REDUCED GENERALIZED NETS WITH CHARACTERISTICS OF THE PLACES

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Abstract: *Generalized Nets with Characteristics of the Places (GNCP) are conservative extensions of the ordinary Generalized Nets (GNs). In the present paper, for the first time algorithm for transition functioning in GNCP is proposed. Some possible applications of GNCP are discussed. It is shown how GNCP can be used for evaluation of the places and to simplify the graphical structure of the net. In analogy with the concept of reduced GNs, reduced GNCP are introduced. It is proved that there exist two minimal reduced GNCP — one with characteristics of the places and one in which only the tokens obtain characteristics. An example of a reduced GNCP that describes the work of a fire company is presented.*

Keywords: *Algorithm for transition functioning, Extensions of generalized nets, Generalized nets, Reduced generalized nets.*

Introduction

Generalized Nets with Characteristics of the Places (GNCP) are conservative extensions of the ordinary Generalized Nets (GNs). They are defined in [Andonov & Atanassov, 2013]. GNCP is the ordered four-tuple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, Y, \Phi, \Psi, b\rangle\rangle \, .$$

All other components except the characteristic functions Y and Ψ are the same as in the standard GNs. For the definition of transition and GN the reader can refer to [Atanassov, 1991; Atanassov, 2007]. The characteristic function Y assigns initial characteristics to the places of the net. The function Ψ assigns characteristics to some of the places when tokens enter them. These characteristics can be the number of tokens in the place, the moment of time when they arrive or other data which is related to the place. The connection between GNCP and the Intuitionistic Fuzzy Generalized Nets of type 1 (IFGN1) and type 2 (IFGN2) is studied in [Andonov, 2013]. Two new extensions that combine the properties of GNCP on one hand, and IFGN1 and IFGN3 on the other, are introduced in [Andonov, 2013]. These are the Intuitionistic Fuzzy Generalized Nets with Characteristics of the Places of type 1(IFGNCP1) and type 3 (IFGNCP3). It is proved that the classes $\Sigma_{IFGNCP1}$ of all IFGNCP1 and $\Sigma_{IFGNCP3}$ of all IFGNCP3 are conservative extensions of the class Σ of the ordinary GNs .

GNCP can be used for evaluation of the work of the places on the basis of their characteristics. For example, let Δ_l denote the set of all good characteristics that can be assigned to place l and Ξ_l denote the set of all bad characteristics. Let $\chi^{l,t} = \langle\chi_1^l, \cdots, \chi_n^l\rangle$ be the n-tuple of the characteristics obtained by place l up to the time moment t. Let

$$I_\Delta^l(x_i^l) = \begin{cases} 1 & , \text{ if } \chi_i^l \in \Delta_l \\ 0 & , \text{ if } \chi_i^l \notin \Delta_l \end{cases}$$

and

$$I_\Xi^l(x_i^l) = \begin{cases} 1 & , \text{ if } \chi_i^l \in \Xi_l \\ 0 & , \text{ if } \chi_i^l \notin \Xi_l \end{cases} \, .$$

Then the characteristic function Ψ can assign to place l the ordered couple $\langle \mu_l^t, \nu_l^t \rangle$ where

$$\mu_l^t = \frac{\sum_{i=1}^{n} I_\Delta^l(\chi_i^l)}{n}$$

and

$$\nu_l^t = \frac{\sum_{i=1}^{n} I_\Xi^l(\chi_i^l)}{n}.$$

Obviously, $\mu_l^t, \nu_l^t \in [0,1]$ and $\mu_l^t + \nu_l^t = 1$. The ordered couple $\langle \mu_l^t, \nu_l^t \rangle$ is a fuzzy evaluation of place l at time t.

In the more general case, we may also have characteristics that are neither good nor bad. Then for the couple $\langle \mu_l^t, \nu_l^t \rangle$ we have $\mu_l^t, \nu_l^t \in [0,1]$ and $\mu_l^t + \nu_l^t \leq 1$. The number $\pi_l^t = 1 - \mu_l^t - \nu_l^t \leq 1$ corresponds to the degree of indeterminacy. In this case $\langle \mu_l^t, \nu_l^t \rangle$ is an intuitionistic fuzzy evaluation of the place. For fuzzy and intuitionistic fuzzy sets see [Atanassov, 2012].

