# NEURAL NETWORK APPROACH TO THE FORMATION MODELS OF MULTIATTRIBUTE UTILITY*

**Stanislav Mikoni**

***Abstract:*** *The inverse problem of choice is formulated as a problem of the settings multiattribute utility model for known-rank alternatives. This problem has the analogy with neural network learning. There is the difference between multiattribute utility model and neural network learning: useful function parameters are subjects for updating instead weight inputs of neurons. It is formulated the main condition alternatives reordering. They must be not comparable in Pareto-dominance analyses. Two alternatives are changed their places due to useful function ratio under its parameters variation. The algorithm is proposed to train the choice model. Algorithm is illustrated by an example.*

***Keywords****: multiattribute utility, inverse problem of choice, utility function, training model, aggregate objective functions.*

***ACM Classification Keywords****: H.4.2 [Information Systems Applications]: Types of Systems decision support*

## Introduction

Rational choice is to solve the problem of multi-criteria optimization based on the input data model and choice conditions model. The choice conditions model contains all the decision-makers preferences. So let's call it a model of preferences. The more preferences the model contains, the more accurate the problem should be solved choice.

The dominant analysis requires the smallest amount of preferences. Its number is limited by specifying optimization direction for each attribute of evaluated alternatives. The multiobjective optimization models occupy an intermediate position. In addition to optimization direction it is given the importance of each criterion, unified criterion scale and function for criteria aggregation. The multiattribute utility model contains maximum amount of preferences. In addition to those preferences are set preferences on the scales of the attributes in the form of non-linear utility functions.

However, accuracy of the solution of choice depends not only on the preferences included in the model choice conditions, but also on the accuracy of their target [Mikoni, Burhakov, 2013]. The transition from a qualitative assessment to quantify preferences has multiple variants. This is the main reason for differences between the choice result and intuitive decision-maker assessment. In connection with this urgent problem setting up a choice conditions model on the evaluation of alternatives, the decision maker made in rank scale. This problem is called the inverse problem of choice as opposed to the direct problem of choice when evaluating alternatives in the rank scale formed on the basis of estimates of utility, measured in a stronger scale.

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Thus, the inverse problem of choice can be seen as the task of teaching choice conditions model on preferences of the decision maker specified in rank scale, similar to training a neural network model with the teacher. Task of preferences model training formulate as follows. It is given the importance and the utility function for the each attribute. The desired order relation is defined on the set of evaluated objects. The task is to sort objects by multiattribute utility functions in accordance with a predetermined order.

## Conditions of Model training

The training of any model with a constant of its structure and functions assumes such change function parameters, which causes a change in the simulation results. The constancy of the structure and function of neural network model assumes the immutability of the chosen architecture and activation functions of neurons of the network. Changeable parameters of neural network model are the weights on the inputs of all neurons of the network. The result of training a neural network model is a set of vectors of weights corresponding to all specified output states network.

Analogue of neural network structure in models of choice is valued attributes structure and formed on the attributes basis criteria reflecting the decision maker preferences. Assuming unchanged criteria importance the only choice conditions model using a utility functions has parameters suitable for training. Models used for the dominant analysis and multiobjective optimization are deprived of such opportunities.

To change the shape of the utility function is necessary to know its formula, the parameters of which are subject to change. Analytical model of the utility function can be obtained by approximating a discrete function, built on the points. Methods for creating the utility function using points were proposed in [Neumann, Morgenstern, 1953] and [Keeney, Raiffa, 1976]. However, the diversity of functions generates a variety of parameters to be changed.

To unify these parameters in [Mikoni, 2013] it was proposed to use standard utility function, reflecting both aversion and propensity for risk of the decision maker. In cases where the target value of the attribute coincides with one of the boundaries of the scale its utility function is accepted, convex upward or downward. In the more general case, when the target value $c_j$ of j-th attribute is set between the boundaries of the scale, the domain of the utility function is divided relative to the target value for desire and forbidden parts [Mikoni 2009]. This separation suggests the possibility of constructing a composite utility function that reflects both aversion and propensity for risk decision-makers. Seems reasonable that relative to forbidden and desire values of attribute scale utility function can reflect propensity and aversion for risk correspondingly. Indeed, the growth in utility forbidden values cannot be large, while towards the target utility increases and then decreases slowly.

If the decision maker arranges the same growth of utility ranges of forbidden and allowed values of the attribute scale, then, in accordance with the above considerations, the schedule of the utility function should not reflect risk propensity in the forbidden values and risk aversion – in the range of allowed values. Such a pattern is displayed logistic utility function (1)

$$u(y) = \frac{1}{1 + e^{-\beta(y-c)}}. \tag{1}$$

The logistic function is presented In Figure 1.

