FUZZY EXPECTED VALUE MODEL WITH INSPECTION ERRORS AND TWO LEVEL OF TRADE CREDIT IN ONE REPLENISHMENT CYCLE

Olha Yegorova

***Abstract**: In this paper economic order quantity model with two level of trade credit in one replenishment cycle, time value of money, inspection errors, planned backorders, and sales returns is offered. In those problem demand rate, selling price for good quality items, selling price for defective items, interest paid rate and interest earned rate consider as fuzzy variable. Following expected value model criterion and a fuzzy expected value model are constructed. Fuzzy simulations are employed to estimate the expected value of fuzzy variable and maximum return on sale. In order to solve the fuzzy expected value model, a genetic algorithms based on the fuzzy simulation is designed.*

***Keywords**: EOQ, imperfect quality, misclassification errors, deterioration items, inventory backordering, trade credit, fuzzy simulation, genetic algorithm.*

***ACM Classification Keywords**: I.6.5 Model Development*

Introduction

Effective inventory management is one of the main issues for any trade company. The traditional economic order quantity model is widely used by decision maker as inventory control tool in real-world environmental. It was tacitly assumed that the buyer must pay for the items purchased as soon as the items are received. Sometimes retailer can't pay at the given period just for economic reasons and uses trade credit. Before the end of the trade credit period, the retailer can sell the goods, accumulate revenue, and earn interest.

During last several years, a great deal of the economic order quantity (EOQ) models with trade credit has been offered in the scientific literature. A lot part of these works shows the main research efforts in the stochastic environmental. For example, an EOQ model under the conditions of permissible delay in payments studied in [Goyal, 1985; Chung, 1998]. Resently, Huang [Huang, 2003] and Sharma et al. [Sharma, 2012] modified this assumption to two level of trade credit. Aggarwal and Jaggi [Aggarwal, 1995], Chung and Liao [Chung, 2004], Shah and Raykundaliya [Shah, 2009], Tripathy and Pradhan [Tripathy, 2012] deal with the problem of determining the economic order quantity for deterioration items under permissible delay in payments. Chang and Dye [Chang, 2001] published the paper for deteriorating items with partial backlogging and permissible delay in payments. Hou and Lin [Hou, 2009] suggested a cash flow oriented EOQ model with deteriorating items under permissible delay in payments.

An analytical review of the research efforts in the fuzzy environment performed on EOQ and economic production quantity models had done by Hsu [Hsu, 2012], Yegorova [Yegorova, 2012]. While the decision variables are similar to the ones in the stochastic environment, the demand and inventory costs are considered fuzzy variables. Similar approach is constrained with constraints such as budget, space, service level.

A careful observation of the given above papers reveals that there are not considered inventory model under the conditions of trade credit with human error in screening process. We notice that Schwaller [Schwaller, 1988] extended the EOQ model by adding the assumptions that defective items of known proportion were present in incoming lots, and that fixed and variable inspection cost were incurred and in finding and removing these defective items. Salameh and Jaber [Salameh, 2000] developed an economic order quantity model for the case where a random proportion of the items in a lot are defective. But human error in the screening process was ignored in their model rather than in Duffuaa and Khan [Duffuaa, 2005]. Later Khan et. al [Khan, 2011] proposed an EOQ with defective items, inspection errors, and sales returns, where shortage are not allowed, defective items are sold at a discounted price at the end of the 100% screening process. More discussions are given in notes by Hsu [Hsu, 2012]. It is clear that production processes are often imperfect. Therefore, in order to provide good service to attract customers and keep them coming back, development of such adequate model is needed with possibility of its clarification and adaption to the changing external terms. Besides, optimal inventory policies that consider both the supplier and retailer viewpoints are more reasonable than those those that consider only from one perspective.

This paper first extends the economic order quantity model for deterioration items with two level of trade credit in one replenishment cycle, inspection errors, planned backorders, and sales returns in both stochastic and fuzzy environments such that the demand, selling prices, interest paid, and interest earned assume a fuzzy nature. As a considered problem has constraints and uncertainty nature, it is necessary to foresee realization of technology which will be allow to shorten the quantity of the analyses cases and optimize the calculation process.

Preliminaries

Possibility theory was proposed by Zadeh [Zadeh, 1978], and developed by many researchers such as Dubois and Prade [Dubois, 1988] and Yager [Yager, 1993]. Recently, B. Liu and Y. K. Liu [Liu, 2002] presented a new measure named credibility measure. Moreover, Liu [Liu, 2004] proposed credibility theory.

Definition 1. Let ξ be a fuzzy variable with membership function $\mu(x)$. Then the possibility, necessity, and credibility measure of the fuzzy event $\xi \geq s$ can be represent, respectively, by [Liu, 2002]

$$Pos\{\xi \geq s\} = \sup_{x \geq s} \mu(x) \tag{1}$$

$$Nec\{\xi \geq s\} = 1 - \sup_{x < s} \mu(x) \tag{2}$$

$$Cr\{\xi \geq s\} = \frac{1}{2}\left[Pos\{\xi \geq s\} + Nec\{\xi \geq s\}\right] \tag{3}$$

Definition 2. The expected value of a fuzzy variable is defined as [Liu, 2002]

$$E[\xi] = \int_0^{\infty} Cr\{\xi \geq s\}ds - \int_{-\infty}^{0} Cr\{\xi \leq s\}ds \tag{4}$$

Definition 3. The optimistic function of α is defined as [Liu, 2004]

$$\xi_{\sup}(\alpha) = \sup\left[s \middle| Cr\{\xi \geq s\} \geq \alpha\right], \quad \alpha \in (0,1]. \tag{5}$$

Definition 4. If $\tilde{\xi} = (a, b, c)$ is a triangular fuzzy number with center *b*, left width $a > 0$, and right width $c > 0$, then its membership functions has the following form

$$\mu(s) = \begin{cases} \dfrac{s-(b-a)}{a}, & b-a \leq s \leq b, \\ \dfrac{(b+c)-s}{c}, & b \leq s \leq b+c, \\ 0, & elsewhere. \end{cases} \tag{6}$$

Definition 5. For the fuzzy variable described in Definition 4, the credibility of the event $Cr\{\xi \le s\}$ is defined based on the definition in (3) as [Liu, 2004]

$$Cr(\xi \le s) = \begin{cases} 0, & s \le b-a, \\ \dfrac{s-(b-a)}{2a}, & b-a \le s \le b, \\ \dfrac{s-(b-c)}{c}, & b \le s \le b+c, \\ 1, & elsewhere. \end{cases} \tag{7}$$

