FORMATION OF PRIORITIES OF NATIONAL MEZOEKONOMICAL POLITICS UNDER THE CONDITIONS OF IMPLEMENTATION ON OF THE PARIS AGREEMENTS

O.Voloshyn, V.Kudin, A.Onyshchenko, L. Khrushch

Abstract. *In the article the modified balanced ecologic-economical model is offered as "input-output" with the Paris agreement set limits on greenhouse gas emissions. The mathematical definition of the device changes the volume of gross output of main and auxiliary production in case of change of branch structure.*

Keywords: *sustainable development, the Paris Agreement, an ecological and economic system, Leontief "input-output" model, Leontief-Ford "input-output" model, simulation modeling.*

ACM Classification Keywords: *I. Computing Methodologies – I.6. Simulation and modeling – I.6.5. Model Development – Modeling Methodologies.*

Introduction

A number of issues related to the country's participation in the Paris Agreement [1] determines necessary, first of all, estimation potential future market volume of ecological services, determination of possible partners, development of economic strategy that would identify priorities for each economic mechanism, proportions of their application in order to attract the maximum amount of ecological investments.

A special role in solving the fundamental problems of nature using, studying values of environmental expenditures considering socio-economic impact and their distribution in a territorial-branch cut [2, 3], belongs to balance ecologic-economical models of "input-output" as well as regional and sectorial models [4,5,6].

Therefore, there is need to build a balance ecologic-economical model that would include the cost of implementing commitments under the Paris Agreement. Thus, inalienability on the maintenance of economic and ecological indicators requires research ofquestion of the productivity of balance model productivity. The last is related to properties of technological matrix model. Changing the sectorial (brunch) structure of ecological and economic system is reflected in the data matrix coefficients influences on production volumes and requires development of algorithms of determination of decision without the decision of model equalizations.

Objectives

Implementation of the set obligations in relation to unexceeding of quotas on the extras of greenhouse gases imposes certain limits on the basic economic indicators of productive activity. As there is dependence between the volumes of gross output and volumes of emissions of CO2, first of all, it concerns the gross output of products, the volume of investments, the final product, their optimal distribution in the system of national wealth. Following the general principle of division of economic researches into micro, meso and macro levels, we will consider the production functioning in the cut of existent industries, that conditioned by complication and manyfactoriness of tasks of reduction of greenhouse gas emissions in the national economy.

As the content of the Paris agreement has ecological and economic character, the realization of its positions requires the use of a multidisciplinary approach. As an effective tool of research in this case the balance ecological and economic model "input-output" [2] can be examined. It has a special role in solving the fundamental problems of the perspective planning taking into account nature using, namely the value of study costs of environmental protection taking into account the socio-economic effect and their distribution in a territorial-branch cut.

The first balance model that covers the relationship of the economy and the environment, was proposed by V.Leontyev D.Ford [3]. It generalizes the classical scheme of inter-branch balance and includes two groups of industries: basic production (material production industry) and auxiliary production (industry to eliminate contaminants). The basic conditions of the model are expressed by the system of equalizations:

$$\begin{aligned} x_1 &= A_{11}x_1 + A_{12}x_2 + y_1, \\ x_2 &= A_{21}x_1 + A_{22}x_2 - y_2. \end{aligned} \tag{1}$$

In the system (1) $x_1 = \left(x_1^1, x_2^1, \dots, x_n^1\right)^T$ – vector-column of production volumes;

$x_2 = \left(x_1^2, x_2^2, \dots, x_m^2\right)^T$ – a vector-column of destroyed pollutants volumes;

$y_1 = \left(y_1^1, y_2^1, \dots, y_n^1\right)^T$ – a vector-column of final production volumes;

$y_2 = \left(y_1^2, y_2^2, \dots, y_m^2\right)^T$ – a vector-column of undestroyed pollution volumes;

$A_{11} = \left(a_{ij}^{11}\right)_1^n$ – a square matrix coefficients of direct costs of production i per unit of production j;

$A_{12} = \left(a_{ig}^{12}\right)_{i,g=1}^{n,m}$ – a rectangular matrix of production costs i per unit of destroying pollutants g;

$A_{21} = \left(a_{kj}^{21}\right)_{k,j=1}^{m,n}$ – a rectangular matrix of pollutants producing k per unit of output j;

$A_{22} = \left(a_{kg}^{22}\right)_{1}^{m}$ – a square matrix of pollutants producing k per unit of destroying pollutants g.

