ANALYSIS FINITELY SMALL VALUES IN COMPUTER SIMULATION

Oleksii Voloshyn, Vladimir Kudin

***Abstract**: We review the development of the infinity concept and continuous mathematics from ancient times to the present day. To the critical analysis of the application of the infinitesimal in the mathematical modeling in XVII-XVIII centuries is given special attention. It is noted necessity of development methods for the analysis of finite small disturbances in computer simulations. The problem of the adequacy of the linear approximation of mathematical and computer models is under discussion. We investigate the influence of finite small disturbances into elements of model to quality of localization solutions of the model which is based on the methodology of options sequential analysis. The results of numerical experiments and taken into account the level of systems conditionality and algorithms (the mantissa length in the numbers representation) are given.*

***Keywords**: continuous and infinite, infinitesimal analysis, mathematical and computer simulation, linear approximation, localization, successive analysis variants, analysis finitely small values*

***ACM Classification Keywords**: H.4.2 Information Systems Applications: Types of Systems: Decision Support.*

Introduction

Together with the invention of the Pythagorean School of incommensurability quantities in mathematics included the concept of infinity [Yushkevich, 1970]. The study of infinite sets and infinite sequences causes the need to solve two major problems that faced the ancient mathematics. This is - the problem of the real number (the modern name of the problem of incommensurability). And the problem measures (problems of actual infinity). Aporia Zeno of Elea (V century BC) raised profound and complex matters that are contained in terms of continuous and infinite, and that never cease to attract the attention of modern scientists and philosophers.

Leave aside all aspects of the problem except one - the problem of the relation of the mathematical model and the real physical world, having agreed with David Hilbert and P. Barnice ("Fundamentals of Mathematics", 1934г.): "... infinity was not given to us, and there was only interpolated or extrapolated by some intellectual process".

Eventually therefore for us the most important issue is the adequacy of the model and the simulated object (process) as well as an intermediate is - a question the correctness of the choice of means of a mathematical model, and in this context - is it permissible to use actual infinite quantities (infinitely large and infinitely small).

Antique mathematics rejected actual infinity - "... the denial of actual infinity does not take away their mathematicians theory, because they do not need such an infinite and do not use it ..." (Aristotle); "... At least there is a something small, but there is always an even smaller" (Anaxagoras). General teaching about relationships and rigorous methods of limiting transitions were created by Eudoxus of Cnidus (406-355 years. BC). "Banishment" of actual infinite quantities was carried out following the axiom (known as the Archimedean axiom): "The quantities are related to each other if they are taken Multiples can outdo each other". Eudoxus relations theory is used by mathematics essentially to the end of the nineteenth century, after announcing on Newton relations numbers [Yushkevich, 1970]. In the second half of the nineteenth century Eudoxus theory was developed by R.Dedekind. Between these theories there is so profound analogy. In one of the letters R.Lipshitz asked R.Dedekind [Yushkevich, 1970, P.97] what he did that was new in comparison with the ancient mathematicians (unlike relations Eudoxus of the modern notion of a real number is a system of relations that does not form a field but the group in terms of formal mathematics).

Difficulties associated with the actual infinity were not overcome the theory of G.Cantor sets (70th years of the nineteenth century.) They only changed the form and appeared as the paradoxes of set theory. In modern times there is a point of view according to which the free handling with the actual infinite sets, even denumerable one, is illegal (authors hold to this point of view as well). Extension of the concept of number was explicitly in Islamic countries in XI-XIII centuries (in Europe in XVI-XVII centuries). In both cases, it is in close connection with the development of computing technology and computers [Yushkevich, 1971]. Principal means of mathematical research in the natural sciences "New time" (XVI-XVII centuries) became infinitesimal methods, although understanding of the problems associated with their use by that time mathematicians.

Some prominent mathematicians concluded (apparently implying G.Leibniz thesis - "Useless paradoxes do not exist"), and that "... the error compensation is the driving force of the infinitesimal analysis and is at the same time strictly scientific method of cognition" [Yushkevich, 1973]. Extreme position in the debate about the infinitesimal was occupied by English philosopher Bishop George Berkeley (1685-1753 years): "... a careful study of affect that in any way is not necessary to use the infinitely small parts or quantities represent them smaller than the smallest felt by." Scientists such rank as Eyler, Lagrange and others had to reckon with witty and largely fair criticism Berkeley [Yushkevich, 1972, P.259].