Definition 6. If $\tilde{\xi} = (a,b,c,d)$ is a trapezoidal fuzzy number with left modal level $b > 0$, right modal level $c > 0$, left width $a > 0$, and right width $d > 0$, then its membership function has the following form

$$\mu(s) = \begin{cases} \dfrac{s-(b-a)}{a}, & b-a \le s \le b, \\ 1, & b \le s \le c, \\ \dfrac{(c+d)-r}{d}, & b \le r \le b+c, \\ 0, & elsewhere. \end{cases} \tag{8}$$

Definition 7. For the fuzzy variable described in Definition 6, the credibility of the event $Cr\{\xi \le s\}$ is defined based on the definition in (3) as [Liu, 2004]

$$\mu(s) = \begin{cases} 0, & s \le b-a, \\ \dfrac{s-(b-a)}{2a}, & b-a \le s \le b, \\ \dfrac{1}{2}, & b \le s \le c, \\ \dfrac{s-c}{2d}, & c \le s \le d, \\ 1, & d+c \le s. \end{cases} \tag{9}$$

In this research, the triangular fuzzy variable is used to model the fuzzy interest paid, interest earned and trapezoidal fuzzy variable is used to model the fuzzy demand, selling price of good quality items and selling price of defective items.

Problem definition

Trade Company, which temporarily does not have working capital, plans to replenish inventory by using of trade credit. Allowable purchasing costs of inventory range from $E_{\min}$ to $E_{\max}$. the supplier offer the retailer a credit period of M -days. During this time, the retailer uses general revenue as investment resource with interest earned per monetary per unit time i_e. At the end of this period, the retailer pays off all units sold, keeping the rest for day-to-day expenses and stars paying for the interest charges on the unsold stock with the interest rate per monetary unit per unit time of i_p. Basic interest rate i_p would be lower to i_w from the date of N, if the retailer pays off all units sold during M to N.

Purchase orders are executed with constant intensity λ. Taking into account the overall dimension of unit of productions b, c, d, for holding inventory it is necessary warehouse with capacity W. Supply items lose their physical characteristics while they were kept stored in the inventory. Defective units may be found during products quality control. To ensure an adequate level of direct consumer service p_s, the maximum allowable number of defective items in the lot should not exceed u units. Inspector could incorrectly classify a non-defective item to be defective or incorrectly classify a defective item to be non-defective. Supplier is to rework (make up deficiency) or replace items with manufacturing imperfection. Production costs of defective items, which was obtained for settle a quality claims, does not include to the purchasing cost of the inventory lot and they will not be returned to the supplier. Other expenditures due to mistakes in incoming goods inspection now pay by the retailer.

Demand D is uniformly distributed on the interval between adjacent replenishment. Shortage is allowed. The items that are classification as defective and those returned by the consumers are kept in stock and sold at the end of the operation cycle. It is supposed that the inflation rate per monetary per unit time will be τ percent.

It is necessity determine the economic order quantity and length of operating cycle with the maximum revenue on sales.

Model formulation

It is known that λ is the replenishment rate per unit per unit time and D is demand rate per unit per unit time. Let us assume that the number of items that are classified as defective included those that are non-defective $(1-\beta)\lambda$, and incorrectly classified as defective (with probability a_1), and those that are defective $\beta\lambda$, and classified as defective (with probability $(1-a_2)$). Thus, we have the number of items that are classified as defective in one cycle

$$((1-\beta)a_1+\beta(1-a_2))\lambda \tag{10}$$

and the number of items that are classified as non-defective in one cycle

$$((1-\beta)(1-a_1)+\beta a_2)\lambda \tag{11}$$

where β – the probability that the item is defective, a_1 – the probability of classifying a non-defective item as defective, a_2 – the probability of classifying a defective item as non-defective.

Similarly, we estimate qualitative structure of the items obtained for settle a quality claims. The number of items that have been obtained for settle a quality claims and classified as defective are

$$((1-\beta_r)a_{r1}+\beta_r(1-a_{r2}))\lambda_r \tag{12}$$

and the number of items that have been obtained for settle a quality claims and classified as non-defective are

$$((1-\beta_r)(1-a_{r1})+\beta_r \cdot a_{r2})\lambda_r \tag{13}$$

where λ_r – replenishment rate of items that have been obtained for settle a quality claims per unit per unit time, β_r – the probability that among obtained for settle a quality claims items is defective, a_{r1} – the probability of classifying a non-defective item among obtained for settle a quality claims as defective, a_{r2} – the probability of classifying a defective item among obtained for settle a quality claims as non-defective.

The proportion of the customers, who would like to accept backlogging at time t, decrease with the waiting time for the next replenishment. In this situation the backlogging rate is defined as $B(t)=1/(1+\delta(t_i-t))$, where t_i – the time at which the i^{th} replenishment is being made, δ – the backlogging parameter.

All cash flow elements will lead to the start of the investment using discounted by continuous scheme for a correct comparison the volume of money involved in the creation of stocks with following income. Determine the net discount rate of the inflation

$$k = \begin{cases} r, & \tau < 5\%, \\ (1+r)(1+\tau)-1, & \tau \geq 5\%, \end{cases} \tag{14}$$

and adjust the components of cash flow with the new rate, where r – current real discount rate in the market. If the amount of money at time t is $v(t)$, then their cost at the time $t = 0$ calculated as

$$v_0 = \int_0^H v(t) e^{-kt} dt \tag{15}$$

where v_0 – the value of money at the beginning of the planning horizon, H – planning horizon.

To determine optimal cash flow oriented inventory management strategies necessity take into account not only the amount of cash flow, but also moments of payments and revenues according to the contract provisions.

The crisp inventory model. The inventory system evolves as follows: units of items arrive at the inventory system with replenishment rate λ per unit per unit time when the shortage level becomes I_b. From Figs. 1, one can see that $[0;t_1)$ is the time taken to fill the backorders with rate $(((1-\beta)(1-a_1)+\beta \cdot a_2)\lambda - D)$. During replenishment period $[t_1;t_2)$ inventory level is declining only due to the demand and deterioration with rate $(((1-\beta)(1-a_1)+\beta \cdot a_2)\lambda - D - \gamma \cdot \theta \cdot I)$, where θ – percentage of items deteriorated per unit time, γ – percentage of deteriorated items screened out from the inventory. As soon as inventory level of serviceable items becomes I_s supplier should default the settlement of claims and maximum serviceable inventory level I_m will be accumulated. After that, inventory level due to the demand and deterioration becomes zero over time interval $[t_3;t_4)$ and backordering period $[t_4;T)$ begins.

