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AN UNCERTAIN CAUCHY PROBLEM OF A NEW CLASS OF FUZZY DIFFERENTIAL EQUATIONS

Alexei Bychkov, Eugene Ivanov, Olha Suprun

Abstract: *The concept of fuzzy perception roaming process is entered. A new integral is built by the fuzzy perception roaming process. Properties of this integral are studied. The new class of differential equalizations is acquired. A theorem of existence and uniqueness of a solution for the new class of fuzzy perception differential equations are proved.*

Keywords: *possibility-theoretical approach, dynamic processes, soft modeling, fuzzy perception processes, Cauchy problem.*

ACM Classification Keywords: *G.1.7 – Ordinary Differential Equations.*

Introduction

It is important to answer a question of how exact the result of design is at non-crisp supervision when we deal with mathematical modeling of complex systems. A stochastic approach is the most widespread to describe a non-crisp at the complex system design. However, such description in probability terms is unnatural for the unique phenomena. It appears that the most suitable for this are the ideas of possibility theory. First, the theory of possibility was most fully presented by Dubois D. Development of ideas [Dubois, 1988] is offered in this article.

There are different approaches to formalize non-crisp dynamics [Aubin, 1990]-[Buckley, 1992], [Friedmanan, 1994]-[Park, 1999], [Seikkala, 1987]. In this article new approaches to adequate description and analysis of non-crisp and uncertain information and dynamics of objects are proposed.

Methods of possibility theory allow to estimate an event truth with respect to other events and to take into account a subjective expert opinion. For example it is very important for prognostication of the social-economic phenomena, for medical diagnostic tasks, for mathematical modeling of human thinking process and other processes.

Preliminaries

Let $(X, \mathbf{A})$ be a measurable space. According to [Pitiev, 2000], [Bychkov, 2005], [Bychkov, 2006] we consider some definitions. A new notations $+, \bullet$ are offered in [Pitiev, 2000]. We shall use them in the further.

Definition 1. Possibility scale is a semi-ring $L = \{[0,1], \leq, +, \bullet\}$, i.e. $[0,1]$ segment with usual order $\leq$ and two operations:

- $a + b = \max\{a, b\}$;

- $a \bullet b = \min\{a, b\}$.

Henceforth we consider only A -measurable functions $f : X \to L$. Let's denote as $L(X)$ a class of functions that satisfies the following conditions:

- $f \in L(X), a \in L \Rightarrow a \bullet f = \min(a, f(x)) \in L(X)$;
- $f, g \in L(X) \Rightarrow f + g = \max(f(x), g(x)) \in L(X)$;

  $f \bullet g = \min(f(x), g(x)) \in L(X)$;
- $f \in L(X) \Rightarrow \neg f \equiv 1 - f(x) \in L(X)$;
- If a sequence of functions $f_1, ..., f_n, ... \in L(X)$, then $\mathop{+}\limits_{n=1}^{\infty} f_n(x) \in L(X)$; $\mathop{\bullet}\limits_{n=1}^{\infty} f_n(x) \in L(X)$.

Definition 2. Function $p : L(X) \to L$:

- $p((a \bullet f)(\cdot) + (b \bullet g)(\cdot)) = (a \bullet p(f(\cdot))) + (b \bullet p(g(\cdot)))$;
- $p\left(\left(\mathop{+}\limits_{n=1}^{\infty} f_n\right)(\cdot)\right) = \mathop{+}\limits_{n=1}^{\infty} p\left(f_n(\cdot)\right)$;
- If $f(\cdot) \equiv 1$, $p(f(\cdot)) = 1$,

is called a possibility measure.

Let's note $\beta(X)$ - set of all subsets X, $\chi_A(\cdot)$ - characteristic function of A.

Definition 3. Let's define the function $P : \beta(X) \to L$ in the following manner: $P(A) \equiv p(\chi_A(\cdot))$. This function is called a possibility of crisp event A. Then let's call a triplet $(X, \mathbf{A}, P)$ a possibility space.

We will use a term **fuzzy perception** (from perceptio – lat.), in future, that will not contradict with term Zadeh L. – fuzzy.

Definition 4. Let $(X, \mathbf{A}, P)$ be a possibility space. Let's call an A -measurable function $\tilde{A} : X \to \beta(Y)$ as fuzzy perception set.

Characteristic function of fuzzy perception set $\tilde{A}$ is calculated as $\mu_{\tilde{A}}(y) = P\{y \in \tilde{A}(x)\} = P(A^{-1}(y))$.

Definition 5. If $\tilde{A}$ is a fuzzy perception set, let's call a number $P(\tilde{A}) \equiv p(\mu_{\tilde{A}}(\cdot))$ as a possibility of fuzzy perception set $\tilde{A}$.

The definition implies the following properties:

- $P(A \cup B) = P(A) + P(B) = \max(P(A), P(B))$;
- $P(A \cap B) \le P(A) \bullet P(B) = \min(P(A), P(B))$.

Definition 6. If $P(A \cap B) = P(A) \bullet P(B)$ the events are called independent.

Definition 7. Conditional possibility of event A in respect to event B is the solution $P\{A \mid B\}$ of the equation

$$\min\{P(A\,|\,B),P(B)\}=P(A\cap B).$$

Theorem 1. [Bychkov, 2007] It is possible to extend possibility P to all the subsets of set X (let's call the extended possibility $\bar{P}:\beta(X)\to L$) and for all $A\in\mathbf{A}$ $\bar{P}(A)=P(A)$.

This theorem allows not to build the enclosed sequence of sigma algebras for the correct construction of fuzzy differential equation solution

Definition 8. With a given possibility space $(X,\mathbf{A},P)$ and a measurable space $(Y,\mathbf{B})$ a fuzzy perception variable is an $(\mathbf{A},\mathbf{B})$- measurable function $\xi:X\to Y$.

Theorem 1 implies that we may define a function $\mu_\xi(y)=\bar{P}\{\xi=y\}$ that is called distribution of a fuzzy perception variable. The possibility that a fuzzy perception variable ξ will fall into set *B* can be expressed by distribution: $P\{\xi\in B\}=\sup_{y\in B}\mu_\xi(y)$.

Definition 9. Fuzzy perception variables ξ and η are independent if distribution of perception vector (ξ,η) equals to $\mu_{\xi\eta}(u,v)=\min\{\mu_\xi(u),\mu_\eta(v)\}$.

Main results. *Modeling the uncertain dynamics*

Let's give some definitions.

Definition 10. [Bychkov, 2005] Fuzzy perception variable ξ (scalar or vector) is normal if its distribution equals to

$$\mu_\xi(u)=\varphi\left\{\left\|\Xi^{-1/2}(u-u_0)\right\|^2\right\},$$

where $\varphi(x)$ – decreasing function, that's specified for $x\geq 0$ such as $\varphi(x)\xrightarrow[x\to\infty]{}0$, $\varphi(0)=1$.

Definition 11. [Bychkov, 2005] Fuzzy perception process is a function $\xi(x,t):X\times R\to Y$.

Definition 12. [Bychkov, 2005]Normal fuzzy perception process $\xi(t)$ is a process of fuzzy perception roaming if the following assumptions exist:

- Under independent increments, i.e. for all moments of time $t_1<t_2\leq t_3<t_4$ fuzzy perception variables $\xi(t_2)-\xi(t_1)$ and $\xi(t_4)-\xi(t_3)$ are independent.

- Under fixed t its transient possibility is $P\left\{\xi(t)=x \mid \xi(t_0)=x_0\right\}=\varphi\left(\dfrac{\left\|\Xi^{-1/2}(x-x_0)\right\|^2}{t-t_0}\right)$.
- $\xi(0)=0$.

Definition 13. A mode (or modal value) of a fuzzy perception variable is a point in which distribution function equals to 1, i.e.

$$\operatorname{mod}\xi=u_0;\quad \mu_\xi(u_0)=1.$$

Definition 14. α-cut of fuzzy perception variable ξ is the following set:

$$[\xi]_\alpha=\left\{y: P\{\xi=y\}\geq\alpha\right\},\ \alpha\in(0,1]$$

An alternative way of specifying fuzzy distribution is to specify its α-cuts.

Definition 15. [Pitiev, 2000] Let's consider a semi-ring $\tilde{L}=\left([0,1];\ \tilde{\leq};\ \tilde{+};\ \tilde{\bullet}\right)$, where $a\tilde{\leq}b$ if $b\geq a$; $a\tilde{+}b=\min(a,b)$; $a\tilde{\bullet}b=\max(a,b)$. We can define a necessity measure $n(f(\cdot)):L(X)\to\tilde{L}$ on $\tilde{L}$ in the same manner as on L. Then for any crisp set $A\in\beta(X)$ necessity of set A is $N(A)=n(\chi_A(\cdot))$. For any fuzzy perception set necessity is $N(\tilde{A})=n(\mu_{\tilde{A}}(\cdot))$.

Definition 16. [Bychkov, 2006] Sequence of fuzzy perception variables ξ_n converges to fuzzy perception variable ξ with necessity 1 ($\underset{n\to\infty}{\mathrm{N1lim}}\,\xi_n=\xi$) if for each $x\in X$ for which $P\left(\{x\}\right)\neq 0$, $\xi_n(x)\to\xi(x)$.
In this case $N\{\xi_n \xrightarrow[n\to\infty]{} \xi\}=1$.

Definition 17. Sequence of fuzzy perception variables ξ_n converges to fuzzy perception variable ξ by possibility ($\underset{n\to\infty}{\mathrm{Plim}}\,\xi_n=\xi$) if for any $c>0$ $P\left\{\left|\xi_n-\xi\right|>c\right\}\xrightarrow[n\to\infty]{}0$.
If any of these limits exists it is unique. $\underset{n\to\infty}{\mathrm{Plim}}\,\xi_n=\xi$ implies $\underset{n\to\infty}{\mathrm{N1lim}}\,\xi_n=\xi$ (unlike probability theory).

Lemma 1. If $\underset{n\to\infty}{\mathrm{N1lim}}\,\xi_n=\xi$ and for arbitrary $\alpha\in(0,1]$ sequence $\xi_n(x)$ is convergent to $\xi(x)$ on the set $[X]_\alpha=\{x\in X: P(\{x\})\geq\alpha\}$ uniformly by x, then $\underset{n\to\infty}{\mathrm{Plim}}\,\xi_n=\xi$. And conversely, if $\underset{n\to\infty}{\mathrm{Plim}}\,\xi_n=\xi$ then for any $\alpha\in(0,1]$ sequence $\xi_n(x)$ converges to $\xi(x)$ uniformly by x on $[X]_\alpha$.

Proof. By definition of uniform convergence, for all $\alpha>0$ and $c>0$ an index $n(\alpha,c)\in N$ exists with following property: if $n'>n(\alpha,c)$ and $P(\{x\})\geq\alpha$ then $\left|\xi_{n'}(x)-\xi(x)\right|\leq c$. Possibility that difference between ξ_n and ξ will exceed c may be estimated as

$$P\{|\xi_n - \xi| > c\} \le \sup\{\alpha : n \le n(\alpha, c)\} \equiv \alpha(n, c).$$

Let's prove that $\alpha(n,c) \xrightarrow[n\to\infty]{} 0$. Let's assume the contrary: $\alpha(n,c) \nrightarrow 0$ when $n \to \infty$. Then such value α_0 exists that for any sequence of indexes $n_k \to \infty$ holds the following: $n_k \le n(\alpha_0, c)$, that's impossible, because $n(\alpha_0, c)$ is finite. The direct lemma is proven.

Let's prove the converse lemma. Let's assume the contrary: such $c > 0$ and $\alpha > 0$ exist there that for any number k, there exist an element of possibility space x and a number $n_k > k$, for which $P\{x\} \ge \alpha$ and $|\xi_{n_k}(x) - \xi(x)| > c$. We found such sequence $n_k \to \infty$ that $P\{|\xi_{n_k} - \xi| > c\} \ge P(\{x\}) \ge \alpha$, i.e. there's no convergence by possibility.

Definition 18. [Bychkov, 2005] Assume that a piecewise-constant function $f : f(t) = \{y_k : \ t_k \le t < t_{k+1}\}$, $k = 0...N-1$, $t_0 = 0$, $t_N = T$ is given, $w(t)$ is a scalar process of fuzzy perception roaming. Denote

$$\int_0^T f(t)dw(t) = \sum_{k=0}^{N-1} y_k (w(t_{k+1}) - w(t_k)).$$

This fuzzy perception variable is called an integral by a process of fuzzy perception roaming of piecewise-constant function.

Lemma 2. For piecewise-constant function $f(t)$ the following equality exists:

$$\left[\int_0^T f(t)dw(t)\right]_\alpha = \left[-\int_0^T |f(t)|dt \cdot \sqrt{\phi^{-1}(\alpha)}, \int_0^T |f(t)|dt \cdot \sqrt{\phi^{-1}(\alpha)}\right].$$

Proof. Let's use mathematical induction.

For $N = 1$ the equality is true by definition of process of fuzzy perception roaming.

Let's assume that the equality is true for N, and prove it for $N+1$.

Fuzzy perception variables $\int_0^{t_N} f(t)dw(t)$ and $\int_{t_N}^{t_{N+1}} f(t)dw(t)$ are independent. Therefore

$$\left[\int_0^{t_{N+1}} f(t)dw(t)\right]_\alpha = \left[\int_0^{t_N} f(t)dw(t)\right]_\alpha + \left[\int_{t_N}^{t_{N+1}} f(t)dw(t)\right]_\alpha =$$

$$= \left[-\int_0^{t_N} |f(t)|dt \cdot \sqrt{\varphi^{-1}(\alpha)}, \int_0^{t_N} |f(t)|dt \cdot \sqrt{\varphi^{-1}(\alpha)}\right] +$$

$$+\left[-\int_{t_N}^{t_{N+1}}|f(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)},\ \int_{t_N}^{t_{N+1}}|f(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)}\right]=$$

$$=\left[-\int_{0}^{t_{N+1}}|f(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)},\ \int_{0}^{t_{N+1}}|f(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)}\right].$$

Corollary 1. Let's assume that a sequence of piecewise-constant functions $f_n(t)\xrightarrow[n\to\infty]{}0$ in the mean. Then $\underset{n\to\infty}{\mathrm{P}\lim}\int_0^T f_n(t)dw(t)=0$.

This corollary implies the following theorem:

Theorem 2. [Bychkov, 2005] A measurable function $f(t)$ is given and two sequences of piecewise-constant functions $f_n(t)$ and $\overline{f}_n(t)$ converge to it in the mean. If the limit $Q=\underset{n\to\infty}{\mathrm{P}\lim}\int_0^T f_n(t)dw(t)$ exists then the limit $\overline{Q}=\underset{n\to\infty}{\mathrm{P}\lim}\int_0^T \overline{f}_n(t)dw(t)$ also exists, and $\overline{Q}=Q$.

Theorem 2 proves correctness of the following definition:

Definition 19. [Bychkov, 2005] Assume that a measurable function $f(t)$ is given and a sequence of piecewise-constant functions $f_n(t)$ converges to it in the mean. ($\int_0^T |f_n(t)-f(t)|\,dt\xrightarrow[n\to\infty]{}0$). If the sequence $\int_0^T f_n(t)dw(t)$ converges by possibility let's call its limit as an integral of a measurable function by the process of fuzzy perception roaming and

$$\int_0^T f(t)dw(t)=\underset{n\to\infty}{\mathrm{P}\lim}\int_0^T f_n(t)dw(t).$$

Definition 20. Sequence of functions $a_n(x)$ is uniformly fundamental by x if for arbitrary $\varepsilon>0$ the independent from x $n_0(\varepsilon)$ exists and such that when $m>n\ge n_0$: $|a_n(x)-a_m(x)|<\varepsilon$.

Lemma 3. If a sequence of functions $a_n(x)$ is uniformly fundamental it is uniformly convergent.

Proof. Let's denote $\lim_{n\to\infty} a_n(x) = a(x)$. From uniform fundamentality for any $\varepsilon > 0$ such n_0 exists that for $m > n \ge n_0$ $|a_n(x) - a_m(x)| < \frac{\varepsilon}{2}$ occurs. Hence, for all $m \ge n_0$ $a_m(x) \in [a_{n_0}(x) - \frac{\varepsilon}{2}, a_{n_0}(x) + \frac{\varepsilon}{2}]$ and $a(x) \in [a_{n_0}(x) - \frac{\varepsilon}{2}, a_{n_0}(x) + \frac{\varepsilon}{2}]$. Therefore, $|a(x) - a_m(x)| < \varepsilon$ that proves uniform convergence.

Lemma 4. For crisp integrable function $f(t)$ α-cuts of the integral also equal to

$$\left[\int_0^T f(t)dw(t)\right]_\alpha = \left[-\int_0^T |f(t)|dt \cdot \sqrt{\varphi^{-1}(\alpha)}, \int_0^T |f(t)|dt \cdot \sqrt{\varphi^{-1}(\alpha)}\right].$$

Proof. As we know from functional analysis such sequence of piecewise-constant functions $f_n(t)$ exists that

$$|f_1(t)| \le |f_2(t)| \le \ldots \le |f_n(t)| \le \ldots \text{ and } \int_0^T f_n(t)dt \xrightarrow[n\to\infty]{} \int_0^T f(t)dt,$$

furthermore $|f_n(t)| \le |f(t)|$ almost everywhere. In such a case $f_n(t) \to f(t)$ in the mean.
Let's prove the existence of the integral.
Let's fix an arbitrary $x_0 \in X$ and consider sequence $\int_0^T f_n(t)dw(t, x_0)$, then

$$\left|\int_0^T f_n(t)dw(t,x_0) - \int_0^T f_m(t)dw(t,x_0)\right| \le \int_0^T |f_n(t) - f_m(t)|\, dw(t,x_0)$$

Since $f_n(t) \xrightarrow[n\to\infty]{} f(t)$ in the mean $|f_n(t) - f_m(t)| \xrightarrow[m,n\to\infty]{} 0$ in the mean and from Corollary 1

$$\operatorname{Plim}_{m,n\to\infty} \int_0^T |f_n(t) - f_m(t)|\, dw(t) = \operatorname{N1lim}_{m,n\to\infty} \int_0^T |f_n(t) - f_m(t)|\, dw(t) = 0.$$

Therefore $\lim_{m,n\to\infty} \int_0^T |f_n(t) - f_m(t)|\, dw(t,x_0) = 0$. So, the sequence $\int_0^T f_n(t)dw(t,x_0)$ is fundamental and hence it is convergent. We proved that $\operatorname{N1lim}_{n\to\infty} \int_0^T f_n(t)dw(t,x) = Q(x)$ exists. Let's prove that $\operatorname{Plim}_{n\to\infty}$ also exists.

By *Lemma 3* for any $x_0 \in X$, for which $P(\{x_0\}) \ge \alpha$ and $m > n$ from choice of $f_n(x)$

$$\left|\int_0^T |f_n(t)-f_m(t)|\,dw(t,x_0)\right| \le \int_0^T |f_n(t)-f_m(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)} \le$$

$$\le \int_0^T |f-f_n(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)}.$$

Holds, i.e. sequence $\int_0^T f_n(t)dw(t,x)$ is uniformly fundamental by x. According to *Lemma 3* this sequence uniformly converges by x. By *Lemma 1* $\mathrm{P}\lim_{n\to\infty}\int_0^T f_n(t)dw(t)$ exists.

We have proven that all Lebesgue integrable functions are also integrable by process of fuzzy roaming. However, the distribution of this integral is unknown. Let's calculate it.

By *Lemma 3*

$$\left[\int_0^T f_n(t)dw(t)\right]_\alpha = \left[-\int_0^T |f_n(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)},\ \int_0^T |f_n(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)}\right] = \overline{I}_n .$$

This segment increases with n, its limit equals to $I=\bigcup_n I_n = \left(-\int_0^T |f(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)},\ \int_0^T |f(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)}\right)$.

Let's denote closure of I as $\overline{I}$. Obviously $\left[\int_0^T f(t)dw(t)\right]_\alpha \subseteq I$. We should prove that it is exactly $\overline{I}$.

For brevity we denote $M(f,\alpha)=\int_0^T |f(t)|\,dt\cdot\sqrt{\varphi^{-1}(\alpha)}$. For arbitrary $\varepsilon>0$ such n_1 exists that for $n>n_1$ $|M(f,\alpha)-M(f_n,\alpha)|<\varepsilon$. Because $\left[\int_0^T f_n(t)dw(t)\right]_\alpha = [-M(f_n,\alpha);M(f_n,\alpha)]$, that for each $n>n_1$ such $x_n\in X$ exists that $P(\{x\})\ge\alpha$ and $\left|\int_0^T f_n(t)dw(x,t)-M(f_n,\alpha)\right|<\varepsilon$. By *Lemma 1* (converse part) such $n_2\ge n_1$ exists that for each $n>n_2$ $\left|\int_0^T f_n(t)dw(x,t)-\int_0^T f(t)dw(x,t)\right|<\varepsilon$ independently of x. It means that for any ε $\left(-M(f_n,\alpha)+3\varepsilon,\ M(f_n,\alpha)-3\varepsilon\right)\subseteq\left[\int_0^T f(t)dw(t)\right]_\alpha$. Therefore $I\subseteq\left[\int_0^T f(t)dw(t)\right]_\alpha$. Because $\pm M(f,\alpha)\in\left[\int_0^T f(t)dw(t)\right]_{\alpha-\delta}$ for any sufficiently small $\delta>0$, $P\left\{\int_0^T f(t)dw(x,t)=\pm M(f,\alpha)\right\}\ge\alpha-\delta$.

Hence, $P\left\{\int_0^T f(t)dw(x,t)=\pm M(f,\alpha)\right\}=\alpha$ and the ends of $\overline{I}$ segment also belong to cut.

Theorem 3. For a fuzzy perception process $\xi(t,\omega)$ which has integrable paths and is independent from $w(t,x)$, holds the following:

$$\left[\int_0^T \xi(t,x)dw(t,x)\right]_\alpha =$$

$$=\left[-\sup_{x:P\{x\}\geq\alpha}\int_0^T |\xi(t,x)|dt\cdot\sqrt{\varphi^{-1}(\alpha)},\ \sup_{x:P\{x\}\geq\alpha}\int_0^T |\xi(t,x)|dt\cdot\sqrt{\varphi^{-1}(\alpha)}\right].$$

Proof. From independence of processes $w(t,x),\xi(t,x)$ follows that:

$$P\{w(t,x)\in A \mid \xi(t,x)\in B\} = P\{w(t,x)\in A\}.$$

Therefore, process $\{w(t,x) \mid \xi(t,x)\in B\}$ is a process of fuzzy perception roaming identical to $w(t,x)$. Fixing $\xi(t,x)$ and using *Lemma 4* conclude proof.

Let's consider an initial-value problem

$$y(t,x) = y_0(x) + \int_{t_0}^{t} a(y(s),s)ds + \int_{t_0}^{t} b(y(s),s)dw(s,x), \quad (1)$$

$$y(t_0,x) = y_0. \quad (2)$$

The solution of the problem (1), (2) is a fuzzy perception process $y(t,x)$ that turns (1), (2) into equalities for each $x\in X$, for which $P(\{x\})\neq 0$.

Main results. *A Cauchy problem for new class of differential equations*

Theorem 4. Let crisp functions $a(y,t)$ and $\mathrm{b}(y,t)$ be continuous by t and satisfy Lipschitz condition by y in the region $D = I\times[t_0, t_0+\Delta t]$, $I=[y_1,y_2]$, i.e.

$$|a(y,t)-a(z,t)|\leq L|y-z|, \qquad |b(y,t)-b(z,t)|\leq L|y-z|$$

for all $x,y\in I$, $t\in[t_0,t_0+\Delta t]$; initial value $y_0(x)$ is any fuzzy perception value.

If for fixed $x_0 \in X$, for which $P(\{x_0\}) \geq \alpha > 0$ the condition $[y_0(x_0) - \Delta y; y_0(x_0) + \Delta y] \subset I$ holds, then for this x_0 the problem (1), (2) has a unique solution in the segment $t \in [t_0; t_0 + h]$, where

$$h = \min\left[k\frac{1}{2L} \cdot \min\left\{1, \frac{1}{\sqrt[4]{\varphi^{-1}(\alpha)}}\right\}, \frac{\Delta y}{\max_t |a(x)| + \sqrt{\varphi^{-1}(\alpha)} \cdot \max_t |b(x)|}, \Delta t\right], \ k \in (0,1).$$

Proof. Let's fix $x_0 \in X$, for which $P(\{x_0\}) \geq \alpha > 0$. The path of the fuzzy perception process $w(t)$ and the initial value y_0 depend on it.