From the perspective of Berkeley, infinite, as sensually imperceptible, have no real existence, such as one ten-thousandth of a piece of inch, and the more there can be no question of the infinite divisibility of any extended value. Two centuries later, in quantum physics, the notion "Planck level" - the smallest physically defined distance (equal to 10^{-35} m). It is known view of Academician A.N. Kolmogorov that in principle there is no number 10^{-100}.

In XIX-th century to construction of models of "wildlife" at research of biological, economic and social processes the analysis of the infinitesimal beginnings to be applied. And if at modeling of objects of "the lifeless nature" (in the physicist and, first of all, in the mechanic) by means of the differential equations the concept of influence on values of function of infinitesimal changes of argument "more or less adequately displays "physical" essence of process at modeling of social and economic processes it basically is incorrect. So, research of influence of infinitesimal change of the price of the goods (10^{-5}\$ or 10^{-35}\$) on demand for it looks as absurdity. Therefore differential models of change of economic parameters are confidants, very "rough". Failures in forecasting of development of social and economic processes appreciably speak this circumstance.

The second factor, which forces to reconsider the relation to infinitesimal models and methods, is computer facilities development in second half XX-th century. Wide use of simulating modeling of the numerical decision of analytical mathematical models have led to that the mathematics has returned "into place" and became, as well as 25 centuries ago, on the substance of final, accuracy of calculations is defined now by length of a mantissa of representation of numbers the computer.

Also does not play a special role what it is - 32, 64 or 1024 digit. Language „ε - δ" here does not work - that from this, what if for $\varepsilon = 0.01$, exists $\delta = 10^{-100}$? How numerically to check up criterion of Teatet (IV century BC) in commensurabilities of two segments [Yushkevich, 1970, C.77]: „If two segments are commensurable, the algorithm of Euclide for final number of steps finds their greatest general measure that is if the algorithm appears infinite segments are incommensurable"? What operating time of the computer to accept for "infinite" - 10, 10^{10}, 10^{100} seconds?

Therefore last decades "the classical" scheme of mathematical modeling "Object - Mathematical model - the Numerical decision" (O-MM-ND) extends at the expense of introduction of "Computer model" (CM) which basically is finit: "O-MM-CM-ND". In 80th years of XX century academician N. N. Moiseev asserted [Moiseyev, 1981] that "… the next decades it is necessary to give the basic attention to research to computer (simulation) models". Now researches of computer models are in a stage of development and are far from successful end.

The analysis finitly small influences in linear models

At the heart of differential and integral calculus (based on infinitesimal methods) the idea of linearization - replacement so difficult as far as possible (nonlinear) change of investigated process by linear one (tangent) is laid. Speaking in images, I.Newton and G.Leibniz „linearized the world". Therefore research of linear models (first of all, systems of the linear algebraic equations and inequalities) was and remains to one of the basic problems in mathematical and computer modeling, and the decision of practical problems causes of consideration of systems of the large dimension, systems with incomplete, inexact, indistinct parameters.

On the other hand, linear models (LM) are only the first approach, local approximation, at research of processes of the real world which is basically nonlinear. For well conditioned problems (even in "a smooth" situation, and in certain cases - and in "rough" [Clark, 1988]) the existing mathematical apparatus allows to estimate influence of small indignations of parameters on properties of model only locally.

However, in many cases approximating linear models are described in a class of systems poorly conditioned (ill-posed) problem, in particular, systems of the linear algebraic equations (SLAE) with the square and strongly filled matrix of restrictions [Samarskiy, 1989; Metjuz, 2001; Demmel, 2001]. Even at small dimension of model and in the absence of structural singularities in a matrix of restrictions of association of a solution from perturbations are approximated insufficiently adequately (It is localized by means of ellipses with axes of essentially various length or parallelepipeds with essentially different boundaries on variables). It is known that the category of poor conditionality is defining in construction of solutions associations (and localization areas) from perturbations in model elements [Samarskiy, 1989].

Experience of a solution of practical problems shows that for mathematical modeling inadequacy of model and real modeled process (appearance simplification, an inaccuracy of the representation of parameters), and for computing experiment - inadequacy of mathematical and computer model (ineradicable errors, errors of digitization, an error of a method, rounding off, truncations, losses of significant figures is characteristic, at performance of operations). The rounding off of numbers to within the fixed value at evaluations is connected with the approached representation of numbers with the final (truncated) digit grid (numbers with fixed and a floating point). "The classical" continuity guarantees existence local somehow a small neighborhood of a solution. (Problems of necessity of definition of a solution with any degree of accuracy now are not considered). In case of computer (discrete) model the solution neighborhood cannot be less than in advance set number so, is not correct local approximation. Round-off errors for such evaluations can be characterized by relative error. During performance of the basic arithmetical operations of an error of rounding off can collect (in большей or a smaller measure). There are certain differences in the scheme of accumulation of errors at performance of evaluations with the fixed and floating numbers. In aggregate they are summarized as an error of evaluations [Samarskiy, 1989; Metjuz, 2001; Demmel, 2001].