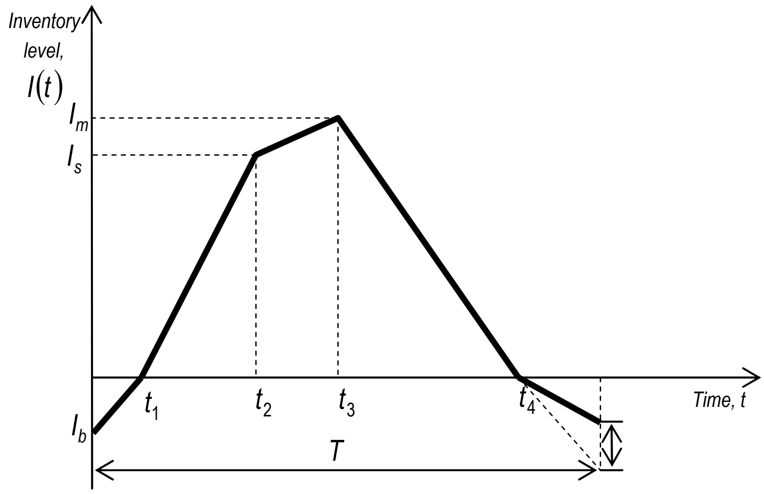

Fig. 1. Behavior of the inventory level over the time

The inventory levels of serviceable items at time t over the five periods in a cycle are determined by following differential equations:

$$\frac{I(t)}{dt}=\begin{cases}((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D, & 0\le t<t_1,\\ ((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D-\gamma\cdot\theta\cdot I, & t_1\le t<t_2,\\ ((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D-\gamma\cdot\theta\cdot I, & t_2\le t<t_3,\\ -D-\gamma\cdot\theta\cdot I, & t_3\le t<t_4,\\ -\dfrac{D}{1+\delta(t-t_4)}, & t_4\le t<T\end{cases}\tag{16}$$

with the boundary conditions $I(0)=I_b$, $I(t_1)=0$, $I(t_2)=I_s$, $I(t_3)=I_m$, $I(t_4)=0$, $I(T)=I_b$. The solutions for the above differential equations (16) are

$$I(t)=\begin{cases}(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)t+c_1, & 0\le t<t_1,\\ \dfrac{1}{\gamma\cdot\theta}(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)+c_2e^{-\gamma\cdot\theta\cdot(t-t_1)}, & t_1\le t<t_2,\\ \dfrac{1}{\gamma\cdot\theta}(((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D)+c_2e^{-\gamma\cdot\theta\cdot(t-t_2)}, & t_2\le t<t_3,\\ -\dfrac{1}{\gamma\cdot\theta}D+c_4e^{-\gamma\cdot\theta\cdot(t-t_3)}, & t_3\le t<t_4,\\ -\dfrac{D}{\delta}\ln\left|1+\delta(t-t_4)\right|+c_5, & t_4\le t<T.\end{cases}\tag{17}$$

The final solutions of above equations are given by

$$I(t)=\begin{cases}(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)t+I_b, & 0\le t<t_1,\\ \dfrac{1}{\gamma\cdot\theta}(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)\left(1-e^{-\gamma\cdot\theta\cdot(t-t_1)}\right), & t_1\le t<t_2,\\ \dfrac{1}{\gamma\cdot\theta}(((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D)\left(1-e^{-\gamma\cdot\theta\cdot(t-t_2)}\right)+I_se^{-\gamma\cdot\theta\cdot(t-t_2)}, & t_2\le t<t_3,\\ \left(I_m+\dfrac{1}{\gamma\cdot\theta}D\right)e^{-\gamma\cdot\theta\cdot(t-t_3)}-\dfrac{1}{\gamma\cdot\theta}D, & t_3\le t<t_4,\\ -\dfrac{D}{\delta}\ln\left|1+\delta(t-t_4)\right|, & t_4\le t<T.\end{cases}\tag{18}$$

Hence, it can be deduced from (18) at $I(t_1)=0$, $I(t_2)=I_s$, $I(t_3)=I_m$, $I(t_4)=0$, $I(T)=I_b$ that

$$t_1=\frac{I_b}{D-\lambda((1-\beta)(1-a_1)+\beta\cdot a_2)}\tag{19}$$

$$t_2=t_1-\frac{1}{\gamma\cdot\theta}\ln\left(1-\frac{\gamma\cdot\theta\cdot I_s}{(((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda-D)}\right)\tag{20}$$

$$t_3=t_2-\frac{1}{\gamma\cdot\theta}\ln\left(\frac{I_m-\dfrac{1}{\gamma\cdot\theta}(((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D)}{I_s-\dfrac{1}{\gamma\cdot\theta}(((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r-D)}\right)\tag{21}$$

$$t_4 = t_3 - \frac{1}{\gamma \cdot \theta} \ln\left(\frac{D}{\gamma \cdot \theta \cdot I_m + D}\right) \tag{22}$$

$$T = t_4 + \frac{1}{\delta}\left(e^{\frac{-I_b \cdot \delta}{D}} - 1\right) \tag{23}$$

where T – length of the cycle.

We may also deduce that economic order quantity is

$$Q = \lambda t_2 \tag{24}$$

Our aim is to maximize return on sale per unit time, which can be expressed as

$$ROS_i(I_b, I_m, I_s) = \frac{(R_s)_i - K - (P_c)_i - C_{si} - C_{rg} - C_h - C_d - C_s - C_{un} - C_{adi} - C_{rnd} - IP_i + IE_i}{(R_s)_i} \to \max, \quad i = 1,2,\ldots,14 \tag{25}$$

subject to the constraints

- investment amount on total production cost cannot be infinity, it may have an upper and lower limits on the maximum investment

$$E_{\min} \le P_C \le E_{\max} \tag{26}$$

- warehouse space where the items are to be stored is limitation

$$b \cdot c \cdot d \cdot \lambda \cdot (t_1 + t_2) \le W \tag{27}$$

- holding cost cannot be more than total production cost

$$C_h < P_C \tag{28}$$

- providing good service of the customers

$$P\left(a_2 \le 1 - \frac{u+1}{\beta \cdot \lambda(t_1 + t_2)}\right) \le p_s \tag{29}$$

