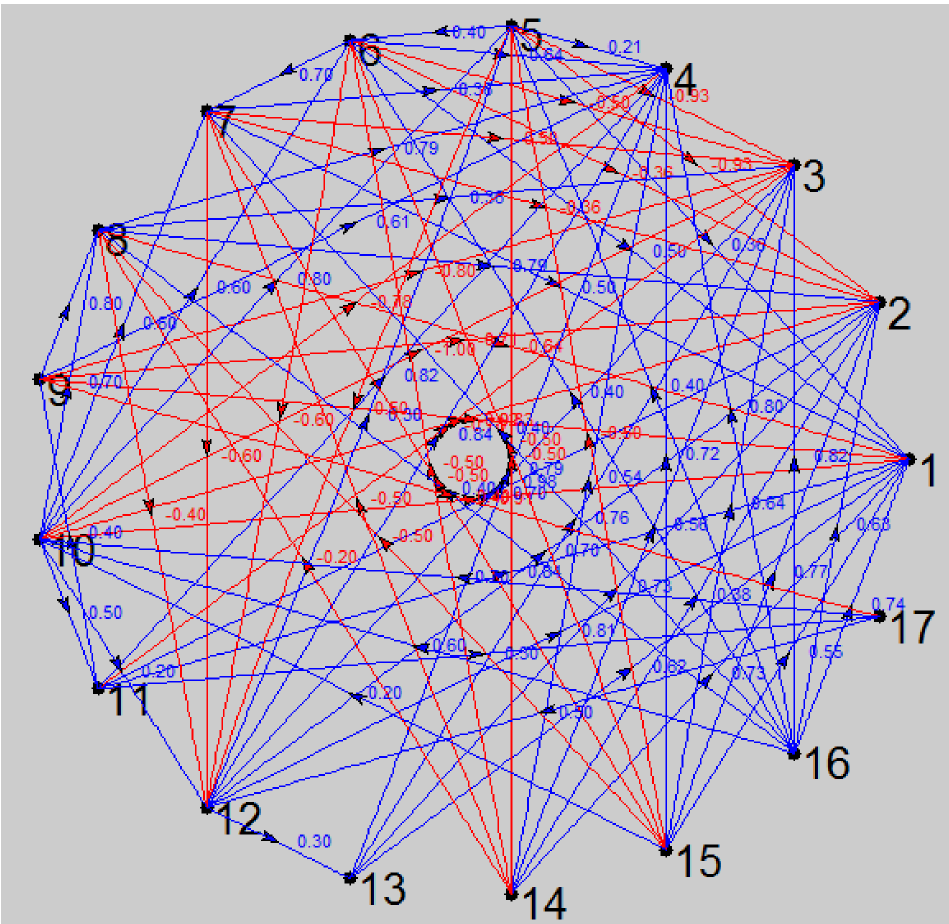

Fig. 4 The cognitive map after the 3rd step of modeling

Table 9. The adjacency matrix after the 4th step of modeling

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17
V1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V5	0.36	-0.93	-0.93	0.21	0.00	0.40	0.00	0.00	0.00	0.00	0.00	-0.50	0.00	0.00	0.00	0.00	0.00
V6	0.50	-0.36	-0.50	0.64	0.00	0.00	0.70	0.00	0.00	0.00	0.00	-0.60	0.00	0.00	0.00	0.00	0.00
V7	0.50	-0.36	-0.50	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.60	0.00	0.00	0.00	0.00	0.00
V8	-0.64	0.79	0.36	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.40	0.00	0.00	0.00	0.00	0.00
V9	-0.83	-0.71	-0.80	0.61	0.00	0.00	0.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.40
V10	-0.91	-0.92	-1.00	-0.78	0.80	0.60	0.60	0.70	0.00	0.00	0.50	0.20	0.00	0.00	0.00	0.00	0.00
V11	0.84	0.70	-0.78	0.82	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10
V12	0.81	0.70	0.98	0.84	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00
V13	0.62	0.73	0.76	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V14	0.73	0.38	0.56	0.54	-0.50	-0.50	-0.50	-0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V15	0.55	0.77	0.64	0.72	-0.50	-0.50	-0.50	-0.50	0.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V16	0.74	0.63	0.82	0.80	0.40	0.40	0.40	0.40	0.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.50	0.00	0.00	0.00	0.00	0.00

Step 4. Paired cycle 11-10-17-10 is remained. In this link the relation between «Trade» and «Global development of IT technologies» is incoherent. Connect these dependencies, as the development of trade, in particular retail, may adversely affect global development of IT-technologies (Internet-shops are developing more slowly). So, the 4th step of modeling we have received a structural stability. The adjacency matrix and the cognitive map are shown in table 9 and figure 5.