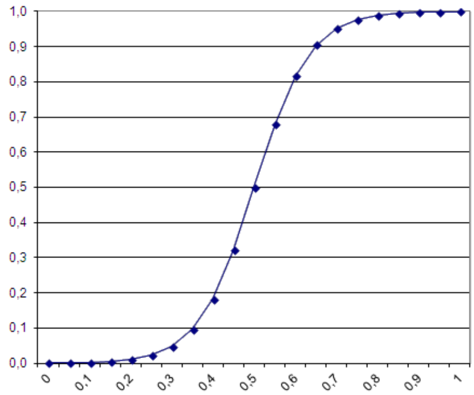


**Figure 1.**

Parameter *c* characterizes the target value and the variable *y* is the actual value of the attribute. Opportunity of utility growth in case of exceeding the target value is created by appointment of 50 percent utility at achieving *c*: $u(c)=0.5$. Measure of risk aversion / risk propensity is adjusted coefficient of $\beta$. It is useful to note the similarity of this function with the sigmoid activation function of the neuron. When $\beta \rightarrow \propto$, logistic function transforms at a threshold function. Such dependence is too presented by Harrington function:

$$u(y)=\exp(-\exp(-(y-c))).$$

In the case where the attribute preferences on a scale of relative values *c* vary in opposite directions with the same speed, the utility function acquires a bell shape with a maximum value at point *c*.

$$u(y) = e^{-\beta \cdot (y-c)^2} \tag{2}$$

The bell shape function is presented In Figure 2.

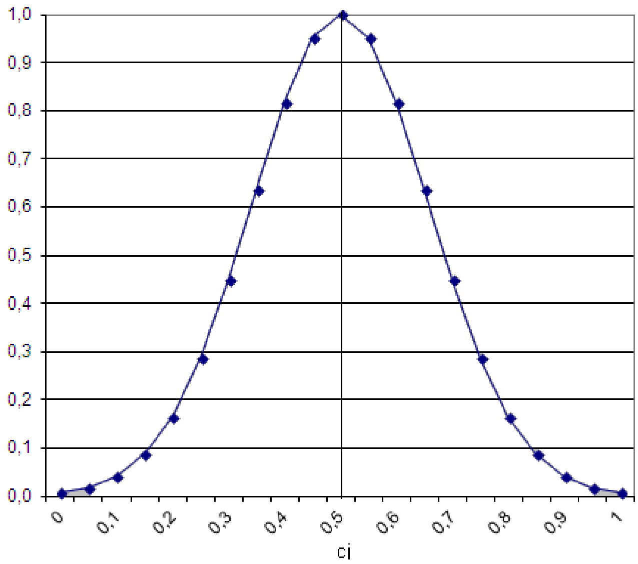


**Figure 2.**

Formulas (1) and (2) have the same parameter that unifies learning utility functions. List of typical utility functions are not restricted to the two above examples.

## Conditions for changing ranks of alternatives on the attributes usefulness

According to useful theory expected utility of alternative assembles expected utility of values of all attributes.

Consider two objects $x_i$ and $x_k$, $x_i$, $x_k \in X$. Evaluate their two numerical attributes $f_1$ and $f_2$ values: $\mathbf{y}(x_i)=(y_{i1}, y_{i2})$ и $\mathbf{y}(x_k)=(y_{k1}, y_{k2})$.

The inequalities $y_{i1} > y_{k1}$, $y_{i2} < y_{k2}$, or $y_{i1} < y_{k1}$, $y_{i2} > y_{k2}$ are the primer condition of alternative ranks change on vector estimates of attributes. Satisfying these conditions objects $x_i$ and $x_k$ are incomparable with respect to Pareto-dominance. They belong to one of the levels ranged graph of domination and therefore have the same place in the general order of objects.

Ordering of disparate objects is possible by converting vector estimates in scalar values using generalizing function. Let it be additive generalizing function. It gives the following scalar evaluation of objects $x_i$ and $x_k$:

$y(x_i)= w_1 \cdot y_{i1} + w_2 \cdot y_{i2}$;

$y(x_k)= w_1 \cdot y_{k1} + w_2 \cdot y_{k2}$.

If $y(x_i)>y(x_k)$, it $\rho(x_i)<\rho(x_k)$, i.e. object $x_i$ has lower rank (the best place) than to $x_k$.

When $w_1 \cdot y_{i1} + w_2 \cdot y_{i2} = w_1 \cdot y_{k1} + w_2 \cdot y_{k2}$ objects have the same scalar evaluation and as a result, occupy the same space. Transform identity in the following expression:

$w_1 \cdot y_{i1} - w_1 \cdot y_{k1} + w_2 \cdot y_{i2} - w_2 \cdot y_{k2} = 0$;

$w_1 \cdot (y_{i1} - y_{k1}) + w_2 \cdot (y_{i2} - y_{k2}) = 0$;

$w_1 \cdot (y_{i1} - y_{k1}) = - w_2 \cdot (y_{i2} - y_{k2})$.