where R_s – sale revenue is given by

$$R_s = \frac{1}{T}\left(\begin{array}{l} p_v\left(\int_0^M e^{-kt}dt\left(\int_{\Xi_1} Ddt\right) + \Psi_0 \int_0^N e^{-kt}dt\left(\int_{\Xi_2} Ddt\right) + \Psi_1 \int_0^{t_4} e^{-kt}dt\left(\int_{\Xi_3} Ddt\right) + \int_0^T e^{-kt}dt\left(\int_{t_4}^T \frac{Ddt}{1+\delta(t-t_4)}\right)\right) + \\ + p_b \int_0^T e^{-kt}dt\left(\beta \cdot a_2 \cdot \lambda \cdot t_2 + \beta_r \cdot a_{r2} \cdot \lambda_r (t_3 - t_2) + ((1-\beta_r)a_{r1} + \beta_r(1-a_{r2}))\lambda_r(t_3 - t_2)\right) \end{array}\right),$$

$$\Xi_1 = \begin{cases} [0;M],\ t_2 \le M < t_3 < T,\ t_3 \le M < t_4 < T,\ t_2 \le M < N < t_3 < T,\ t_3 \le M < N < t_4 < T, \\ \qquad t_2 \le M < t_3 < N < t_4 < T,\ t_2 \le M < t_3 < t_4 < N < T,\ t_3 \le M < t_4 < N < T, \\ \qquad t_2 \le M < t_3 < t_4 < T < N,\ t_3 \le M < t_4 < T < N, \\ [0;t_4],\ t_4 \le M < T,\ M > T,\ t_4 \le M < N < T,\ N > M > T,\ t_4 \le M < T < N; \end{cases}$$

$$\Psi_0 = \begin{cases} 1,\ t_2 \le M < N < t_3 < T,\ t_3 \le M < N < t_4 < T,\ t_2 \le M < t_3 < N < t_4 < T,\ t_2 \le M < t_3 < t_4 < N < T, \\ \quad t_3 \le M < t_4 < N < T,\ t_2 \le M < t_3 < t_4 < T < N,\ t_3 \le M < t_4 < T < N, \\ 0,\ otherwise; \end{cases}$$

$$\Xi_2 = \begin{cases} [M;N], \; t_2 \le M < N < t_3 < T, \; t_3 \le M < N < t_4 < T, t_2 \le M < t_3 < N < t_4 < T, \\ [M;t_4], t_2 \le M < t_3 < t_4 < N < T, t_3 \le M < t_4 < N < T, \; t_2 \le M < t_3 < t_4 < T < N, \; t_3 \le M < t_4 < T < N \\ 0, \; otherwise; \end{cases}$$

$$\Psi_1 = \begin{cases} 1, t_2 \le M < t_3 < T, \; t_3 \le M < t_4 < T, \; t_2 \le M < N < t_3 < t_4 < T, \; t_3 \le M < N < t_4 < T, \\ \quad t_2 \le M < t_3 < N < t_4 < T, \\ 0, otherwise; \end{cases}$$

$$\Xi_3 = \begin{cases} [M;t_4], \; t_2 \le M < t_3 < T, \; t_3 \le M < t_4 < T, \\ [N;t_4], t_2 \le M < N < t_3 < t_4 < T, \; t_3 \le M < N < t_4 < T, \; t_2 \le M < t_3 < N < t_4 < T, \\ 0, \; otherwise; \end{cases}$$

K – the ordering cost, P_c – the purchase cost is given by

$$P_c = \frac{p_c}{T}\left(\begin{array}{l} \int\limits_0^M e^{-kt}dt \left(\begin{array}{l} \int\limits_{\Xi_1} Ddt + ((((1-\beta)(1-a_1)+\beta a_2)\lambda - D)(t_2 - t_1) - I_s) + \\ + ((((1-\beta_r)(1-a_{r1})+\beta_r a_{r2})\lambda_r - D)\cdot \Xi_4 - (\Phi - I_s)) + \mathrm{H} \end{array} \right) + \\ + \Psi_0 \int\limits_0^N e^{-kt}dt \left(\int\limits_{\Xi_2} Ddt + \mathrm{H}_1 \right) + \Psi_1 \int\limits_0^{t_4} e^{-kt}dt \left(\int\limits_{\Xi_3} Ddt + \mathrm{H}_2 \right) \end{array} \right),$$

$$\Xi_4 = \begin{cases} (M - t_2), t_2 \le M < t_3 < T, t_2 \le M < N < t_3 < T, t_2 \le M < t_3 < N < t_4 < T, \\ \qquad t_2 \le M < t_3 < t_4 < N < T, t_2 \le M < t_3 < t_4 < T < N, \\ (t_3 - t_2), t_3 \le M < t_4 < T, t_4 \le M < T, M > T, \; t_3 \le M < N < t_4 < T, t_4 \le M < N < T, N > M > T, \\ \qquad t_3 \le M < t_4 < N < T, \; t_3 \le M < t_4 < T < N, t_4 \le M < T < N; \end{cases}$$

$$\Phi = \begin{cases} I_m, \; t_3 \le M < t_4 < T, \; t_4 \le M < T, M > T, \; t_3 \le M < N < t_4 < T, \; t_4 \le M < N < T, \; N > M > T, \\ \qquad t_3 \le M < t_4 < N < T, t_3 \le M < t_4 < T < N, \; t_4 \le M < T < N, \\ I(M), otherwise; \end{cases}$$

$$\mathrm{H} = \begin{cases} (I_m - D(t_4 - t_3)), t_4 \le M < T, \; M > T, \; t_4 \le M < N < T, \;\; N > M > T, t_4 \le M < T < N, \\ (I_m - I(M) - D(M - t_3)), \; t_3 \le M < t_4 < T, t_3 \le M < N < t_4 < T, t_3 \le M < t_4 < N < T, t_3 \le M < t_4 < T, \\ 0, \; otherwise; \end{cases}$$