Let's prove that for given x_0, $y_0(x_0)$ the unique solution exists in $[t_0, t_0 + h]$. Let's consider such space of all continuous functions $f(t)$, $t \in [t_0, t_0 + h]$ that $f(t_0) = y_0(x_0)$. Let's define a distance in this space:

$$\rho(f,g) = \max_{t \in [t_0; t_0 + \Delta t]} |f(t) - g(t)|.$$

We consider the following mapping:

$$\Phi(f(t)) = y_0(x_0) + \int_{t_0}^{t} a(f(s),s)ds + \int_{t_0}^{t} b(g(s),s)dw(s,x_0).$$

Let's prove that it's contracting. Lipschitz condition implies

$$\rho(\Phi(f),\Phi(g)) =$$

$$= \max_{t \in [t_0, t_0 + h]} \left| \int_{t_0}^{t} \left(a(f(s),s) - a(g(s),s)\right) ds + \int_{t_0}^{t} \left(b(f(s),s) - b(g(s),s)\right) dw(s,x_0) \right| \leq$$

$$= \max_{t \in [t_0, t_0 + h]} \left(\int_{t_0}^{t} \left|a\left(f(s),s\right) - a\left(g(s),s\right)\right| ds + \int_{t_0}^{t} \left|b\left(f(s),s\right) - b\left(g(s),s\right)\right| dw(s,x_0) \right) \leq$$

$$\leq Lh\left(1 + \int_{t_0}^{t_0 + h} dw(s,x_0)\right)\rho(f,g) = Lh(1 + (w(t_0 + h, x_0) - w(t_0, x_0)))\rho(f,g) \cdot$$

Because of $P(\{x\}) \geq \alpha$, $w(t_0 + h, x_0) - w(t_0, x_0) \leq \sqrt{\varphi^{-1}(\alpha)}h$, where $\varphi(y) : [0; +\infty) \to [0,1]$ is the function that corresponds to distribution of possibilities of the process $w(t)$. We obtain:

$$\rho\left(\Phi(f), \Phi(g)\right) \leq$$

$$\leq Lh\left(1 + \sqrt{\varphi^{-1}(\alpha)} \cdot h\right) \cdot \rho(f,g) \leq 2L \cdot \max\left\{h, \sqrt{\varphi^{-1}(\alpha)} \cdot h^2\right\} \cdot \rho(f,g).$$

So, with $h \le k\frac{1}{2L}\cdot\min\left\{1,\frac{1}{\sqrt[4]{\varphi(\alpha)}}\right\}$, $k\in(0,1)$, the mapping is contracting and according to a contraction principle it has a unique fixed point. Obviously, any fixed point of mapping Φ is a solution of initial-value problem for given x_0.

At last, let's obtain conditions for h, which provide that the solution will remain within the domain D. First of all $h\le\Delta t$. Then with $h\le\frac{\Delta y}{\max\limits_t|a(x)|+\sqrt{\varphi^{-1}(\alpha)}\cdot\max\limits_t|b(x)|}$ we can estimate $y(t,x_0)$:

$$\left|y(t)-y(t_0)\right|=\left|\int_{t_0}^{t}a(y(s),s)\,ds+\int_{t_0}^{t}b(y(s),s)\,dw(s)\right|\le$$

$$\le h\cdot\left[\max_t|a(t)|+\max_t|b(t)|\cdot\sqrt{\varphi^{-1}(\alpha)}\right]\le\Delta t,$$

i.e. with these assumptions $y(t)\in I$.

If the functions $a(y,t)$ and $b(y,t)$ are defined in a finite domain D it is possible that for any $h>0$ such $x_0\in X$ exists that $P(\{x_0\})\ne 0$ and in the time segment $[t_0,t_0+h]$ for $x=x_0$ the solution of (1), (2) will go beyond the bounds of D. In other words, for arbitrary small h it is impossible to rely on a solution existence for all possible $x\in X$ in $[t_0,t_0+h]$ segment. Let's prove the theorem that is a sufficient condition of a solution existence for all $t\ge t_0$.

Theorem 5. If fuzzy perception functions $a(y,t,x)$ and $b(y,t,x)$, where $y,t\in R$, $x\in X$, for arbitrary $x\in X$ are continuous by t and satisfy local Lipschitz condition by y on R, i.e. for arbitrary segment I

$$|a(y,t,x)-a(z,t,x)|\le L(I)|y-z|,\qquad |b(y,t,x)-b(z,t,x)|\le L(I)|y-z| \text{ for } x,y\in I,$$

where $L(I)$ is a finite value that depends on chosen segment I, and growth condition

$$|a(y,t,x)|^2\le K\left(1+|y|^2\right),\qquad |b(y,t,x)|^2\le K\left(1+|y|^2\right),$$

then for any fuzzy y_0 the problem (1),(2) has a unique solution for $t\in[t_0;+\infty)$.

Proof. Crispness or fuzziness of functions a and b doesn't influence the proof.

Like in Theorem 4, we fix $x_0 \in X$, for which $P(\{x_0\}) \geq \alpha$. Let's take arbitrary great $T > 0$ and prove that on the segment $[t_0, t_0 + T]$ solution of the problem (1), (2) exists.

Although Theorem 4 proves that the solution exists for $t \in [t_0, t_0 + h]$, we can extend this solution beyond these limits. Indeed, let's consider the following problem

$$y(t, x_0) = y(t_0 + h, x_0) + \int_{t_0+h}^{t} a(y(s), s)ds + \int_{t_0+h}^{t} b(y(s), s)dw(s, x_0).$$

According to Theorem 4, this problem has a solution on $[t_0 + h, t_0 + h']$. Substitute $y(t_0 + h, x_0)$ with

$$y(t_0 + h_1, x_0) = y(t_0, x_0) + \int_{t_0}^{t_0+h_1} a(y(s), s)ds + \int_{t_0}^{t_0+h_1} b(y(s), s)dw(s, x_0)$$

and obtain that

$$y(t, x_0) = y(t_0, x_0) + \int_{t_0}^{t} a(y(s), s)ds + \int_{t_0}^{t} b(y(s), s)dw(s, x_0).$$

Hence, we got a solution of the system on $[t_0, t_0 + h + h']$ segment. This operation can be repeated as many times as needed. We must show that the solution will not run to infinity for finite t.

Assume that the solution of the system exists on the segment $[t_0, t_0 + T]$. Estimate $y(t, x_0)$:

$$|y(t, x_0) - y(t_0, x_0)| \leq \int_{t_0}^{t} |a(y(s), s)| ds + \int_{t_0}^{t} |b(y(s), s)| dw(s, x_0).$$

Growth condition implies that

$$|y(t, x_0) - y(t_0, x_0)| \leq |z(t, x_0) - y(t_0, x_0)|, \tag{3}$$

where $z(t, x_0)$ is a solution of crisp Cauchy problem

$$z(t, x_0) = y(t, x_0) + \left(1 + \sqrt{\varphi^{-1}(\alpha)}\right) \int_{t_0}^{t} \left(1 + K|z(s, x_0)|^2\right)^{1/2} ds.$$

This problem has a solution for all $t \geq t_0$.

Thus, having fixed x_0, we fix the segment $I_T = [y(t_0, x_0) - 2\Delta; y(t_0, x_0) + 2\Delta]$, where $\Delta = |z(t_0 + T, x_0) - y(t_0, x_0)|$. According to Theorem 4, the solution of (1), (2) exists on $[t_0, t_0 + h_T]$, where $h_T \leq k \frac{1}{2L(I_T)} \cdot \min\left\{1, \frac{1}{\sqrt[4]{\varphi^{-1}(\alpha)}}\right\}$. Extend this solution to $[t_0; t_0 + 2h_T]$, etc., until we cover the whole segment $[t_0, t_0 + T]$ or cross the bounds of I_T.

Let's assume that we cross the bounds for some $t_1 < t_0 + T$. For this t_1 (3) doesn't hold and that's impossible. Theorem 4 ensures uniqueness of the solution, that concludes the proof.

Conclusions

In this article, we consider the problem of modeling non crisp dynamics. A new formalism for the design of uncertain dynamics – differential equation by fuzzy perception roaming processes is offered. Correctness of construction of this equation is shown and a Cauchy problem theorem is proved.

Local Lipschitz condition provides uniqueness of the solution; growth condition ensures that it won't run to infinity for finite t .

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Index matrices with function-type of elements. Part 2

Krassimir T. Atanassov

Abstract. *Index Matrices (IMs) are extensions of the ordinary matrices. Some of the operations and relations defined over IMs are analogous of the standard ones, while there have been defined new ones, as well. Operators that do not have analogues in matrix theory have been defined, as well. In general, the elements of an IM can be real or complex numbers, as well as propositions or predicates. In the present paper, new operation over IMs with elements being functions, is defined, and some of its properties are discussed.*

Keywords: *Function, Index matrix, Operation, Operator, Relation.*

AMS Classification: *11C20*

1 Introduction

The concept of Index Matrix (IM) was introduced in 1984, but the full description of the research over them was published in [1] exactly 30 years later. In [2], the concept of IM with elements being functions (IMFEs), is defined, and two specific operations over these IMs are introduced. Here, we introduce a new operation over IMEFs.

Firstly, we give the definition of a standard IM with elements being real (or complex) numbers.

Let $\mathcal{I}$ be a fixed set of indices and $\mathcal{R}$ be the set of the real numbers. By IM with index sets K and L $(K, L \subset \mathcal{I})$, we denote the object:

$$[K, L, \{f_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\},\ L = \{l_1, l_2, ..., l_n\},$ for $1 \le i \le m$, and $1 \le j \le n : a_{k_i,l_j} \in \mathcal{R}$.

Six operations, six relations and a lot of operators are defined over IMs in [1].

2 Definition of the index matrix with function-type of elements

Let the set of all used functions be $\mathcal{F}$.

The research over IMs with function-type of elements develops in the following two cases:

- each function of set $\mathcal{F}$ has one argument and it is exactly x (i.e., it is not possible that one of the functions has argument x and another function has argument y) – let us mark the set of these functions by $\mathcal{F}_x^1$;
- each function of set $\mathcal{F}$ has one argument, but that argument might be different for the different functions or the different functions of set $\mathcal{F}$ have different numbers of arguments.

The IM with Function-type of Elements (IMFE) has the form (see, [2])

$$[K, L, \{f_{k_i,l_j}\}] \equiv \begin{array}{c|ccccc} & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & f_{k_1,l_1} & \dots & f_{k_1,l_j} & \dots & f_{k_1,l_n} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_i & f_{k_i,l_1} & \dots & f_{k_i,l_j} & \dots & f_{k_i,l_n} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & f_{k_m,l_1} & \dots & f_{k_m,l_j} & \dots & f_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\},\ L = \{l_1, l_2, ..., l_n\},$ for $1 \le i \le m$, and $1 \le j \le n : f_{k_i,l_j} \in \mathcal{F}_x^1$.

The IMFE has this form independently of the form of its elements. They can be functions from $\mathcal{F}_x^1$ having one, exactly determined argument (e.g., x), as well as functions with a lot of arguments. The set of n-argument functions will be marked by $\mathcal{F}^n$.

3 Standard operations over IMFEs

Here we give the definitions of the operations over IMFEs. The first three of them are in more general form than in [2].

Let the IMFEs $A = [K, L, \{f_{k_i,l_j}\}], B = [P, Q, \{g_{p_r,q_s}\}]$ be given. Then

Addition:

$$A \oplus_{(\circ)} B = [K \cup P, L \cup Q, \{h_{t_u,v_w}\}],$$

where

$$h_{t_u,v_w} = \begin{cases} f_{k_i,l_j}, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ g_{p_r,q_s}, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ f_{k_i,l_j} \circ g_{p_r,q_s}, & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q; \\ \perp, & \text{otherwise} \end{cases},$$

where here and below, symbol "$\perp$" denotes the lack of operation in the respective place and $\circ \in \{+, \times, \max, \min, ...\}$.

Termwise multiplication

$$A \otimes_{(\circ)} B = [K \cap P, L \cap Q, \{h_{t_u,v_w}\}],$$

where

$$h_{t_u,v_w} = f_{k_i,l_j} \circ g_{p_r,q_s},$$

for $t_u = k_i = p_r \in K \cap P$ and $v_w = l_j = q_s \in L \cap Q$.

Multiplication

$$A \odot_{(\circ,*)} B = [K \cup (P - L), Q \cup (L - P), \{c_{t_u,v_w}\}],$$

where

$$h_{t_u,v_w} = \begin{cases} f_{k_i,l_j}, & \text{if } t_u = k_i \in K \\ & \text{and } v_w = l_j \in L - P - Q \\ g_{p_r,q_s}, & \text{if } t_u = p_r \in P - L - K \\ & \text{and } v_w = q_s \in Q \\ \underset{l_j = p_r \in L \cap P}{\circ} (f_{k_i,l_j} * g_{p_r,q_s}), & \text{if } t_u = k_i \in K \text{ and } v_w = q_s \in Q \\ \perp, & \text{otherwise} \end{cases},$$

where $(\circ, *) \in \{(+, \times), (\max, \min), (\min, \max), ...\}$.

Structural subtraction: $A \ominus B = [K - P, L - Q, \{c_{t_u,v_w}\}]$, where "–" is the set–theoretic difference operation and

$$h_{t_u,v_w} = f_{k_i,l_j}, \text{ for } t_u = k_i \in K - P \text{ and } v_w = l_j \in L - Q.$$

Multiplication with a constant: $\alpha.A = [K, L, \{\alpha.f_{k_i,l_j}\}]$, where α is a constant.

Termwise subtraction: $A - B = A \oplus_{+} (-1).B$.

The problem with the “zero"-IMFE is similar to the “zero-IM": if $f_{k_i,l_j} \in \mathcal{F}$, then

$$I_\emptyset = [\emptyset, \emptyset, \{f_{k_i,l_j}\}].$$

4 Operations over IMFEs and IMs

Let the IM $A = [K, L, \{a_{k_i,l_j}\}]$, where $a_{k_i,l_j} \in \mathcal{R}$ and IMFE $F = [P, Q, \{f_{p_r,q_s}\}]$ be given. Then, we can define:

(a) $A \boxplus F = [K \cup P, L \cup Q, \{h_{t_u,v_w}\}]$, where

$$h_{t_u,v_w} = \begin{cases} a_{k_i,l_j}.f_{p_r,q_s}, & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q; \\ \perp, & \text{otherwise} \end{cases},$$

with elements of $\mathcal{F}^1$;

(b) $A \boxtimes F = [K \cap P, L \cap Q, \{h_{t_u,v_w}\}]$, where

$$h_{t_u,v_w} = a_{k_i,l_j}.f_{p_r,q_s},$$

for $t_u = k_i = p_r \in K \cap P$ and $v_w = l_j = q_s \in L \cap Q$ with elements of $\mathcal{F}^1$;

(c) $F \oplus A = [K \cup P, L \cup Q, \{h_{t_u,v_w}\}]$, where

$$h_{t_u,v_w} = \begin{cases} f_{p_r,q_s}(a_{k_i,l_j}), & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q \\ \perp, & \text{otherwise} \end{cases}$$

with elements of $\mathcal{R}$;

(d) $F \otimes A = [K \cap P, L \cap Q, \{h_{t_u,v_w}\}]$, where

$$h_{t_u,v_w} = f_{p_r,q_s}(a_{k_i,l_j}),$$

for $t_u = k_i = p_r \in K \cap P$ and $v_w = l_j = q_s \in L \cap Q$ with elements of $\mathcal{R}$.

Let the IM $A = [K, L, \{\langle a^1_{k_i,l_j}, ..., a^n_{k_i,l_j}\rangle\}]$, for the natural number $n \geq 2$, where $a^1_{k_i,l_j}, ..., a^n_{k_i,l_j} \in \mathcal{R}$ and IMFE $F = [P, Q, \{f_{p_r,q_s}\}]$, where $f_{p_r,q_s} : \mathcal{F}^n \to \mathcal{F}$ be given. Then

(e) $F \Diamond_\oplus A = [K \cup P, L \cup Q, \{h_{t_u,v_w}\}]$, where

$$h_{t_u,v_w} = \begin{cases} f_{p_r,q_s}(\langle a^1_{k_i,l_j}, ..., a^n_{k_i,l_j}\rangle), & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q \\ \perp, & \text{otherwise} \end{cases}$$

with elements of $\mathcal{R}$;

(f) $F \Diamond_\otimes A = [K \cap P, L \cap Q, \{h_{t_u,v_w}\}]$, where

$$h_{t_u,v_w} = f_{p_r,q_s}(\langle a^1_{k_i,l_j}, ..., a^n_{k_i,l_j}\rangle),$$

for $t_u = k_i = p_r \in K \cap P$ and $v_w = l_j = q_s \in L \cap Q$ with elements of $\mathcal{R}$.

Obviously, in some sense, operators $\boxplus$ and $\boxtimes$ are modifications/extensions of operation Multiplication with a constant.

An interesting **Open problem** is: Can there be defined new operations/operators that are modifications/extensions of operations Multiplication and Structural subtraction.

5 New operations over IMFEs

Let the IMFEs $F = [K, L, \{f_{k_i,l_j}\}]$ and $G = [P, Q, \{g_{p_r,q_s}\}]$ be given, where f_{k_i,l_j}, g_{p_r,q_s} are functions.

(g) $F \Diamond_{\oplus} G = [K \cup P, L \cup Q, \{h_{t_u,v_w}\}]$, where

$$h_{t_u,v_w} = \begin{cases} f_{k_i,l_j}, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ \\ g_{p_r,q_s}, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ \\ f_{k_i,l_j}(g_{p_r,q_s}), & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q; \\ \\ \bot, & \text{otherwise} \end{cases}.$$

The elements of this IM are functions, i.e., $F \Diamond_{\oplus} G$ is an IMFE.

(h) $F \Diamond_{\otimes} G = [K \cap P, L \cap Q, \{h_{t_u,v_w}\}]$, where

$$h_{t_u,v_w} = f_{p_r,q_s}(g_{p_r,q_s}),$$

for $t_u = k_i = p_r \in K \cap P$ and $v_w = l_j = q_s \in L \cap Q$.

Therefore, the elements of this IM are functions, too.

Now, we can see that for every three IMFE $F = [K, L, \{f_{k_i,l_j}\}]$, $G = [P, Q, \{g_{p_r,q_s}\}]$ and $H = [T, U, \{h_{t_u,v_w}\}]$ operations $\boxplus, \boxplus, \oplus, \otimes, \Diamond_{\oplus}, \Diamond_{\otimes}$ are not commutative.

Theorem. For the above three IMFEs F, G, H and for two IMs A and B with elements real numbers:

(a) $(F \Diamond_{\oplus} G) \Diamond_{\oplus} H = F \Diamond_{\oplus} (G \Diamond_{\oplus} H)$,

(b) $(F \Diamond_{\otimes} G) \Diamond_{\otimes} H = F \Diamond_{\otimes} (G \Diamond_{\otimes} H)$,

(c) $(F \Diamond_{\oplus} G) \bigotimes A = F \Diamond_{\oplus} (G \bigotimes A)$,

(d) $(F \Diamond_{\otimes} G) \bigotimes A = F \Diamond_{\oplus} (G \bigotimes A)$,

(e) $(A \oplus_{\times} B) \boxplus F = A \boxplus (B \boxplus F)$,

(f) $(A \oplus_{\times} B) \boxtimes F = A \boxtimes (B \boxtimes F)$,

(g) $(F \oplus_{\circ} G) \Diamond_{\otimes} H = (F \Diamond_{\otimes} H) \oplus_{\circ} (G \Diamond_{\otimes} H)$.

Proof. Let IMFEs F, G, H be given. Then (g) is valid, because

$$(F \oplus_{\circ} G) \Diamond_{\otimes} H = [K \cap P, L \cap Q, \{f_{k_i,l_j} \circ g_{p_r,q_s}\}] \Diamond_{\otimes} H$$

$$= [K \cap P \cap T, L \cap Q \cap V, \{(f_{k_i,l_j} \circ g_{p_r,q_s})(h_{t_u,v_w})\}]$$

$$= [(K \cap T) \cap (P \cap T), (L \cap V) \cap (Q \cap V), \{f_{k_i,l_j}(h_{t_u,v_w}) \circ g_{p_r,q_s}(h_{t_u,v_w})\}]$$

$$= [(K \cap T), (L \cap V), \{f_{k_i,l_j}(h_{t_u,v_w})\}] \oplus_{\circ} [(P \cap T), (Q \cap V), \{g_{p_r,q_s}(h_{t_u,v_w})\}]$$

$$= (F \Diamond_{\otimes} H) \oplus_{\circ} (G \Diamond_{\otimes} H).$$

6 Conclusion

We will use the new operations for description of some components of the artificial intelligence. For example, in [3] it was shown that the neural networks can be described in the form of IMs. On the basis of this paper, a new extension of the concept of neural networks, was introduced in [4].

In future, the new type of IM will be used for description of other components of the artificial intelligence, e.g., genetic algorithms.

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AN APPROACH TO BUSINESS PROCESSES REENGINEERING BASED ON INTEGRATION OF THE PROCESS MINING METHODS AND DOMAIN SPECIFIC MODELING TOOLS

Renata Ayzatullova, Lyudmila Lyadova, Irina Shalyaeva

***Abstract**: An approach to reengineering business processes through the integration of the domain specific modeling platform and Process Mining tools is described. An analysis of the existing approaches to business processes improvement is presented and restrictions are shown. The Process Mining methods are related to business process reengineering stages and tasks. Comparative analysis of Process Mining tools is executed. The advantages of the using of domain specific modeling tools (language workbenches, DSM platforms) are substantiated. Brief comparison of various visual languages notations and model transformation examples are described. The DSM platform ensures mutual understanding between specialists. The MetaLanguage DSM platform is the basis of integration tools. Some DSL (metamodels) are described and transformations are illustrated. The implementation of integrated tools reduces the complexity of analyst's work.*

***Keywords**: business processes reengineering, domain specific modeling, DSM, modeling languages, DSL, language toolkits, DSM platform, model transformations, business process analysis, Process Mining.*

***ACM Classification Keywords**: H.4 INFORMATION SYSTEMS APPLICATIONS: H.4.1 Office Automation –Workflow management; H.4.2 Types of Systems – Decision support (e.g., MIS). I.6 SIMULATION AND MODELING: I.6.2 Simulation Languages; I.6.4 Model Validation and Analysis; I.6.5 Model Development Modeling methodologies.*

Introduction

One of the most important directions in modern management is business process reengineering.

In accordance with the definition [Hammer, 1993], [Kotler, 2010] "*Business process reengineering* is the fundamental rethinking and radical redesign of business processes to achieve dramatic improvements in critical, contemporary measures of performance such as cost, quality, service and speed". Business process reengineering (BPR) allows to achieve the maximum effect of production, commercial, financial and economic activities, issued by the relevant regulatory and administrative documents. Reengineering uses the specific means of problem information representation and processing, clear to both managers and developers of information systems.

Business process reengineering assumes the solution of several tasks, which are carried out step by step, in particular, it is necessary to carry out:

1) modeling and analysis of existing business processes, defining key performance indicators, and identifying processes that require change;
2) rethinking and development of basically new business processes, definition of the "ideal" embodiment of each business process in accordance with the selected indicators;
3) optimization of the existing business processes translation to "ideal";
4) actually reengineering, implementation of new ("ideal") business processes;
5) monitoring and continuous processes improvement based on key performance indicators.

Business process reengineering is often associated with the introduction of new information technology (IT), the establishment of information systems (IS), automating business processes or individual labor-intensive operation. However, automation of "imperfect" business process usually has quite low efficiency, reduces the utility of IT implementation: automated system will repeat the shortcomings of existing processes and the cost of correcting errors in the implementation of automated business processes, increases many times. The most effective approach is to introduce the IS, which is accompanied by a reengineering of business processes.

Business processes reengineering main problems are the lack of integrated tools, complexity and immensity of the issue to solve, leading to high requirements for professionals, engaged in reengineering, qualification and experience.