Algorithms which realize concrete methods, can be steady (errors during evaluations collect slightly) and, accordingly, otherwise - unstable. To algorithms which realize methods, it is possible to show two groups of criteria: criteria of adequacy of models (discrete computer and mathematical continuous) and criteria of convergence of computing algorithms at magnification of number of the equations in exposition of mathematical model, speed, accuracy of evaluations. „A phenomenon of Runge" [Metjuz, 2001] gives representation about problems of inadequacy of mathematical and machine models. It consists of volume that at interpolating the approximation error grows at magnification of an amount of knots [Voloshyn, 2010].

Such concept is connected with convergence of algorithms as a correctness of a numerical method of solution (continuous association on input datas, uniform concerning an amount of the equations). In particular, a correctness as the stability at modifications of input datas in linear systems, is characterised as a stability from input datas of right members (conditionality) and from all elements of model. The measure of a correctness of a problem is quantitatively described by a condition number [Metjuz, 2001]. It is necessary to notice that the category of a correctness of linear system (conditionality of problems) is exhibited on each pitch of a repetitive process.

In a continuous case the basic criterion of convergence is the monotonicity and boundedness of estimations of a solution, in case of computer model the "numerical" monotonicity and boundedness of "computer" time does not guarantee lack of "ejections" in "the real [Voloshyn, 2011].

In the conditions of an incorrectness (for the Hadamard) [Samarskiy, 1989] small inaccuracies of representation of mathematical model, in particular, at level of computing algorithm, can essentially influence quantitative and qualitative performances of a received solution at use of a concrete method (algorithm) [Voloshyn, Kudin, 2010; Bogaenko, Kudin, 2010]. Such inaccuracies are caused by boundedness of length of a mantissa at representation of numbers from a floating point (an error of truncation, a rounding off) more often. Despite of presence of the COMPUTER with the effective organisation of operation of a rounding off, to the full to avoid them, or to improve known theoretical estimations, it is not possible. It is important to notice that carrying out of evaluations with low accuracy ("roughly") smoothes ("hides") some important details.

In such cases there is an illusion of non-uniqueness of a solution (an incomplete rank) for SLAE or compatibility of area of solutions for SLAU. High-precision evaluations uncover details and more often "give" a unique solution (a full grade of a matrix) for Slough and incompatibility for SLAU. At carrying out of the analysis of evaluations it is important to co-ordinate with different degree of accuracy (to co-ordinate among themselves) properties of rough and high-precision evaluations.

Sequential analysis of variants

In the given work outcomes of research of association of quality of localization of area of a membership of solutions from magnitude of perturbations in model elements, from a condition number (as defining factor) and the organizations of evaluations (from length of a mantissa representation of numbers) are reduced. The attention to complexities of construction of the sets localizing set of solutions is focused. The methodology of a sequential analysis of variants (SAV) [Voloshyn, 1987] and a method of basis matrixes (MBM), as its concrete realization [Kudin, 2002] are in the heart of researches. The general formalism of SAV is offered by V.S.Mihalevich in [Mihalevich, 1965] where the general scheme of SAV on the basis of generalization of idea of the theory of consecutive solutions of A.Vald and R.Bellman dynamic programming is described. The scheme of SAV is reduced to the following sequence of procedures which repeat:

1. The partition of set of variants of solutions of a problem on some subsets, each of which has any specific properties;
2. Use of these specific properties for determination of logic inconsistencies in exposition of separate subsets;
3. Elimination of the further reviewing of those subsets of variants of a solution, in which exposition logic inconsistencies.

The technique of a sequential analysis and elimination of variants consists in such mode of construction of research which allows to "eliminate" initial segments of variants before their full construction are unpromising.

Thus, the considerable economy of computing expenditures as at elimination of unpromising initial parts of variants all possible sets sub variant their prolongations are eliminated also is carried out.