$$\mathrm{H}_1 = \begin{cases} (((1-\beta_r)(1-a_{r1})+\beta_r a_{r2})\lambda_r - D)(N - M) - (I(N) - I(M)), t_2 \le M < N < t_3 < T, \\ (I(M) - I(N) - D(N - M)), \; t_3 \le M < N < t_4 < T, \\ (((1-\beta_r)(1-a_{r1})+\beta_r a_{r2})\lambda_r - D)(t_3 - M) - (I_m - I(M)) + (I_m - I(N) - D(N - t_3)), \; t_2 \le M < t_3 < N < t_4 < T, \\ (((1-\beta_r)(1-a_{r1})+\beta_r a_{r2})\lambda_r - D)(t_3 - M) - (I_m - I(M)) + (I_m - D(t_4 - t_3)), \; t_2 \le M < t_3 < t_4 < N < T, \\ \qquad\qquad t_2 \le M < t_3 < t_4 < T < N, \\ (I(M) - D(t_4 - M)), t_3 \le M < t_4 < N < T, \; t_3 \le M < t_4 < T < N, \\ 0, \; otherwise; \end{cases}$$

$$\mathrm{H}_2 = \begin{cases} \left(\left(\left(1-\beta_r\right)\left(1-a_{r1}\right)+\beta_r a_{r2}\right)\lambda_r - D\right)\left(t_3 - M\right) - \left(I_m - I(M)\right) + \left(I_m - D\left(t_4 - t_3\right)\right), \ t_2 \le M < t_3 < T, \\ \left(I(M) - D\left(t_4 - M\right)\right), \ t_3 \le M < t_4 < T, \\ \left(\left(\left(1-\beta_r\right)\left(1-a_{r1}\right)+\beta_r a_{r2}\right)\lambda_r - D\right)\left(t_3 - N\right) - \left(I_m - I(N)\right) + \left(I_m - D\left(t_4 - t_3\right)\right), \ t_2 \le M < N < t_3 < T, \\ \left(I(N) - D\left(t_4 - N\right)\right), \ t_3 \le M < N < t_4 < T, \ t_2 \le M < t_3 < N < t_4 < T, \\ 0, \ otherwise; \end{cases}$$

C_{si} – the (inspection) screening cost is given by

$$C_{si} = \frac{c_{sc}}{T}\left(\lambda t_2 + \lambda_r\left(t_3 - t_2\right)\right)\int_0^{t_3} e^{-kt}dt,$$

C_{rg} – the cost for return the rejection items to supplier is given by

$$C_{rg} = \frac{c_r}{T}\left(\left(\left(1-\beta\right)a_1 + \beta\left(1-a_2\right)\right)\left(\lambda t_1 \int_0^{t_1} e^{-kt}dt + \lambda\left(t_2 - t_1\right)\int_0^{t_2} e^{-kt}dt\right)\right),$$

C_h – the inventory holding cost per cycle is given by

$$C_h = \frac{1}{T}\left(h_s\left(\int_{t_1}^{t_4} I(t)dt \int_0^{t_3} e^{-kt}dt\right) + h_r \left(\begin{array}{l} \frac{1}{2}\left(\left(1-\beta\right)a_1 + \beta\left(1-a_2\right)\right)\lambda t_1^2 \int_0^{t_1} e^{-kt}dt + \\ + \frac{1}{2}\left(\left(1-\beta\right)a_1 + \beta\left(1-a_2\right)\right)\lambda\left(t_2 - t_1\right)t_2 \int_0^{t_2} e^{-kt}dt + \\ + \frac{1}{2}\left(\left(1-\beta_r\right)a_{r1} + \beta_r\left(1-a_{r2}\right)\right)\lambda_r\left(t_3 - t_2\right)\left(T - t_2\right)\int_0^{T} e^{-kt}dt + \\ \frac{1}{2}\left(\beta a_2 \lambda t_2 + \beta_r a_{r2}\lambda_r\left(t_3 - t_2\right)\right)T\int_0^{T} e^{-kt}dt \end{array} \right)\right),$$

C_d – the deterioration cost is given by

$$C_d = \frac{1}{T}\left(c_{dg} + \frac{c_{out}\left(1-\gamma\right)}{\gamma}\right)\left(\begin{array}{l} \left(\left(\left(\left(1-\beta\right)\left(1-a_1\right)+\beta a_2\right)\lambda - D\right)\left(t_2 - t_1\right) - I_s\right) + \\ + \left(\left(\left(\left(1-\beta_r\right)\left(1-a_{r1}\right)+\beta_r a_{r2}\right)\lambda_r - D\right)\left(t_3 - t_2\right) - \left(I_m - I_s\right)\right) + \left(I_m - D\left(t_4 - t_3\right)\right) \end{array} \right)\int_0^{t_4} e^{-kt}dt,$$

C_s – the shortage cost per cycle due to backlog is given by

$$C_s = \frac{c_{sh}}{T}\left(\int_0^{t_1} -I(t)e^{-kt}dt + \int_{t_4}^{T} -I(t)dt \int_0^{T} e^{-kt}dt\right),$$

C_{un} – the opportunity cost due to lost sale per cycle is given by

$$C_{un} = \frac{c_u D}{T}\left(\int_{t_4}^{T}\left(1 - \frac{1}{1+\delta\left(t - t_4\right)}\right)dt \int_0^{T} e^{-kt}dt\right),$$

C_{adi} – the cost of accepting a defective items

$$C_{adi} = \frac{c_{ad}}{T}\left(\left(1-\beta\right)a_1 \lambda t_2 \int_0^{t_3} e^{-kt}dt\right),$$

C_{rnd} – the cost of rejection a non-defective items

$$C_{rnd} = \frac{c_{nd}}{T}\left(\left(\beta a_2 \lambda t_2 + \beta_r a_{r2} \lambda_r (t_3 - t_2)\right) \int_0^T e^{-kt} dt \right),$$

IP – interest paid is given by

$$IP = \frac{1}{T}\left(p_c \left(\Phi_1 \int_0^{t_4} e^{-kt} dt \int_{\Xi_3} I(t) dt + \Psi_0 \cdot i_p \int_0^N e^{-kt} dt \left(\mathrm{H}_3 \int_{\Xi_2} I(t) dt \right)\right)\right),$$

$$\Phi_1 = \begin{cases} i_p, \ t_2 \le M < t_3 < T, \ t_3 \le M < t_4 < T, \\ i_w, \ t_2 \le M < N < t_3 < t_4 < T, \ t_3 \le M < N < t_4 < T, \ t_2 \le M < t_3 < N < t_4 < T, \\ 0, \ otherwise; \end{cases}$$

$$\mathrm{H}_3 = \begin{cases} (1 + N - t_4), \ t_2 \le M < t_3 < t_4 < N < T, \ t_3 \le M < t_4 < N < T, \ t_2 \le M < t_3 < t_4 < T < N, \ t_3 \le M < t_4 < T < N, \\ 0, \ otherwise; \end{cases}$$