Reengineering is impossible without the use of business process modeling tools. At that, modeling is considering both as a process of formal models construction, and the process of their research.

It is accepted to develop two models of business processes: the existing (model “as is”) and future (model “as to be”). In the "standart" business process optimization existing model is considered as a basis for future processe models, and in case of reengineering – model of the future processes is developed "from scratch".

Building the model “as is” allow to illustrate

– redundant operation, which can be eliminated in the optimization process;
– possibilities for process parallelization;
– business process “hot spots” – transactions which can cause faults;
– possibilities for automating business processes or individual operations.

Analysis of the existing business processes and development of models are labor-intensive tasks which traditionally require interviewing staff at all levels, monitoring of their work, document analysis, the study of critical cases, etc. Partially automation of the time-consuming work to create formal models of existing business processes is possible by means of Process Mining tools.

Applying different analysis methods is often connected with usage of instruments that support such methods, but choice of instruments is based on languages for model development.

Present-day modelling instruments usually supports different notations and business process analysis methods, but neither of them provides complex solution of tasks. Integration of modelling tools can be realized on the basis of model transformation. It means conversion of models according to tasks for which different tools based on different modelling notations can be applied.

During realization of reengineering specific attention is on team forming and organizing interaction with specialists engaged in business process execution. Different specialists can be engaged in team, they have access to instruments which must be understandable for different specialist categories. Right framework for development of such instruments can be domain specific modelling (DSM) instruments (DSM platforms).

Research goal is to study possibility to apply DSM platform as basis for development of complex instrument to integrate Process Mining and DSM tools in order to solve business process reengineering tasks.

Approaches to Business Process Improvement

There are different approaches to organize business process reengineering on the basis of present-day information technologies.

A lot of research works [Aleksandrov, 2009], [Andreeva, 2004], [Landsberg, 1998], [Ostankov, 2008] offer to solve separate reengineering tasks, on the basis of applying mtehod chosen during research, in particular tasks can be include information system development and implementation for business processes automation. Other researchers [Konnova, 2008], [Panarin, 2006] offer methodologies based on the applying complex instrument for implementation of all reengineering stages, unity of approaches and succession for tasks solving during different methods integration.

The other research [Aksenov, 2015] includes analysis of existing reengineering methods and shows that solution of labor-intensive tasks and reengineering costs, high requirements to analyst qualification level is actual. Nowadays there is no general method to implement business process reengineering for different object domains effectively. Even the most popular software for business modelling does not provides advanced tools for end-to-end automation of all reengineering tasks. Generally such software is oriented to model development, documentation and visualization.

Need to solve listed problems determines actuality of different approaches development, improvement and integration as well as development of successive methods for reengineering tasks solution and realization of instrument to automate this kind of activity.

Researcher [Aksenov, 2015] compares his own analysis method and “bottle neck” of multi agent process of resources converting with methods considered at [Aleksandrov, 2009], [Konnova, 2008], [Filippovich, 2015].

Researchers [Konnova, 2008], [Panarin, 2006] determine reengineering as set of strategic actions for complex improvement of management system, technologies of activity and interaction. Research area concerns consulting. Note that main problems with which enterprises face during implementation of reengineering concern with absence of complex methodic instrument and united reengineering conception. In addition it is necessary to consider complication and scope of tasks to solve as well as high requirements to qualification and experience of analysts involved in reengineering. Consulting companies specifying on business process optimization help to solve problems. Reengineering methodologies offered by consulting companies are similar at the levels of main stages. In this conditions quality and effectiveness of projects realization depend on adaptability of methods for specific tasks solution.

There was no formal methods of business process reengineering until the last years, the approaches described changes only on an intuitive level without any strict formal models of business processes. But now there are methods based on the use of certain mathematical theories and aimed to solve a certain class of problems.

Business processes reengineering metods based on the integration of structural analysis methods, expert systems and formal grammars, proposed in [Konnova, 2008], [Filippovich, 2015] are based on the theoretical work [Kalyanov, 1996], [Telnov, 2003, a], [Telnov, 2003, a], [Telnov, 2004], [Telnov, 2005]. The structural approach is used for the formal business processes description. The focus is on information flows and binding resources to the company organizational structure. For the analysis of business processes various scenarios an expert system based on the "classic" reengineering rules is used, the quantitative analysis of the process dynamic characteristics is not carried out. This method allows to automate and construct the algoritm for evaluation and analysis of the initial business processes. Results of the analysis are used to find alternatives for their implementation, satisfying the basic principles of reengineering.

The method has limitations, which are noted as in [Aksenov, 2015]:

1) lack of tools for analyzing the dynamic characteristics of the processes;
2) lack of tools for "bottlenecks" identification;
3) inability to conduct parametric and structural process models changes;
4) lack of tools for simulation analysis of the constructed models, their comparisons;
5) inability to formalize the decision-making scenarios;
6) possibility of change in the business process models only at the organizational structure level.

Modeling methodology for distributed business processes management systems [Aleksandrov, 2009] is focused on solving the problems of distributed information systems designing and business processes modeling and progress monitoring.

Author's method of "tactical business process reengineering" isn't focused on large-scale processes models transformations. This technique mostly focused on the adjustment of the organizational structure project and related documents (function diagram, staff schedule, regulations on structural units, job descriptions, contracts and so on.). This methodology is based on the modeling of structures on the basis of models in IDEF0. Analysis of business process models and calculation of resources for their implementation is based on the «Bill of Material» (BOM) approach, which is applicable in the operational management for the ERP systems implementation. To solve the problems of analysis and eliminate bottlenecks by means of simulation modeling colored Petri nets are used.

In [Aksenov, 2015] for the simulation and analysis of the technological, logistical and organizational processes in the automated system of manufacture of steel products (VMP AS) researcher used multi-agent model of the process of conversion of resources (MPPR). The main elements of the model are MPPR operation, agents, sources and receivers of resources, processes, intersections, resources, tools, applications. Multiagent resource conversion process is built in the form of multi-channel queuing system. The model allows to analyze and eliminate the bottlenecks of the processes. The method is implemented in an automated system of manufacture of steel products.

Analysis has shown that the task of creating an integrated tool for solving reengineering problems remains relevant. In particular, in considered tools the problem of automation of formal models construction, and integration created tools with other systems, or they are hesitant to limited conditions.

Applying Process Mining Tools for Solving Business Process Reengineering Tasks

Generally different business process reengineering methods require to solve following tasks (*stages*):

1. *Development of reeingeeniring project and identification of business to reform* (project objective, tasks and reengineering approach are determined, project team are formed and it is useful to include specialists of different areas as well as representatives of external environment in project team).
2. *Documenting business processes* (graphical models of business processes are developed based on the documentation methods and business process operations are timed).
3. *Comparative analysis of business processes* (business processes are analyzed and compared with progressive and advanced business processes of enterprise divisions or competitor organizations).
4. *Development of future organization model* (ideology system is formulated for new organization an according to its aims and possibilities).
5. *Analysis of existing problems and reengineering of processes and technologies* (identification of problems of existing technologies and business processes, development of new processes models).
6. *Implementation of new business processes and technologies*, *evaluation of results* (comparison of reengineering results, performance evaluation of system operation according to specific criteria).

Process mining tools can be used for solving every tasks.

Most of data that can be used for reengineering tasks solution are not structured or structured poorly. That is why serious processing problems of business processes identification and analysis tasks can occur. One of the key approaches to solve such problem is to extract useful information about processes from data stored in informational systems. Such approach is realized in Process Mining tools, which unite process data mining instruments [Tiwari, 2005], [Aalst, 2007], [Aalst, 2011], [Aalst, 2012], [Wen, 2009].

Process Mining is powerful instrument to use for rationalization of business process reengineering on the different stages. Process Mining tools can be used for business process effectiveness increase at the enterprises and organization of different industries.

Two key factors determine potential and need of the method:

1. There is a lot of information stored in corporate informational systems. Such information is history of processes, in particular, information technologies are used for process implementation. Events are recorded in logs of operational systems and DBMS, databases and software. Although abundance of event data, process identification and analysis as well as problem diagnostic are often based on the expert notion, subjective conclusions and propositions, but not on facts.
2. Software for using by analysts is not customized to reengineering tasks: most tools of business analytics are focused on the data, reports generation, analysis results visualization, but don't describe business process essence and form complex understanding of all company processes by users.

Basis of Process Mining is methods and technologies of business process modelling and Data Mining. However, other methods can be applied for solving tasks in particular statistic methods, genetic algorithms and so on [Medeiros, 2006].

Process Mining includes three *main approaches*:

– *Discovery* is identification of processes and development of business process formal model based log records about real events in system.
– *Conformance checking* is checking correspondence of business process models with real business processes to determine where process deviates from expected behavior and why such deviations occur.
- *Enhancement* is search of response for a question that model changes to improve some business processes indicators (search of fast and effective means to implement business processes).

In summary, we can propose that application of advanced instruments of Process Mining provides to solve more laborious and complex tasks of reengineering. Process Mining provides not only to identify processes and build their formal models but also check compliance of models with real situation and determine deviations by comparison of event data in system with process model. It supports decision making and gives recommendations to improve processes.

Central element of Process Mining is support of connection between process models and reality represented as event logs.

Traditional tools for business analysis don’t correspond for deep process analysis, therefore new software is developed and focused on Process Mining tasks solution. The system ProM is more popular framework with open code, it supports different Process Mining methods. Analysis tools are completed by large number of plugins to extend possibilities of the system [Shershakov, 2013]. At the market, there is range of commercial tools for deep process analysis (Futura Reflect, Disco and so on.) In summary traditional reengineering tools (tools for interactive graphics, visual modelling of business processes, simulation etc.) are extended by means for deep mining of business processes.

Nowadays it is actual to use Process Mining tools by users who are not IT-specialists (business development managers, business analysts etc.). Such approach can largely simplify business process reengineering. One of the important features of Process Mining tools is possibility to work with not only event logs of some formats but also business process models of different notations. Contradiction is that system for speedup and simplification of analysts work leads to problems connected with processing of business process models by hands and presents analysis results in unclear form for users. It leads to wrong interpretation of process analysis results and wrong conclusions. Additionally important feature is integration of used tools with other instruments, informational and analytical systems for complete analysis of processes under investigation and development of new system to realize models developed by analysts.

The general BPM scheme including business process reengineering stages is changed too (Fig. 1). Monitoring results can be used by analysts for specification of model realized.

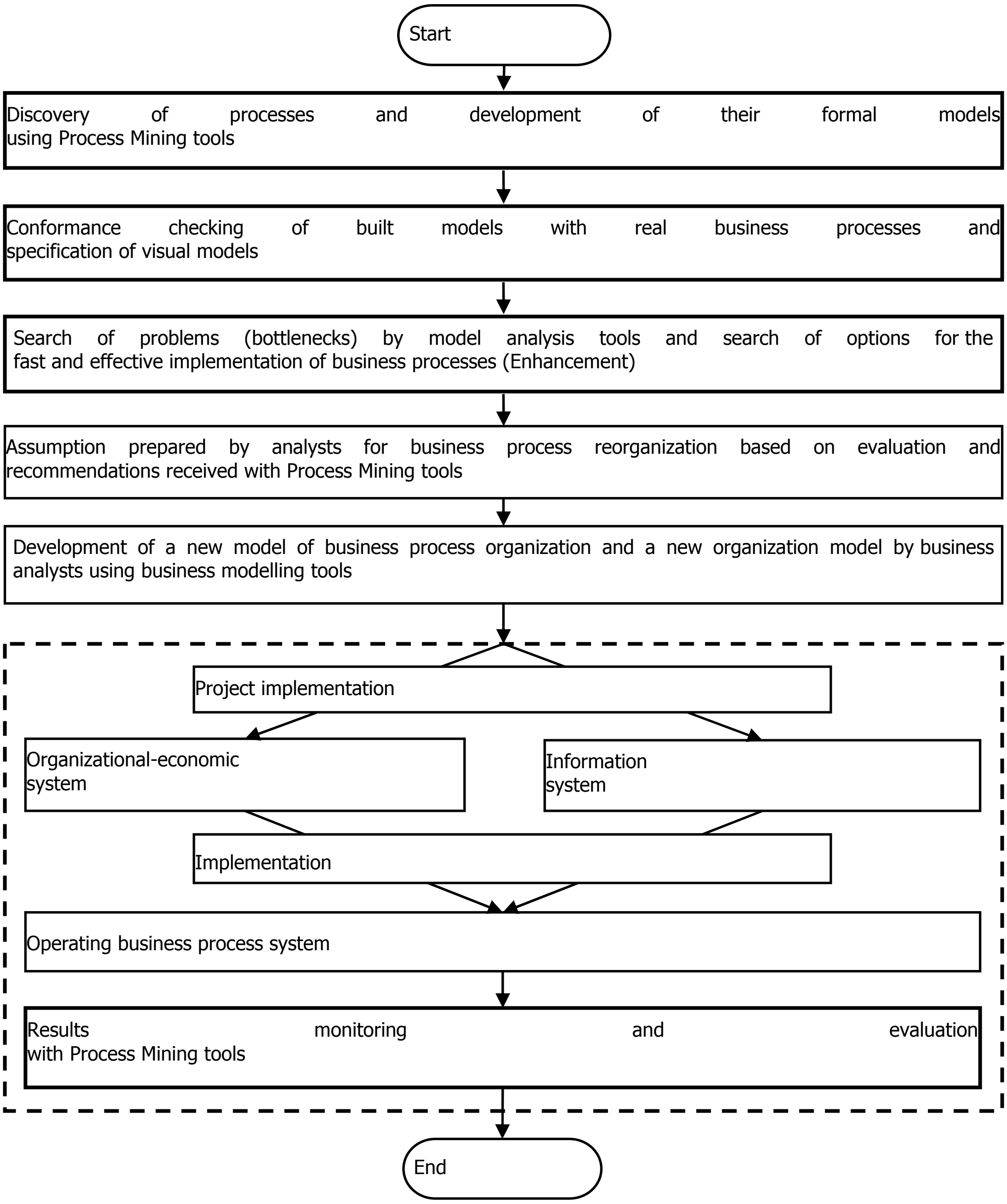

Figure 1. Business process reengineering stages using Process Mining

Table 1 includes results of Process Mining instruments comparing. Systems are evaluated by scale from 0 to 10, where 10 is maximum mark. Thus, ProM system is leader on all indicators (maximal performance is achieved by system openness).

Table 1. Comparative analysis of Process Mining tools

Process Mining System	System integration	Import model support	System functionality	Sum of marks
ARIS Process Performance Manager	6	6 (supports to use model developed by other ARIS tools)	8	20
ProcessAnalyzer	4	–	5	9
ProM	8	7	10	35
Disco	8	7 (due to compatibility with ProM)	6	28

Many of ideas realized in ProM were introduced in such commercial products as Disco, Perceptive Process Mining, Celonis and QPR ProcessAnalyzer.

ARIS Process Performance Manager (Software AG) is the most convenient, user friendly and understandable for user system from all described above. However, such system is very expensive. Disco is more efficient to use by small and medium business. Such system is convenient, ease of use and oriented on clients. Although ProM open system supports to extend and add system functionality for users, conduct research and solve actual Data Mining problems. Additionally one of key criteria to choose process analysis tools is possibility to enter not only event logs but also graphic process models. Such feature is realized in ProM system thought developed plugins [Shershakov, 2013].

DSM Platforms help to provide model interoperability and possibility to work with them for different user categories, as well as integrate Process Mining tools with other business analysis and system development cases. DSM platform also offer to develop models of DSL notations, implement transformation and export them to external information systems.

DSM Platform Usage for Solving Business Process Reengineering Tasks

All domain-specific modeling systems for models creation involve the use of domain-specific languages (DSL), allowing modelers (domain experts, analysts etc.) to work in the familiar terms of subject areas, using tools designed to solve specific professional problems for development and analysis of models. Language toolkits, or DSM platform, provide the ability to create new languages at minimal cost.

To solve described problems DSM platform should except opportunity to create new languages for modeling business processes, also provide ability to perform model transformations (both vertical and horizontal). These particular features allow to consider the DSM platform as a basis for integration of various tools (used in various stages of reengineering by different categories of users) for business modeling and information systems creation. The results of a DSM platforms comparative analysis is shown in Table 2 [Suhov, 2012].

Table 2. DSM-platforms comparison results

System feature	**MetaEdit+**	**DSL Tools, State Machine Designer**	**Eclipse GMF**	**QReal**	**Meta-Language**
Creation a DSL for a wide range of areas	+	+	+	+	+
Means of describing an abstract syntax	GORP	MOF	Encore	MOF	+
The possibility of modifying the meta-language	+	–	–	–	+
The multi-level modeling possibility	+	–	–	–	+
Changing the metamodel without code regeneration	+	–	–	–	+
The ability to "manual" DSL adaptation	–	+	+	+	+
The ability of horizontal transformation	–	–	By plugins	–	+

The MetaLanguage system is based on the use of *metalanguage* the basic elements of which are following [Suhov, 2013, a], [Suhov, 2013, b]: *entities* – domain objects that are essential in terms of the current task; *relationships* represent relationships between entities (inheritance relationships, association and aggregation); *restrictions* allow to describe rules of domain models creation through the restrictions imposed on entities and relations. *Transformer* is one of the central MetaLanguage components, the operation of which is based on the use of graph grammars. Transformation component allows the conversion of user-created graphical models (diagrams) into text or visual models, described in other graphical notation [Sukhov, 2014].

A simplified DSM-platform usage diagram for solving business processes modeling and analysis tasks during reengineering is shown in Fig. 2.

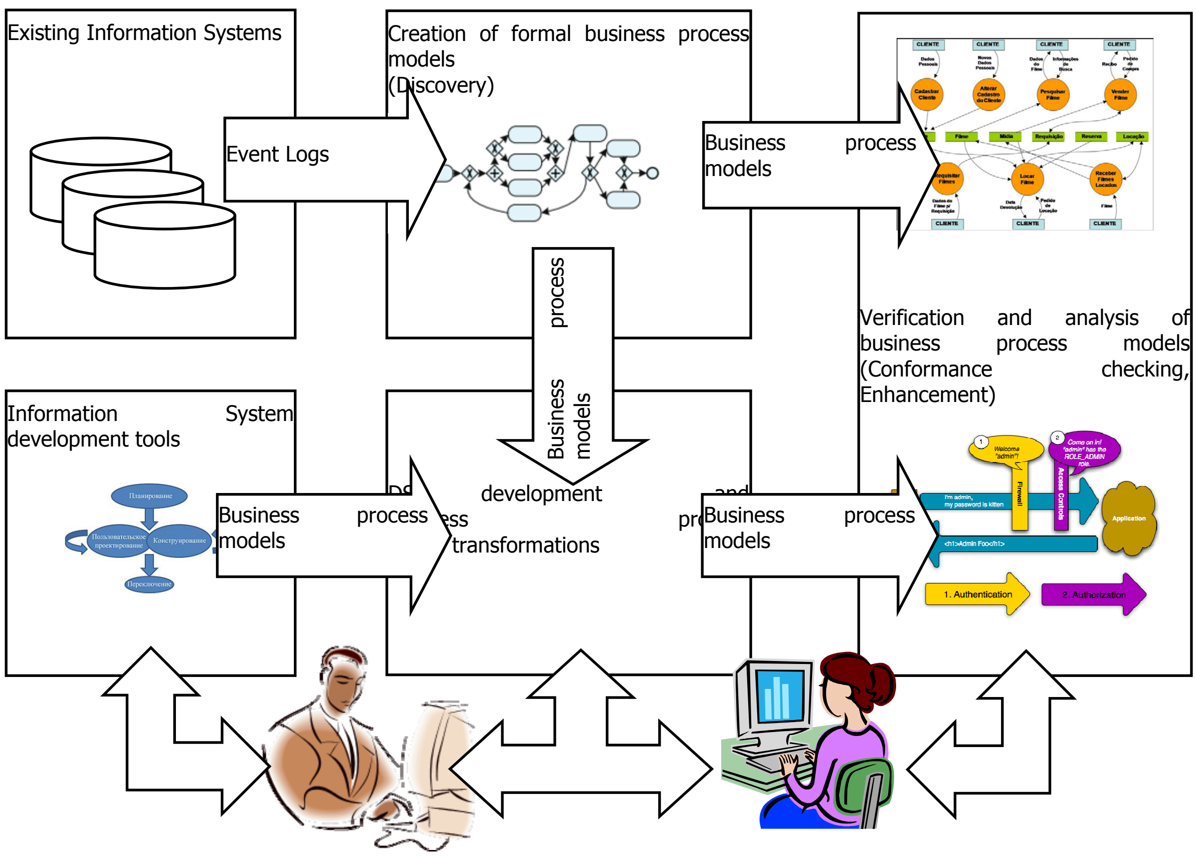

Figure 2. The solution of business process reengineering tasks with usage DSM platform

Various notations used in business analysis systems include the "standard" elements allowing to model business processes in solving reengineering tasks. Table 3 demonstrate the comparison of some widely used business process modeling notations, their elements (as exemplified by solving the tasks of refinery holding's business process reengineering). The given data can be used to develop model transformation rules. The results of implemented in MetaLanguage rules were analyzed to identified "losses", the elimination of which requires "manual" improvements and model's completions.

The main criterion for evaluating the performance of transformation is to keep in converted models:

– functional structure of processes (it is necessary that the target model has retained all the features of the original model);
– logical structure (after the transformation the model must preserve the logic of the functions);
– organizational structure (responsible for the performance of functions rests with the organizational elements involved in the business process, so this information should be communicated to the target model);
– information flows (after models transformation the input data and function results stream must be monitored).

Some transformation analysis results are shown in Table. 4.

Table 3. A comparison of business process modeling visual language notations

Notation	**IDEF0**	**UML Activity Diagram**	**AnyLogic**	**ARIS eEPC**	**ARIS VAD**
Function	+	+	+	+	+
Organizational element	+	+	+	+	+
Event	–	+	+	+	not required
Information object	+	+	–	+	+
External business process	–	–	not required	+	not required
Means of function performance	+	–	+	+	not required
Branch	–	+	+	+	not required
Merging	–	+	–	+	not required

Table 4. Evaluation of models transformation

Transformations	Functional structure	Logical structure	Organizational structure	Information flows
ARIS eEPC – AnyLogic	Completely preserved in the target model	Partially preserved in the target model	Completely preserved in the target model	Not required
UML Activity Diagram – ARIS eEPC	Completely preserved in the target model	Completely preserved in the target model	Completely preserved in the target model	Completely preserved in the target model
ARIS VAD – IDEF0	Completely preserved in the target model	Completely preserved in the target model	Necessary to complete the target model	Not preserved in the target model
ARIS eEPC – IDEF0	Completely preserved in the target model	Not preserved in the target model	Not preserved in the target model	Partially preserved in the target model

Although the process of transformation rules developing is quite labour-consuming and requires time, all rules are developed only once and then can be used by analysts repeatedly. It should be noted that, if necessary, the analysts themselves can create the necessary metamodels for new modeling languages and develop rules for their transformation or supplement the existing rules with the help of visual modeling and model transformation platform – MetaLanguage. During the reengineering of business processes by companies specialized in providing consulting services and using corresponding business process modeling tools to perform their work, the use of DSM platform can significantly reduce the complexity of the work and reduce the time spent on the models construction.

An example of model transformation is described below. The metamodel of the ARIS eEPC visual language in MetaLanguage system is shown in Fig. 3. The metamodel of the AnyLogic language is presented in Fig. 4. The model transformation result is shown in Fig. 5. This model requires "manual" improvements and model completions. The result of these improvements is presented in Fig. 6.