GNCP can also be used to simplify the graphical structure of the net. Oftentimes, in a GN we have transitions for which a place is both input and output. If tokens loop in this place or other tokens from other input places can enter it but no tokens can be transferred from this place to other output places, then we can exclude this place from the set of input places of the transition. The characteristics of the tokens that loop in this place can be assigned to the place instead. For the transition Z in Figure 1 we have two such places — l_5 and l_6. If we use characteristics for these two places, the functioning of Z can be represented by the transition Z^* on the right in Figure 1. The two circles denote that the place can obtain characteristics.

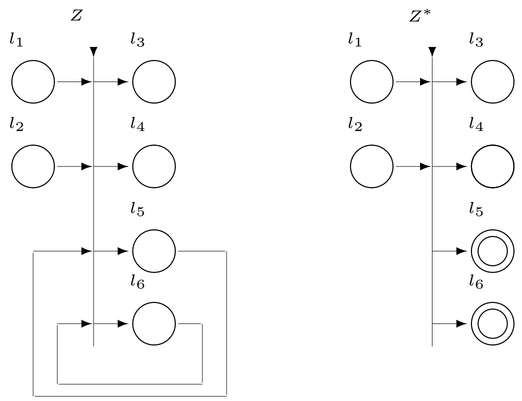

Figure 1

Algorithm for transition functioning in GNCP

The algorithms for transition and net functioning in the standard GNs can be found in [Atanassov, 2007]. Here for the first time we propose the algorithm for transition functioning in GNCP.

Algorithm A'

(A'01) The input and output places are ordered by their priorities.

(A'02) For every input place two lists are compounded. One with all tokens in the place ordered by their priorities and an empty list.

(A'03) An empty index matrix R which corresponds to the index matrix of the predicates r is generated. A value 0 (corresponding to truth-value "false") is asigned to all elements of R which:

- are in a row corresponding to empty input place;

- are in a column corresponding to full output place;
- are placed in a position (i, j) for which the current capacity of the arc between the i-th input and j-th output place is 0.

(A'04) The places are passed sequentially by order of their priorities starting with the place with the highest priority for which transfer has not occurred on the current time step and which has at least one token. For the token with highest priority from the first group we determine if it can split or not. The predicates in the row corresponding to the current input place are checked. If the token cannot split the checking of the predicates stops with the first predicate whose truth value is not 0. If the token can split, the truth values of all predicates in the row for which the elements of R are not equal to 0 are evaluated.

(A'05) Depending on the execution of the operator for permission or prohibition of tokens' splitting, the token from **(A'04)** is transferred either to all permitted output places or to the place with the highest priority for which the truth value of the corresponding predicate is 1. If a token cannot be transferred at the current time step, it is moved to the second group of the corresponding input place. The tokens which have been transferred are moved into the second group of the output places. The tokens which have entered the input place after the activation of the transition are moved to the second group too.

(A'06) The current number of tokens in every output place is increased with 1 for each token that has entered the place at the current time step. If the maximum number of tokens in an output place has been reached, the elements in the corresponding column of R are assigned value 0.

(A'07) The number of tokens in the input place is decreased by 1.

(A'08) The capacities of all arcs through which a token has passed decrement with 1. If the capacity of an arc has reached 0, value 0 is assigned to the element from the index matrix R that corresponds to this arc.

(A'09) The values of the characteristic function Φ for the corresponding output places (one or more) in which tokens have entered according to **(A'05)** are calculated. These values are assigned to the tokens.

(A'10) If there are more input places at the current time step from which tokens can be transferred, the algorithm proceeds to **(A'04)**, otherwise it proceeds to **(A'11)**.

(A'11) The values of the characteristic function Ψ for all output places to which tokens have been transferred are calculated. These values are assigned to the places.

(A'12) The current model time t' is increased with t^0.

(A'13) Is the current time moment equal or greater than $t^1 + t^2$? If the answer to the question is "no", go to go to **(A'04)**. Otherwise, terminate the functioning of the transition.

The algorithm for GNCP's functioning is the same as the general algorithm for GN's functioning denoted by *algorithm B* (see [Atanassov, 2007]), with the exception that the algorithm for transition functioning in GNCP is applied over the abstract transition. It is important to mention that in GNCP characteristics are assigned only to those output places to which tokens have been transferred at the current time step. This requirement justifies the use of characteristics of the places as a convenient way to track the changes in the places during the functioning of the net.