IE – interest earned is given by

$$IE = \frac{1}{T}\left(p_v \cdot i_e \left(\int_0^M e^{-kt} dt \left(\int_{\Xi_1} Dt dt + \Theta_1 \int_{\Xi_1} D dt \right) + \Psi_0 \int_0^N e^{-kt} dt \left(\int_{\Xi_2} Dt dt + \Theta_2 \int_{\Xi_2} D dt \right)\right)\right),$$

$$\Theta_1 = \begin{cases} (M - t_4), \ t_4 \le M < T, \ M > T, \ t_4 \le M < N < T, \ N > M > T, \ t_4 \le M < T < N, \\ 0, \ otherwise; \end{cases}$$

$$\Theta_2 = \begin{cases} (N - t_4), \ t_2 \le M < t_3 < t_4 < N < T, \ t_3 \le M < t_4 < N < T, \ t_2 \le M < t_3 < t_4 < T < N, \\ \qquad t_3 \le M < t_4 < T < N, \\ 0, \ otherwise; \end{cases}$$

p_v – selling price per unit of good quality items, p_b – selling price per unit for defective items, p_c – purchasing cost per unit, c_{sc} – screening (inspection) cost per unit, c_r – the cost for return the rejection item to supplier per monetary per unit, h_s – holding cost of serviceable items per monetary per unit per unit time, h_r – holding cost of imperfect quality items per monetary per unit per unit time, c_{dg} – deterioration cost per monetary per unit, c_{out} – penalty cost of selling deteriorated items to customer per monetary per unit, c_{sh} – unit shortage cost per monetary per unit per unit time, c_u – cost of lost sale per monetary per unit, c_{ad} – cost of accepting a defective item, c_{nd} – cost of rejection a non-defective items.

Hence our crisp problem is

$$\max\ ROS_i(l_b,l_m,l_s)=\begin{cases} ROS_1(l_b,l_m,l_s), & t_2 \le M < t_3 < T, \\ ROS_2(l_b,l_m,l_s), & t_3 \le M < t_4 < T, \\ ROS_3(l_b,l_m,l_s), & t_4 \le M < T, \\ ROS_4(l_b,l_m,l_s), & M > T, \\ ROS_5(l_b,l_m,l_s), & t_2 \le M < N < t_3 < T, \\ ROS_6(l_b,l_m,l_s), & t_3 \le M < N < t_4 < T, \\ ROS_7(l_b,l_m,l_s), & t_4 \le M < N < T, \\ ROS_8(l_b,l_m,l_s), & N > M > T, \\ ROS_9(l_b,l_m,l_s), & t_2 \le M < t_3 < N < t_4 < T, \\ ROS_{10}(l_b,l_m,l_s), & t_2 \le M < t_3 < t_4 < N < T, \\ ROS_{11}(l_b,l_m,l_s), & t_3 \le M < t_4 < N < T, \\ ROS_{12}(l_b,l_m,l_s), & t_2 \le M < t_3 < t_4 < T < N, \\ ROS_{13}(l_b,l_m,l_s), & t_3 \le M < t_4 < T < N, \\ ROS_{14}(l_b,l_m,l_s), & t_4 \le M < T < N. \end{cases} \tag{30}$$

subject to the constraints

$$E_{\min} \le P_C \le E_{\max}\ ,$$

$$b \cdot c \cdot d \cdot \lambda \cdot (t_1 + t_2) \le W\ ,$$

$$C_h < P_C\ ,$$

$$P\left(a_2 \le 1 - \frac{u+1}{\beta \cdot \lambda (t_1 + t_2)}\right) \le p_s\ ,$$

where $ROS_i(l_b,l_m,l_s)$, $i = 1,2,\ldots,14$ is given by Eq. (25).

Fuzzy expected value inventory model. In this paper, we have considered the demand rate, selling prices, interest paid rate and interest earned rate as fuzzy variables to tackle the reality in more effective way. When the parameters $\tilde{D}$, $\tilde{p}_v$ $\tilde{p}_b$, $\tilde{i}_p$, $\tilde{i}_w$, $\tilde{i}_e$ (as per assumption) treated as fuzzy variables, the above inventory expressions become fuzzy and thereby the return on sales becomes fuzzy variables on the credibility space $(X, P(X), \mathrm{Cr})$. Let $\tilde{D}$, $\tilde{p}_v$, $\tilde{p}_b$, $\tilde{i}_p$, $\tilde{i}_w$ and $\tilde{i}_e$ be defined by trapezoidal and triangular fuzzy numbers such that $\tilde{D} = [D_1, D_2, D_3, D_4]$, $\tilde{p}_v = [p_{v_1}, p_{v_2}, p_{v_3}, p_{v_4}]$, $\tilde{p}_b = [p_{b_1}, p_{b_2}, p_{b_3}, p_{b_4}]$, $\tilde{i}_p = [i_{p_1}, i_{p_2}, i_{p_3}]$, $\tilde{i}_w = [i_{w_1}, i_{w_2}, i_{w_3}]$ and $\tilde{i}_e = [i_{e_1}, i_{e_2}, i_{e_3}]$, where $(D_1 < D_2 < D_3 < D_4)$, $(p_{v_1} < p_{v_2} < p_{v_3} < p_{v_4})$, $(p_{b_1} < p_{b_2} < p_{b_3} < p_{b_4})$, $(i_{p_1} < i_{p_2} < i_{p_3})$, $(i_{w_1} < i_{w_2} < i_{w_3})$ and $(i_{e_1} < i_{e_2} < i_{e_3})$ based on the subjective judgments. If the decision maker wants to determine optimal pricing and inventory police such that fuzzy expected value of the return on sale is maximal, a fuzzy EVM can be constructed as follows,