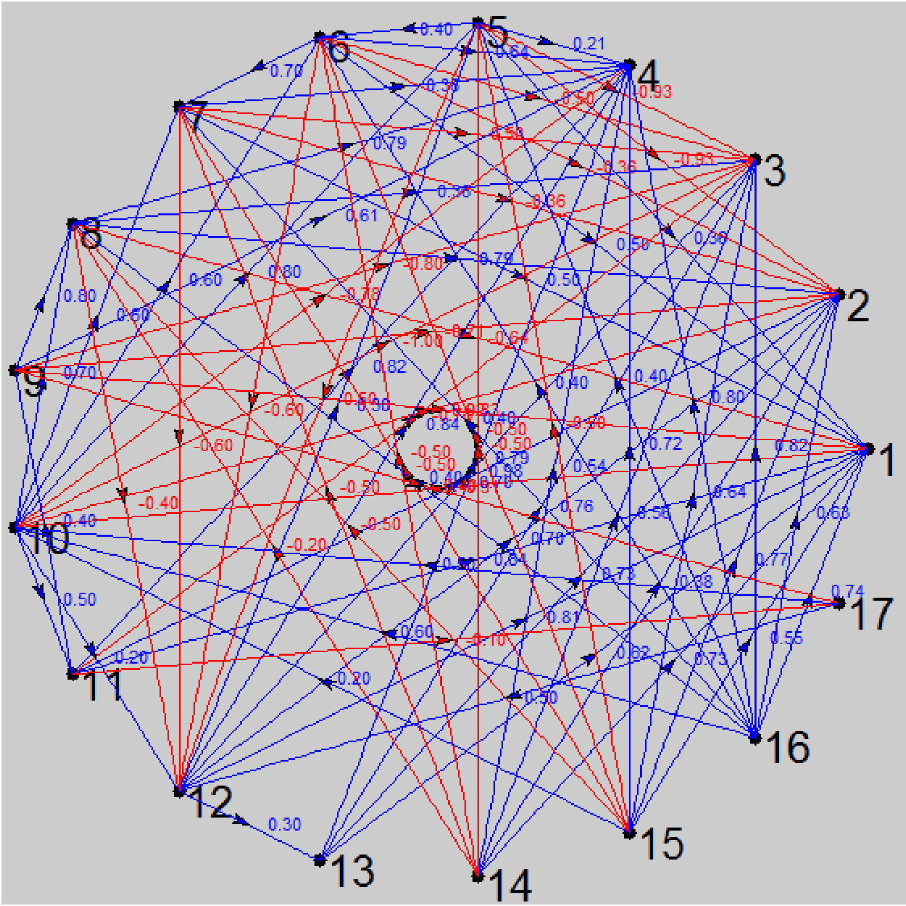

Fig. 5 Stable cognitive map

Thus, in the process of modeling a stable cognitive map, which is reasonable scenario of the appropriate innovative development of the recreational sector of the South coast, is received.

Conclusion

It is shown that the solution of complex unstructured problems must be created on the basis of synthesis of foresight methodologies and cognitive modeling.

Involvement of the process of modeling of foresight methodology at the first stage, which is sufficiently formalized, allows with the help of expert evaluation to identify the critical technologies and build the alternative scenarios with quantitative characteristic values. The obtained characteristics are the initial data for the initial iteration of cognitive modeling.

Using a model of cognitive graphs is expedient for obtaining reasonable decisions of a complex system behavior for the strategic prospects with a large number of interconnections and interdependencies. Of course, a real model, used in the framework of strategic planning, will include in consideration a greater number of important for the market factors and will require achieving its stability dozens of iterations of the modeling. A key advantage of cognitive graph models is the ability to use them for creation a reasonable scenario, which makes them an indispensable tool set in analytical support of strategic planning and development at the company, mega-polis and region level.

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