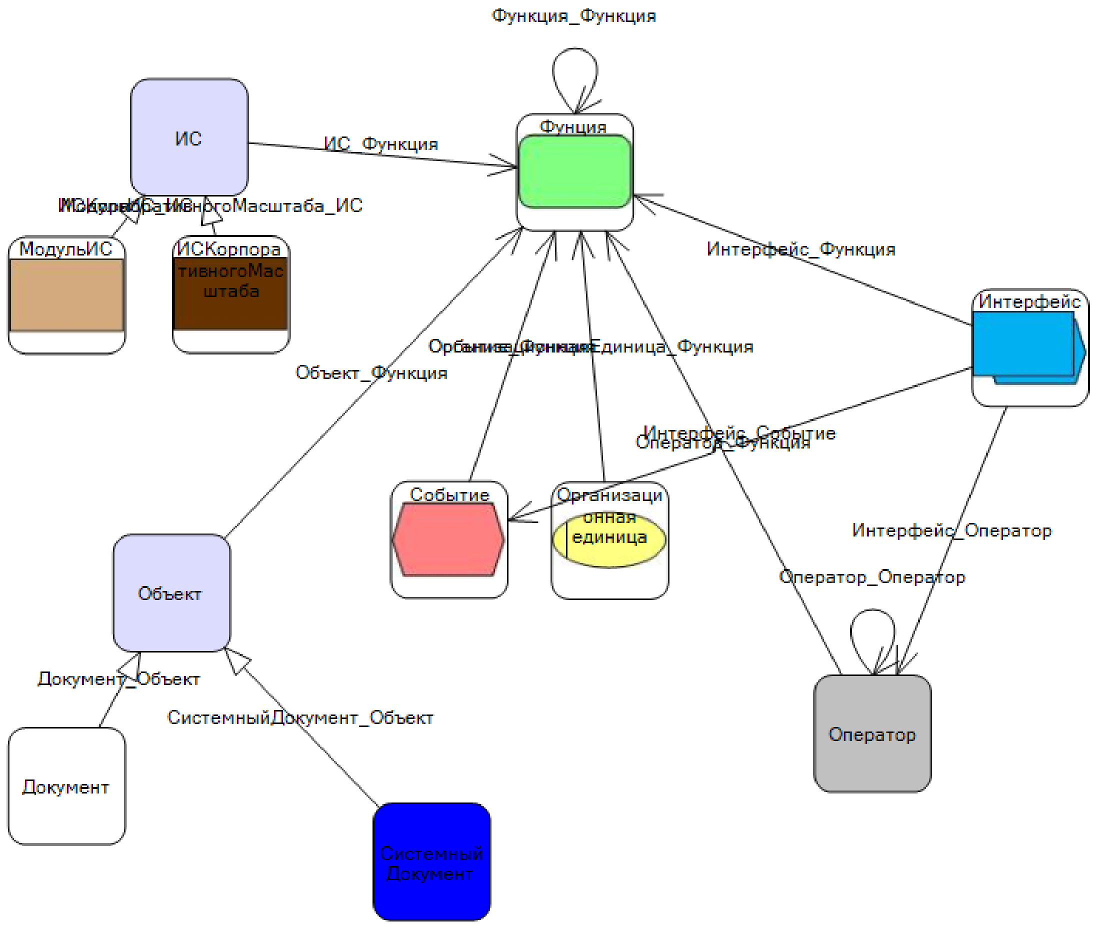

Figure 3. The metamodel of the ARIS eEPC language in MetaLanguage system

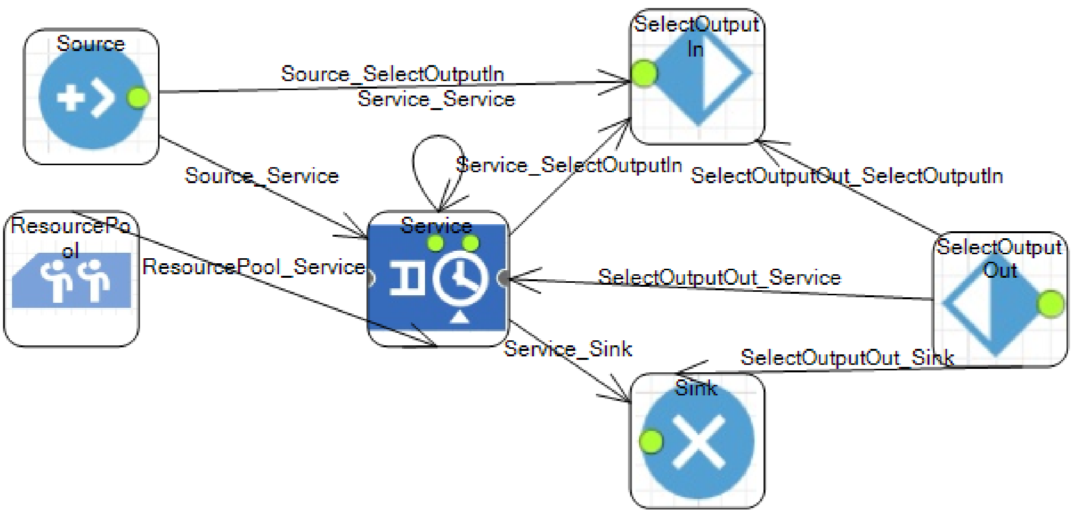

Figure 4. The metamodel of the AnyLogic simulation system language in MetaLanguage system

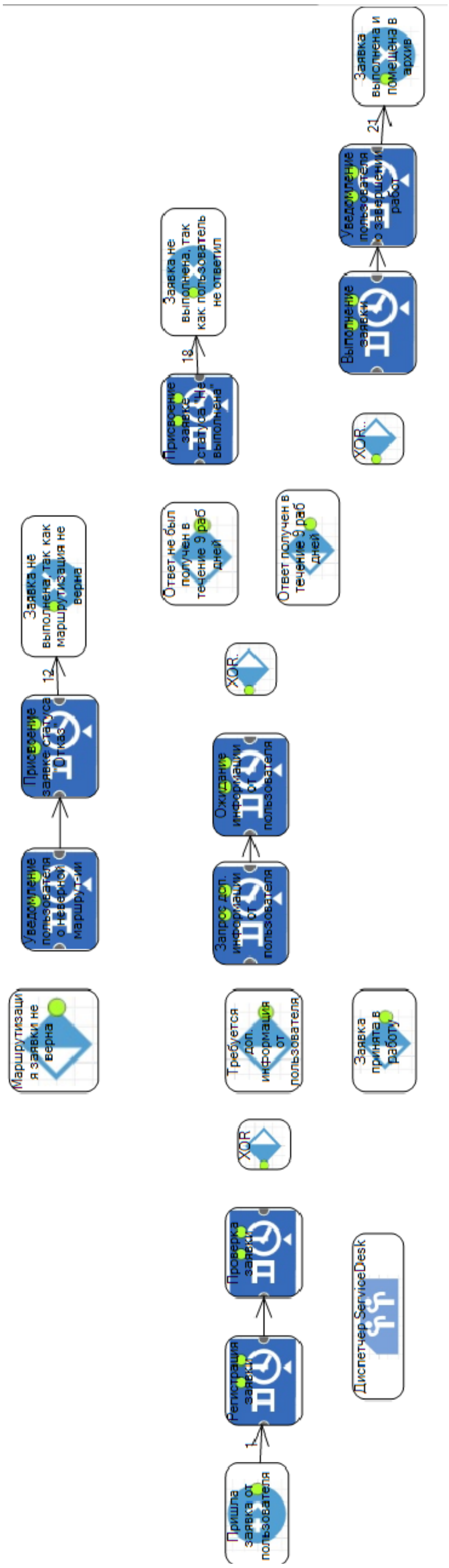

Figure 5. The result of automatic transformation of the business process model from ARIS eEPC notation into AnyLogic model

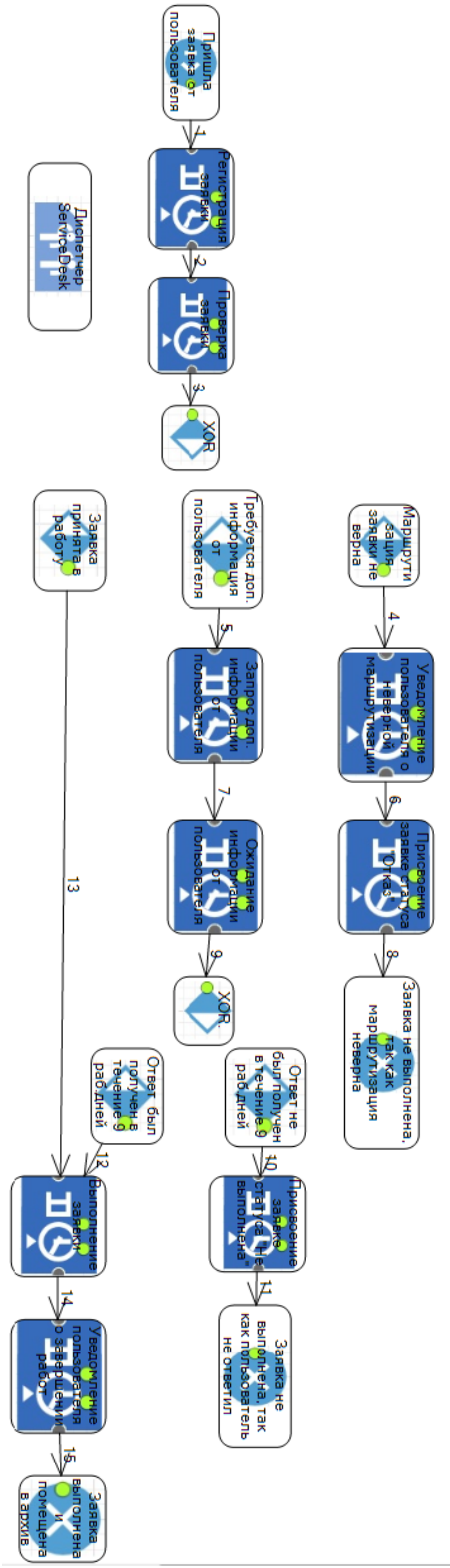

Figure 6. The result of "manual" improvements of the transformed model in AnyLogic simulation system

Conclusion

Studies have shown the promise of an integrated tool, designed to meet the challenges of reengineering including the language tools and means of in-depth mining business processes. Maximum efficiency is achieved when it is the integration of tools to automate the steps of the construction of formal models, analysis and transformation, DSM-implemented platforms and tools Process Mining, and traditional media business modeling [Lyadova, 2012], [Lyadova, 2014].

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PROPERTIES PROOF METHOD IN IPCL APPLICATION TO REAL-WORLD SYSTEM CORRECTNESS PROOF

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***Abstract**: The correctness proof for programs with parallelism, and interleaving concurrency with shared memory in particular, is complicated problem because the state of separate execution thread can be changed even in ready-to-run (waiting) time due to possible inference of one execution path over the other via shared data or messaging mechanics. Classical methods like Floyd-Hoare cannot be applied directly to this case and new non-trivial methods are required to cope with such a complexity. The safety property proof of real-world system using method for software correctness proof in Interleaving Parallel Composition Language (for defined custom class of programs – namely server software for Symmetric Multi-Processing architecture like DB-server or Web-server) is the subject of this article. Operational semantics of the system is defined in terms of state transitions. Program Invariant as well as Pre- and Post- conditions are formulated in accordance with the methodology. Conclusions about adequacy of the Method usage for such a kind of tasks (thanks to flexibility of composition-nominative platform) and its practicality as well as ease of use for real-world systems have been made based on this and other authors' works.*

***Keywords**: software correctness, safety proof, concurrent program, interleaving, invariant, IPCL, composition approach, composition-nominative languages, formal verification.*

***ACM Classification Keywords**: F.3.1 Theory of Computation - LOGICS AND MEANINGS OF PROGRAMS - Specifying and Verifying and Reasoning about Programs, D.2.4 Software - SOFTWARE ENGINEERING - Software/Program Verification.*

Introduction

The problem of software correctness proof is quite relevant nowadays. There are a lot of scientific researches and methods devoted to program verification, but nevertheless the problem stays relevant, because most of methods are either too complicated for practical application or too theorized (which makes them impractical – and here emerges the question of transferring these theoretical results into a more practical field), or simply unable to cope with real tasks. At the same time, contemporary software becomes more and more parallel (the increase in processors' core clock frequency started to slow down, forcing the increase in number of them, therefore stimulating code parallelization), but classical formal methods of verification are not well suited for such kind of tasks, where mutual in influence between parallel processes is present [Panchenko, 2006, Panchenko, 2007, Panchenko, 2008, Panchenko2, 2007]. Special interest is devoted to systems based on shared memory concurrency which are less investigated [Panchenko, 2006]. Those are supercomputers with UMA and NUMA memory architecture, SMP-based computer hardware architectures, operating systems, database management systems (DBMS), centralized databases and data warehouses (for example, in Business Intelligence systems), server-side software in client-server environments, etc.

Regarding the necessity to prove the correctness of programs, it is mostly related to so-called safety-critical systems – systems, whose failure or malfunction may result in death or injury to people health, loss or severe damage to property and/or equipment or environmental harm. According to Trusted Computer System Evaluation Criteria [DD, 1985] (the famous “Orange book”), formal specification and verification of programs that are claimed to be of the highest level of reliability is needed. In a more contemporary of Computer System Evaluation Criteria, which is standard ISO/IEC 15408 [ISO, 2005] (“Common Criteria”), formal verification is demanded for 3 out of 7 levels of reliability – EAL5-EAL7 (Evaluation Assurance Level), which means that requirements have strengthened even more.

The Problem

System Description

In this work we will prove the safety property of partial correctness of an Infosoft e-Detailing 1.0 software system. This software is designed for making (almost) synchronous presentations by one speaker (manager) to a numerous audience (client). The usage of this system basically lies in switching slides on a manager's device which is almost immediately followed by an automatic switching to the same slide on each of the clients' devices. This product is commercial, hence we are not going to include the source code, but we are including the same (slightly simplified) code written in compositional language IPCL [Panchenko, 2004], which is going to be used in proofs. Compositional nominative languages IPCL provide means of working with any kind of parallelism [Panchenko2, 2008] and cover the most common class of parallel programs – MIMD architecture, according to Flynn's taxonomy.

The most important from the correct functioning point of view is to make sure that every client will see the same slide that manager has switched to. Work of the system consists of cycles, namely switching a slide on manager's device and then switching a slide on all of clients' devices. The amount of such cycles is unlimited, the only stopping criteria is that everyone has left their presentation session. Typical cycle would look like this: manager sends to the server, and clients are reading from it, the index of a current slide (currently using HTTP + AJAX + Long Poll technologies) – in such a way the asynchronous slide replication is achieved on all the devices.

The problem statement is to prove the correctness of one typical slide switching cycle. More precisely: if manager has switched to a new slide (this slide has index $slideM$) and notified the server about it (S common variable on the server, which holds the current slide index for every client to read, and at the beginning $S \neq slideM$, in other words manager has switched to a slide that is different from the previous one), then eventually all the clients will have their own slide index (for each client i – $sldieC_i$) equal to the slide index that manager has switched to, i.e. for each client we have:

$$\forall i(valueOf(slideC_i) = valueOf(S) = valueOf(slideM))$$

All presentations by default begin from the first slide, in other words at the beginning we have:

$$valueOf(slideM) = valueOf(slideC_i) = valueOf(S) = 1$$

Stages in Correctness Proof

The sources of manager and client programs in IPCL and verification using method for program's properties correctness proof [Panchenko, 2006, Panchenko, 2008, Panchenko, 2004] are given below, namely:

- The notion of a state is specified;
- Transition system is constructed (model of execution of the program **manager || client** $^{\boldsymbol{n}}$);
- Starting and final states as well as precondition and postcondition are specified;
- Invariant of the software system is introduced, and it is proven that each of macrotransitions keeps it true, and that precondition on starting states implies invariant, and invariant on final states implies postcondition.

System Formal Model

Sources of manager and client programs with labels (in accordance with the method for program's properties correctness proof):

manager ≡ **[M1]** S := slideM **[M2]**

client ≡ **[C1]** newSlide := S;

while **[C2]** (slideC = newSlide) do

    [C3] newSlide := S;

[C4] slideC := newSlide **[C5]**

The whole software system will have the following structure:

$$\textbf{program} = \textbf{manager} \,||\, \textbf{client}^{\,n}.$$

The power here is understood in a sense [Panchenko, 2006, Panchenko, 2007, Panchenko, 2008, Panchenko2, 2007, Panchenko, 2004], i.e. parallel execution of n instances of a program in an interleaving manner.

States of the System

The state of such program will have the following structure:

$$State = (M, CS, [S \mapsto S_0], [slideM \mapsto SM], CD)$$

where $M \in$ {**M1**,**M2**} – manager's labels, $CS = (s_1,\ldots,s_n)$ – clients' labels, where n – number of clients, $\forall i$ ($s_i \in$ {**C1**,**C2**,**C3**,**C4**,**C5**}), $[S \mapsto S_0]$ – global (common) data, $[slideM \mapsto SM]$ – manager's local data, $CD = (d_1,\ldots,d_n)$ – clients' local data, where $\forall i$ ($d_i = [newSlide \mapsto NSi, slideC \mapsto SCi]$), $\forall i$ ($\{S_0, SM, NS_i, SC_i\} \subset \mathbf{N}$) , – slide indices. $States$ is a set of all possible states.

Here we will use standard in composition-nominative approach denomination composition $A \Rightarrow$ [Redko, 1978, Nikitchenko, 1998], which returns the value of variable with name A over the data d:

$$A\Rightarrow (d) = w \Leftrightarrow [A \mapsto w] \in d$$

Also we will denote $s_i(S) = Pr_i(Pr_2(S))$ and $d_i(S) = Pr_i(Pr_5(S))$.

Transition System

The transition system will have following macro-transitions (the scheme of transitions):

$$Transitions = \{ S_1 \to S_2 \mid S_1, S_2 \in States \wedge (Tr_1(S_1,S_2) \vee Tr_2(S_1,S_2) \vee Tr_3(S_1,S_2) \vee Tr_4(S_1,S_2) \vee Tr_5(S_1,S_2) \vee Tr_6(S_1,S_2)) \}$$

where each of $Tr_i(S_1,S_2)$ corresponds to some of possible program atomic steps (which will be executed in interleaving manner somehow due to concurrent environment during runtime execution path) and defines the semantics of such a step (i.e. corresponding transition between states), namely:

1) for **manager**'s move from label **M1** to label **M2**:

$Tr_1(S_1,S_2) = (Pr_1(S_1) = \mathbf{M1}) \wedge (Pr_1(S_2) = \mathbf{M2}) \wedge (Pr_2(S_1) = Pr_2(S_2)) \wedge (Pr_3(S_1) = d) \wedge (Pr_3(S_2) = d \nabla [S \mapsto SM]) \wedge (Pr_4(S_1) = Pr_4(S_2) = [slideM \mapsto SM]) \wedge (Pr_5(S_1) = Pr_5(S_2))$

2) for **client**'s move **C1** → **C2** (pre-while-cycle assignment):

$Tr_2(S_1,S_2) = (Pr_1(S_1) = Pr_1(S_2)) \wedge (Pr_3(S_1) = Pr_3(S_2) = [S \mapsto S_0]) \wedge (Pr_4(S_1) = Pr_4(S_2)) \wedge \exists j (s_j(S_1) = \mathbf{C1} \wedge s_j(S_2) = \mathbf{C2} \wedge \forall i\ (i \neq j \rightarrow s_i(S_1) = s_i(S_2) \wedge d_i(S_1) = d_i(S_2)) \wedge (d_j(S_1) = d) \wedge (d_j(S_2) = d \nabla [newSlide \mapsto S_0]))$

3) for **client**'s move **C2** → **C4** ($false$ value of while-cycle condition):

$Tr_3(S_1,S_2) = (Pr_1(S_1) = Pr_1(S_2)) \wedge (Pr_3(S_1) = Pr_3(S_2)) \wedge (Pr_4(S_1) = Pr_4(S_2)) \wedge (Pr_5(S_1) = Pr_5(S_2)) \wedge \exists j\ (s_j(S_1) = \mathbf{C2} \wedge s_j(S_2) = \mathbf{C4} \wedge \forall i\ (i \neq j \rightarrow s_i(S_1) = s_i(S_2)) \wedge slideC \Rightarrow (d_j(S_1)) \neq newSlide \Rightarrow (d_j(S_1)))$

4) for **client**'s move **C2** → **C3** ($true$ value of while-cycle condition):

$Tr_4(S_1,S_2) = (Pr_1(S_1) = Pr_1(S_2)) \wedge (Pr_3(S_1) = Pr_3(S_2)) \wedge (Pr_4(S_1) = Pr_4(S_2)) \wedge (Pr_5(S_1) = Pr_5(S_2)) \wedge \exists j\ (s_j(S_1) = \mathbf{C2} \wedge s_j(S_2) = \mathbf{C3} \wedge \forall i\ (i \neq j \rightarrow s_i(S_1) = s_i(S_2)) \wedge slideC \Rightarrow (d_j(S_1)) = newSlide \Rightarrow (d_j(S_1)))$

5) for **client**'s move **C3** → **C2** (while-cycle body assignment statement execution):

$Tr_5(S_1,S_2) = (Pr_1(S_1) = Pr_1(S_2)) \wedge (Pr_3(S_1) = Pr_3(S_2) = [S \mapsto S_0]) \wedge (Pr_4(S_1) = Pr_4(S_2)) \wedge \exists j$ $(s_j(S_1) = \mathbf{C3} \wedge s_j(S_2) = \mathbf{C2} \wedge \forall i\ (i \neq j \rightarrow s_i(S_1) = s_i(S_2) \wedge d_i(S_1) = d_i(S_2)) \wedge (d_j(S_1) = d) \wedge (d_j(S_2) =$ $d\ \nabla\ [newSlide \mapsto S_0]))$

6) for **client**'s move **C4** → **C5** (post-while-cycle assignment):

$Tr_6(S_1,S_2) = (Pr_1(S_1) = Pr_1(S_2)) \wedge (Pr_3(S_1) = Pr_3(S_2)) \wedge (Pr_4(S_1) = Pr_4(S_2)) \wedge \exists j\ (s_j(S_1) = \mathbf{C4} \wedge$ $s_j(S_2) = \mathbf{C5} \wedge \forall i\ (i \neq j \rightarrow s_i(S_1) = s_i(S_2) \wedge d_i(S_1) = d_i(S_2)) \wedge (d_j(S_1) = d) \wedge (d_j(S_2) =$ $d\ \nabla\ [slideC \mapsto newSlide \Rightarrow (d_j(S_1))]))$

Invariant of the Program

Now let us fix Starting states for the transition system described:

$$StartStates=\{S \in States | Pr_1(S)=\mathbf{M1} \wedge \forall i\ (s_i(S)=\mathbf{C1})\}$$

and Final states for this system are:

$$StopStates=\{S \in States | Pr_1(S)=\mathbf{M2} \wedge \forall i\ (s_i(S)=\mathbf{C5})\}$$

To prove the safety condition we formulate Precondition:

$$PreCond(S) = \forall i\ (slideC \Rightarrow (d_i(S)) = S \Rightarrow (Pr_3(S))) \wedge (slideM \Rightarrow (Pr_4(S)) \neq S \Rightarrow (Pr_3(S)))$$

and Postcondition:

$$PostCond(S) = \forall i\ (slideC \Rightarrow (d_i(S)) = S \Rightarrow (Pr_3(S))) \wedge (slideM \Rightarrow (Pr_4(S)) = S \Rightarrow (Pr_3(S)))$$

The invariant of **program** system:

$Inv(S) = I_1(S) \wedge I_2(S) \wedge I_3(S) \wedge I_4(S) \wedge I_5(S)$, where

$$I_1(S) = (Pr_1(S) = M2 \rightarrow S \Rightarrow (Pr_3(S)) = slideM \Rightarrow (Pr_4(S))),$$

$$I_2(S) = (Pr_1(S) = M1 \rightarrow (S \Rightarrow \left(Pr_3(S)\right) \neq slideM \Rightarrow \left(Pr_4(S)\right) \wedge \forall i \, (slideC \Rightarrow \left(d_i(S)\right) = S \Rightarrow \left(Pr_3(S)\right) \wedge s_i(S) \in \{C1, C2, C3\}))),$$

$$I_3(S) = \forall i \, (s_i(S) = C4 \rightarrow (slideC \Rightarrow \left(d_i(S)\right) \neq S \Rightarrow \left(Pr_3(S)\right) \wedge newSlide \Rightarrow (d_i(S)) = S \Rightarrow (Pr_3(S)))),$$

$$I_4(S) = \forall i \, (s_i(S) = C5 \rightarrow slideC \Rightarrow (d_i(S)) = S \Rightarrow (Pr_3(S))),$$

$$I_5(S) = \forall i \, (s_i(S) = C1 \vee slideC \Rightarrow \left(d_i(S)\right) = newSlide \Rightarrow \left(d_i(S)\right) \vee Pr_1(S) = M2 \wedge newSlide \Rightarrow (d_i(S)) = S \Rightarrow (Pr_3(S))).$$