$$\max \quad E[ROS_i(I_b, I_m, I_s)] = \begin{cases} E[ROS_1(I_b, I_m, I_s)], & t_2 \le M < t_3 < T, \\ E[ROS_2(I_b, I_m, I_s)], & t_3 \le M < t_4 < T, \\ E[ROS_3(I_b, I_m, I_s)], & t_4 \le M < T, \\ E[ROS_4(I_b, I_m, I_s)], & M > T, \\ E[ROS_5(I_b, I_m, I_s)], & t_2 \le M < N < t_3 < T, \\ E[ROS_6(I_b, I_m, I_s)], & t_3 \le M < N < t_4 < T, \\ E[ROS_7(I_b, I_m, I_s)], & t_4 \le M < N < T, \\ E[ROS_8(I_b, I_m, I_s)], & N > M > T, \\ E[ROS_9(I_b, I_m, I_s)], & t_2 \le M < t_3 < N < t_4 < T, \\ E[ROS_{10}(I_b, I_m, I_s)], & t_2 \le M < t_3 < t_4 < N < T, \\ E[ROS_{11}(I_b, I_m, I_s)], & t_3 \le M < t_4 < N < T, \\ E[ROS_{12}(I_b, I_m, I_s)], & t_2 \le M < t_3 < t_4 < T < N, \\ E[ROS_{13}(I_b, I_m, I_s)], & t_3 \le M < t_4 < T < N, \\ E[ROS_{14}(I_b, I_m, I_s)], & t_4 \le M < T < N, \end{cases} \tag{31}$$

subject to the constraints

$$E_{\min} \le E\left[\tilde{P}_C\right] \le E_{\max},$$

$$b \cdot c \cdot d \cdot \lambda \cdot \left(E\left[\tilde{t}_1\right] + E\left[\tilde{t}_2\right]\right) \le W,$$

$$E\left[\tilde{C}_h\right] < E\left[\tilde{P}_C\right],$$

$$P\left(a_2 \le 1 - \frac{u+1}{\beta \cdot \lambda \left(E\left[\tilde{t}_1\right] + E\left[\tilde{t}_2\right]\right)}\right) \le p_s,$$

where

$$E[ROS_i(I_b, I_m, I_s)] = E\left[\frac{\left(\tilde{R}_s\right)_i - K - \left(\tilde{P}_c\right)_i - \tilde{C}_{si} - \tilde{C}_{rg} - \tilde{C}_h - \tilde{C}_d - \tilde{C}_s - \tilde{C}_{un} - \tilde{C}_{adi} - \tilde{C}_{rnd} - I\tilde{P}_i + I\tilde{E}_i}{\left(\tilde{R}_s\right)_i}\right],$$

$i = 1,2,...,14$.

Next sections carried out the simulation technique to estimate the fuzzy parameters and solution methodology for fuzzy expected value model along with theoretical results to identify global optimal solution for (I_b, I_m, I_s).

Fuzzy simulation technique

In above-mention inventory system, frequently, the decision maker wants to control the fuzzy revenue on sales in each replenishment cycle such that the critical value of the fuzzy revenue on sales is maximal. Similarly, in model (31) it needs to find the appropriate vector (I_b, I_m, I_s) such that satisfies the constraints and reaches it maximal value. For the fixed value of (I_b, I_m, I_s) a fuzzy simulation technique [Taleizadeh, 2013] is employed to estimate the fuzzy parameters such as demand, selling prices, interest paid rate and interest earned rate.

Step 1. Set $E = 0$ and initialized G and O.

Step 2. We randomly generate sequences $\left(D^g, p_v^g, p_b^g, i_p^g, i_w^g, i_e^g\right)$ from the α-level sets of fuzzy variables $\tilde{D}$, $\tilde{p}_v$, $\tilde{p}_b$, $\tilde{i}_p$, $\tilde{i}_w$, $\tilde{i}_e$, $g = 1,2,...,G$, where α is a sufficiently small positive number.

Step 3. Calculate $ROS_i\left(l_b, l_m, l_s, D^g, p_v^g, p_b^g, i_p^g, i_w^g, i_e^g\right)$ for $g = 1,2,\ldots,G,\ \ i = 1,2,\ldots,14$.

Step 4. Set $a_i = ROS_i\left(l_b, l_m, l_s, D^1, p_v^1, p_b^1, i_p^1, i_w^1, i_e^1\right) \wedge \ldots \wedge ROS_i\left(l_b, l_m, l_s, D^g, p_v^g, p_b^g, i_p^g, i_w^g, i_e^g\right)$,

$$b_i = ROS_i\left(l_b, l_m, l_s, D^1, p_v^1, p_b^1, i_p^1, i_w^1, i_e^1\right) \vee \ldots \vee ROS_i\left(l_b, l_m, l_s, D^g, p_v^g, p_b^g, i_p^g, i_w^g, i_e^g\right).$$

Step 5. Randomly generate s_i from $\left[a_i, b_i\right]$.

Step 6. If $s_i \ge 0$, then $E_i \leftarrow E_i + Cr\left\{ROS_i\left(l_b, l_m, l_s, \widetilde{D}, \widetilde{p}_v, \widetilde{p}_b, \widetilde{i}_p, \widetilde{i}_w, \widetilde{i}_e\right) \ge s_i\right\}$,

where

$$Cr\left\{ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) \ge s_i\right\} = \frac{1}{2}\left(\begin{array}{l} \max\limits_{j=1,2,\ldots,O}\left\{\mu_{ij} \middle| ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) \ge s_i\right\} + 1 - \\ - \max\limits_{j=1,2,\ldots,O}\left\{\mu_{ij} \middle| ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) < s_i\right\} \end{array}\right)$$

and $\mu_i\left(D^g, p_v^g, p_b^g, i_p^g, i_e^g\right) = \mu_i\left(D^g\right) \wedge \mu_i\left(p_v^g\right) \wedge \mu_i\left(p_b^g\right) \wedge \mu_i\left(i_p^g\right) \wedge \mu_i\left(i_w^g\right) \wedge \mu_i\left(i_e^g\right)$.

Otherwise, $E_i \leftarrow E_i - Cr\left\{ROS_i\left(l_b, l_m, l_s, \widetilde{D}, \widetilde{p}_v, \widetilde{p}_b, \widetilde{i}_p, \widetilde{i}_w, \widetilde{i}_e\right) \le s_i\right\}$,

where

$$Cr\left\{ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) \le s_i\right\} = \frac{1}{2}\left(\begin{array}{l} \max\limits_{j=1,2,\ldots,O}\left\{\mu_{ij} \middle| ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) \le s_i\right\} + 1 - \\ - \max\limits_{j=1,2,\ldots,O}\left\{\mu_{ij} \middle| ROS_i\left(l_b, l_m, l_s, D_j, p_{vj}, p_{bj}, i_{pj}, i_{wj}, i_{ej}\right) > s_i\right\} \end{array}\right).$$

Step 7. Repeat 5 and 6 for O times.