The general scheme of SAV is rendered concrete for various classes of multiple problems in the form of algorithms „a sequential analysis, eliminations and designing of variants" [Voloshyn, 1987] widely known algorithm „the Kiev broom" [Mihalevich, Shor, 1961]. „The principle of monotone recursiveness", related to criterion of an optimality of a dynamic programming of R.Bellman was the key rule of elimination of unpromising variants in these algorithms.

Simultaneously with known advantages, algorithms of step by step designing have also certain shortages. In development of the general concepts of SAV in a series of works of V.S.Mihalevich, V.L.Volkovich and A.F.Voloshyn procedures of parallel elimination подвариантов, in particular, known algorithm W (Volkovich-Voloshyn) [are offered Volkovich, Voloshyn, 1978].

Thus there is a problem of designing of a full variant which dares by consecutive input of restrictions on values of an objective functional and the problem of designing of a full variant is reduced to construction of procedures of the analysis and elimination under variants. Efficiency of the algorithms based on offered principles, proves to be true computing experiments, theoretical estimations and a solution of practical problems [Voloshyn, 2013].

The basic concept in application of algorithmic schemes, are based on decomposition schemes of SAV [Voloshin, 1987] which can be described in spaces of the various nature (values of variables and/or the criteria, the limited amount of variants of the solution, the limited amount of restrictions of a problem, etc. [Voloshyn, 2013]) consists in "localization" of area of an optimum of a target functional.

The method of basis matrices (MBM)

MBM allows to consider the effect of small perturbations on the solving the SLAE. In contrast to the classical iterative methods (such as the simple method) MBM finds a solution in two stages: 1) selection of basis matrix; 2) finding the appropriate selection of basis matrix solving the linear analysis and conditioning system that allows the situation includes the accumulation of errors.

Discusses the linear model:

$$Au = C, \tag{1}$$

where the matrix A (with rows $a_1, a_2, \dots, a_m$, $a_j = (a_{j1}, a_{j2}, \dots, a_{jm}),\ j = \overline{1,m}$) is square with dimension $(m \times m)$, in which the vectors of variables $u = (u_1, u_2, .., u_m)^T$ and constraints $C = (c_1, c_2, \dots, c_m)^T$ have dimension m. Constraint matrix of the SLAE are characterized, in general, large sizes, heavily stocked and poor conditioning. MBM is based on the idea of the base matrix. Basis matrices during the iterations vary sequential substitution lines of an auxiliary base matrix (auxiliary SLAE) rows limitations of the model (1). Consider the vectors $a_{i1}, a_{i2}, \dots, a_{im}$ it's normal limits (referred to as strings), $a_j u_б^T \le c_j$, $j \in J$, which form a matrix $A_б$ where $J_б = \{i_1, i_2, \dots, i_m\}$ is the indices of restrictions.

Definition 1. Square matrix $A_б$ consisting of m linearly independent rows of an auxiliary linear algebraic equation (SLAE) is called an artificial base and the solution corresponding system of equations $A_б u = C^0$, where $C^0 = (c_{i_1}, c_{i_2}, \dots, c_{i_m})^T$ is artificial basic solution.

Definition 2. Two basic matrices that differ only in one line are called adjacent.

Let e_{ri} is the elements of the matrix $A_б^{-1}$, which is inverse to $A_б$ and $e_{kб} = (A_k^{-1})$ is k- th column of the inverse matrix. $u_0 = (u_{01}, u_{02}, ..., u_{0m})^T$ is basic solution. $\alpha_r = (\alpha_{r1}, \alpha_{r2}, ..., \alpha_{rm})$ is a decomposition vector of normal restrictions $a_r u \le c_r$ on $a_r u \le c_r$ the basic lines of the matrix $A_б$. $\Delta_r = a_r u_0 - c_r$ is the residual r-th constraints u_0. The line a_l (normal limits $a_l u \le c_l$, $l \notin J_б$). $\alpha_l = (\alpha_{l1}, \alpha_{l2}, ..., \alpha_{lm})$ is a vector of decomposition a_l row by rows of basis matrix $A_б$, which is a vector of relations $a_l = \alpha_l A_б$.