Reduced GNCP

GNs may or may not have some of the components in their definition. GNs which do not have some of the components form special classes called reduced GNs. For more details about the concept of reduced GNs see [Atanassov, 1991]. Let

$$\Omega = \{A, \pi_A, \pi_L, c, f, \theta_1, \theta_2, K, \pi_K, \theta_K, T, t^0, t^*, X, \Phi, b\} \cup \{A_i | 1 \leq i \leq 7\},$$

where $A_i = pr_i A$ is the i-th projection of the set A of the transitions of the net, i.e. $A_i \in \{L', L'', t_1, t_2, r, M, \square\}$. If Σ is the class of all GNs and $Y \in \Omega$, then by Σ^Y we denote the class of all GNs that do not have component

Y. They are called Y-*reduced GNs*. In [Atanassov, 1991] many assertions for the different classes of reduced GNs are proved.

Let Σ_1 and Σ_2 be subclasses of Σ. We will need the following definitions:

• $\Sigma_1 \vdash \Sigma_2$ *iff* the functioning and the results of the work of every element of Σ_2 can be described by an element of Σ_1.

• $\Sigma_1 \equiv \Sigma_2$ *iff* $\Sigma_1 \vdash \Sigma_2$ & $\Sigma_2 \vdash \Sigma_1$.

The class $\Sigma^* = \Sigma^{A_3,A_4,A_6,A_7,\pi_A,\pi_L,c,\theta_1,\theta_2,\pi_K,\theta_K,T,t^0,t^*,b}$ is the class of minimal reduced GNs (**-GNs*). The minimal reduced GNs have the form

$$E' = \langle\langle A', *, *, *, *, *, *\rangle, \langle K, *, *\rangle, *, \langle X, \Phi, *, *\rangle\rangle\,,$$

where

$$A' = \{Z' | Z' = \langle L', L'', *, *, r, *, *\rangle \& Z = \langle L', L'', t_1, t_2, r, M, \square\rangle \in A\}\,.$$

For the minimal reduced GNs the following notation is also used

$$E' = \langle A', K, X, \Phi\rangle\,.$$

The minimal elements of Σ^* are denoted by

$$E^* = \langle A^*, K^*, X^*, \Phi^*\rangle\,,$$

where A^* is the set of transitions of the form $Z^* = \langle L', L'', r'\rangle$.

The following theorem is proved in [Atanassov, 1991].

Theorem 1. $\Sigma \equiv \Sigma^*$.

To introduce the concept of reduced GNCP we need the following notation:

$$\Omega_{CP} = \{A, \pi_A, \pi_L, c, f, \theta_1, \theta_2, K, \pi_K, \theta_K, T, t^0, t^*, X, Y, \Phi, \Psi, b\} \cup \{A_i | 1 \le i \le 7\}\,,$$

where again $A_i = pr_i A$. By Σ^Y_{CP} we denote the class of those GNCP which do not have Y-component. In analogy to the standard reduced GNs they will be called Y-reduced GNCP. Again, as in the case of the reduced ordinary GNs we have

$$\Sigma^A_{CP} = \Sigma^{A_1}_{CP} = \Sigma^{A_2}_{CP} = \Sigma^K = \emptyset\,.$$

More generally, if $Y_1, Y_2, \cdots, Y_k \in \Omega_{CP}$ where $k \ge 1$ is natural number, then $\Sigma^{Y_1,Y_2,\cdots,Y_k}_{CP}$ will be called $(Y_1, Y_2, \cdots, Y_k)$-reduced class of GNCP. Now we have two classes of minimal reduced GNCP. In one of them only the tokens receive characteristics, while in the other characteristics are assigned only to the places. The class $\Sigma^{A_3,A_4,A_6,A_7,\pi_A,\pi_L,c,\theta_1,\theta_2,\pi_K,\theta_K,T,t^0,t^*,Y,\Psi,b}_{CP}$ coincides with the class Σ^*, i.e. all nets from this class have the form $E' = \langle A', K, X, \Phi\rangle$. With Σ^*_{CP} we will denote the class of minimal reduced GNCP in which only the places obtain characteristics. Formally,

$$\Sigma^*_{CP} = \Sigma^{A_3,A_4,A_6,A_7,\pi_A,\pi_L,c,\theta_1,\theta_2,\pi_K,\theta_K,T,t^0,t^*,X,\Phi,b}_{CP}\,.$$

The following theorems specify the connection between Σ, Σ_{CP}, Σ^* and Σ^*_{CP}.