It is unobviously assumed in the system (1), that coefficients $a_{ij}^{11} \geq 0$, $a_{ig}^{12} \geq 0$, $a_{kj}^{21} \geq 0$, $a_{kg}^{22} \geq 0$ distribute on all types of productiveactivity (material production and elimination of pollutants) the hypotheses of basic model of inter-branch balance: the amount of technological methods equals the amount of types of products and in every technological method produces only one product type. In the future we will consider matrixes A_{11}, A_{12}, A_{21}, A_{22} inalienable: $A_{11} \geq 0$, $A_{12} \geq 0$, $A_{21} \geq 0$, $A_{22} \geq 0$. The economic content of the Leontiev-Ford model requires that all its variables were inalienable, that is, $x_i^1 \geq 0$, $x_k^2 \geq 0$, $y_i^1 \geq 0$, $y_k^2 \geq 0$.

Research results

Let's put the problem on the basis of the above balance circuit "input-output" considering the costs of complying with restrictions on the Paris agreement. Solving this problem involves solving of the whole complex of the modern science fundamental problems. It includes, for example, the development of reliable methods for predicting environmental parameters and criteria of quality that can provide a quantitative measuring of the degree of the human needs satisfaction in a cleanness and natural variety; creation of scientifically reasonable methodology of determination of economic loss is from the pollution of environment; construction of the system of models of cooperation of different components of natural complexes taking into account natural and anthropogenic factors and conditions.

It is suggested to take into account the costs on implementation of emission limitations of greenhouse gases in the structure of industries of basic production in the form:

$$\begin{cases} x_1 = A_{11}x_1 + A_{12}x_2 + Cy_2 + y_1, \\ x_2 = A_{21}x_1 + A_{22}x_2 - y_2, \end{cases} \tag{2}$$

here Cy_2 – the costs of greenhouse gas emissions (that is maintenance costs of greenhouse gas emissions, including a payment for allowances);

$C = \left(c_{ig}^{12}\right)_{i,g=1}^{n,m}$ – rectangular matrix of production costs i per unit of pollutant emissions g;

In vector-matrix form model (2) can be represented as:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} E_1 & C \\ 0 & -E_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix},$$

here E_1 and E_2 – are corresponding single diagonal matrixes.

The first equalization of the offered model represents the economic balance – the distribution of the branch grossproducing of products on the productive consumption of basic and auxiliary productions, final consumption ofbasic production and charges related to fulfiling commitment on the Paris agreement. The second equalization represents physical balance ofgreenhouse gases, as a sum of the emissions caused byactivity of basic and auxiliary productions, and their undestroyed volumes.

The economic content of the variables of the model (2) requires the consideration of their inalienable values. The latter is closely connected with the question of balance model production to talk about the actual functioning of the production system that can provide intermediate consumption, positive volume of the final product and implementation of the set limitations from the extras of greenhouse gases.

In of order to study the issue of solutions express inseparable x_2 from the second equalization and substitute it in the first place:

$$x_1 = (E_1 - A_1)^{-1}\left(y_1 + Cy_2 - A_{12}(E_2 - A_{22})^{-1} y_2\right),$$

here $A_1 = A_{11} + A_{12}(E_2 - A_{22})^{-1} A_{21}$ – a square matrix of n - order.

Let's also express x_1 from the first equalization and substitute it into the second equalization:

$$x_2 = (E_2 - A_2)^{-1}\left(A_{21}(E_1 - A_{11})^{-1} y_1 + A_{21}(E_1 - A_{11})^{-1} Cy_2 - y_2\right),$$

here $A_2 = A_{22} + A_{21}(E_1 - A_{11})^{-1} A_{12}$ – a square matrix of m - order.

Thus, the formal solution of system (2) can be written as:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} (E_1 - A_1)^{-1} & (E_1 - A_1)^{-1}\left(A_{12}(E_2 - A_{22})^{-1} - C\right) \\ (E_2 - A_2)^{-1} A_{21}(E_1 - A_{11})^{-1} & (E_2 - A_2)^{-1}\left(E_2 - A_{21}(E_1 - A_{11})^{-1} C\right) \end{pmatrix} \begin{pmatrix} y_1 \\ -y_2 \end{pmatrix}.$$

According to the methodology proposed in [3] we can generalize the concept of productivity in the case of block matrix with the inalienable elements:

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \geq 0. \tag{3}$$

Let's consider an inalienable block matrix productive, if productive are matrixes A_{11}, A_{12}, A_1 and A_2. The productivity of matrixes A_1 and A_2 means profitability of basic and auxiliary productions after the complete cycle of production of goods and after the complete cycle of greenhouse gases elimination. If matrixes A_{11}, A_{12}, A_1 та A_2 – are productive, then matrixes

$$(E_1 - A_{11})^{-1} \geq 0,\ (E_2 - A_{22})^{-1} \geq 0,\ (E_1 - A_1)^{-1} \geq 0,\ (E_2 - A_2)^{-1} \geq 0$$

exist and have inalienable elements.