We have the relations between the expansion coefficients of the normal restrictions (1) of the basic lines of the artificial matrix and elements: of inverse matrices, of basic solutions, of residuals of constraints (1) in two adjacent basis matrices replacing k-th row of the base matrix $A_б$, and of a_l string [Voloshyn, Kudin, 2010]:

$$\overline{\alpha}_{rk} = \frac{\alpha_{rk}}{\alpha_{lk}}, \ \overline{\alpha}_{ri} = \alpha_{ri} - \frac{\alpha_{rk}}{\alpha_{lk}}\alpha_{li}, \quad r = \overline{1,m}; \ i = \overline{1,m}; \ i \ne k; \tag{2}$$

$$\overline{e}_{rk} = \frac{e_{rk}}{\alpha_{lk}}, \ \overline{e}_{ri} = e_{ri} - \frac{e_{rk}}{\alpha_{lk}}\alpha_{li}, \quad r = \overline{1,m}; \quad i = \overline{1,m}; \ i \ne k; \tag{3}$$

$$\overline{u}_{0j} = u_{0j} - \frac{e_{jk}}{\alpha_{1k}}\Delta_l, \quad j = \overline{1,m}, \tag{4}$$

$$\overline{\Delta}_k = -\frac{\Delta_l}{\alpha_{lk}}, \ \overline{\Delta}_r = \Delta_r - \frac{\alpha_{rk}}{\alpha_{lk}}\Delta_l, \ r = \overline{1,n}; \quad r \ne k. \tag{5}$$

It should be noted condition of a regularity basis matrix by replacing a_l line to k-th row of the base matrix $A_б$ is under condition of the inequality $\alpha_{lk} \ne 0$. For the existence of a unique solution of (1) is necessary and sufficient to be carried out the condition $\alpha_{lk}^{(i)} \ne 0, \ i = \overline{1,m}$; where $\alpha_{lk}^{(i)}$ is the key elements of the operation of substitution strings basis matrix normal constraints (1). A matrix of the system (1) is non-degenerate if $\alpha_{lk}^{(i)} \ne 0, \ i = \overline{1,m}$.

In the vector form of the formula (3) is: $\overline{e}_k = \dfrac{e_k}{\alpha_{lk}}$, $\overline{e}_i = e_i - \dfrac{e_k}{\alpha_{lk}}\alpha_{li}, \quad i = \overline{1,m}; \ i \ne k$.

Key stage algorithmic scheme finding the value of the machine rank basis matrix and solutions (1) based on the known properties of the trivial linear algebraic equation (SLAE) of the same dimension it has:

1) Perform simplex iteration substitution strings trivial basis matrix elements with known limitations of the method limited system (1) according to the relations (2) - (5);
2) Check the performance of conditions of regularity at each iteration;
3) Find the corresponding elements of the vector decomposition method: basis matrices by rows limitations of (1), the inverse basis matrix, the discrepancy limitations, artificial basic solutions $u_0^{(r)}$ where k is the number of iteration;
4) Control the number of iterations of substitution strings auxiliary system of the host (1), for which the nonsingularity conditions.

If the number of iterations for which $\alpha_{kk}^{(k)} \neq 0$, as well it is equal m, the only solution is got from the ratio of $A_{\delta}^{-1} \cdot C = u^{0}$. If not, then if the conditions $k < m$ for the linear algebraic equation (1) is not satisfied the condition uniqueness of the solution. In this case, the model needs further analysis.

The computational process of translation of the inverse matrix is holding two stages of matrix operations:

5) Dividing the leading k-th column $e_k = (A_{\delta}^{-1})_k$ on the value of the leading element $\alpha_{kk}^{(k)} \neq 0$ $e_k^{k+1} = e_k^k / \alpha_{kk}$;
6) To calculate on the k-th iteration for the i-th column of the inverse matrix ($i \neq k,\ i \in I$) of a new column $e_i^{k+1} = e_i^k - e_k^{k+1} \times \alpha_{ki}$, the quality of the localization field perturbation solutions (or approximations) essentially depends on the condition number (as a determining factor) and of algorithms (the length of the mantissa).

Features representations of numbers in the computer

Recently, the most commonly used standard IEEE. This is a standard binary arithmetic. This standard provides a well-defined data types along the length of the mantissa and the value of the order. For example, the mantissa 23, the order of 8 (32 characters), the mantissa order 52 11 (64 characters). This imposes a restriction on the threshold of "machine (computer) zero" and "machine (computer) infinity." For example, in the first case we have the machine zero threshold 2^{-126}, the threshold machine infinity 2^{128}.

In accordance with [Demmel, 2001] regardless of the length of the mantissa and the order of the following quality representation of real numbers in a computer:

- Machine zero (2^{-126});
- Subnormalized numbers $(-2^{-128}, 2^{-126})$;
- Normalized positive nonzero numbers $(2^{-128}, 2^{126})$;
- Normalized negative nonzero numbers $(-2^{-128}, -2^{-126})$;
- Positive overflow threshold (2^{127});
- Negative overflow threshold (-2^{128}).