Step 8. Calculate

$$E\left[ROS_i\left(l_b, l_m, l_s, \widetilde{D}, \widetilde{p}_v, \widetilde{p}_b, \widetilde{i}_p, \widetilde{i}_w, \widetilde{i}_e\right)\right] = \int_0^{\infty} Cr\left\{ROS_i\left(l_b, l_m, l_s, \widetilde{D}, \widetilde{p}_v, \widetilde{p}_b, \widetilde{i}_p, \widetilde{i}_w, \widetilde{i}_e\right) \ge s_i\right\} ds - \\ - \int_{-\infty}^{0} Cr\left\{ROS_i\left(l_b, l_m, l_s, \widetilde{D}, \widetilde{p}_v, \widetilde{p}_b, \widetilde{i}_p, \widetilde{i}_w, \widetilde{i}_e\right) \le s_i\right\} ds$$

as $E\left[ROS_i\left(l_b, l_m, l_s, \widetilde{D}, \widetilde{p}_v, \widetilde{p}_b, \widetilde{i}_p, \widetilde{i}_w, \widetilde{i}_e\right)\right] = a_i \vee 0 + b_i \wedge 0 + E_i \dfrac{b_i - a_i}{O}$.

So far, we have constructed EOQ in fuzzy sense, and known that the EVM can be solved with generic approaches. However, the model needs to be solved with the heuristics algorithm owing the complexity of problem. There are several heuristic algorithms inspired from evolution of nature, such as Genetic Algorithms [Goldberg, 1989], Evolutionary Strategies [Rechenberg, 1994] and Ant Colony Algorithm [Dorigo, 1996]. Here in order to solve the EVM, we choose Genetic Algorithm as the foundation to design an algorithm which integrates fuzzy simulation and Genetic Algorithm, where the fuzzy simulation is employed to estimate the maximal revenue on sales, and Genetic Algorithm is used to find the optimal solution.

Genetic algorithms based on the fuzzy simulation

Genetic algorithm is a member of the wide category of evolutionary algorithms, which was originally proposed by Goldberg [Goldberg, 1989]. It is a heuristic optimization technology based on such natural evolution mechanisms as crossover, mutation and natural selection. Till now, GA has been successfully applied to a wide range of applications. The comprehensive survey of GA and its application to solve inventory control problems can be found in [Taleizadeh, 2013].

In what follows the main characteristics of the genetic algorithm employed in this research are described. In this paper, the chromosomes are the strings of the unfilled order backlog I_b, inventory level of serviceable items I_s and maximum inventory level of serviceable items I_m. Therefore, a binary vector as a chromosome to represent real values of the variables I_b, I_s and I_m. The length of the vector depends on the required precision. Moreover, infeasible chromosomes, the ones that do not satisfy the constraints of the model in (31) are not considered.

The crossover operator creates two new chromosomes from a pair of selected chromosomes of the parents' generation. There are a number of special crossover operations for binary-coded data: one-point crossover, two-point crossover, n-point crossover, uniform crossover. In this research, a single point crossover with different probabilities P_c of 0.5, 0.6, 0.7 is utilized.

The mutation operator changes the value of a random selected element of the chromosome. To do this, a random number RN between $(0,1)$ is generated for each gene. If RN is less than a predetermined mutation probability P_m, than the mutation occurs in the gene. Otherwise, it does not.

In a maximization problem, the more adequate the solution, the greater the objective function (fitness value) will be. Therefore, the fittest chromosomes will take part in offspring generation with a large probability. The fuzzy simulation is used to evaluate the objective function of this research.

The selection operator ensures the selection of well-performing chromosomes for the reproduction process. The most popular techniques to duplicates of good individuals are tournament and elitism. The practical design witnessed advantage of exactly elitism as at him optimum vectors-decisions are not lost. From all types of selection only for elitism it is proved [Harti, 1990] in a theory that the iterative search process of optimum decision meets. Also there are other selection methods such as panmictic, inbreeding, outbreeding and proportional selection.

Stopping condition for the model optimization can be defined in a few different ways:

- By entering the total number of the generations;
- Achievement of necessary value of fitness-function;
- Selective population consists of identical elements;
- By introducing a positive number ε such that the optimization is stopped, if the condition $f_{\max} - f_{\min} < \varepsilon$ is fulfilled, $f_{\max}$ and $f_{\min}$ being maximum and minimum objective function value, respectively.

Before starting the optimization algorithm, it is necessary select the value ε in the stopping condition (or the number of generation), the population size L, and using the penalty function method converted constrained problem to an unconstrained.

In short, the steps involved in the GA algorithm used in this research are

1. Set the parameters P_c, P_m and randomly create the initial population of size L (the individuals should satisfy the constraints).
2. Calculate the fitness value for each chromosome.
3. Select an individual for mating pool by tournament selection method using elitism.
4. Apply a crossover operator to each selected pair of chromosome with probability P_c.
5. Apply a mutation operator to a randomly selected chromosome with probability P_m.

6. Replace the current population by the resulting new feasible population.
7. Calculate the objective function.
8. If the stopping conditions are met, stop. Otherwise, go to step 4.

Conclusion

In this paper, a stochastic replenishment inventory model was development. Nonlinear programming model for deterioration items with two level of trade credit in one replenishment cycle, time value of money, inspection errors, planned backorders, and sales returns have been proposed. We have considered the EOQ inventory problem by characterizing the demand rate, the selling price for good quality items, the selling price for defective items, interest paid rate and interest earned rate the as fuzzy variable and constructed the fuzzy expected value model. Applying the fuzzy simulation technique, we have estimated the expected value of fuzzy cost parameters and interest rates. Whereas the fuzzy expected value model is hard to solve with analytic methods, and we have designed a genetic algorithm based on the fuzzy simulation to solve it.

Some avenues for future works follow:

- the holding cost or other parameters of the problem may take uncertain forms (stochastic or rough) as well;
- some other deterioration function rather than exponential may be considered for the deterioration rate;
- some other meta-heuristic algorithms such as particle swarm may be employed to solve the problem;
- Evolutionary Technologies of Directed Optimization [Snytyuk, 2004] can be considered as an effective technique to solve the problem.

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Authors' Information

Olha Yegorova *– assistant; Cherkasy State Technological University, Shevchenko Blvd., bl.460, Cherkasy, 18006, Ukraine; e-mail: yegorovaov@gmail.com*

Major Fields of Scientific Research: intelligence computations, fuzzy modeling technologies and decision making in economics