The productivity of the block matrix (3) does not guarantee inalienability of solutions of the system (2). Let's analyze the got expressions for x_1 and x_2. From the system (2) we get

$$x_1 = (E_1 - A_{11})^{-1}(A_{12}x_2 + Cy_2 + y_1).$$

From this equalization it appears, that when $x_2 \geq 0$, $y_1 \geq 0$, $y_2 \geq 0$ the condition $x_1 \geq 0$ is completed

Thus, a necessary and sufficient condition of inalienability of the model solutions (2) when the block matrix (3) is productive and when $y_1 \geq 0$, $y_2 \geq 0$ the condition $x_2 \geq 0$, will be completed, that is

$$(E_2 - A_2)^{-1}\left(A_{21}(E_1 - A_{11})^{-1} y_1 + A_{21}(E_1 - A_{11})^{-1} Cy_2 - y_2\right) \geq 0.$$

From the last inequality we get the sufficient condition of existence of inalienable solutions:

$$A_{21}(E_1 - A_{11})^{-1}(y_1 + Cy_2) \geq y_2,$$

that can be replaced yet by more hard sufficient condition:

$$A_{21}(y_1 + Cy_2) \geq y_2.$$

The last inequality means that a sufficient operating condition of basic and auxiliary productions is unexceeding of volume of unutilized of greenhouse gas emissions above complete emissions of greenhouse gases that arise up at the production of final product and costs sent to the service obligations according to the Paris Agreement.

Another actual scientific problem is that the basis of the proposed model (2) is a schematic diagram of inter-branch balance, the first quadrant of which are inter-branch streams that are built to the functional and structural industry connections. A branch plays a crucial role in providing progressive structural changes in the economic system, accelerating development and improving the technical level of production. It creates socio-economic background of deep transformations in the field of labour, service and householding. The interbranch economy relations are multilateral, the scale of their development determine the volume of trade and its structure. Features and change of the branch complexes operating conditions determine the necessity of their taking into account during the planning and forecasting calculations of gross branch production, of the basic and auxiliary productions, of the final product, of the volume of greenhouse gas emission reductions, of the interbranch ecological and economic connections.

The structural change of the indicated indexes is determined by the elements of technological matrixes of model (3) that stipulates the necessity of development of the estimation algorithms of the change influence in matrix structure system on the solution of the system of equalizations.

The model (2) can be also represented as:

$$Au = C. \tag{4}$$

Where

$$A = \begin{pmatrix} E_1 - A_{11} & -A_{12} \\ -A_{21} & E_2 - A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad u = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - (n+m)\text{-dimensional vector,}$$

$$C = \begin{pmatrix} E_1 & C \\ 0 & -E_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \ E_1,\ E_2 - \text{corresponding dimension block unit matrixes,} \tag{5}$$

0 – block zero matrix.

Let us also consider the system perturbations (in matrixes A_{11}, A_{12}, A_{21}, A_{22}, C elements) to linear algebraic equations system (5):

$$\bar{A}u = \bar{C}. \tag{6}$$

Where A, C - are corresponding perturbed matrix. Let us suppose that for the system (4) the basic solution and inverse matrix were found. Then there is the following theorem [Kudin, 2007].

Theorem 1. There are the following ratio for vectors normal restrictions expansion coefficients on matrix basic lines, inverse matrices elements, basic solutions and restriction residuals in two related basic solutions:

$$\bar{\alpha}_{rk} = \frac{\alpha_{rk}}{\alpha_{lk}}, \ \bar{\alpha}_{ri} = \alpha_{ri} - \frac{\alpha_{rk}}{\alpha_{lk}}\alpha_{li}, \ r = \overline{1, n+m}, \ i = \overline{1, n+m}, \ i \neq k. \tag{7}$$

$$\bar{e}_{rk} = \frac{e_{rk}}{\alpha_{lk}}, \ \bar{e}_{ri} = e_{ri} - \frac{e_{rk}}{\alpha_{lk}}\alpha_{li}, \ r = \overline{1, n+m}, \ i = \overline{1, n+m}, \ i \neq k. \tag{8}$$

$$\bar{u}_{0j} = u_{0j} - \frac{e_{jk}}{\alpha_{lk}}\Delta_l, \ j = \overline{1, n+m}. \tag{9}$$

$$\overline{\Delta}_k = -\frac{\Delta_l}{\alpha_{lk}},\ \overline{\Delta}_r = \Delta_r - \frac{\alpha_{rk}}{\alpha_{lk}}\Delta_l,\ r = \overline{1, n+m},\ r \neq k. \tag{10}$$

The matrix condition for being basis when entering the normal vector a_l restrictions $a_l u \leq c_l$ for k-y basic matrix position A is the inequality fulfillment: $\alpha_{lk} \neq 0$.

Based on the reduced ratio we can build algorithmic scheme of study (6) (when the model has changes). The algorithm will be based on simplex method ideology [Voloshyn, Kudin, 2015], involving some iterative process features. In particular, the transition from the system (4) to the system (6) will be carried consecutively by relevant perturbed rows ($i, i+1, i+2, \ldots, i+i_0$) replacement.