Theorem 2. $\Sigma_{CP} \equiv \Sigma^*$.

Proof. In [Andonov & Atanassov, 2013] it is proved that $\Sigma_{CP} \equiv \Sigma$. From Theorem 1 we have $\Sigma \equiv \Sigma^*$. Therefore $\Sigma_{CP} \equiv \Sigma^*$.

A more detailed constructive proof, analogous to the proof of Theorem 2.2.1 in [Atanassov, 1991], would show how we can actually construct a minimal reduced GN given a GNCP.

Theorem 3. $\Sigma^* \equiv \Sigma^*_{CP}$.

Proof. First we will show that $\Sigma^*_{CP} \vdash \Sigma^*$. Let E be arbitrary minimal reduced generalized net from the class Σ^*.

$$E = \langle A, K, X, \Phi \rangle .$$

Every transition of E has the form $Z = \langle L', L'', r' \rangle$. Let Z be arbitrary transition of E. For simplicity, we shall consider that characteristics of the tokens are not used in the predicates of the transitions and in the characteristic function Φ. We construct a transition Z_{CP} with the same graphic structure as Z, i.e. the same number of input and output places and the same index matrix of the transitions condition. Let A_{CP} be the set of transitions obtained after repeating this procedure for all transitions of E. We will prove that the minimal reduced GNCP

$$G = \langle A_{CP}, *, *, *, *, *, * \rangle, \langle K_{CP}, *, * \rangle, *, \langle *, Y, *, \Psi, * \rangle$$

represents the functioning and the results of work of E. The set of tokens K_{CP} of G consists of the same number and types of tokens, i.e. for every token $\alpha' \in K_{CP}$ there is a corresponding token $\alpha \in K$. The function Y assigns the initial characteristics of the tokens in E to the places in which their corresponding tokens in G enter the net, i.e. if l and l_{CP} are two corresponding input places of E and G respectively, then $X_l = Y_{l_{CP}}$. The characteristic function Φ of E can be written in the form

$$\Phi = \bigcup_{l \in L - Q^I} \Phi_l = \bigcup_{Z \in A} (\bigcup_{l \in pr_2 Z} \Phi_l) ,$$

and similarly

$$\Psi = \bigcup_{l_{CP} \in L_{CP} - Q^I_{CP}} \Psi_{l_{CP}} = \bigcup_{Z_{CP} \in A_{CP}} (\bigcup_{l_{CP} \in pr_2 Z_{CP}} \Psi_{l_{CP}}) ,$$

where Q^I_{CP} is the set of input places of G and L_{CP} is the set of all places of G. The function $\Psi_{l_{CP}}$ assigns to the places of G a list of all tokens that have entered the places together with the characteristics that their corresponding tokens of E receive through the function Φ of E. If $\alpha^1_{CP}, \alpha^2_{CP}, ..., \alpha^k_{CP}$ are the tokens that have entered place l''_{CP} in G, then the characteristic of place l''_{CP} has the form

$$\text{“}\langle \langle \alpha^1_{CP}, \Phi_{l''}(\alpha^1) \rangle, \langle \alpha^2_{CP}, \Phi_{l''}(\alpha^2) \rangle, ..., \langle \alpha^k_{CP}, \Phi_{l''}(\alpha^k) \rangle \rangle\text{”} .$$

To prove that the so constructed reduced GNCP G represents the funtioning and the results of work of E we take two correspondig transitions $Z \in pr_1 pr_1 E$ and $Z_{CP} \in pr_1 pr_1 G$. Let $\alpha \in K$ and $\alpha_{CP} \in K_{CP}$ be two corresponding tokens that are respectively in places l' and l'_{CP}. If the token α is tranferred to output place l'', then α_{CP} is tranferred to the corresponding output place l''_{CP}. The characteristic obtained by α in l'' is assigned by the function Ψ to place l''_{CP}. If the token α cannot be transferred to any output place of Z, then α_{CP} also will not be tranferred to any output place. The case in which splitting of tokens is allowed can be verified in a similar way. From the theorem for the completeness of the GN transition it follows that $\Sigma^*_{CP} \vdash \Sigma^*$.