The number is called normalized if its mantissa significant bit of nonzero.

Let is $a \odot b$ accurate results of calculations using the operation $\odot$, $fl(a \odot b)$ approximately computed floating-point number. If the number is closest to the calculated, then we believe that arithmetic rounding is correct (IEEE rounding correctly). For equidistant from rounding to less (the last sign of the mantissa is 0) - round to nearest even number.

When absenteeism beyond the allowable index order and correct rounding can be written: $fl(a \odot b) = (a \odot b)(1 + \delta)$, where - $|\delta|$ does not exceed the number of ε (machine epsilon) or machine precision (ε). You can read that the maximum relative error is equal to a number. Then the machine zero is maximum of $|\delta|$, at which the above mentioned ratio is carried out.

Subnormal number is calling, when it is located between zero and the smallest normalized floating-point number (smallest order number). For any two such numbers x, y value $fl(x - y)$ cannot be machine zero. An important feature is the fact that only in this case.

In the IEEE standard also provides for the number NAN (Not a number) overflow (as a result of division by zero). It follows that depending on the embodiment of submission to IEEE standard real accurate zero (as a natural number) in its calculations will respond zone subnormal numbers (fixed neighborhood of zero).

In calculations with different accuracy, we will have their own versions of the quantitative values of the zero-non-null, which can significantly affect the accuracy of the calculations.

Computer experiment in LM of low-dimensional

The effect of small perturbations, in the "small" LM quality localization solutions, is illustrated on a test study. It was organized as a structured multi-step algorithmic procedure. It consists in constructing a sequence of low-dimensional model problems that have properties irregularities starting with some simple steps. Just find items based on MBM method, calculate the condition numbers and graphical presentation of the main functional dependencies [Voloshyn, 2011].

Below are the resulting experiment symbols sets accessories making (localization).

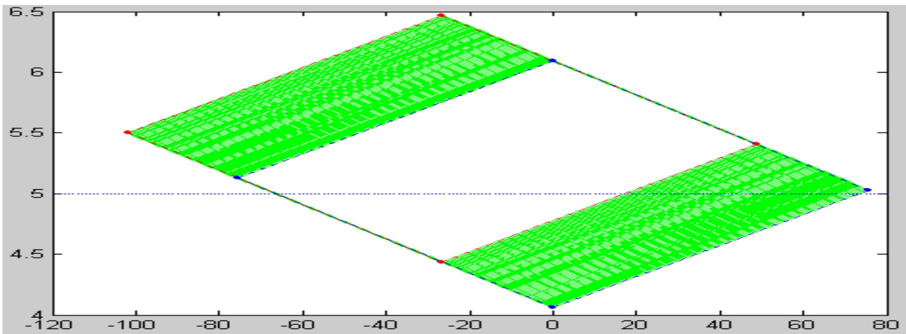

Figure 1. Rhomboid shape evaluation sets accessories decisions on iteration method (axes components of solutions) under perturbations of "parallel transport"

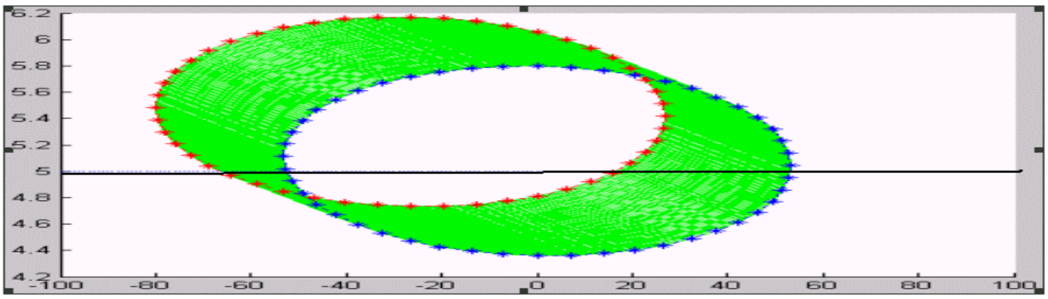

Figure 2. Ellipses accessories solutions on iteration method under perturbations vector of the right sides of sphere limitations of small diameter

Graphic images of ellipsoid shaped figures and rhombs (localization) are given disproportionate respect minimum and maximum abscissa and ordinate axes. Ellipsoid shaped pieces and rhombs images perturbation set of solutions (for small perturbations of the model elements) are elongated relative to the variety solutions boundary system of not full rank. Ellipsoid shaped pieces and rhombs images perturbation set of solutions (for small perturbations of the model elements) are elongated relative to the variety solutions boundary system full rank.