This means that the normal vectors hyperplanes that form the basis matrix lines and the corresponding inverse matrix will be replaced by appropriate “perturbed” normal vectors. Following basic solutions and inverse matrixes will be recalculated based on simplex relations (7)-(10). While maintaining the basic properties on replacement iterations, system (6) solution would be found by i_0 iterations. The result is a new base solution and the inverse matrix.

During the realization of iterations to clarify the elements the models can have some changes and clarifications not onlyseparate elements and lines of matrix of limitations, but also columns. The above formulas do not clearly show the influence of such changes on solutions (6). Let’s study the impact of changes k column of the matrix restrictions A as $\overline{A}_k = A_k + A_k^{'}$ to the solution u_0, where $A_k = (a_{1k}, a_{2k}, \ldots, a_{mk})^T$, $A_k^{'} = (a_{1k}^{'}, a_{2k}^{'}, \ldots, a_{mk}^{'})^T$, that is in such a form A is replaced by $\overline{A}$.

On basis of (6) let us build the next auxiliary linear system:

$$\max C^T u, \tag{11}$$

$$Au = C \tag{12}$$

$$u \geq 0 \tag{13}$$

$$\min C^T x, \tag{14}$$

$$A^T x > C \tag{15}$$

By the structure it is the dual pair of linear programming with some peculiarities of the vector coinciding - normals of objective function and vector type, the matrix of restrictions tasks A, A^T - are square. Let’s consider that for (12) $u_0,\ A = A_b,\ A_b^{-1}$ - are known - the basic solution, basic line and inverse matrixes, and a column A_k is a subject of indignation in the next kind $\overline{A}_k = A_k + A_k^{'}$ (accordingly $\overline{A}_k^T = A_k^T + A_k^{'T}$ - is an indignant row k of the matrix A^T). It should be noted that for (15) $A^T = A_b^T$, $\left(A_b^T\right)^{-1}$ - accordingly basic and inverse basic matrixes in the context of formulas (11) -(13). Let us draw a connection according to formulas (11) - (13) elements basic matrixes method in two contiguous basic matrixes the research values of component vector normal C^T decomposition of the objective function (14) by the substitution in the basic matrix A^T line k by a line $\overline{A}_k^T = A_k^T + A_k^{'T}$ and the conditions of nondegenerate preservation of matrix limits. Let's also establish their links with the problem (11) - (13). Where as, $A_b \times A_b^{-1} = E,\ \left(A_b^T\right)^{-1} \times A_b^T = E$, for the expansion component of the vector-line C^T we can write down

$$C^T = L_0 \times A_b^T,\ L_0 = C^T \times \left(A_b^T\right)^{-1} = A_b^{-1} \times C\ , \tag{16}$$

here $L_0 = (L_{01}, L_{02}, ..., L_{0m})$ - is a vector of separation of the vector C^T according to the lines of the basic matrix A_b^T . It is not hard to persuade, that $A_b^{-1} \times C$ coinsides with u_0 , that is $u_0 = A_b^{-1} \times C$ for (12) (or (6)).

Similarly, while considering the indignant matrix $\overline{A}_b^T$ ($\overline{A}^T = \overline{A}_b^T$, $\left(\overline{A}_b^T\right)^{-1}$), that is constructed by the substitution of the line k of matrix A^T by the line $\overline{A}_k^T = A_k^T + A_k^{'T}$ the separation of vector C^T will look like

$$C^T = \overline{L}_0 \times \overline{A}_b^T,\ \overline{L}_0 = C^T \times \left(\overline{A}_b^T\right)^{-1} = \overline{A}_b^{-1} \times C\ , \tag{17}$$

where $\overline{L}_0 = (\overline{L}_{01}, \overline{L}_{02}, ..., \overline{L}_{0m})$ - is a vector of separation of the vector C^T according to the row of the basic matrix $\overline{A}_b^T$.

It is not hard to persuade, that $\overline{A}_b^{-1} \times C$ coinsides with $\overline{u}_0$, that is $\overline{u}_0 = \overline{A}_b^{-1} \times C$ for (12) (or (6)). Here the matrix $\overline{A}_b^{-1}$ is inversed to $\overline{A}_b = \overline{A}$, that is inversed to the matrix A , in which the substitution of k -column in matrix limits A_k by the column $\overline{A}_k = A_k + A_k^{'}$ is made. According to the positions of the basic matrix method (to the problem (14) - (15)) the connection is a component of the separation vector of normal vector C^T (L_0 and $\overline{L}_0$) at two contiguous basic matrixes (different by the k - row) is given in the co-relation

$$\overline{L}_{0i} = L_{0i} - \frac{L_{0k}}{\overline{L}_{kk}} \times \overline{L}_{ki},\ i \neq k \tag{18}$$