In the beginning of the proof we assumed that characteristics of tokens are not used in the predicates of the transition Z and in the characteristic function Φ. If such characteristics are used, then they must be substituted with the corresponding characteristics of the places in Z_{CP}. Let now G be arbitrary minimal reduced GNCP.

$$G = \langle A_{CP}, *, *, *, *, *, * \rangle, \langle K_{CP}, *, * \rangle, *, \langle *, Y, *, \Psi, * \rangle .$$

We will contruct a minimal reduced GN which represents the functioning and the results of the work of G. For every transition $Z_{CP} \in pr_1 pr_1 G$ (see Figure 2) we construct a corresponding transition Z (see Figure 3).

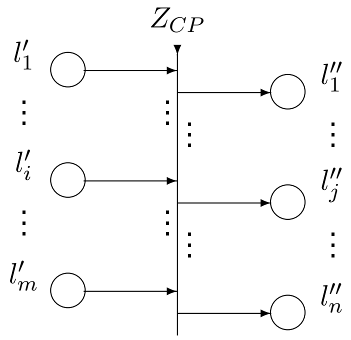

Figure 2

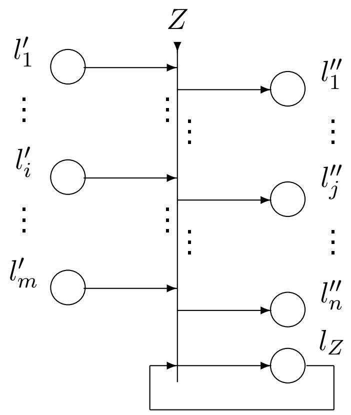

Figure 3

Again, we shall consider that characteristics of places are not used in the predicates of Z_{CP} and in the characteristic function Ψ. If $Z_{CP} = \langle L'_{CP}, L''_{CP}, r^{CP} \rangle$, then $Z = \langle L', L'', r \rangle$ where

$$L' = L'_{CP} \cup \{l_Z\},$$

$$L'' = L''_{CP} \cup \{l_Z\}.$$

To every transition we add additional place which is input and output for the transition where a token α_Z will loop and keep as characteristics the characteristics assigned to the corresponding output places of Z_{CP}. The initial characteristic of the token α_Z is a list of the initial characteristics of the places of the transition Z_{CP}. Place l_Z has the lowest priority among the places of the transition.

If

$$r^{CP} = pr_5 Z_{CP} = [L'_{CP}, L''_{CP}, \{r^{CP}_{l_i,l_j}\}]$$

has the form of an IM, then

$$r = pr_5 Z = [L'_{CP} \cup \{l_Z\}, L''_{CP} \cup \{l_Z\}, \{r_{l_i,l_j}\}],$$

where

$$(\forall l_i \in L'_{CP})(\forall l_j \in L''_{CP})(r_{l_i,l_j} = r^{CP}_{l_i,l_j})$$

$$(\forall l_i \in L')(\forall l_j \in L'')(r_{l_i,l_Z} = r_{l_Z,l_j} = \text{“}false\text{”}),$$

$$r_{l_Z,l_Z} = \text{“}true\text{”}.$$

Let A be the set of transitions obtained after repeating the above procedure for all transitions of G. We will prove that the minimal reduced GN

$$E = \langle A, K, X, \Phi \rangle$$

represents the functioning and the results of work of G.

$$K = K_{CP} \bigcup \{\alpha_Z | Z \in A_{CP}\}.$$

The characteristic function X assigns initial characteristic $x_0^{\alpha_Z}$ only to the α_Z tokens and it is a list of all places of the transition Z and their initial characteristics in G:

$$\text{“}\langle l'_1, Y(l'_{CP,1})\rangle, \langle l'_2, Y(l'_{CP,2})\rangle, ..., \langle l''_n, Y(l''_{CP,n})\rangle\text{”}.$$

The characteristic function Φ assigns to the α_Z-tokens a list with the output places of the transition and the characteristics of their corresponding output places of G in the form

$$\Phi_{\{l_Z|Z\in A\}}(\alpha_Z) = \text{“}\{\langle l''_j, \Psi(l''_{CP,j})\rangle | l''_j \in L''\}\text{”}.$$

The proof that the so constructed minimal reduced GN represents the functioning and the results of the work of G is similar to the proof that $\Sigma^*_{CP} \vdash \Sigma^*$. Now all characteristics of the places of G are kept as characteristics of the α_Z tokens. Thus we obtain $\Sigma^* \equiv \Sigma^*_{CP}$.

In the beginning of the proof, we assumed that characteristics of the places are not used in the predicates of the transition Z_{CP} and in the characteristic function Ψ. If such characteristics are used, then they must be substituted with the corresponding characteristics of the tokens in Z. □

From Theorem 2 and Theorem 3 we obtain

Theorem 4. $\Sigma_{CP} \equiv \Sigma^*_{CP}$.