The Proof

To proof the safety condition of the program we need to be sure that Precondition (S) implies Invariant $Inv(S)$ over all $S \in StartStates$, Postcondition $PostCont(S)$ follows from Invariant $Inv(S)$ over all $S \in StopStates$ and also the $Inv(S)$ preserves $true$ value over all possible $Transitions$. In other words, this needs to be proven in accordance with the Method:

$$\forall S \in StartStates \left(PreCond(S) \rightarrow Inv(S)\right) \wedge$$
$$\forall S \in StopStates \left(Inv(S) \rightarrow PostCond(S)\right) \wedge$$
$$\forall (S_1 \rightarrow S_2) \in Transitions \wedge S_1, S_2 \in States \, (Inv(S_1) \rightarrow Inv(S_2)).$$

To make the proof simple, let's prove this in terms of first order logic, using Gödel's completeness theorem:

1. $PreCond(S) = true \vDash Inv(S) = true, S \in StartStates$

$a.\ S \in StartStates \vDash (I_1(S) \wedge I_3(S) \wedge I_4(S) \wedge I_5(S))$

$b.\ (S \in StartStates \wedge \forall\, i\, (slideC \Rightarrow (d_i) = S \Rightarrow (Pr_3(S))) \wedge slideM \Rightarrow (Pr_4(S))$
$\neq S \Rightarrow (Pr_3(S)))$

$\vDash (S \Rightarrow (Pr\_3\ (S)) \neq slideM \Rightarrow (Pr\_4\ (S)) \wedge \forall\, i\, (slideC \Rightarrow (d\_i\ (S)) = S$
$\Rightarrow (Pr\_3\ (S)) \wedge s\_i\ (S) \in \{C1, C2, C3\})) \vDash I\_2\ (S)$

$\left(I_1(S) \wedge I_2(S) \wedge I_3(S) \wedge I_4(S) \wedge I_5(S)\right) \vDash Inv(S)$ ■

2. $Inv(S) = true \vDash PostCond(S) = true, S \in StopStates$

$a.\ \left(S \in StopStates \wedge I_4(S)\right) \vDash \forall\, i\, (slideC \Rightarrow (d_i) = S \Rightarrow (Pr_3(S)))$

$b.\ \left(S \in StopStates \wedge I_1(S)\right) \vDash S \Rightarrow \left(Pr_3(S)\right) = slideM \Rightarrow (Pr_4(S))$

$c.\ (\forall\, i\, (slideC \Rightarrow (d_i) = S \Rightarrow (Pr_3(S))) \wedge S \Rightarrow (Pr_3(S)) = slideM$
$\Rightarrow (Pr_4(S))) \vDash PostCond(S)$ ■

3. $Tr_1(S_1, S_2) = true \wedge Inv(S_1) = true \vDash Inv(S_2) = true$

$a.\ (Pr_3(S_1) = d \wedge Pr_3(S_2) = d \nabla [S \mapsto SM] \wedge Pr_4(S_1) = Pr_4(S_2)$
$= [slideM \mapsto SM]) \vDash S \Rightarrow \left(Pr_3(S_2)\right) = slideM \Rightarrow \left(Pr_4(S_2)\right)$
$\vDash I_1(S_2)$

$b.\ Pr_1(S_2) = M2 \vDash I_2(S_2)$

$c.\ (I_2(S_1) \wedge Pr_2(S_1) = Pr_2(S_2)) \vDash \forall\, i\, (s_i(S_2) \notin \{C4, C5\}) \vDash (I_3(S_2) \wedge I_4(S_2))$

$d.\ (Pr_1(S_1) = M1 \wedge I_5(S_1)) \vDash \forall\, i\, (slideC \Rightarrow (d_i(S_1)) = newSlide$
$\Rightarrow (d_i(S_1)))$

$(\forall\, i\, (slideC \Rightarrow (d_i(S_1)) = newSlide \Rightarrow (d_i(S_1))) \wedge Pr_5(S_1) = Pr_5(S_2))$
$\vDash \forall\, i\, (slideC \Rightarrow (d_i(S_2)) = newSlide \Rightarrow (d_i(S_2))) \vDash I_5(S_2)$

$\left(I_1(S) \wedge I_2(S) \wedge I_3(S) \wedge I_4(S) \wedge I_5(S)\right) \vDash Inv(S)$ ■

4. $Tr_2(S_1, S_2) = true \wedge Inv(S_1) = true \vDash Inv(S_2) = true$

$a.\ (I_1(S_1) \wedge Pr_1(S_1) = Pr_1(S_2) \wedge Pr_3(S_1) = Pr_3(S_2) \wedge Pr_4(S_1) = Pr_4(S_2)) \vDash I_1(S_2)$

$b.\ (Pr_1(S_1) = Pr_1(S_2) \wedge Pr_3(S_1) = Pr_3(S_2) \wedge Pr_4(S_1) = Pr_4(S_2) \wedge d_j(S_1) = d \wedge d_j(S_2) = d \nabla [newSlide \mapsto S_0] \wedge \forall i = \overline{1,n} \wedge i \neq j \left(d_i(S_1) = d_i(S_2)\right) \wedge s_j(S_1) = C1 \wedge s_j(S_2) = C2 \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_1) = s_i(S_2))) \vDash I_2(S_2)$

$c.\ (d_j(S_1) = d \wedge d_j(S_2) = d \nabla [newSlide \mapsto S_0] \wedge \forall i = \overline{1,n} \wedge i \neq j\ (d_i(S_1) = d_i(S_2)) \wedge Pr_3(S_1) = Pr_3(S_2)) \vDash \forall i\ (slideC \Rightarrow (d_i(S_1)) = slideC \Rightarrow (d_i(S_2)))$

$(I_3(S_1) \wedge I_4(S_1) \wedge \forall i\ (slideC \Rightarrow (d_i(S_1)) = slideC \Rightarrow (d_i(S_2))) \wedge d_j(S_1) = d \wedge d_j(S_2) = d \nabla [newSlide \mapsto S_0] \wedge \forall i = \overline{1,n} \wedge i \neq j\ (d_i(S_1) = d_i(S_2))) \vDash (I_3(S_2) \wedge I_4(S_2))$

$d.\ \left(I_1(S_2) \wedge I_2(S_2) \wedge I_5(S_1) \wedge d_j(S_1) = d \wedge d_j(S_2) = d \nabla [newSlide \mapsto S_0] \wedge \forall i = \overline{1,n} \wedge i \neq j \left(d_i(S_1) = d_i(S_2)\right) \wedge Pr_3(S_2) = [S \mapsto S_0]\right) \vDash I_5(S_2)$

$\left(I_1(S) \wedge I_2(S) \wedge I_3(S) \wedge I_4(S) \wedge I_5(S)\right) \vDash Inv(S)$ ■

5. $Tr_3(S_1, S_2) = true \wedge Inv(S_1) = true \vDash Inv(S_2) = true$

$a.\ (I_1(S_1) \wedge I_5(S_1) \wedge Pr_1(S_1) = Pr_1(S_2) \wedge Pr_3(S_1) = Pr_3(S_2) \wedge Pr_4(S_1) = Pr_4(S_2) \wedge Pr_5(S_1) = Pr_5(S_2)) \vDash (I_1(S_2) \wedge I_5(S_2))$

$b.\ (slideC \Rightarrow (d_j(S_2)) \neq newSlide \Rightarrow (d_j(S_2)) \wedge Pr_5(S_1) = Pr_5(S_2) \wedge I_5(S_1)) \vDash Pr_1(S_1) = M2$

$(Pr_1(S_1) = Pr_1(S_2) \wedge Pr_1(S_1) = M2) \vDash Pr_1(S_2) = M2 \vDash I_2(S_2)$

$c.\ (Pr_3(S_1) = Pr_3(S_2) \wedge Pr_5(S_1) = Pr_5\ (S_2) \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_1) = s_i(S_2))) \vDash (\forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C4 \rightarrow (slideC \Rightarrow \left(d_i(S_2)\right) \neq S \Rightarrow (Pr_3(S_2)) \wedge newSlide \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2)))) \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C5 \rightarrow slideC \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2))))$

$s_j(S_2) = C4 \vDash (s_j(S_2) = C5 \rightarrow slideC \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2)))$

$(Pr_3(S_1) = Pr_3(S_2) \wedge Pr_5(S_1) = Pr_5(S_2) \wedge s_j(S_2) = C4 \wedge slideC \Rightarrow (d_j(S_2)) \neq newSlide \Rightarrow (d_j(S_2)) \wedge I_5(S_2)) \vDash (s_j(S_2) = C4 \rightarrow (slideC \Rightarrow (d_j(S_2)) \neq S \Rightarrow (Pr_3(S_2)) \wedge newSlide \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2))))$

$(\forall\, i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C4 \rightarrow (slideC \Rightarrow \left(d_i(S_2)\right) \neq S \Rightarrow \left(Pr_3(S_2)\right) \wedge newSlide \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2)))) \wedge \forall\, i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C5 \rightarrow slideC \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2))) \wedge (s_j(S_2) = C4 \rightarrow (slideC \Rightarrow \left(d_j(S_2)\right) \neq S \Rightarrow (Pr_3(S_2)) \wedge newSlide \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2)))) \wedge (s_j(S_2) = C5 \rightarrow slideC \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2)))) \vDash (I_3(S_2) \wedge I_4(S_2))$

$\left(I_1(S) \wedge I_2(S) \wedge I_3(S) \wedge I_4(S) \wedge I_5(S)\right) \vDash Inv(S)$ ■

6. $Tr_4(S_1, S_2) = true \wedge Inv(S_1) = true \vDash Inv(S_2) = true$

$a.$ $(I_1(S_1) \wedge I_5(S_1) \wedge Pr_1(S_1) = Pr_1(S_2) \wedge Pr_3(S_1) = Pr_3(S_2) \wedge Pr_4(S_1) = Pr_4(S_2) \wedge Pr_5(S_1) = Pr_5(S_2)) \vDash (I_1(S_2) \wedge I_5(S_2))$

$b.$ $(Pr_1(S_1) = Pr_1(S_2) \wedge Pr_3(S_1) = Pr_3(S_2) \wedge Pr_4(S_1) = Pr_4(S_2) \wedge Pr_5(S_1) = Pr_5(S_2) \wedge s_j(S_1) = C2 \wedge s_j(S_2) = C3 \wedge \forall\, i = \overline{1,n} \wedge i \neq j\ (s_i(S_1) = s_i(S_2))) \vDash I_2(S_2)$

$c.$ $(Pr_3(S_1) = Pr_3(S_2) \wedge Pr_5(S_1) = Pr_5(S_2) \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_1) = s_i(S_2))) \vDash (\forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C4 \rightarrow (slideC \Rightarrow (d_i(S_2)) \neq S \Rightarrow (Pr_3(S_2)) \wedge newSlide \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2)))) \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C5 \rightarrow slideC \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2))))$

$\left(Pr_3(S_1) = Pr_3(S_2) \wedge Pr_5(S_1) = Pr_5(S_2) \wedge s_j(S_2) = C3\right) \vDash ((s_j(S_2) = C4 \rightarrow (slideC \Rightarrow (d_j(S_2)) \neq S \Rightarrow (Pr_3(S_2)) \wedge newSlide \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2)))) \wedge (s_j(S_2) = C5 \rightarrow slideC \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2))))$

$(\forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C4 \rightarrow (slideC \Rightarrow \left(d_i(S_2)\right) \neq S \Rightarrow (Pr_3(S_2)) \wedge newSlide \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2)))) \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C5 \rightarrow slideC \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2))) \wedge (s_j(S_2) = C4 \rightarrow (slideC \Rightarrow (d_j(S_2)) \neq S \Rightarrow (Pr_3(S_2)) \wedge newSlide \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2)))) \wedge (s_j(S_2) = C5 \rightarrow slideC \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2)))) \vDash (I_3(S_2) \wedge I_4(S_2))$

$\left(I_1(S) \wedge I_2(S) \wedge I_3(S) \wedge I_4(S) \wedge I_5(S)\right) \vDash Inv(S)$ ■

7. $Tr_5(S_1, S_2) = true \wedge Inv(S_1) = true \vDash Inv(S_2) = true$

a. $(I_1(S_1) \wedge Pr_1(S_1) = Pr_1(S_2) \wedge Pr_3(S_1) = Pr_3(S_2) \wedge Pr_4(S_1) = Pr_4(S_2)) \vDash I_1(S_2)$

b. $(Pr_1(S_1) = Pr_1(S_2) \wedge Pr_3(S_1) = Pr_3(S_2) \wedge Pr_4(S_1) = Pr_4(S_2) \wedge d_j(S_1) = d \wedge d_j(S_2) = d\, \nabla\, [newSlide \mapsto S_0] \wedge \forall i = \overline{1,n} \wedge i \neq j\ \left(d_i(S_1) = d_i(S_2)\right) \wedge s_j(S_1) = C3 \wedge s_j(S_2) = C2 \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_1) = s_i(S_2))) \vDash I_2(S_2)$

c. $(d_j(S_1) = d \wedge d_j(S_2) = d\, \nabla\, [newSlide \mapsto S_0] \wedge \forall i = \overline{1,n} \wedge i \neq j\ (d_i(S_1) = d_i(S_2)) \wedge Pr_3(S_1) = Pr_3(S_2)) \vDash \forall i\ (slideC \Rightarrow (d_i(S_1)) = slideC \Rightarrow (d_i(S_2)))$

$(I_3(S_1) \wedge I_4(S_1) \wedge \forall i\ (slideC \Rightarrow (d_i(S_1)) = slideC \Rightarrow (d_i(S_2))) \wedge d_j(S_1) = d \wedge d_j(S_2) = d\, \nabla\, [newSlide \mapsto S_0] \wedge \forall i = \overline{1,n} \wedge i \neq j\ (d_i(S_1) = d_i(S_2))) \vDash (I_3(S_2) \wedge I_4(S_2))$

d. $(I_1(S_2) \wedge I_2(S_2) \wedge I_5(S_1) \wedge d_j(S_1) = d \wedge d_j(S_2) = d\, \nabla\, [newSlide \mapsto S_0] \wedge \forall i = \overline{1,n} \wedge i \neq j\ (d_i(S_1) = d_i(S_2)) \wedge Pr_3(S_2) = [S \mapsto S_0]) \vDash I_5(S_2)$

$\left(I_1(S) \wedge I_2(S) \wedge I_3(S) \wedge I_4(S) \wedge I_5(S)\right) \vDash Inv(S)$ ■

8. $Tr_6(S_1, S_2) = true \wedge Inv(S_1) = true \vDash Inv(S_2) = true$

$$a.\ (I_1(S_1) \wedge Pr_1(S_1) = Pr_1(S_2) \wedge Pr_3(S_1) = Pr_3(S_2) \wedge Pr_4(S_1) = Pr_4(S_2)) \vDash I_1(S_2)$$

$$b.\ (I_5(S_1) \wedge Pr_1(S_1) = Pr_1(S_2) \wedge Pr_3(S_1) = Pr_3(S_2) \wedge d_j(S_1) = d \wedge d_j(S_2) = d \nabla [slideC \mapsto newSlide \Rightarrow d_j(S_2)] \wedge \forall i = \overline{1,n} \wedge i \neq j\ (d_i(S_1) = d_i(S_2))) \vDash I_5(S_2)$$

$$c.\ (Pr_3(S_1) = Pr_3(S_2) \wedge \forall i = \overline{1,n} \wedge i \neq j\ (d_i(S_1) = d_i(S_2)) \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_1) = s_i(S_2))) \vDash (\forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C4 \rightarrow (slideC \Rightarrow (d_i(S_2)) \neq S \Rightarrow (Pr_3(S_2)) \wedge newSlide \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2))))) \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C5 \rightarrow slideC \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2))))$$

$$s_j(S_2) = C5 \vDash (s_j(S_2) = C4 \rightarrow slideC \Rightarrow (d_j(S_2)) \neq S \Rightarrow (Pr_3(S_2)))$$
$$\big(I_3(S_1) \wedge S_j(S_1) = C4\big) \vDash newSlide \Rightarrow (d_j(S_1)) = S \Rightarrow (Pr_3(S_1))$$
$$(d_j(S_2) = d \nabla [slideC \mapsto newSlide \Rightarrow d_j(S_2)] \wedge newSlide \Rightarrow (d_j(S_1)) = S \Rightarrow (Pr_3(S_1))) \vDash (s_j(S_2) = C5 \rightarrow slideC \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2)))$$

$$(\forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C4 \rightarrow slideC \Rightarrow \big(d_i(S_2)\big) \neq S \Rightarrow (Pr_3(S_2))) \wedge \forall i = \overline{1,n} \wedge i \neq j\ (s_i(S_2) = C5 \rightarrow slideC \Rightarrow (d_i(S_2)) = S \Rightarrow (Pr_3(S_2))) \wedge s_j(S_2) = C4 \rightarrow slideC \Rightarrow (d_j(S_2)) \neq S \Rightarrow (Pr_3(S_2)) \wedge s_j(S_2) = C5 \rightarrow slideC \Rightarrow (d_j(S_2)) = S \Rightarrow (Pr_3(S_2))) \vDash (I_3(S_2) \wedge I_4(S_2))$$

$$d.\ (I_2(S_1) \wedge s_j(S_1) = C4 \wedge Pr_1(S_1) = Pr_1(S_2)) \vDash Pr_1(S_1) = Pr_1(S_2) = M2$$
$$Pr_1(S_2) = M2 \vDash I_2(S_2)$$

$$\big(I_1(S) \wedge I_2(S) \wedge I_3(S) \wedge I_4(S) \wedge I_5(S)\big) \vDash Inv(S)\ \blacksquare$$

Conclusion

Partial correctness of the software system, namely Infosoft e-Detailing 1.0, according to an initial problem statement has been proven using Correctness Proof Methodology [Panchenko, 2006, Panchenko, 2008, Panchenko, 2004] in an IPCL language [Panchenko, 2004]. Considering the difficulties in the process of such proof in parallel environments, we can state:

- Correctness Proof Method in IPCL is well suited for the verification of parallel programs or the software correctness proof in terms of safety properties;
- The Method allows shortening the proof at the expense of choosing an adequate abstraction level [Nikitchenko, 1998] due to universality of a compositional nominative approach [Nikitchenko, 1998, Redko, 1978] and by fixing the appropriate basic function set of semantic algebra.

Taking into account flexibility of the Methodology, existence of Simplified State Model reasoning in some cases [Panchenko2, 2007], and universal nature of the approach [Panchenko2, 2008], it can be recommended for program properties proof (particularly safety property or partial correctness) for wide range of software which is executed in interleaving concurrency environment with shared memory, primarily for server-side software of client-server complexes.

The same conclusion is obtained in [Polishchuk, 2015] also.

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CONSTRUCTING AN OPTIMAL INVESTMENT PORTFOLIO BY USING FUZZY SETS THEORY

Yuri Zaychenko, Inna Sydoruk

Abstract: *The problem of constructing an optimal portfolio of securities under uncertainty was considered.*

The global market crisis of recent years has shown that the existing theory of optimization of investment portfolios and forecasting stock indices exhausted and the revision of the basic theory of portfolio management is strongly needed. Therefore the fuzzy sets theory was used for getting an optimal portfolio.

In this paper direct, dual and multicriteria problems with the use of triangular membership functions work were considered. The problem of portfolio optimization during the time period also was described in this article. In direct task we define structure of a portfolio, which will provide the maximum profitableness at the set risk level. In dual task we define structure of a portfolio, which will provide the minimum risk level at the set level of critical profitableness. In multicriteria problem we simultaneously maximize profitability and minimize risk level. The input data for the optimization system were predicted by using the Fuzzy Group Method of Data Handling (FGMDH). The optimal portfolios for assets were determined. The comparative analysis of optimal portfolios obtained by different methods and approaches was fulfilled.

Keywords: *membership function, fuzzy sets theory, optimal portfolio, investments, stock securities, fuzzy number, FGMDH*

ACM Classification Keywords: *G.1.0 Mathematics of Computing– General – Error analysis; G.1.6 Mathematics of Computing – Numerical Analysis – Optimization - Gradient methods, Least squares methods; I.2.3 Computing Methodologies - Artificial Intelligence - Uncertainty, "fuzzy", and probabilistic reasoning.*

Introduction

The problem of investment in securities arouse with appearance of the first stock markets. The main objective of portfolio investment is to improve the investment environment, giving securities such investment characteristics that are only possible in their combination. Careful processing and accounting of investment risks have become an integral and important part of the success of each company. However, more and more companies have to make decisions under uncertainty, which may lead to unintended consequences and, therefore, undesirable results. Particularly serious consequences may have the wrong decisions at long-term investments. Therefore, early detection, adequate and the most accurate assessment of risk is one of the biggest problems of modern investment analysis.

Historically, the first and the most common way to take account of uncertainty is the use of probability theory. The beginning of modern investment theory was in the article H. Markowitz, "Portfolio Selection", which was released in 1952. In this article mathematical model of optimal portfolio of securities was first proposed. Methods of constructing such portfolios under certain conditions are based on theoretical and probabilistic formalization of the concept profitability and risk. For many years the classical theory of Markowitz was the main theoretical tool for optimal investment portfolio construction, after which most of the novel theories were only modifications of the basic theory.

However, the global market crisis of recent years has shown that the existing theory of investment portfolio optimization and forecasting stock indices exhausted itself and a revision of the basic theory of portfolio management is strongly needed.

New approach in the problem of investment portfolio construction under uncertainty is connected with fuzzy sets theory. Fuzzy sets theory was created about half a century ago in the fundamental work of Lotfi Zadeh [Zadeh, 1999]. This theory came into use in the economy in the late 70's. By using fuzzy numbers in the forecast parameters decision - making person was not required to form probability estimates.

The application of fuzzy sets technique enabled to create a novel theory of fuzzy portfolio optimization under uncertainty and risk deprived of drawbacks of classical portfolio theory by Markovitz. In this work we use fuzzy sets theory for getting an optimal investment portfolio. Firstly, portfolio optimization problem in this formulation was considered by O.A. Nedosekin [Nedosekin, 2002]. But in his work only direct problem was considered. The investigations were continued by Esfandiyarfard Maliheh. In [Zaychenko & Maliheh, 2008] the direct optimization problem using different membership functions was considered. However, in these studies, the investor can’t determine an optimal portfolio during the time period. Therefore, in this study multiobjective optimization problem in which an investor can prefer risk or profitability using weights coefficients and solve portfolio optimization problem at the chosen time period is considered and analyzed.

Problem statement of portfolio optimization

The purpose of the analysis and optimization of an investment portfolio is research in area of portfolio optimization, and also the comparative analysis of structure of the effective portfolios received using the model Markovitz and fuzzy-set model of a share portfolio optimization.

Let us consider a share portfolio from N components and its expected behavior at time interval $[0, T]$. Each of a portfolio component $i = \overline{1, N}$ at the moment T is characterized by its financial profitableness r_i (evaluated at a point T as a relative increase in the price of the asset for the period) [Zaychenko, 2008].

The holder of a share portfolio – the private investor, the investment company, mutual fund – operates the investments, being guided by certain reasons. On the one hand, the investor tries to maximize the profitableness. On the other hand, it fixes maximum permissible risk of an inefficiency of the investments. We will assume the capital of the investor be equal 1. The problem of optimization of a share portfolio consists in a finding of a vector of share price distribution of papers in a portfolio $x = \{x_i\}, i = \overline{1, N}$ of the investor maximizing the income at the set risk level (obviously, that $\sum_{i=1}^{N} x_i = 1$).

In process of practical application of Markovitz model its lacks were found out:

- The hypothesis about normality profitableness distributions in practice does not prove to be true;
- Stationary of price processes also not always is confirmed in practice;
- At last, the risk of assets is considered as a dispersion (or standard deviation) of the prices of securities from expected value i.e. as decrease in profitableness of securities in relation to expected value, and profitableness increase in relation to an average are estimated absolutely the same.

Though for the proprietor of securities these events are absolutely different.