$$\overline{L}_{0k} = \frac{L_{0k}}{\overline{L}_{kk}},\ \ i = k \tag{19}$$

where for (14)-(15)

$$\overline{L}_{ki} = \overline{A}_k^T \times (A_b^T)_i^{-1}$$
$$\overline{L}_{ki} = (A_k^T + A_k^{''T}) \times (A_b^T)_i^{-1} = A_k^T \times (A_b^T)_i^{-1} + A_k^{''T} \times (A_b^T)_i^{-1}$$
$$\overline{L}_{ki} = L_{ki} + L_{ki}^{'} = 0 + L_{ki}^{'}$$

$$\overline{L}_{kk} = \overline{A}_k^T \times (A_b^T)_k^{-1} = \overline{A}_k^T \times (A_b^T)_k^{-1}$$
$$\overline{L}_{kk} = (A_k^T + A_k^{''T}) \times (A_b^T)_k^{-1} = A_k^T \times (A_b^T)_k^{-1} + A_k^{''T} \times (A_b^T)_k^{-1}$$
$$\overline{L}_{kk} = L_{kk} + L_{kk}^{'} = 1 + L_{kk}^{'}$$

accordingly, for (11)-(13)

$$\overline{L}_{ki} = \overline{A}_k^T \times (A_b^T)_i^{-1} = (A_b)_i^{-1} \times \overline{A}_k$$
$$\overline{L}_{ki} = (A_k^T + A_k^{''T}) \times (A_b^T)_i^{-1} = (A_b)_i^{-1} \times (A_k + A_k^{''})$$
$$\overline{L}_{ki} = (A_b)_i^{-1} \times (A_k + A_k^{''}) = (A_b)_i^{-1} \times A_k + (A_b)_i^{-1} \times A_k^{''} =$$
$$\overline{L}_{ki} = L_{ki} + L_{ki}^{'} = 0 + L_{ki}^{'}$$

$$\overline{L}_{kk} = \overline{A}_k^T \times (A_b^T)_k^{-1} = (A_b)_k^{-1} \times \overline{A}_k$$
$$\overline{L}_{kk} = (A_k^T + A_k^{''T}) \times (A_b^T)_k^{-1} = (A_b)_k^{-1} \times (A_k + A_k^{''})$$
$$\overline{L}_{kk} = (A_b)_k^{-1} \times (A_k + A_k^{''}) = (A_b)_k^{-1} \times A_k + (A_b)_k^{-1} \times A_k^{''} =$$
$$\overline{L}_{kk} = L_{kk} + L_{ki}^{'} = 1 + L_{ki}^{'}$$

here $(A_b^T)_i^{-1}$ - a column i of the matrix $(A_b^T)^{-1}$, $(A_b)_k^{-1}$ - a row k of the matrix $(A_b)^{-1}$ coinside with the coincide with the components of the basic solutions u_0 and $\overline{u}_0$ problems (11)-(13) with the substitution of k -column in matrix limits A_k by the column $\overline{A}_k = A_k + A_k^{'}$.

Statement 1. The evolution of solutions (6) at the indignation of the matrix columns in $\overline{A}_k = A_k + A_k^{'}$ coincides with the coefficients changes of the normal vector C^T separation(14) by changing the row k of the matrix A_b^T in $\overline{A}_k^T = A_k^T + A_k^{'T}$ according to the scheme of the basic matrixes method.

Statement 2. Each component connection of solutions vectors (4) u_0 and $\overline{u}_0$ (6) during the substitution of k -column in matrix limits A_k by the column $\overline{A}_k = A_k + A_k^{'}$ is described in the co-relation

$$\overline{u}_{0k} = \frac{u_{0k}}{1 + (A_b^{-1})_k \times A_k^{'}}, \ i = k \quad (20)$$

$$\overline{u}_{0i} = u_{0i} - \frac{u_{0k}}{1 + (A_b^{-1})_k \times A_k^{'}} \times \left[(A_b^{-1})_i \times A_k^{'} \right], \ i \neq k \quad (21)$$

thus the condition of preserving nondegenerate of solution is implementation of condition $1 + (A_b^{-1})_k \times A_k^{'} \neq 0$.

Consequence 1. During realization of calculations it ispossible to apply more general formula of calculation of the components of the vector solution (6) as a result of indignation of vector of column of matrix limits (the condition of the statement 2) is the next

$$\bar{u}_0 = u_0 - \frac{u_{0k}}{1+(A_b^{-1})_k \times A_k^{'}} \times \left[(A_b^{-1}) \times A_k^{'} \right], \tag{23}$$

or

$$\bar{u}_0 = u_0 - \frac{u_{0k}}{\bar{L}_{kk}} \times L_k^{'} ,$$

here

$$\bar{L}_{kk} = 1 + (A_b^{-1})_k \times A_k^{'}, \quad L_k^{'} = A_b^{-1} \times A_k^{'}$$

In the statement 2 it is established, that $\bar{u}_{0i} = u_{0i} - \frac{u_{0k}}{1+(A_b^{-1})_k \times A_k^{'}} \times \left[(A_b^{-1})_i \times A_k^{'} \right],\ i \neq k$.