Example of a reduced GNCP

A GN model that represents the work of a fire company is presented in Figure 4.

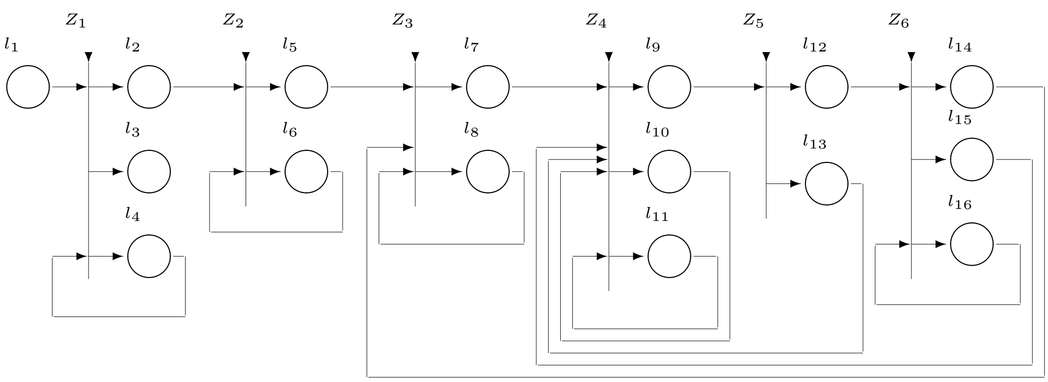

Figure 4. GN model of the process of fire extinguishing by a fire company.

Here we will contruct a reduced GNCP that describes the process of fire extinguishing by a fire company. The graphical representation of the net is in Figure 5.

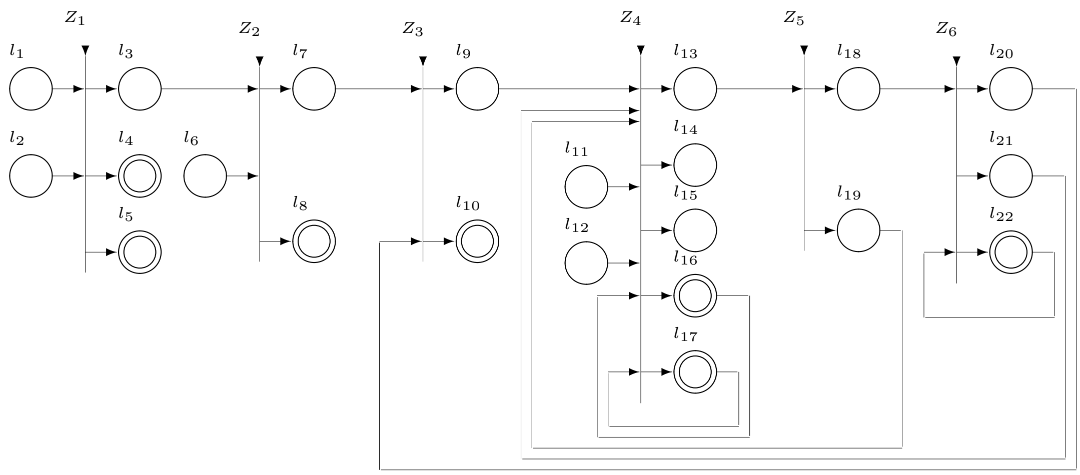

Figure 5. GNCP model of the process of fire extinguishing by a fire company.

The GNCP model presented in Figure 5 consists of six transitions and twenty two places.

- In Z_1 the alarm messages are filtered according to certain criteria.
- In Z_2 all available data for the place where a wildfire has been reported is collected. This can be coordinates, meteorological data, terrain profile etc.
- In Z_3 all available history for the place is collected.
- Z_4 represents the stuff and fire-fighting machinery.
- In Z_5 a decision is taken whether the available resources are enough to cope with the fire.
- Z_6 represents the place of the fire.

In the model we have five different types of tokens:

- α represents the alarm message;
- β represents the criteria for the correctness of the alarm messages;
- γ represents the database with data about the place where a fire is reported;
- δ represents all machinery available to the fire company;
- ϵ represents the fire-fighting stuff of the company.

The α tokens enter the net in place l_1 with initial characteristic

"alarm message".