These weaknesses of Markovitz theory define necessity of use of essentially new approach of definition of an optimum investment portfolio.

Let review the main principles and idea of a method.

The risk of a portfolio is not its volatility, but possibility that expected profitableness of a portfolio will appear below some pre established planned value:

- Correlation of assets in a portfolio is not considered and not accounted;
- Profitableness of each asset is not random, but a fuzzy number. Similarly, restriction on extremely low level of profitableness can be both usual scalar and fuzzy number of any kind. Therefore optimize a portfolio in such statement may mean, in that specific case, the requirement to maximize expected profitableness of a portfolio in a point of time T at the fixed risk level of a portfolio;

— Profitableness of a security on termination of ownership term is expected to be equal r and is in a settlement range. For i *-th* security let's denote:

- $\overline{r_i}$ – Expected profitableness of *i-th* security;
- r_{1i} – The lower border of profitableness of *i-th* security;
- r_{2i} – The upper border of profitableness of *i-th* security;
- $r_i = \left(r_{1i}, \overline{r_i}, r_{2i}\right)$ – Profitableness of *i-th* security is triangular fuzzy number.

Then profitableness of a portfolio:

$$r = (r_{\min} = \sum_{i=1}^{N} x_i r_{1i}; \overline{r} = \sum_{i=1}^{N} x_i \overline{r_i}; r_{\max} = \sum_{i=1}^{N} x_i r_{2i}) \quad (1)$$

where x_i is a weight of i *-th* asset in portfolio, and

$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i = \overline{1, N} \quad (2)$$

Critical level of profitableness of a portfolio at the moment of T may be fuzzy triangular type number $r^* = \left(r_1^*; \overline{r}^*; r_2^*\right)$.

Direct portfolio optimization problem with triangular membership functions

To define structure of a portfolio which will provide the maximum profitableness at the set risk level, it is required to solve the following problem (3):

$$\{x_{opt}\} = \{x\} \mid r \to \max,\ \ \beta = const \quad (3)$$

where r is profitableness, β is a desired risk, vector's components x satisfy (2).

The most expected value risk degree of a portfolio is defined:

$$\beta = \begin{cases} 0, \ if \ \ r^* < r_{\min} \\ R\left(1+\dfrac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), \ if \ \ r_{\min} \le r^* \le \tilde{r} \\ 1-(1-R)\left(1+\dfrac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), \ if \ \ \tilde{r} \le r^* < \ r_{\max} \\ 1, \ if \ \ r^* \ge r_{\max} \end{cases} \tag{4}$$

where

$$R = \begin{cases} \dfrac{r^* - r_{\min}}{r_{\max} - r_{\min}}, \ \ if \ \ r^* < r_{\max} \\ 1, \ \ \ if \ \ r^* \ge \mathrm{r}_{\max} \end{cases}$$

$$\alpha_1 = \begin{cases} 0, \ \ \ if \ \ \ r^* < r_{\min} \\ \dfrac{r^* - r_{\min}}{\tilde{r} - r_{\min}}, \ \ if \ \ \ r_{\min} \le r^* < \tilde{r} \\ 1, \ \ \ \ \ \ \ \ \ \ if \ \ r^* = \tilde{r} \\ \dfrac{r_{\max} - r^*}{r_{\max} - \tilde{r}}, \ \ if \ \ \ \tilde{r} < r^* < r_{\max} \\ 0, \ \ \ \ \ \ \ \ \ \ if \ \ \ r^* \ge r_{\max} \end{cases} \tag{5}$$

Having recollected also, that profitableness of a portfolio is:

$$r = (r_{\min} = \sum_{i=1}^{N} x_i r_{1i}; \ \widetilde{r} = \sum_{i=1}^{N} x_i \widetilde{r}_i; \ r_{\min} = \sum x_i r_{2i}),$$

where $(r_{1i}, \overline{r_i}, r_{2i})$ is the profitableness of the i *-th* security, we obtain the following problem of optimization (6) - (8):

$$\widetilde{r} = \sum_{i=1}^{N} x_i \widetilde{r}_i \to \max \tag{6}$$

$$\beta = const \tag{7}$$

$$\sum_{i=1}^{N} x_i = 1, \ x_i \ge 0, \ i = \overline{1, N} \tag{8}$$

At a risk level β variation 3 cases are possible. We'll consider in detail each of them.

1. $\beta = 0$

From (4) it is evident, that this case is possible when $r^* < \sum_{i=1}^{N} x_i r_{i1}$.

We obtain the following problem of linear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max \tag{9}$$

$$\sum_{i=1}^{N} x_i r_{i1} > r^* \tag{10}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N} \tag{11}$$

Found result of the problem solution (9) - (11) vector $x = \{x_i\},\ i = \overline{1, N}$ is a required structure of an optimum portfolio for the given risk level.

2. $\beta = 1$

From (4) it follows, that this case is possible when $r^* \geq \sum_{i=1}^{N} x_i r_{i2}$.

We obtain the following problem

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max,\quad \sum_{i=1}^{N} x_i r_{i2} \leq r^*,$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N}$$

Found result of the problem decision (9) - (11) vector $x = \{x_i\},\ i = \overline{1, N}$ is a required structure of an optimum portfolio for the given risk level.

3. $0<\beta<1$

From (4) it is evident, that this case is possible when $\sum_{i=1}^{N} x_i r_{i1} \le r^* \le \sum_{i=1}^{N} x_i \tilde{r}_i$, or when $\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* \le \sum_{i=1}^{N} x_i r_{i2}$.

a) Let $\sum_{i=1}^{N} x_i r_{i1} \le r^* \le \sum_{i=1}^{N} x_i \tilde{r}_i$ .Then using (4) - (5) problem (6) - (8) is reduced to the following problem of nonlinear programming [Zaychenko, 2006]:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max ,$$

$$\left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) + \left(\sum_{i=1}^{N} x_i \tilde{r}_i - r^* \right) \cdot \ln \left(\frac{\sum_{i=1}^{N} x_i \tilde{r}_i - r^*}{\sum_{i=1}^{N} x_i \tilde{r}_i - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) \cdot \tag{12}$$

$$\cdot \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta, \tag{13}$$

$$\sum_{i=1}^{N} x_i r_{i1} \le r^* \tag{14}$$

$$\sum_{i=1}^{N} x_i \tilde{r}_i > r^* \tag{15}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i = \overline{1,N} . \tag{16}$$

б) Let $\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* \le \sum_{i=1}^{N} x_i r_{i2}$. Then the problem (6) - (8) is reduced to the following problem of nonlinear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max \tag{17}$$

$$\left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) - \left(r^* - \sum_{i=1}^{N} x_i \tilde{r}_i \right) \cdot \ln \left(\frac{r^* - \sum_{i=1}^{N} x_i \tilde{r}_i}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) \cdot \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta \qquad (18)$$

$$\sum_{i=1}^{N} x_i r_{i2} > r^* \qquad (19)$$

$$\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* \qquad (20)$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i = \overline{1,N} \qquad (21)$$

The R-algorithm of minimization of not differentiated functions is applied to the decision of problems (12) - (16) and (17) - (21). Let both problems: (12) - (16) and (17) - (21) solvable. Then to the structure of a required optimum portfolio will correspond a vector – $x = \{x_i\},\ i = \overline{1,N}$ the decision of that problem (12) - (16), (17) - (21) the criterion function value of which will be greater.

The dual optimization problem

It is necessary to determine the structure of the portfolio, which will provide a minimum level of risk for a given level of the portfolio profitability.

We obtain the following optimization problem:

$$\min \beta(x),\ \tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \ge r_{зад} = \mathrm{r}^*,$$
$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i = \overline{1,N}, \qquad (22)$$

where r and β is determined by the used membership function.

Consider optimization problem with the triangular MF.

It is necessary to solve optimization problem (22) where $\beta(x)$ is determined from the formula (4) and (5).

Multicriteria optimization problem

Now consider multicriteria fuzzy portfolio optimization problem in which portfolio profitableness should be maximized and risk should be minimized.

In order to find the structure of corresponding fuzzy portfolio the following problem is to be solved:

$$\{x_{opt}\} = \{x\} \quad | \quad r \to \max, \quad \beta \to \min \, , \tag{23}$$

where r and β are determined by formulas (4) and (5) and vector *X* components satisfy (8).

For simplifying problem solution transfer it to single criterion. Normalize the value of profitableness as follows:

$$\tilde{r}_н = \frac{r_{\max} - \tilde{r}}{r_{\max} - r_{\min}}, \quad \tilde{r}_н \in [0;1] \, , \tag{24}$$

Using formulas (23) and (24), we obtain the optimization problem in such form:

$$\{w_1 \tilde{r}_н + w_2 \beta(x)\} \to \min$$
$$w_1 \ge 0, \; w_2 \ge 0, \; w_1 \ne w_2, \; w_1 + w_2 = 1$$
$$\sum_{i=1}^{N} x_i = 1, \; x_i \ge 0, \; i = \overline{1, N}$$

Optimization problem during the time period

In this case we must define the structure of the portfolio, which provides the maximum average return for a given level of risk. So we calculate the profitableness from (3) as:

$$\tilde{r} = \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} x_{ti} \tilde{r}_{ti} \, , \tag{25}$$

where $\tilde{r}_{ti}$ - the expected profitableness of *i-th* security in time unit t. T - the length of time period. We should find an optimal portfolio from the (3), using (4), (5), (25).

Analysis of the results

The profitableness values of leading companies in the period from 01.12.2014 to 10.04.2015 were used as the input data. The companies: Google Inc (GOOGL), Walt Disney Co (DIS), The Coca Cola Co

(KO), Kimberly Clark Corp (KMB), Seagate Technology PLC (STX), Tesla Motors Inc (TSLA). The corresponding data is presented in the Table 1.

Table 1. The profitableness

Companies / Dates	**GOOGL**	**DIS**	**KO**	**KMB**	**STX**	**TSLA**
05.12.2014	-0,0214	0,0114	-0,0229	-0,0161	-0,0192	-0,0418
12.12.2014	-0,0174	-0,0246	-0,0517	-0,0126	-0,0312	-0,0143
12.12.2014	0,0081	0,0219	0,0340	0,0311	0,0847	0,0389
26.12.2014	0,0173	0,0088	0,0144	0,0157	-0,0067	-0,0284
02.01.2015	-0,0144	-0,0183	-0,0168	-0,0161	-0,0235	-0,0163
09.01.2015	-0,0361	0,0202	0,0211	0,0091	0,0185	-0,0452
16.01.2015	0,0270	0,0076	-0,0026	0,0163	-0,0207	0,0488
23.01.2015	0,0628	-0,0002	0,0035	-0,0500	0,0136	-0,0143
30.01.2015	0,0015	-0,0422	-0,0426	-0,0172	-0,0444	0,0304
06.02.2015	0,0032	0,1098	-0,0034	-0,0096	0,0370	-0,0630
13.02.2015	0,0413	0,0240	0,0184	0,0257	0,0238	0,0624
20.02.2015	-0,0059	0,0041	0,0038	-0,0028	0,0168	-0,0173
27.02.2015	0,0516	-0,0087	0,0346	-0,0120	-0,0140	-0,0186
06.03.2015	-0,0037	-0,0195	-0,0389	-0,0325	-0,0757	-0,0174
13.03.2015	-0,0368	0,0112	-0,0358	-0,0115	-0,0565	-0,0115
20.03.2015	0,0059	0,0099	0,0089	0,0193	0,0305	0,0124
27.03.2015	**-0,0138**	**-0,0253**	**-0,0133**	**-0,0240**	**-0,0553**	**-0,0733**
02.04.2015	**-0,0353**	**-0,0011**	**0,0042**	**-0,0067**	**-0,0119**	**0,0023**
10.04.2015	**0,0084**	**0,0125**	**-0,0070**	**-0,0056**	**0,0538**	**0,0384**

For getting an input data for the optimization system we used the Fuzzy GMDH method [Zaychenko, 2000] with triangular membership functions, linear partial descriptions, training sample of 70% size. The profitableness values were forecasted for each of 3 weeks (Table 2).

Table 2. Forecasted profitableness

Companies	Profitableness				MAPE test sample	MSE test sample
	Real value	Low bound	Forecasted value	Upper bound		
27.03.2015						
GOOGL	-0,0138	-0,0408	-0,0116	0,0176	1,5116	0,0265
DIS	-0,0253	-0,0130	0,0087	0,0304	2,0415	0,0412
KO	-0,0133	-0,0629	-0,0160	0,0309	1,0619	0,0131
KMB	-0,0240	-0,1520	-0,0303	0,0914	2,3122	0,0136
STX	-0,0553	-0,0618	-0,0344	-0,0070	2,5215	0,0462
TSLA	-0,0733	-0,1919	-0,0391	0,1137	1,459	0,0078
02.04.2015						
GOOGL	-0,0353	-0,0672	-0,0328	0,0016	2,0537	0,0226
DIS	-0,0011	0,0082	0,0299	0,0516	1,7141	0,0371
KO	0,0042	-0,0418	0,0021	0,0460	1,9743	0,0159
KMB	-0,0067	-0,1342	-0,0125	0,1092	1,7452	0,0245
STX	-0,0119	-0,0164	0,0090	0,0344	1,8243	0,0172
TSLA	0,0023	-0,0999	0,0349	0,1697	2,101	0,0241

10.04.2015						
GOOGL	0,0084	-0,0215	0,0109	0,0433	1,439	0,0162
DIS	0,0125	0,0308	0,0425	0,0542	3,103	0,0215
KO	-0,007	-0,0506	-0,0087	0,0332	1,037	0,0107
KMB	-0,0056	-0,1136	-0,0119	0,0898	2,014	0,0135
STX	0,0538	0,0483	0,0747	0,1011	2,855	0,0178
TSLA	0,0384	0,0362	0,069	0,1018	2,014	0,0194

In this way the portfolio optimization system stops to be dependent on factor of expert subjectivity. Besides, we can get data for this method automatically, without expert's estimates.

Let the critical profitableness level set by 7,5 %. Varying the risk level we obtain the following results for triangular MF (10.04.2015). The results are presented in the Tables 3, 4 and Figure 1.

Table 3. Direct problem - distribution of components of the optimal portfolio with critical level r *=7,5 %

GOOGL	DIS	KO	KMB	STX	TSLA
0,00065	0,00025	0,00137	0,00423	0,99017	0,00333
0,00159	0,00157	0,00066	0,00323	0,9923	0,00065
0,00435	0,00307	0,00058	0,00271	0,98533	0,00396
0,00018	0,00008	0,00082	0,00291	0,99363	0,00238
0,0049	0,00394	0,00071	0,00244	0,97964	0,00837
0,00287	0,00525	0,00296	0,00506	0,97678	0,00708
0,0033	0,00187	0,00279	0,00301	0,9707	0,01833

0,00188	0,00232	0,00153	0,0125	0,97369	0,00808
0,00439	0,00297	0,00238	0,0152	0,94177	0,03329
0,00314	0,00317	0,0036	0,02553	0,93466	0,0299
0,00339	0,00579	0,00405	0,03727	0,93124	0,01826

Table 4. Direct problem - parameters of the optimal portfolio with critical level r *=7,5 %

Low bound	Expected profitableness	Upper bound	Risk level
4,732	**7,412**	**10,093**	**0,25**
4,756	7,421	10,085	0,3
4,74	7,402	10,064	0,35
4,77	7,435	10,1	0,4
4,732	7,394	10,056	0,45
4,681	7,362	10,043	0,5
4,705	7,383	10,061	0,55
4,586	7,325	10,064	0,6
4,484	7,262	10,04	0,65
4,317	7,172	10,026	0,7
4,131	7,063	9,995	0,75

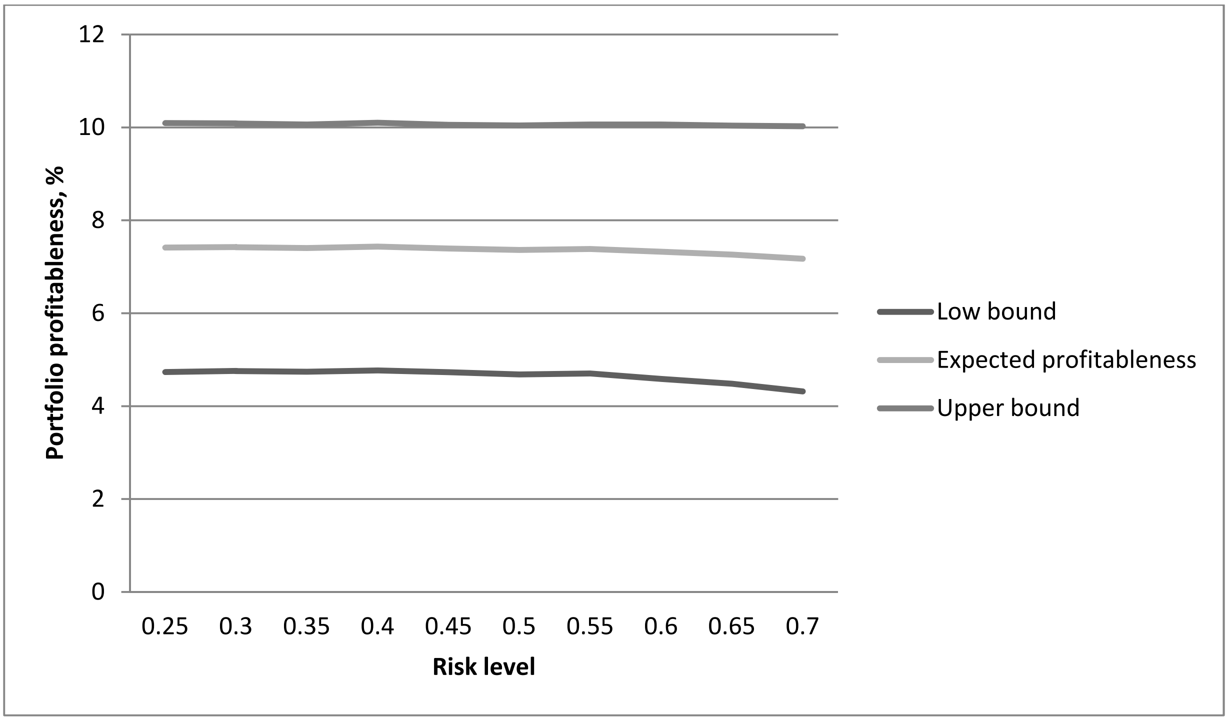

Figure 1. Dependence of expected portfolio profitableness on risk level

As we can see on Figure 1 the dependence profitableness - risk has a descending type, the greater risk the lesser is profitableness opposite to classical probabilistic methods. It may be explained so that at fuzzy approach by risk is meant the situation when the expected profitableness happens to be less than the given criteria level. When the expected profitableness decreases, the risk grows.

The profitableness of the real portfolio is 5,34 %.This value falls in results calculated corridor of profitableness [4,732; 7,412; 10,093], indicating the high quality of the forecast. The main portfolio portion in this case goes to company Seagate Technology PLC that can be explained by the high level of its profitableness in comparison with other companies.

Now consider in Tables 5, 6 and Figure 2, the results gotten by solving multicriteria problem.

Table 5. Multicriteria problem - distribution of components of the optimal portfolio with critical level r *=7,5 %

GOOGL	DIS	KO	KMB	STX	TSLA
0,01069	0,02289	0,00316	0,00117	0,92946	0,03263
0,0121	0,02234	0,0058	0,00414	0,92512	0,0305
0,01265	0,02131	0,00733	0,00596	0,92451	0,02824
0,01418	0,02124	0,00982	0,00871	0,91908	0,02697
0,01478	0,02049	0,01124	0,01038	0,918	0,02511
0,0154	0,01989	0,01265	0,01199	0,91656	0,02351
0,01603	0,01929	0,01402	0,01355	0,91518	0,02193
0,01668	0,0188	0,01537	0,01505	0,91356	0,02054
0,01733	0,01838	0,01669	0,01655	0,91182	0,01923
0,01069	0,02289	0,00316	0,00117	0,92946	0,03263
0,0121	0,02234	0,0058	0,00414	0,92512	0,0305

Table 6. Multicriteria problem - parameters of the optimal portfolio with critical level r *=7,5 %

Low bound	Expected profitableness	Upper bound	Risk level	w1
4,626	**7,273**	**9,92**	**0,65253**	**0,1**
4,545	7,219	9,893	0,67539	0,2

4,501	7,192	9,882	0,69878	0,3
4,423	7,138	9,854	0,71007	0,4
4,381	7,112	9,842	0,72732	0,5
4,34	7,085	9,83	0,74142	0,6
4,3	7,059	9,818	0,75483	0,7
4,26	7,033	9,806	0,76619	0,8
4,221	7,007	9,793	0,77641	0,9

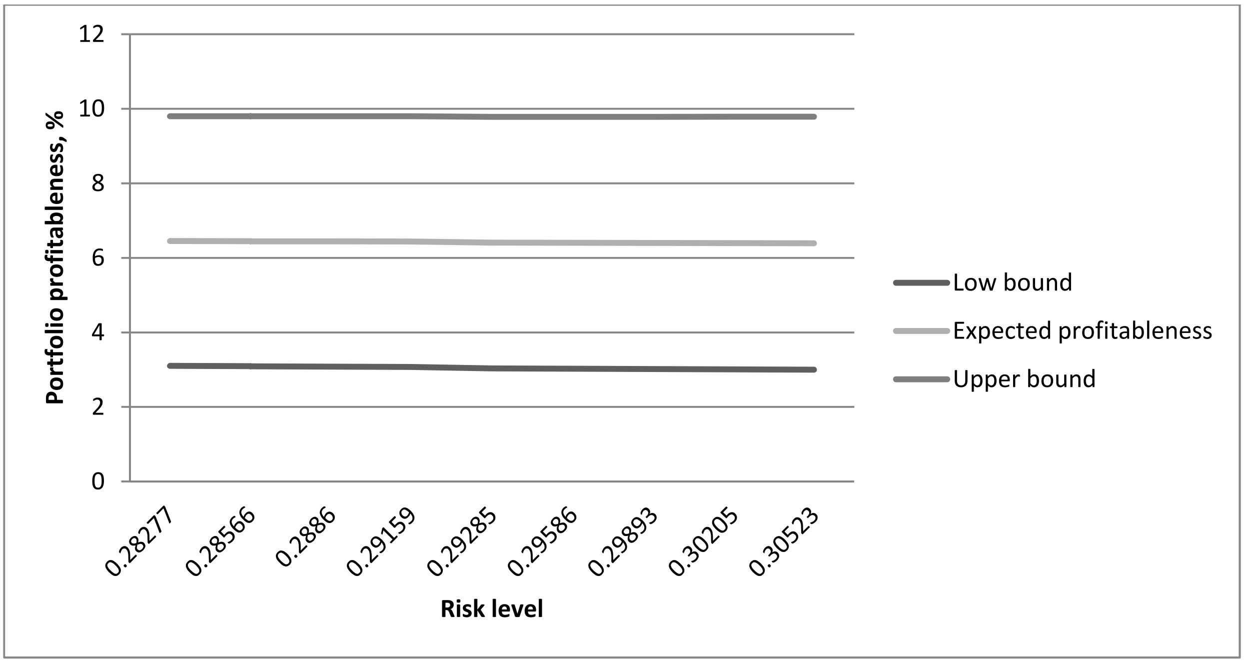

Figure 2. Dependence of expected portfolio profitableness on risk level

The profitableness of the real portfolio is 5,16 %. This value falls in results calculated corridor of profitableness [4,626; 7,273; 9,92]. The dependence profitableness - risk also has descending type.

Let's consider an optimization portfolio at the time interval of 3 weeks. We have got profitableness values for all weeks by using the Fuzzy GMDH (Table 2). The results of using direct problem during the period are presented in Table 7.