Then:

$$\frac{y_{i1} - y_{k1}}{y_{i2} - y_{k2}} = -\frac{w_2}{w_1} \quad (3)$$

In expression (3) difference values features $f_1$ and $f_2$ are constants. Consequently, their contributions to the overall estimations of objects can be influenced only through the change in the ratio of weights of attributes.

The magnitude of weight $w_2$, which involves alignment of contributions on both attributes, determined by the formula:

$$w_2 = -w_1 \cdot \frac{y_{i1} - y_{k1}}{y_{i2} - y_{k2}} \quad (4)$$

The magnitude of weight $w_1$, which involves alignment of contributions on both attributes, determined by the formula:

$$w_1 = -w_2 \cdot \frac{y_{i2} - y_{k2}}{y_{i1} - y_{k1}} \quad (5)$$

Unlike linear scales of attributes utility functions scales are non-linear: $u_j = \varphi(y_j)$, $j = \overline{1,n}$. This means that the scale equal intervals correspond unequal lengths thereof utility scale [0, 1], which is clearly shown in Figure 3.

If we take $y_i = 6$, and $y_k = 4$, the difference values utility increases from the left to the logistic function neighboring features:

$u_i' - u_k' = 0{,}73 - 0{,}27 = 0{,}46 \rightarrow u_i'' - u_k'' = 0{,}88 - 0{,}12 = 0{,}76$.

Difference of utility values is changed to 0.3.

This effect makes it possible to change the ratio of overall ratings utility objects by changing the steepness of nonlinear utility functions.

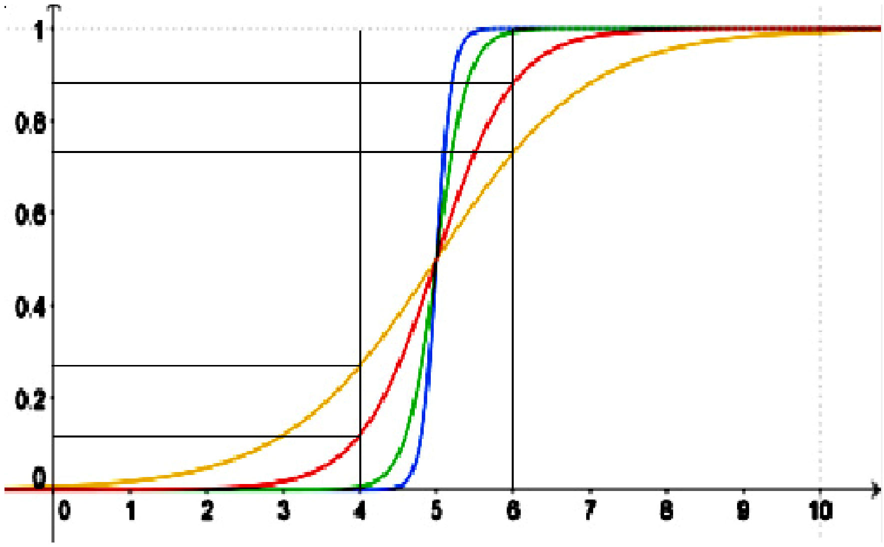


**Figure 3.**

Substitute in the formula (3) the utility of attribute values

$$\frac{u_{i1} - u_{k1}}{u_{i2} - u_{k2}} = -\frac{w_2}{w_1} \tag{6}$$

changing of the steepness of non-linear utility function fronts. Alignment condition overall ratings of the second attribute are the basis of the expression:

$$u_{i1} - u_{k1} = -\frac{w_2}{w_1} \cdot (u_{i2} - u_{k2}) \tag{7}$$

Alignment condition overall ratings of objects at the first attribute are the expression:

$$u_{i2} - u_{k2} = -\frac{w_1}{w_2} \cdot (u_{i1} - u_{k1}) \tag{8}$$

Violation of the identities (3) and (4) leads to a change of alternatives ranks on the opposite ones. Effect of change in utility is reached at a scale factor greater than 1 when the numerator is more than the denominator – $w_2 > w_1$ in (3) and $w_1 > w_2$ in expression (4).

Based on the examination of the conditions change ranks of objects by their usefulness, it should be an opportunity to train the model multiattribute utility by changing the parameters of the utility functions. Under a given object ordering training procedure is similar to training a neural network with the teacher. The difference is that the training of the neural network is the selection of the weighting values of neurons and training multiattribute utility model is realized by finding the values of the parameters of the utility functions.

## The algorithm of objects ordering relative to the sample

The initial data for solving the problem are vector estimations of objects in the feature space, the importance of attributes and the required order of objects.