During the functioning of the net β tokens may enter place l_2 with characteristic

"new criteria for correctness of the alarm messages".

During the functioning of the net γ tokens may enter place l_6 with initial characteristic

“new data about the coordinates, terrain profiles, meteorological conditions etc.".

During the functioning of the net δ tokens may enter place l_{11} with initial characteristic

“new machinery, type, number".

During the functioning of the net ϵ tokens may enter place l_{12} with initial characteristic

“stuff, name, decisions taken".

Token δ stays in place l_{16} in the initial time moment with characteristic

“machinery, type, number".

Token ϵ stays in place l_{17} in the initial time moment with characteristic

“stuff, name, decisions taken".

The places in Figure 5 represented by two concentric circles receive characteristics during the functioning of the net when tokens enter them. The initial characteristic of place l_5 is

“criteria for correctness of the alarm messages".

Place l_8 has initial characteristic

“database with coordinates, terrain profiles, meteorological conditions etc.".

Place l_{10} has initial characteristic

“data about previous fires".

Place l_{16} has initial characteristic

“machinery, type, number".

Place l_{17} has initial characteristic

“stuff, name, decisions taken" .

Place l_{22} does not have initial characteristic.

What follows is a formal description of the transitions of the net.

$$Z_1 = \langle \{l_1, l_2\}, \{l_3, l_4, l_5\}, r_1, \square_1 \rangle ,$$

where

$$r_1 = \begin{array}{c|ccc} & l_3 & l_4 & l_5 \\ \hline l_1 & W_{1,3} & W_{1,4} & false \\ l_2 & false & false & true \end{array}$$

and
$W_{1,3} =$*"The criterion shows that the alarm message is correct"*;
$W_{1,4} = \neg W_{1,3}$.

$$\Box_1 = \vee(l_1, l_2)\,.$$

If according to the criteria the alarm message is false, the α token enters place l_3 without new characteristic. Place l_4 obtains the characteristic

"false alarm, source of the alarm".

When the truth value of the predicate $W_{1,3}$ is "true" the α token enters place l_2 without new characteristic. Token β enters place l_5 without characteristic. Place l_5 obtains the characteristic

"new criteria for the correctness of the signals".

$$Z_2 = \langle \{l_3, l_6\}, \{l_7, l_8\}, r_2, \Box_2 \rangle\,,$$

where

$$r_2 = \begin{array}{c|cc} & l_7 & l_8 \\ \hline l_3 & true & false \\ l_6 & false & true \end{array}$$

$$\Box_2 = \vee(l_3, l_6)\,.$$

Upon entering place l_7 the α token obtains the characteristic

"coordinates of the fire, meteorological conditions, terrain profile".

The γ tokens enter place l_8 without new characteristic. Instead place l_8 receives the characteristic of the γ token.

$$Z_3 = \langle \{l_7, l_{20}\}, \{l_9, l_{10}\}, r_3, \Box_3 \rangle\,,$$

where

$$r_3 = \begin{array}{c|cc} & l_9 & l_{10} \\ \hline l_7 & true & false \\ l_{20} & false & true \end{array}$$

$$\Box_3 = \vee(l_7, l_{20})\,.$$

The token coming from place l_{20} enters place l_{10} without characteristc. Place l_{10} receives the characteristic of the token that has entered it. Upon entering place l_9 the α token obtains the characteristic

"data about previous fires at the place".

$$Z_4 = \langle \{l_9, l_{11}, l_{12}, l_{16}, l_{17}, l_{19}, l_{21}\}, \{l_{13}, l_{14}, l_{15}, l_{16}, l_{17}\}, r_4, \Box_4 \rangle,$$

where

$$r_4 = \begin{array}{c|ccccc} & l_{13} & l_{14} & l_{15} & l_{16} & l_{17} \\ \hline l_9 & true & false & false & false & false \\ l_{11} & false & false & false & true & false \\ l_{12} & false & false & false & false & true \\ l_{16} & W_{16,13} & W_{16,14} & false & true & false \\ l_{17} & W_{17,13} & false & W_{17,15} & false & false \\ l_{19} & W_{19,13} & false & false & false & false \\ l_{21} & W_{21,13} & false & false & false & false \end{array}$$

and
$W_{16,13}$ =*"A decision to send machinery is taken"*;
$W_{16,14}$ =*"There is damaged machinery"*;
$W_{17,13}$ =*"A decision to send more people is taken"*;
$W_{17,15}$ =*"The current stuff on duty must be changed"*.

$$\Box_4 = \wedge(\wedge(l_{16}, l_{17}), \vee(l_9, l_{11}, l_{12}, l_{19}, l_{21})).$$

When the truth value of the predicate $W_{17,13}$ is "true" token ϵ splits into two tokens — the original ϵ which remains in place l_{17} and ϵ' which enters place l_{13} with characteristic

"names of the fire fighting stuff sent to the place of the fire".