Table 7. Optimization problem during the period - distribution of components of the optimal portfolio for triangular MF with critical level r *=7,5 %

GOOGL	DIS	KO	KMB	STX	TSLA
0,01964	0,92474	0,02535	0,00102	0,02878	0,00047
0,01965	**0,92309**	**0,02536**	**0,00287**	**0,02876**	**0,00027**
0,02066	0,92203	0,02637	0,00066	0,02975	0,00053
0,01968	0,92274	0,02538	0,00069	0,02873	0,00278
0,01969	0,92329	0,02539	0,00206	0,02871	0,00086
0,0197	0,92068	0,0254	0,00208	0,02869	0,00345
0,01972	0,92355	0,02541	0,00231	0,02867	0,00034
0,01973	0,9228	0,02542	0,00223	0,02865	0,00117
0,02075	0,9212	0,02643	0,00124	0,02962	0,00076
0,01976	0,92152	0,02544	0,00205	0,0286	0,00263

Table 8. Optimization problem during the period - parameters of the optimal portfolio for triangular MF with critical level r *=7,5 %

Low bound	Expected profitableness	Upper bound	Risk level
4,732	7,412	10,093	0,25
4,756	**7,421**	**10,085**	**0,3**
4,74	7,402	10,064	0,35
4,77	7,435	10,1	0,4
4,732	7,394	10,056	0,45
4,681	7,362	10,043	0,5
4,705	7,383	10,061	0,55
4,586	7,325	10,064	0,6
4,484	7,262	10,04	0,65
4,317	7,172	10,026	0,7

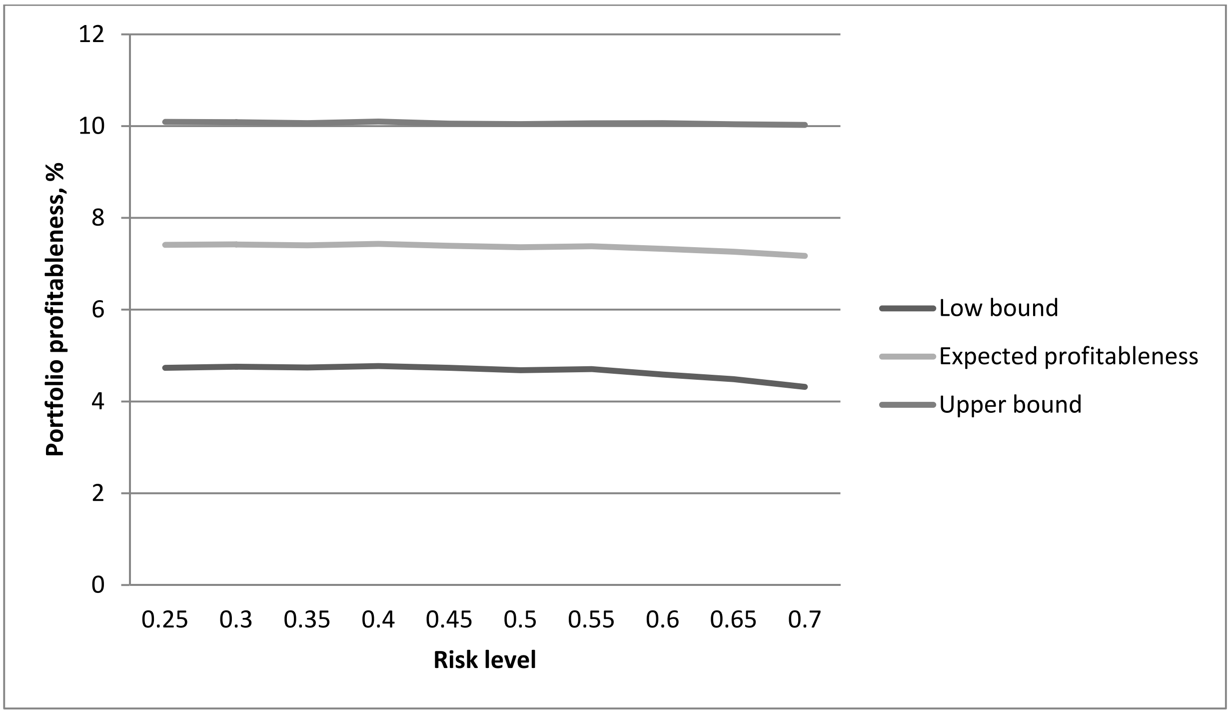

Figure 3. Dependence of expected portfolio profitableness on risk level

Let's consider the results obtained by solving the dual problem using triangular MF. In this case, the investor sets the rate of return, and the problem is to minimize the risk. The main portfolio portion in this case goes to company Walt Disney Co that can be explained by the high level of its average profitableness in comparison with other companies.

The optimal portfolio is listed in Tables 9, 10 and Figure 4.

Table 9. Dual problem. Distribution of components of the optimal portfolio

GOOGL	DIS	KO	KMB	STX	TSLA
0,01756	0,01918	0,01664	0,01503	0,91202	0,01956
0,01752	0,02002	0,01609	0,01389	0,91166	0,02082
0,01666	0,02021	0,01459	0,01194	0,91494	0,02166
0,01606	0,02093	0,01318	0,01032	0,91606	0,02345
0,01404	0,02119	0,00973	0,00691	0,92231	0,02582
0,00768	0,02313	0,00185	0,00015	0,93191	0,03528
0,00951	0,02323	0,0009	0,0006	0,93027	0,0355

Table 10. Dual problem. Parameters of the optimal portfolio

Low bound	Expected profitableness	Upper bound	Risk level	Critical rate of return
0,07016	0,04242	0,0979	0,01821	5
0,07027	0,04263	0,09791	0,05381	5,5
0,07061	0,04315	0,09808	0,11501	6
0,07088	0,04356	0,0982	0,21157	6,5

0,07157	0,04456	0,09857	0,37038	7
0,0731	0,04673	0,09947	0,6253	7,5
0,07302	0,04661	0,09942	0,87555	8

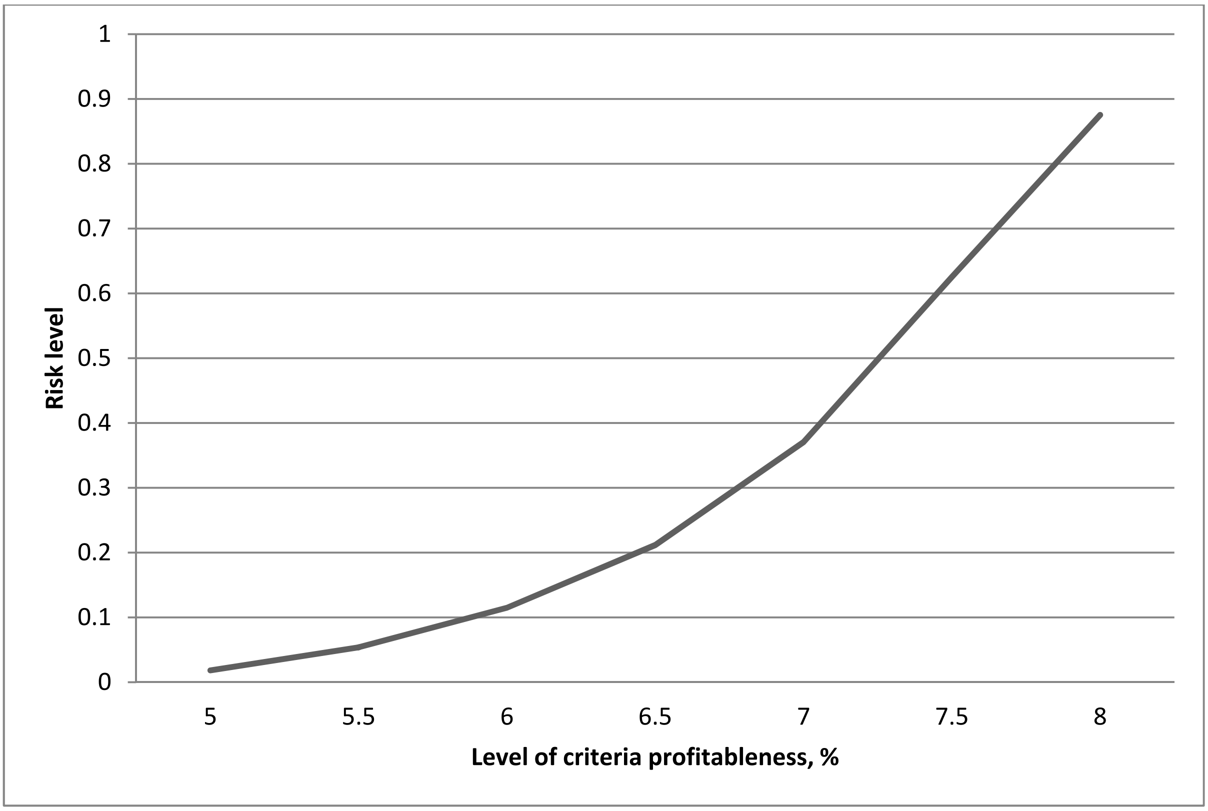

Figure 4. Dependence of the risk level on a given critical return

From these results we can see that the dependence risk - given critical level of profitability takes a growing character, because at the growth of the critical profitability the probability that the expected return will be lower than a given critical value also increases.

Conclusion

In this work the research in the field of portfolio management was carried out. Fuzzy sets theory was used as a tool for getting an optimal portfolio. As a result of research the mathematical model based on the fuzzy-set approach for a finding of structure of the optimal investment portfolio has been obtained.

On the basis of the theory of fuzzy sets the algorithm of optimization of a share portfolio has been developed. As a result of research the following conclusions were made:

- The dependence profitableness - risk has a descending type, the greater risk the lesser is profitableness opposite from classical probabilistic methods. It may be explained so that at fuzzy approach by risk is meant the situation when the expected profitableness happens to be less than the given criteria level. When the expected profitableness decreases, the risk grows;
- Portfolios during the time period and at the end of period have different structure and characteristics, that can be explained by the variations of average profitableness;
- For improving the accuracy of the suggested fuzzy portfolio model, the fuzzy GMDH method was applied for profitableness forecasting. The experimental investigations have proved its high efficiency.
- The dependence risk - given critical level of profitability has a growing character in the dual task, because with the growth of the critical profitability increases the probability that the expected return will be lower than a given critical value.

Thus, we have developed a system that not only automates the search of the optimal portfolio, but also provides a flexible and effective management of portfolio investments.

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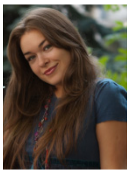

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THE EXPERIENCE OF THE AGENT-BASED SIMULATION SYSTEM DEVELOPING

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***Abstract**: This paper discusses the problems of the agent-based simulation system design. It is well known that agent models extend the capabilities of simulation for solving some problems that can't be solved by the methods of system dynamics and discrete event simulation. Particular attention in the design and the implementation of agent-based simulation authors pay to the problems of distributed simulation and the problems of intelligent agent's implementation and the use of ontologies at all stages of the simulation experiments.*

***Keywords**: simulation, agent-based model, distributed simulation, intelligent agent*

***ACM Classification Keywords**: I.6 SIMULATION AND MODELING I.6.8 Types of Simulation - Distributed : I.2 ARTIFICIAL INTELLIGENCE I.2.5 Programming Languages and Software - Expert system tools and techniques*

Introduction

The problems encountered in everyday life (in industry, commerce, management, etc.) become more complex, so it is necessary to develop new and to improve existing methods for solving these problems. One of the methods for solving complex problems in various areas of activity is a simulation method.

It is well known that simulation is very important in the investigations of complex dynamical systems. Simulation methods application are rational if it is very difficult to formalize a problem or in the case when analytical and numerical methods require strong abstractions to avoid some sufficient details.

Two classes of simulation systems exist: continuous and discrete-event simulation (DES). But some problems can't be described by continuous and discrete-event simulation models. This is true with respect to the economic-mathematical models for example (these models describe the processes occurring in the, ecological and economic systems in the form of equations and inequalities). Modern models are dynamic and they have large dimensions. So designed simulation models may be valid but contradictory.

So the application of new techniques that allow us to solve the problem of managing the complex objects becomes relevant. One of the methods is the method of agent-based modeling. The essence of the agent-based modeling is that the local behavior of agents operating under their own rules defines the global behavior of the whole system (the concept of designing "bottom-up"). This differs from traditional approaches to a simulation model design. The traditional approach supposes «top-down» design. Investigator defines a set of global laws of the system behavior and the elements of this system operate on basis of these global laws. But global functioning of the agent-based simulation system is not known to the investigator. Two or three simple rules can already lead to very diverse forms of behavior in a group of agents. An example is a boids-modeling and cellular automata theory [Macal & North, 2005; Macal & North, 2009].

The need for the development, analysis and a business process reengineering is another argument in favor of the agent-based simulation [Klyshinskii, 2000]. Some operations in a business process are executed by the objects that determine their own behavior (decision makers, automated and robotic systems, objects that are managed by humans). The behavior of the object is determined by a set of rules. The behavior of this object may differ due to particular situation: it should consider the impact of other objects and the impact of the external environment.

So let us entertain the hypothesis that an agent is an independent (autonomous) system having the opportunity to take effect from the outside world, to determine its reaction to this effect and to carry out its reaction, and an intelligent agent - an agent which has some knowledge about itself and the world around it, at that its reasonable behavior is determined by its knowledge. There are many works that show the relevance of the application of agent-based approach in the marketing of [Ivashkin, 2003], in the simulation of situations that occur in the auction [Gribanova, 2006], inventory management, supply chains, etc.

Currently there are many software systems (most often - the software libraries) that implement agent-based simulation. We'll discuss the various software systems below. These systems are both domestic and foreign, both paid and freeware. This paper considers the specific features of architecture of agent-based simulation systems and suggests some approaches to improve the adaptability of the software product, the effectiveness and reliability of the agent-based simulation experiment.

Review of existing agent-based simulation systems

Let us discuss the specific characteristics of agent-based simulation systems and special libraries for the creation these systems:

- ***Anylogic.*** AnyLogic [Borshchev, 2014, Borshchev et al, 2012] is one of the famous agent-based simulation systems in Russia, and it is well known all over the world. AnyLogic permits to use not only continuous modelling but discrete-oriented and agent-based modelling too. It has graphical interface and the set of standard libraries. These tools make the process of simulation model design simpler and allow designing simulation models for more wide class of problems. AnyLogic was implemented on the base of Java language (so it is possible to say that Anylogic is a cross-platform system), moreover the power of Anylogic may be extended if the investigator includes into a simulation model some modules developed with the help of Java.

  It is possible to develop Java-applets. These applets can be opened by various browsers.

  So the program tools of AnyLogic allow designing, implementing and documenting the simulation models, it is possible to collect a set of statistical data during the simulation run, to analyze a simulation model and to optimize it. But intellectual agents not implemented in AnyLogic and it isn't a distributed one (we must notice that some publications [Borshchev et al, 2012] concerning the developing of program tools supporting distributed simulation in AnyLogic appeared).

- ***BPSim***. BPSIM – it is another agent-based simulation system [Aksienov, 2013]. This simulation system is dedicated to the simulation of business processes. BPSIM is the program realization of the resource consumption multi-agent conversion model. The agents control the process objects of resource conversion. The agents analyze a current situation, refer to the knowledge base, find a solution, control the purpose gaining and exchange messages.

  The program tools BPSIM allow to develop a conceptual model, dynamic model, to carry out a simulation experiment and to export the results to EXCEL. The system contains (a) reactive agents, (b) reactive-intelligent agents, (c) intelligent agents (their behavior is described by the planning system, the knowledge is stored in the knowledge frame base, applied for construction of complicated advising expert system and so on), (d) hybrid agents (construction of complicated planning systems).

  But one can hardly speak that BPSim is cross-platform software (code is generated in Delphi). Moreover BPSim software does not support a distributed (parallel) simulation.

- ***REPAST:*** REcursive Porous Agent Simulation Toolkit (Repast) [Repast, 2015] – it is an open source and freely available libraries for large-scale agent-based modeling. Repast supports the development of flexible agent-based models and it may be used in modeling of social processes, in marketing and logistics. A modeler builds simulation model including components from open source libraries into his program. One can use Visual Repast. There are three versions of Repast: Repast for Python (Repast Py), Repast for Java (Repast J) and Repast for the Microsoft.NET framework (Repast .NET). Repast use MatLab, SQL, Excel for the processing of the results of simulation experiments. Repast is very powerful program tool but user must be skilled programmer.

- ***NetLogo:***NetLogo [NetLogo, 2015] **-**simulation environment designed to create models describing the natural and social phenomena. This system allows the models to operate "on the fly" dynamically changing the behavior of the system under the influence of various conditions. The system is quite simple and it allows the users without programming skill to open and run models saved in libraries. These models can be used again, or modified. Moreover users may build their own simulation models. NetLogo modeling environment is cross-platform.

- ***Mason:*** Agent-oriented simulation system Mason [Mason, 2015] - is a multi-agent simulation environment and it is represented as a set of libraries in Java (cross-platform). Mason supports discrete-oriented modeling paradigm (released under Academic Free License, version 3.0). Mason includes powerful visualization software (2D, 3D). MASON system is protected from cyber-attacks. However Mason does not support distributed simulation (but currently developers try to create a distributed version). Moreover users have to know programming language Java rather seriously, so Mason is not convenient for the inexperienced users.

- ***Ascape:*** Simulation system Ascape [Ascape, 2015] is another cross-platform agent-based system, written in Java and it is freely available. Users without programming skills may learn many aspects of modeling.

- ***Swarm:*** Swarm [Swarm, 2015] was first program tool for the creation of agent-based applications and was implemented in 1994. Moreover Swarm attempts to create distributed platform.

The requirement for the agent-based simulation systems

So the review being mentioned above suggests that not only users skilled in programming can create the simulation model but the ordinary users skilled in specific domains may do it too. Due to this fact it is advisable to develop special software and visual languages to create and change a simulation model. Moreover it is advisable that agent-based simulation system should have the following characteristics: (a) agent-based simulation system should be cross-platform (the developers of simulation software use Java and C languages in order to achieve this property), (b) agents are developed as the objects sending messages from one to another, (c) users have a possibility to change adjusting parameters of a simulation model during a simulation run [Vlasov, 2013; Zamyatina, 2012].

The authors of this paper discuss the developing of agent-based simulation systems. The authors attempt to take into account the experience of the other developers. Their agent-based simulation system has to meet the following requirements:

- ***Operations with a simulation model.*** It is advisable to have a possibility to change a set of interacting agents and to change links between agents.
- ***Hierarchical representation of simulation model.*** Simulation model should be hierarchical because it is necessary to consider model in details or in contrary to change the group of agents by one agent. This new agent must simulate the behavior of a group of agents [19,20,21].
- ***The highlighting of the structure of collaborating agents.*** It is advisable to develop software for highlighting structure of agent's connections in the simulation model. One can carry out the analysis of highlighting structure of the agents, their relationships, examining its structural characteristics (it is possible to use the methods of graph theory) [Mikov, 1995; Zamyatina & Mikov, 2012].
- ***Data collection and data processing.*** The statistical data collection and data processing must be carried out by special agents that act as sensors, monitors and form "the researching algorithm". This algorithm should be separated from most of the simulation models [Mikov, 1995].
- ***Verification and validation of simulation models.*** Most of simulation systems either do not have the special program components for the verification and validation of simulation models or the functionality of these components is limited. So it is advisable to develop special software for debugging and testing the simulation models in order to obtain a reliable and valid model [Mikov et al, 2013]
- ***Adaptability.*** Simulation systems solve the problems dealt with various specific domains. Moreover simulation model may be described in various «terms». Indeed, let us consider computer networks. One may investigate computer network using the theory of graphs, or the

theory of Petri Nets, or theory of queueing network. So it is advisable to describe computer network as a queuing network or as a graph (a set of nodes and a set of arcs connecting these nodes) or as a Petri Net (bipartite multigraph: a set of nodes (the nodes of two types: positions and transitions) and a set of arcs connecting these nodes (the nodes of different types). It is useful to carry out the description of simulation model using appropriate domain specific language (DSL). The designers of simulation systems apply ontological approach to adjust it to the particular specific domain. One may read the related papers in [Mikov, 2009; Mikov et al.,2009; Sukhov, 2013; Zamyatina et al, 2013].

- ***Reducing the time of the simulation experiment.*** It is well known that simulation systems deal with complicate time consuming problems, so it is necessary to reduce the time of simulation experiment and to use a set of compute nodes of supercomputer or cluster or computer network, so it is useful to provide parallel (distributed) simulation, hence optimistic or conservative algorithms providing causality of events of distributed simulation experiments must be implemented [Fujimoto, 2003]. Load balancing is the second method to reduce the time of distributed simulation experiments [Wilson, 1998].
- ***Safety and fault tolerance of simulation systems.*** Distributed simulation experiment demands additional software. This software provides safety and fault tolerance of a simulation system. It is assumed that software have to find failed compute node and to transfer agents to alive compute node.
- ***Remote access.*** It is advisable to provide the remote access to the simulation system by developing the appropriate Web-services. Remote access will not only allow users to get quickly simulation results and create models using graphical or text editor, but also collaborates users who are geographically located at a remote distance from each other.
- ***Processing of simulation results.*** The user receives a large number of unstructured information as a result of simulation experiment. It is advisable in this case to perform additional processing of the simulation results, using the methods of Data Mining and Knowledge Discovery. Additional processing of the simulation results will optimize simulation experiment, identifying dependencies and relationships between the input parameters [Kolevatov & Zamyatina, 2012].
- ***The intelligent agents.*** In order to reflect more adequately the simulated processes in economics, marketing, logistics it is very important to implement the intelligent agents. Most of the above systems provide the user with the tools to describe the reactive behavior or quite simple one. But it is essential to teach the intelligent agents following the changes in external environments, to pursue goals, to choose a particular strategy depending on the role these agents.

  In this version of the agent-based simulation system the authors focused on the development of tools that support distributed modeling (distributed simulation experiment) and on the development of intelligent agents.

An implementation of agent-based simulation system

The current version of the multi-agent simulation system is made in the language Scala [Odersky, 2015]. Scala language was chosen as the programming language to implement agent-based modeling platform. Scala language is object-functional programming language that combines functional programming and OOP (object-oriented programming) and specializes in building easily scalable component-oriented software. Scala was founded by Martin Odersky in the University EPFL (Lausanne, Switzerland) in 2004. It is currently available for the Java platform and the .NET Framework.

The Scala language includes the following key features: a single object model, the presence of traits, the method of pattern matching, lambda-calculus, type views bounds, type linearization, parametric and functional polymorphism, variation of types, case classes and etc. Moreover there are numerous tools to create new language constructs and list processing. Generalized block diagram of the simulator is shown in Figure 1.

Thus the architecture of the simulator is not a multi-component but multi-layered. This is achieved due to the fact that a module that provides some basic functionality may be "mixed" with other components extending the structure, behavior and semantics.