The objects are ordered by Pareto dominance relation. The initial strict order of object is determined by multiattribute optimization. The difference reveals between received and required order of objects. The pairs of objects are defined to be reordered. Each pair is analyzed with respect to the identity of Pareto dominance. If one object from the pair dominates another one reordering of the pair is impossible. In the absence of Pareto dominance an attribute is defined with maximum difference of useful function. The front steepness of the useful function is tuned. It is the iterative process. It is completed when the ratio multiattribute utility functions compared objects is reversed. The algorithm consists of the following steps.

1. While not considered all pairs of objects, selected a pair of objects $x_i$, $x_k \in X$.
2. If one need to reordering pair ($x_i$, $x_k$) to a pair of ($x_k$, $x_i$), the possibility of the operation is analyzed. If the objects $x_i$ and $x_k$ are related by Pareto dominance, when $x_i \underset{P}{\succ} x_k$ or $x_k \underset{P}{\succ} x_i$, then they cannot be reordered, go to 1.
3. The utility difference $\Delta u_{ik,j} = u_j(x_i) - u_j(x_k)$ is calculated between objects $x_i$ and $x_k$ in all attributes, $j = \overline{1,n}$.
4. The maximum utility difference $\Delta u_{j,\max} = \max\limits_{j} (\Delta u_{ik,j})$ is found.
5. If $\Delta u_{j,\max}$ >0, decrease the front steepness on the $\Delta M$, otherwise increase one on the $\Delta M$.
6. The next objects estimates $u_j(x_i)$ and $u_j(x_k)$ are determined.
7. If $u_j(x_i) > u_j(x_k)$ and the front steepness can be changed, go to 5, otherwise go to 1.

The presented algorithm has been tested on the instrumental system choice and ranking by utility SVIR-U.

The simulation results showed the legitimacy of the proposed approach.

We illustrate the process of changing places of objects on the example of ranking five apartments on three attributes: total area (TA), price and kitchen area (KA). For these attributes expert appointed the following utility function: bell-shaped, decreasing logistics and increasing logistics respectively. The initial values of these functions for the assessed apartments listed on the left side table. 1. Weighted Average utility estimations of these attributes are shown in the column "Utility". The apartments ranking is performed by this estimations. According to a predetermined order of apartment 1 cannot be favored apartments 5. To change the order of these apartments their utility difference is analyzed for each attribute. The largest difference occurs in "Price" attribute. To decrease difference it is necessary to reduce the steepness of the decreasing logistic function by parameter M. Instead initial value of function parameter M=0.3333 was found value M=0.3350, which reduces the difference between the apartments utility by price (see right column "Price"). As a result of changes in the ratio of multiattribute utility of apartments 5 and 1 its ranks are exchanged (see right column "Rank").

**Table 1.** Rating apartments for utility

| | TA | Price | KA | Utility | Rank | TA | Price | KA | Utility | Rank |
|---|---|---|---|---|---|---|---|---|---|---|
| №1 | 0,699 | **0,881** | 0,269 | 0,62 | 4 | 0,699 | **0,795** | 0,269 | 0,588 | 5 |
| №2 | 0,902 | 0,761 | 0,378 | 0,68 | 2 | 0,902 | 0,692 | 0,378 | 0,657 | 2 |
| №3 | 1,000 | 0,592 | 0,500 | 0,70 | 1 | 1,000 | 0,567 | 0,500 | 0,689 | 1 |
| №4 | 0,902 | 0,408 | 0,622 | 0,64 | 3 | 0,902 | 0,433 | 0,622 | 0,653 | 3 |
| №5 | 0,699 | **0,239** | 0,777 | 0,57 | 5 | 0,699 | **0,308** | 0,777 | 0,595 | 4 |

## Conclusion

There are many methods for ordering of alternatives. One of them can be accept for predominant one. Then the question arises – is it possible to setting some choice model for known-rank alternatives? In the paper there was shown such possibility for multiattribute utility model of choice. That possibility exists due to nonlinear utility functions. Increase of attribute value on equal measures leads to Increase of utility function value on not equal ones. That effect permits to change ratio of values of multiattribute useful functions for reordered alternatives. So its ranks change too. Places exchange is possible for only not comparable alternatives in Pareto-dominance analyses. The algorithm is proposed to train the choice model. It is based on choosing of standard utility functions for each attribute and its parameters turning with the purpose of alternatives reordering. The algorithm is illustrated by an example. Program system choice and ranking SVIR, produced at Saint Petersburg State Transport University in Russia (www.mcd-svir.ru/refer07.html) can be applied to calculating the results.

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## Authors' Information


***Stanislav Mikoni*** *– professor of St. Petersburg State Transport University, St. Petersburg 190031, Russia; e-mail: svm@sm4265.spb.edu*

*Major Fields of Scientific Research: System analysis, Multicriteria choice, Intelligent technologies, Simulation*