Let's make transformation of co-relation $\bar{u}_{0k} = \frac{u_{0k}}{1+(A_b^{-1})_k \times A_k^{'}},\ i = k$ to the view that is close to the previous

$$\bar{u}_{0k} = \frac{u_{0k}}{1+(A_b^{-1})_k \times A_k^{'}} = u_{0k} - u_{0k} + \frac{u_{0k}}{1+(A_b^{-1})_k \times A_k^{'}} = .$$

$$= u_{0k} - \left[u_{0k} - \frac{u_{0k}}{1+(A_b^{-1})_k \times A_k^{'}} \right] =$$

$$= u_{0k} - \left[\frac{u_{0k} + u_{0k} \times (A_b^{-1})_k \times A_k^{'} - u_{0k}}{1+(A_b^{-1})_k \times A_k^{'}} \right] =$$

$$= u_{0k} - \frac{u_{0k} \times (A_b^{-1})_k \times A_k^{'}}{1+(A_b^{-1})_k \times A_k^{'}} = u_{0k} - \frac{u_{0k}}{1+(A_b^{-1})_k \times A_k^{'}} \times (A_b^{-1})_k \times A_k^{'} .$$

From these we have got the next

$$\bar{u}_0 = u_0 - \frac{u_{0k}}{1+(A_b^{-1})_k \times A_k^{'}} \times \left[(A_b^{-1}) \times A_k \right] \quad .$$

The statement is brought out from the previously established correlations, that connect the problems (11)-(13) and (14)-(15), inparticular, $u_0 = L_0$, $\bar{u}_0 = \bar{L}_0$, formulas (7), connections of lines, inverse and transpose matrixes in the scheme of basic matrixes method.

The Remark. The structure of formula $\bar{u}_0 = u_0 - \frac{u_{0k}}{\bar{L}_{kk}} \times L_k^{'}$ indicates that this presentation of the line equalization in parametric form, where u_0 - is an initial vector, $L_k^{'}$ - is a normal vector, $-\frac{u_{0k}}{\bar{L}_{kk}}$ - is the meaning of parameter of displacement along the vector $L_k^{'}$ (from u_0). The value of vector component $L_k^{'}$ and $-\frac{u_{0k}}{\bar{L}_{kk}}$ is influenced by the size of indignation in the column with the number k.

Here are the basic steps of the algorithm of indignant solution component (6) - as a result of changes of the column system (4) A_k by the column $\bar{A}_k = A_k + A_k^{'}$

1. We have got a known vector $u_0 = (u_{01}, u_{02}, ..., u_{0m})^T$, A_b, A_b^{-1} - line and inverse basic matrix (4).

2. Let conduct the substitution of the k - column in the matrix limits A_k by the column $\bar{A}_k = A_k + A_k^{'}$.

We find the vector

$$L_k^{'} = (L_{k1}^{'}, L_{k2}^{'}, ..., L_{km}^{'}) = A_b^{-1} \times A_k^{'},\ L_{kk}^{'} = \left(A_b^{-1} \right)_k \times A_k^{'} \ ,$$

$$\bar{L}_{kk} = 1 + L_{kk}^{'} = 1 + \left(A_b^{-1} \right)_k \times A_k^{'} \ ,$$

here $\left(A_b^{-1} \right)_k$ - k -row matrix A_b^{-1}.

3. Let us make the new solutions $\bar{u}_0 = u_0 - \frac{u_{0k}}{\bar{L}_{kk}} \times L_k^{'}$ according to the formula (23).

The steps of the algorithm of indignant solution component (6) - as a result of changes of the column system (4) A_k by the column $\overline{A}_k$

1. We have got a known vector $u_0 = (u_{01}, u_{02}, ..., u_{0m})^T$, $A_b,\ A_b^{-1}$ - line and inverse basic matrix (4).

2. Let conduct the substitution of the k - column in the matrix limits A_k by the column $\overline{A}_k$.

We find the vector $\overline{L}_k = (L_{k1}, L'_{k2}, ..., L_{km}) = A_b^{-1} \times \overline{A}_k$.

3. We make a new solution based on (20) (21)

$$\overline{u}_{0k} = \frac{u_{0k}}{1 + (A_b^{-1})_k \times A'_k} = \frac{u_{0k}}{\overline{L}_{kk}},\ i = k$$

$$\overline{u}_{0i} = u_{0i} - \frac{u_{0k}}{1 + (A_b^{-1})_k \times A'_k} \times \left[(A_b^{-1})_i \times A'_k \right] = u_{0i} - \frac{u_{0k}}{\overline{L}_{kk}} \times \overline{L}_{ki},\ i \neq k$$

.