When the truth value of the predicate $W_{16,13}$ is "true" token δ splits into two tokens - the original δ which remains in place l_{16} and δ' which enters place l_{13} with characteristic

"type and numbers of the machinery which is sent to the place of the fire".

All tokens entering place l_{13} unite and generate a new token $\alpha_{\delta,\epsilon}$. The tokens from places l_{11} and l_{12} enter respectively places l_{16} and l_{17} where they unite with the δ and ϵ tokens. Place l_{16} obtains the characteristic

"relevant data about the machinery".

Place l_{17} obtains characteristic

"fire fighting stuff, names, decisions taken etc.".

When the truth value of the predicate $W_{16,14}$ is "true" the δ token in place l_{16} splits into two tokens - the original δ which reamains in l_{16} and a new δ'' which enters l_{14} with characteristic

"type of machinery, reason for leaving, time etc.".

When the truth value of the predicate $W_{17,15}$ is "true" the ϵ token in place l_{17} splits into two tokens - the original ϵ which reamains in l_{17} and a new ϵ'' which enters l_{15} with characteristic

"names of the people from the stuff, reason for leaving, time etc.".

$$Z_5 = \langle \{l_{13}\}, \{l_{18}, l_{19}\}, r_5, \square_5 \rangle ,$$

where

$$r_5 = \begin{array}{c|cc} & l_{18} & l_{19} \\ \hline l_{13} & true & W_{13,19} \end{array}$$

where
$W_{13,19}$ =*"The resources being sent are not enough"*.

$$\square_5 = \vee(l_{13}).$$

When the truth value of the predicate $W_{13,19}$ becomes "true" the $\alpha_{\delta,\epsilon}$ token splits into two tokens - the original which enters place l_{18} and a new token $\alpha'_{\delta,\epsilon}$ which enters place l_{19}. In place l_{18} the tokens do not obtain new characteristics. In place l_{19} the tokens obtain the characteristic

"Number of the additional stuff and machinery which is needed; type of machinery".

$$Z_6 = \langle \{l_{18}, l_{22}\}, \{l_{20}, l_{21}, l_{22}\}, r_6, \square_6 \rangle ,$$

where

$$r_6 = \begin{array}{c|ccc} & l_{20} & l_{21} & l_{22} \\ \hline l_{18} & false & false & true \\ l_{22} & W_{22,20} & W_{22,21} & true \end{array}$$

and
$W_{22,20}$ =*"The fire is extinguished"*;
$W_{22,21}$ =*"The resources at the place of the fire are not sufficient"*.

$$\square_6 = \vee(l_{18}, l_{22}).$$

In place l_{22} the tokens receive the characteristic

"current state of the fire".

Upon entering place l_{20} the $\alpha_{\epsilon,\zeta}$ token obtains the characteristic

"total burnt area, duration of the wildfire, estimated damages".

When the truth value of the predicate $W_{22,21}$ is "true" the $\alpha_{\delta,\epsilon}$ tokens splits into two tokens the original which remains in place l_{22} and $\alpha''_{\delta,\epsilon}$ which enters place l_{21} with characteristic

"number of the additional stuff and machinery which is needed; type of machinery".

Place l_{22} receives characteristic

"current state of the fire, weather conditions etc".

Conclusion

Most GN models developed so far are reduced ones. The reduced GNCP proposed in this paper are convenient tool for modelling of real processes. In comparison to the reduced ordinary GNs the reduced GNCP allow us to keep all data which is relevant to some of the places in the form of characteristics of these places. We have shown that there exist two minimal reduced classes of GNs — one with characteristics of the places and one with characteristics of the tokens. In the proof of Theorem 2 we used the already established result $\Sigma \equiv \Sigma^*$ but a direct constructive proof of this result would also show how we can construct reduced GN given a GNCP. We obtained Theorem 4 as a consequence from Theorem 2 and Theorem 3. A direct constructive proof of this result should also be presented in future.

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