Figures include" major axis "(diagonal) and "minor axis" structurally. Major axis corresponds to a larger eigenvalue along the horizontal axis, is small (with small values of the diagonal axis of ordinates) - responds to small eigenvalues.

Figures 1 and 2 to a certain extent confirms the conjecture one of the co-authors [Voloshyn, 2006], on the validity of the Heisenberg uncertainty principle in computer research. This is of course the influence of small perturbations, namely the reduction of uncertainty ("improvement" localization) in one variable leads to an increase in uncertainty ("deterioration" localization) on the other.

Computer experiment on the analysis of small perturbations in the middle dimension LM

For computational experiment were elected three procedures MBM: without specifying solutions (0), with a one-step (1) and two-stage specification (2) [Bogaenko, 2012; Voloshyn, Kudin, 2010]. Having used different types of data: floating-point numbers (double precision (Double), 128-bit number (Dd) and 256 - bit numbers (Qd), and computation of exact numbers. Worth noting that on the same computing platform the ratio of the speed of algorithms that use different types of data will be permanent.

These computational experiments showed that the test platform for the chosen algorithm, which uses a 128-bit number, was about 35 times slower than an algorithm which uses 64-bit number, and if the 256-bit number - ~ 450 times in slowly. Such a significant slowdown due to the fact that operations with floating point High resolution is not implemented in hardware, in contrast to the operations on 64-bit numbers.

To build up relationships between the heuristic solutions and condition number was carried out a series of computational experiments. Conducted decision SLAE dimension 256x256 different algorithms MBM. As a criterion for the accuracy of machine precision was taken u0 solutions of (1) compared with the analytical (exact) solution. The experimental data presented in [Voloshyn, 2010].

Computer experiment established close to a linear dependence:

1) The accuracy of the solution (and matrix inversion) of the condition number (for fixed data type and algorithm) and the dimension of the model;
2) The accuracy of the solution on the data type (for a fixed number of conditionality and algorithm).

This allows you to build:

1) Interpolation polynomials - depending on the accuracy of the solution of conditionality;
2) Approximating the accuracy of solutions sets accessories (including ellipses minimum area);
3) Depending on extrapolation (for accuracy making algorithms) of the condition number.

Methodology SAV [Voloshyn, 1987] aims to conduct research to establish the status of the component model, the analysis of inclusions (exclusions) component model on a bounded and closed (i.e., the quality of the localization). As follows from the results of computer simulation, even minor quantitative changes in the components of such models can qualitatively change the status of the component model, and as a consequence, the structure of the set of solutions of the problem.

In the study of stability of the problem is important to identify a quantitative measure of changes in the source data (correctness) in which this property is stored (lost). Identify the parameter values for which there are qualitative changes in the properties of the system. "Catch and evaluate" such structural quality parameters depending on the quantity (in the small) between the practical problems of mathematical description of the model and engine. For example, in the language" $(\varepsilon - \delta)$" - of the deviation or change solutions - from data changes within the (category of stability) is not only difficult, but more often and it is impossible even for ill-conditioned problems of small dimension.

In conclusion, it can be noted the need to: inclusion in the process of numerical experiment of the decision; analyze the impact of different strategies for computing (data types, algorithms, level of system conditioning) on the basic parameters of the solution (the accuracy of the solutions and matrix inversion, speed, volume calculations); develop efficient computational methods and algorithms schemes evaluating conditioning system during the experiment on the given criteria for models of different dimension.

In particular, we consider the properties of the basic methods and algorithms for finding the exact solution of linear algebraic equation in MAPLE environment. The features of the calculations with different length of the mantissa, the dimension of the constraint matrix, with the "bad" structural properties and verified the reliability of some "non-strict" evaluation of errors of calculation formulas. Dependency analysis of the relative errors solutions on the dimension of the constraint matrix in an ill-conditioned LM middle dimension shown in Figure 3, with the length of the mantissa m = 5, 10, 16 etc.