Let's illustrate the offered algorithm of the determination volumes of gross branch output in case of technological inter-branch changes on conditional data. Let the coefficients of technological matrixes of ecological-and economic model (3) have the following meanings:

$$A_{11} = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{pmatrix},\ A_{12} = \begin{pmatrix} 0.1 & 0.2 \\ 0.1 & 0.2 \end{pmatrix},\ A_{21} = \begin{pmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{pmatrix},\ A_{22} = \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.1 \end{pmatrix},$$

the matrix of charges on maintenance of greenhouse gases emissions and vectors of the branch final producing and limitation after the extrass of greenhousegases accordingly:

$$C = \begin{pmatrix} 0.3 & 0.2 \\ 0.1 & 0.5 \end{pmatrix},\ y_1 = \begin{pmatrix} 12 \\ 23 \end{pmatrix},\ y_2 = \begin{pmatrix} 5 \\ 8 \end{pmatrix}.$$

Let's check implementation of the productivity condition for ecological and economic system in the case of selected numerical data. The block matrix A

$$A = \begin{pmatrix} 0.2 & 0.1 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 & 0.3 \\ 0.2 & 0.3 & 0.3 & 0.1 \end{pmatrix}$$

obviously is productive after the sufficient condition of the productivity of technological matrixes of balance models of Leontiev type. Except that the considered higher sufficient condition of the productivity model(3) – inequality $A_{21}(y_1 + Cy_2) \geq y_2$:

$$\begin{pmatrix} 9.76 \\ 11.27 \end{pmatrix} \geq \begin{pmatrix} 5 \\ 8 \end{pmatrix}.$$

Let us pass to the step by step algorithm realization

1. Find the solution of the original system (4) and the block inverse matrix technology:

$$u_0 = \begin{pmatrix} 38.17 \\ 60.43 \\ 32.67 \\ 30.62 \end{pmatrix}, A = \begin{pmatrix} 0.2 & 0.1 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 & 0.3 \\ 0.2 & 0.3 & 0.3 & 0.1 \end{pmatrix},$$

$$A^{-1} = A_b^{-1} = \begin{pmatrix} 1.25 & 3.125 & -3.125 & 0.625 \\ -11.25 & 6.875 & 3.125 & -0.625 \\ 8.75 & -8.125 & -1.875 & 4.375 \\ 5.0 & -2.5 & 2.5 & -2.5 \end{pmatrix}.$$

2. We assume that the that indignation in a model(6) tests the third column ($k = 3$): let’s make the substitution of k -column in the matrix limit $A_3 = \begin{pmatrix} 0.1 \\ 0.1 \\ 0.2 \\ 0.3 \end{pmatrix}$ by the column $\overline{A}_3 = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.1 \\ 0.1 \end{pmatrix}$. Find the vector

$\overline{L}_k = (L_{k1}, L'_{k2}, ..., L_{km}) = A_b^{-1} \times \overline{A}_k$:

$$\overline{L}_3 = \begin{pmatrix} 1.25 & 3.125 & -3.125 & 0.625 \\ -11.25 & 6.875 & 3.125 & -0.625 \\ 8.75 & -8.125 & -1.875 & 4.375 \\ 5.0 & -2.5 & 2.5 & -2.5 \end{pmatrix} \cdot \begin{pmatrix} 0.2 \\ 0.2 \\ 0.1 \\ 0.1 \end{pmatrix} = \begin{pmatrix} 0.625 & -0.625 & 0.375 & 0.5 \end{pmatrix}$$

3. Let's find the solutions:

$$\overline{u}_1 = 38.17 - \frac{32.67}{0.375} \cdot 0.625 = -16.28$$

$$\overline{u}_2 = 60.43 - \frac{32.67}{0.375} \cdot (-0.625) = 114.88$$

$$\overline{u}_3 = \frac{32.67}{0.375} = 87.12$$

$$\overline{u}_4 = 30.62 - \frac{32.67}{0.375} \cdot 0.5 = -12.94$$

The got solutions coincide with the solutions got by the direct calculations what are easily seen and specify on a substantial change in functioning of auxiliary production, in particular, negative indexes require a change in the structure of technological matrixes of both basic and auxiliary spectrum of industries.

Conclusion

The necessity of taking into account of ecological factor in the modern system of further development of civilization causes the actuality of consideration of productive activity of society within a single socio-ecological-economic system. Thus, the important requirement of her existence is a necessity of the balanced interests each of the indicated subsystems. An effective instrument is a balance method and corresponding models worked out on its basis serve as for this purpose, model of taking into account of charges offered in particular in the article on realization of projects of reduction of greenhouse gases emissions. With the aim of its effective use the terms of the productivity are set and the algorithm of determination of volumes of the gross branch producing is offered in case ofchange of technological branch structure. Further researches should be performed towards the including of additional economic and ecological limitations, and change of classic original assumptions in relation to the technological structure of the offered model.