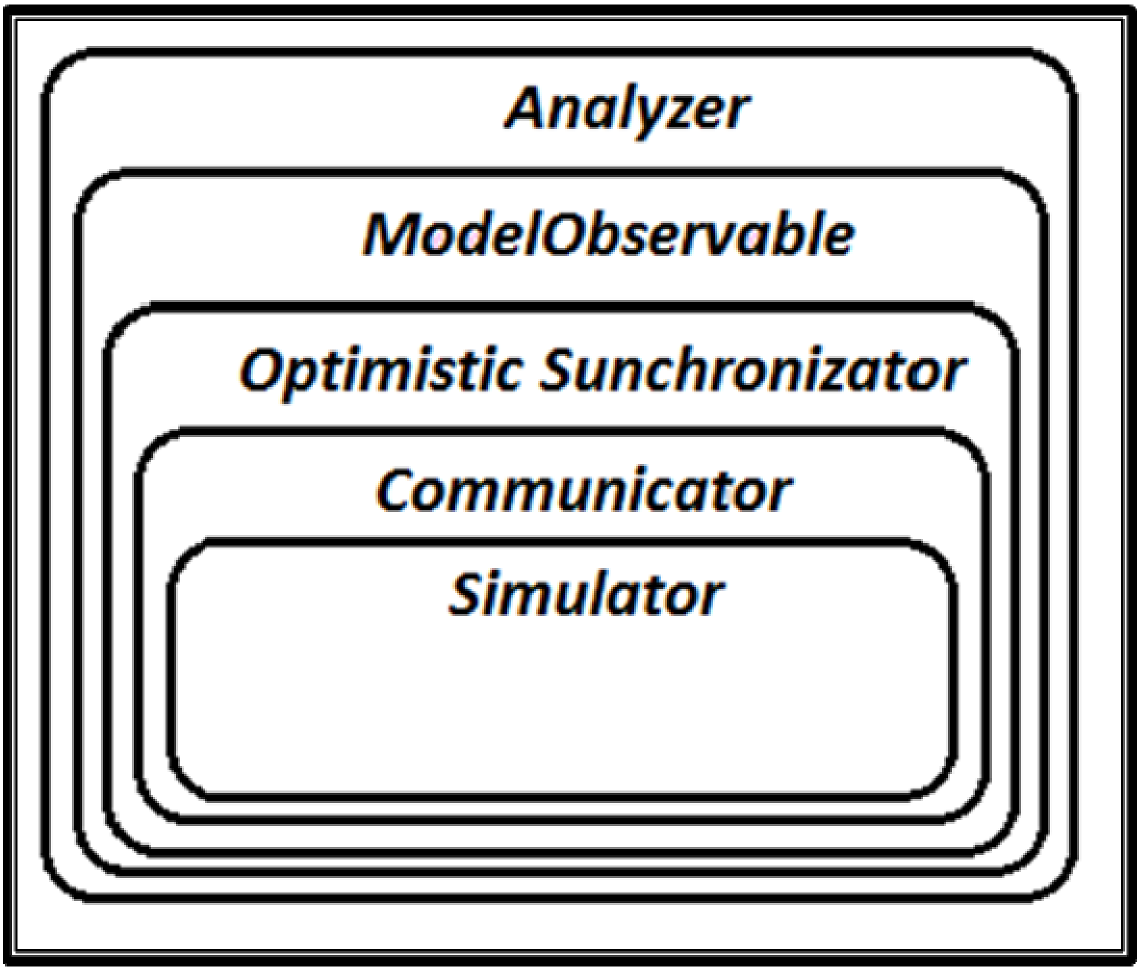

Figure 1. A Multilayer architecture of the simulator

Trait Simulator is the base module, which is interfered with the other elements, specifying it's behavior. Actually, Simulator, - is a frame for a logical process. Traits Communicator is a module for applying the system of actors. It contains a copy of the actor engaged in sending/receiving messages. Traits OptimisticSynchronizator is a key element for the organization of parallel (distributed) discrete-event simulation (PDES). It implements an optimistic synchronization algorithm Time Warp.

Trait ModelObservable is intended to implement "information" procedures – program unit for data collection during a simulation experiment.

Трейт Analyzer is intended to make the synchronization algorithm more "intellectual".

Particular attention is paid to the synchronization algorithm of the developed distributed simulation system. It is known that the distributed model is a collection of logical processes executing on the different computing nodes of high-performance computing system. Logic processes exchange messages among each other. It is important to maintain the causality of events in the distributed systems. The causality of events is supported by classic distributed algorithms. These algorithms are divided into two groups: the conservative and optimistic. Conservative algorithms involve the promotion time only after it becomes apparent that the next event is safe. The event is safe, if you are sure that no other event will be with less "time stamp" [Fujimoto, 2003; Bryant, 1977,Chandy & Misra, 1979]. Optimistic algorithm is as follows: the process is performed by moving from one event to another for as long as it does not receive a message from another process with a smaller time stamp than the time stamp of the next event [Jefferson, 1985].

An optimistic synchronization algorithm based on knowledge of the simulation model is implemented in present simulation system (KBASA). There is a number of works [Zamyatina & Ermakov, 2011], which use knowledge of the simulation model in order to reduce the execution time of distributed simulation experiment.

The authors implemented efficient synchronization algorithm controlled by expert system based on rules in order to reduce the runtime of distributed algorithm. On the other hand the reduction of time is obtained through the load balancing [Wilson, ;Zheng, 2015; Mikov & Zamyatina, 2010].

Classic algorithms also use "knowledge" about the model: *lookahead* in conservative algorithms, *lookback* in the algorithm Work Flow, etc. Using the knowledge of the agent-based simulation model derived from ontologies, have yielded good results in the implementation of synchronization algorithms. Experiments were performed with the same model many times having the same parameters. The total duration of the simulation was 300 (± 15) units of modeling time.

The results of the experiments have shown that the metrics of the algorithm KBASA, based on knowledge of the model are substantially reduced in comparison with the metrics of the optimistic algorithm Time Warp. In fact, (a) the number of rollbacks decreased from 71.2 to 3.8 (Fig. 2). (b) The total number of not deep rollbacks reduced almost to zero (on average 0.1-0.2 vs. 7-11). (c) The number of deep rollbacks also decreased from 19.4 to 3.2. (d) The number of antimessages decreased from 108.4 to 12.8.

The implementation of intelligent agent

It is well known that the creation of intelligent agents is a very complex task that requires a theoretical foundation. There are various models of intelligent agents. These models describe knowledge, the methods of reasoning, the strategy of the behavior and actions of agents in various ways. One may consider these models from two points of view: in terms of the analysis of the properties and behavior of agents during their functioning; from the point of view of the design of the properties of the agents and their internal processes (acquisition of knowledge, goals definition, decision making, etc.).

There are three types of architectures: (a) deliberative architecture and model; (b) reactive architecture and model; (c) hybrid architecture and model.

Reactive approach allows the application of a set of rather simple scenarios. We mean the scenario of the agents behavior. These scenarios are a reaction to the appearance of an event in the external environment.

Deliberative model allows the application of rigorous formal methods and well-proven technologies of traditional artificial intelligence, makes it easy to represent knowledge in a symbolic form and use them in the agent-based systems. The hybrid type of architecture combines the advantages of the architectures mentioned above. Thus, the intelligent agent has a high-level inference engine and low-level reactive abilities.

Thus the development of agent-based simulation system involves the development of a set of base classes to represent the intelligent agents. It was decided to present the architecture of the agents in the form of hybrid schemes and deliberative one.

The inference engine is implemented with the help of production systems and neural networks. It is known that neural networks have the ability to self-learning, which is important for the implementation of intelligent agents, which have to adapt to changes in the external environment and change their behavior in order to make a decision. Three types of neural networks were used: multilayer perceptron, Hopfield network and the Hamming network [Khaikin, 2006; Kruglov, 2002]. A genetic algorithm is proposed for training the neural network.

The developed programming tools were tested. The task of searching of attractions in the park [Zamyatina & Chudinov, 2010; Dubiel & Thimsini, 2005] and "Artificial Life" were chosen in order to test implemented agents based on deliberate and hybrid schemes.

The problem of searching the attractions may be formulated as following: a person is looking for some attractions in the park and trying to get information about his location with the help of maps and informers. Having received information from an informer, a person will go to tram station, rather than solely on foot to get to the sights of interest. When the source data card is used, the person gets to the destination on foot, etc. The problem of "artificial life" is well known.

The results of the test tasks "Searching of attractions" are in the table below. It has been made 100 test runs of the model. The simulation results were obtained indicating the possibility of using this type of agents (see Table 1).

Agent model based on neural networks was implemented as a 2-layer perceptron having 6 neurons of input level and 2 neurons of output layer. Following input data for the input layer of the network was presented: (a) the fact that an informer is in sight of person looking for attractions (info); (b) the distance to the informer (when there are not only one informer it is necessary to take into account the distance to the nearest one) (dinf); (c) the distance to the target (df) and etc.

100 test runs were carried out during verification and validation of the agent-based simulation model. The simulation results were obtained indicating the possibility of using this type of agents (see Table 2).

Table 1. The results of simulation experiment “searching the attractions in the park” (agents with production rules”

The Number of agents-informers	**Average time to search the target (in seconds)**
1	29.6
3	27.9
5	19.5

Table 2. The results of simulation experiment “searching the attractions in the park” (agent - neural network)

The Number of agents-informers	**Average time to search the target (in seconds)**
1	37.6
3	33.5
5	25.1

Conclusion

So, authors designed and implemented the prototype of an agent-oriented simulation system. Authors tried to follow all the recommendations for the designers of the agent-based simulation systems (cross-platform, hierarchical representation of simulation model, adaptability, operations with simulation model (add new agent, delete agent, add new links connecting agents), distributed simulation experiment). Developed simulation system is cross-platform because authors used Scala language as a base language.

Particular attention was paid to the development of the synchronization algorithm, which supports distributed simulation. It is known that the distributed model is a collection of logical processes located on different computing nodes. Logic processes must be synchronized to ensure the causality of the events in a distributed model. Authors developed synchronization algorithm based on classical optimistic algorithm. The synchronization algorithm is knowledge-based because it uses the knowledge about simulation model. The investigations have shown that the use of knowledge about the model significantly increases the efficiency of the algorithm. Authors used a model of actors as a formal model of an agent-based simulation system, because it is the most appropriate model showing the behavior of agents in a distributed (parallel) environment.

Another important element in building an agent-based simulation system is the development of intelligent agents. Most agent-based simulation systems do not have the modeling tools that support the functioning of intelligent agents (these system support only the functioning of reactive agent which can't adapt to the changing external environment. The presence of intelligent agents in the simulation system makes it possible to build more appropriate models of real processes and situations.

Intelligent agents which were implemented in agent-based system are based on the production rules and artificial neural networks. Two test models were built (searching of attractions in the park and well known "artificial life") and almost 100 simulation runs were carried out in order to test the functioning of each model. The results of experiments showed the correctness of the developed tools and the decisions taken in their design.

Implementation of works on the design of agent-based simulation system is relevant, because now the question of a wide practical application of simulation expertise [Vlasov et al, 2013] is discussed. It is believed that any design work on the establishment and modernization of any systems (objects) must be preceded by simulation studies "to give practical conclusions and guidelines regarding the appropriateness of existence, building, functioning and modernization of the system."

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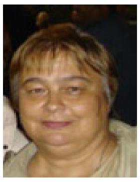

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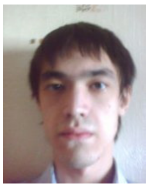
Fields of Scientific Research: simulation, artificial intelligence, ontology, domain specific languages (DSL)



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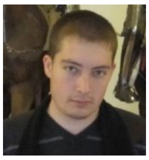

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PODCASTS: A BRIDGE FROM E-LEARNING TO M-LEARNING

Larisa Savyuk, Oleksiy Voychenko

Abstract*: For educational organizations offering e-learning, intensive flow of technological innovations may be considered as both blessing and curse at the same time.*

On the one hand, many challenging tasks, such as creation of multimedia content, become easier with new technologies and tools, so e-learning creation, support and delivery is currently affordable in sense of investments and human resources as a part of common educational practice.

On the other, large scale technological changes, such as transfer to a new platform, require meticulous analysis of potential benefits and drawbacks due to the need for personnel training, compatibility problems or unexpected development costs. Introduction of mobile learning as an option for distance learning delivery is a typical example. Nowadays e-learning services are available globally, so high technical quality and pedagogical soundness of learning offers are the issue of survival for educational organization.

Technology innovations, as m-learning, cannot be ignored, taking into account wide availability of not only mobile devices but also mobile applications and services. Smartphone's and tablets are widely used by the youth for various forms of communication, including email, Skype and specific apps, for playing games, or for internet access to news, data and content - browsing/skimming, listening to favorite music or watching HD video. Therefore, e-learning organizations have no other choice than offer m-learning for their customers - a learning content in a mobile-accessible mode - as it enhances flexibility, extends "learning hours" by time spent on commuting, waiting, and unexpected breaks, and thus makes e-learning offer more attractive.

Several approaches to introduce m-learning are known, and each of them has both benefits and drawbacks further discussed in the paper. Thus each organization has to make its own decision based on short- and long-term analysis of its resources, personnel skills and qualifications, potential audience, and many other factors.

There is no universal method to evaluate all of them, and related error cost may be significant. Moreover, due to specifics of mobile devices and environment in which mobile learning takes place, as well as some implementation issues, mobile mode may be rather an option for e-learning delivery and cover some but not all learning experiences offered in a course. Therefore, introduction of m-learning should not be costly or time-consuming, and should not require changes of the established e-learning *processes. Considering that m-learning is a new delivery method, we decide to offer it in a trial mode to evaluate customers' readiness and preferences and adjust related mechanisms and resources accordingly. Based on the analysis of the approaches and requirements to mobile delivery, we suggest generating of podcasts as a first step in m-learning implementation. Further, the implementation of podcasts is described and its relevance is discussed.*

***Keywords**: e-learning, m-learning, podcasting, LMS, content*

***ACM Classification Keywords**: D. m Miscellaneous*

Introduction

Nowadays highly qualified specialist cannot work a lifetime using once obtained knowledge. Almost any knowledge in rapidly developing areas after a while no longer meets the requirements [Savyuk, 2014]. To stay qualified a person has to constantly go through trainings, acquire new knowledge, get new professional skills, etc.

As a reply to this challenge raised new paradigm - Lifelong Learning - learning throughout life when professional activity is combined with educational activities, which means to work and study at the same time. This area is primarily concerned with adult education. Modern adult learning has its specifics.

The traditional solution to this problem is the use of distance learning technologies [Voychenko & Synytsya, 2011]. However, technological changes lead to an increasing demand for educational services in a mobile format.

Efficient m-learning often requires redesign of content, activities, interface, as well as access rules to cope with different presentation frame, non-learning environment challenges and input specifics. At initial stage of m-learning, an e-learning organization should evaluate its own potential as well as expected benefits from mobile delivery to select an appropriate approach.

Thus each organization has to make its own decision based on short- and long-term analysis of its resources, personnel skills and qualifications, potential audience, and many other factors. Implementation of m-learning solution may be a costly and sophisticated process

E-learning and m-learning

The appropriate use of modern technologies for mobile learning still generates a large number of questions. One important aspect in the implementation of mobile learning in the educational practices of the institution is a clear understanding of the fact that at this stage of technological development the mobile learning is not a competitor to the traditional distance learning but rather its organic complement.

As an analogy, we may consider a comparison of a Smartphone and a laptop: the majority of students and actively use both, depending on the problem, but cases of complete rejection of a laptop in favor of a Smartphone or vice versa never occur. In most cases, both devices are used in a complementary manner. For example, photographs taken by a Smartphone then are loaded on a laptop for further processing in an image editor, and audio files are copied from the laptop to a Smartphone for listening them at any time.

Returning from this analogy to the problem of determining the place of mobile learning in the educational process and the selection of the optimal strategy for its implementation, it is easy to notice that the most popular services of mobile learning will be in situations when access to a PC (laptop) is difficult or not possible. For example, when driving the use of a laptop is rather problematic, while a Smartphone may be successfully used for listening learning audios.

Another important issue is the understanding of Bring your own device (BYOD) policy [Wikipedia] in terms of m-learning. Students, especially adult ones are more likely to use their owned mobile devices for all tasks including learning rather than purchase new ones for particular learning purposes. So when planning the implementation of m-learning it is important to provide maximum interoperability of offered con-tent/services with wide range of learnrs' devices.

There are a few approaches usually considered for m-learning implementation: adaptation of existing delivery methods to serve mobile devices, creation of mobile applications for a particular platform (native apps) or web apps.

Adaptation of e-learning solutions to mobile devices is offered in latest versions of LMS, such as MOODLE. In general, this is rather complicated and costly task, as it requires creation of mobile interfaces and modification of both LMS interface and learning content. Change of interface alone does not solve the problem, as learning content created for "traditional" computers may be displayed incorrectly or be missing on mobile devices. Courses designed according to the slides metaphor are intended for some range of screen sizes, so adaptation would require reconstruction of the content or dropping some part of it.

Interactive courses or their elements based on Adobe Flash technology do not work on Apple iOS mobile devices, and only some non-interactive fragments like videos could be converted into MP4 for mobile delivery. So, this approach in some cases leads to need of substantial changes in learning content and still gives no warranty that it would work well on the majority of mobile devices used by the learners.

Native apps for mobile devices are more popular as an efficient m-learning instrument, allowing learners to use the whole functionality of their mobile devices. Based on the features of a certain platform and device, they usually offer interactive activities and engaging content. However, their creation requires specific programming skills, and resulting product is deliverable to the chosen platform only. In contrast to corporate training, educational organizations usually rely on BYOD (bring your own device) or rather “use your own device” principle and thus should be able to serve their audience using various platforms – at least Android, iOS, Windows Phone and may be more. The amount of resources necessary to deliver an hour of training depends on the number of supported platforms and availability of tools to replicate typical scenarios, but even in case of a single platform e-learning content production is more efficient and easy due to availability of authoring tools and producers’ skills requirements.

Web apps are an alternative to native ones based on HTML5 technology that ensures that the product will run on different platforms and behave in a similar way. It is also easier to find qualified developers, as HTML5 is gaining popularity. Cross-platform applications rely on browser’s interpretation of HTML5 and do not have access out-side the browser environment, for example to media gallery, camera, gyroscope, etc. The last two approaches are similar in a sense that programming efforts in creating a learning app exceed that of instructional design, so high expenses may be justified by creation of unique learning experience rather than repackaging existing learning content.

Podcasting

A podcast is a sequence of digital media fragments called episodes that are published on some resource (website) and available for download or streaming for the sub-scribed users.

Podcasting is based on the RSS format with a specific element "enclosure" describing media content (a document, video or audio file) of an episode.

Podcasting is a popular service for sharing information and news on a range of topics, such as sport, politics, healthcare, entertainment etc., so learning podcasts clients may have already been subscribed to other types of podcasts. According to Apple, subscriptions of podcasts on iTunes reached the 1-billion mark last year and a study by Edison Research shows that 39 million Americans listen to some podcasts monthly [Tynan & Colbran, 2006].

Here we consider three basic content types for podcasts: text, audio, video. There is a lot of free software available for creating and modifying digital content of these types.

While text processing is not a problem in the most cases, the production of audio and video content may be confusing for some users.

We recommend focusing on the simple approaches like creating media content using the disposal Smartphone. Use of particular software depends on the Smartphone platform but the general approach is almost uniform.

There are many services allowing authors to create and host their podcasts.

Probably the most popular one is Soundcloud (www.soundcloud.com). However it has certain limitations. As it becomes clear from its name, the service is dedicated for dealing only with audio content. Users are not allowed to use in their podcasts content other than audio files.

Moreover free accounts provide hosting for only 180 minutes of audio that is in most cases insufficient to host an educational podcast with audio recordings of several lectures. Paid options extend hosting capacity, but even in this case such solution seems to be reasonable only in case of providing just single authored audio-only pod-cast.

Another popular service is Archive.org. It allows hosting any kind of content and does not limit users in a disk quota used, but some users report problems with access to podcasts hosted there from iTunes, the most popular podcast player.

Another problem is file management as Archive.org is aimed to keep all previously uploaded data; the deletion of previously updated digital content may cause some difficulties.

Amazon Simple Storage Service (S3) provides good set of features to host podcasts. It charges reasonable prices for its service. However, the total cost depends on popularity of the content (number of downloads), which is probably appropriate choice for companies and individuals promoting commercial content like music or talk shows but does not seem to be a good choice for educational institutions. As it can be seen from a short overview above, most of the podcast services are not free, and many of them set some limitations.

If a podcast service supports only certain type of media, organization would need to use two services to distribute both audio and video materials, and text documents still need to be cared of.

The organization may choose to install its own server for generating necessary pod-casts instead of using available cloud services; however, it involves both initial investments and ongoing costs for administration of the server, content management, users support, etc.

Selection of free podcast players’ software is available for all major mobile platforms as well as for PCs.

Podcasts may be considered as one of the first successful technologies used for mobile content delivery in a form of audio information. Its potential for learning was built on a long-term practice of broadcasting through educational video and audio channels and users’ habits of downloading music or audio books to listen during commuting to school or university.

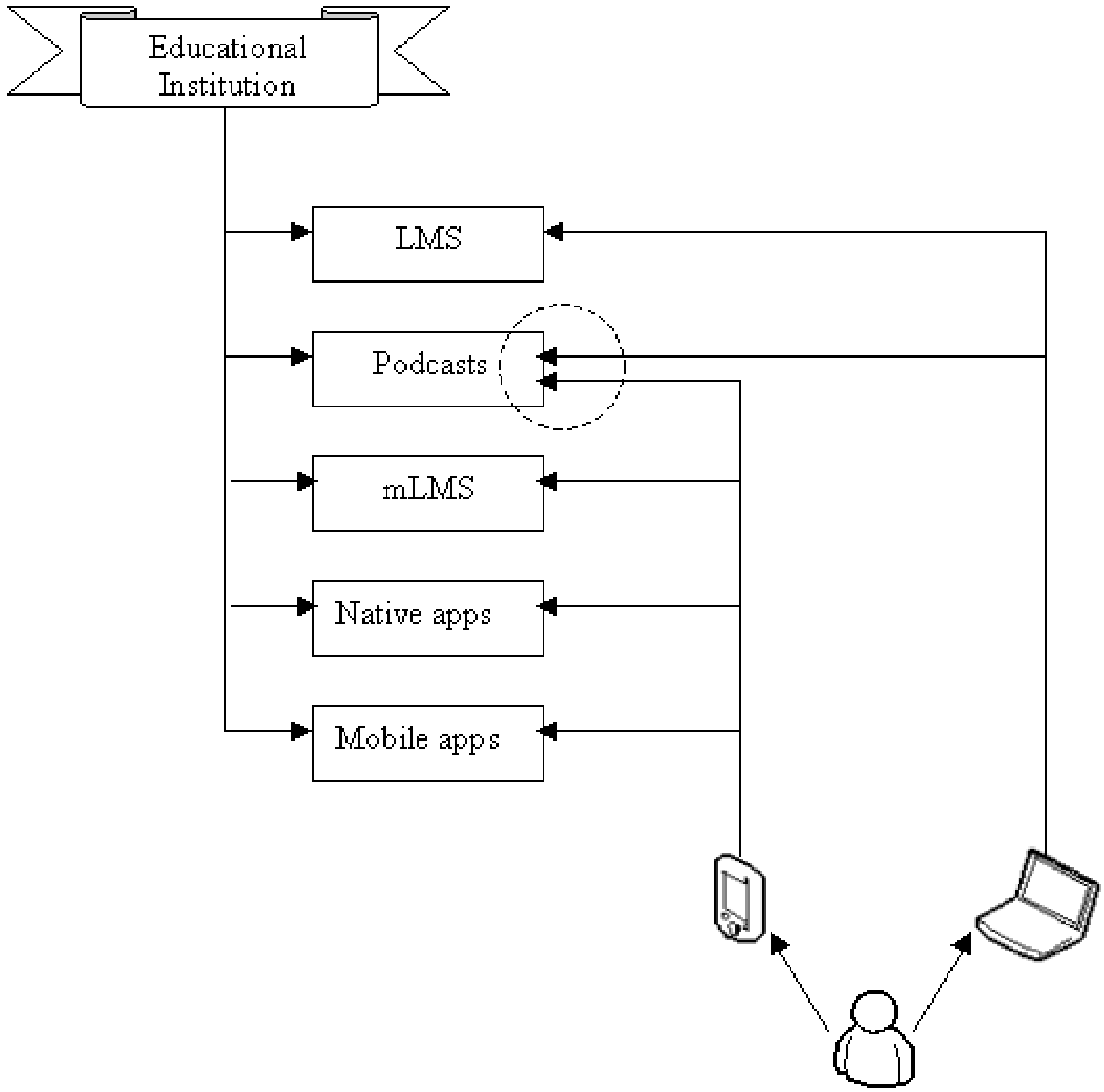

Figure 1. Options for learning content access from various devices

Figure 1 illustrates options for learning content access from various devices.

Experiments with podcasting in education were inspired by the idea of giving students access to the learning resource which could be re-played and operated by the student in contrast to a real lecture or tutorial. Indeed, the research findings indicate benefits of podcasting perceived by the students, such as increased flexibility in time management, ability to listen to the missed lecture, get back to important fragments, or stop the audio while checking associated information [Tynan & Colbran, 2006; Evans, 2008].

Conclusion

Suggested approach to introduce m-learning elements as a part of m-learning development strategy for educational organization met all requirements. It does not involve additional investments or operational costs, staff training or significant changes in routine procedures of the tutors. It does not require a preparatory stage or involvement of external experts. It extends access options for existing learning materials and thus eliminates issues of uneven opportunities.

The use of podcasts from mobile devices may be a good indicator of the audience's overall interest to m-learning, and willingness to extend learning habits. A pilot project based on podcasts may be a reliable source of information concerning mobile devices used by the students and practice of their use. Statistics and a short questionnaire can save a lot of efforts by helping identify what type of learning content would be most popular on mobile devices without jumping into development, transformation or adaptation of the course for mobile delivery.

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