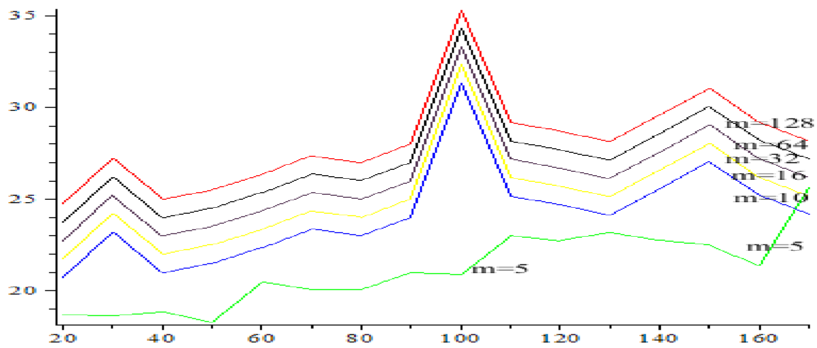

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Figure 3.

One of the empirical formulas accuracy of calculations given in [Khimich, 2007] as follows: if Masheps = 2.2 * 10 ^ (-16), M-condition number (order), the calculation error may be filed as d = Masheps - M (order).

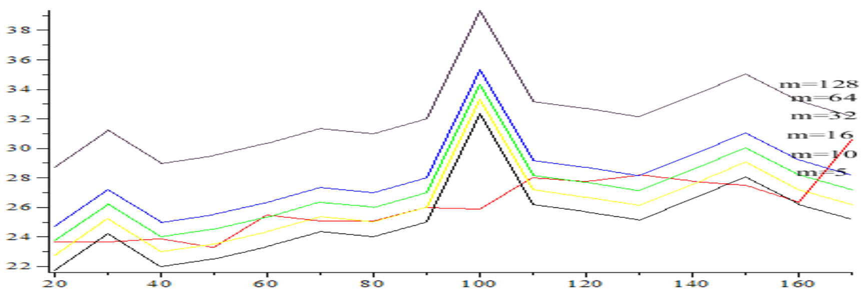

Figure 4.

Other empirical formula estimating the order of accuracy of the calculations [Alfeld, 1987]:

if the number is written in the form of floating-point, where m - mantissa, b - base power. e - the procedure is usually base 2. If there is an algorithm that runs in machine interval arithmetic with a mantissa length t1 and mantissa length t2, where t2 = t1 + ll> 0, then the limit of absolute and relative error will decrease in time bl.

Computer experiment set: 1) a significant influence of the structural properties of the constraint matrix, the dimension of the model, the length of the mantissa conditionality computational properties of algorithms; 2) The computational complexity of the condition number; 3) Loss of credibility evaluation formulas with "growth" ill-conditioned, the dimension of the model; 4) The special influence of the leading elements and standards leading columns on the quality of evaluations.

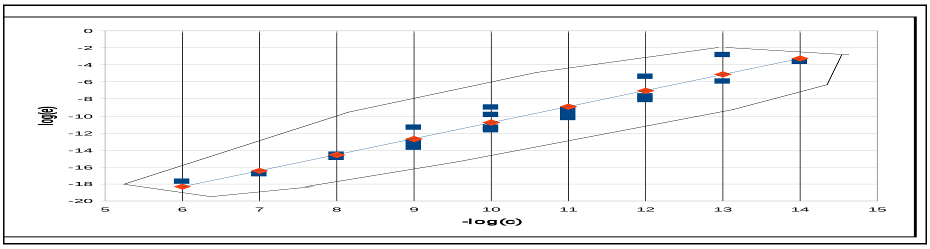

Figure 5. Option piecewise linear approximation (localization) of the range of the order of accuracy in the coordinate system of values and the order of accuracy of the order of the condition number.

Computer experiment also found close to a linear dependence [Bogatenko, 2012]:

1) The accuracy of the solution (and matrix inversion) of the condition number (for fixed data type and algorithm) and the dimension of the model; 2) The accuracy of the solution on the data type (for a fixed number of conditionality and algorithm); 3) close to the polynomial speed depending on the dimension of the problem (for fixed data types, including conditionality and dimension model); 4) the need to develop new algorithms for analysis of linear systems , in which there are " evaluators " condition numbers during calculations.

Conclusion

This work continues (and in some sense generalizes) study the influence of small perturbations of course linear models [Voloshyn, Kudin, 2010 - 2012] and emphasizes the need to develop "analysis certainly small" analysis tools like computer models, in the first place, for the localization of solutions systems of linear algebraic equations. Subsequent publications will address the challenges of "computer convergence" of iterative processes and ways of solving them by using proposed in this paper (and previous publications) approaches based on "analysis certainly small".

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