Bibliography

[KYOTO] KYOTO PROTOCOL TO THE UNITED NATIONS FRAMEWORK CONVENTION ON CLIMATE CHANGE http://unfccc.int/kyoto_protocol/items/2830.php

[Leontief, 1986] Leontief, Wassily W. Input-Output Economics. 2nd ed., New York: Oxford University Press, 1986.

[Lyashenko, 1999] Lyashenko I.M. Ekonomiko–matematychni metody ta modeli stalogo rozvytku. – K.: Vyshha shkola, 1999. – 236 s.(in Ukr.)

[Khrushch, 2007] Khrushch L. Z. Ekoloho-ekonomichni vyrobnychi funktsii (pobudova ta zastosuvannia u prognozuvanni) // Modeluvannia ta informaciini systemy v ekonomici. Zbirnyk naukovykh prac. – Vypusk 76. – Kiev, 2007. – S. 115-165. (In Ukr)

[Lyashenko, Khrushch, 2008] Lyashenko I.M., Khrushch L. Z. Uzahalnennia modeli "Vytraty-vypusk" Leontieva-Forda na mnozhynu tekhnolohichnykh sposobiv // Ekonomika: problemy teorii ta praktyky/ Zbirnyk naukovykh prac. – Vypusk 239. Tom 3. – Dnipropetrovsk: DNU, 2008. – № 4. – S. 240-244. (In Ukr)

[Khrushch, Korzhevska, 2012] Khrushch L. Z. , Korzhevska O.P. Rozshyrennia mizhhaluzevoi ekoloho-ekonomichnoi modeli Leontieva-Forda // Biznes inform. Naukovyi zhurnal. – № 3, 2012. – S. 75-78. (In Ukr)

[Kudin, 2007] Kudin V.I., Lyashko S.I., Hritonenko N.V., Yatsenko Yu.P. Analiz svoystv lineynoy sistemyi metodom psevdobazisnyih matrits // Kibernetika i sistemnyiy analiz. – 2007. – N 4. –.S. 119–127.

[Lyashenko, 2009] Lyashenko I.M., Onyshhenko A.M. Pryami ta dvoyisti balansovi modeli „vytraty-vypusk" // Ekonomichna kibernetyka. – 2009. – #1-2. – S. 14–18. (in Ukr.)

[Mechanisms] The Mechanisms under the Kyoto Protocol: Clean development mechanism, joint implementation and emissions trading http://unfccc.int/kyoto_protocol/mechanisms/items/1673.php

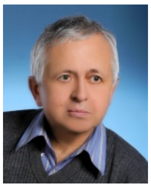
[Onyshhenko, 2011] Onyshhenko A. M., Lyashenko I.M. Metodologiya matematychnogo modelyuvannya ekonomiko–ekologichnoyi vzayemodiyi v umovax realizaciyi Kiotskogo protokolu // Ekonomichna kibernetyka. – 2011. – #4–6(70-72) – S. 17-26. (in Ukr.)

[Voloshin, 2010] Voloshyn A., Maschenko S. Individually optimal principles of distribution of greenhouse gas emission quatas // "Information Models of Knowledge", ITHEA, Kiev-Sofia, 2010, 470 p. - P. 209-214.

[Voloshin, Kudin, 2013] Voloshin O.F., Gnatienko G.M., Kudin V.I. Poslidovniy analiz variantiv: Technologiy ta zastosuvannya // Monografiya.-K.: Stilos.-2013,-304s (in Ukr.)

[Voloshin, Kudin, 2015] Voloshin O.F., Kudin V.I. Poslidovniy analiz variantov v zadachach isledovaniya i proektirovaniya system // Monografiya.-K.: Izdatelsko-poligraficheskiy centr "Kievskiy universitet" - Naukova dumka, - 2015. - 351s. (in Rus.)

Authors Information



Oleksii Voloshyn *- Dr. Sc., Professor, Kyiv National Taras Shevchenko University, Faculty of Computer Science and Cybernetics ; e-mail: olvoloshyn@ukr.net*

Research interests*: decision-making theory, decision support systems, information technology, expert systems, mathematical economics, fuzzy analysis, e-learning*

Vladimir Kudin *- Dr. Sc., Professor, Kyiv National Taras Shevchenko University; Faculty of Information Technology; e-mail: V_I_Kudin@mail.ru*

Research interests*: optimization techniques, systems analysis, computational method, decision-making, mathematical economics*

Andrii Onyshchenko *– Dr. Sc., Professor, Kyiv National Taras Shevchenko University; Faculty of Information Technology; e-mail: onyshchenko@yandex.ru*

Research interests*: mathematical economics, eco-economic modeling, optimal control, data maining*

Lesya Khrushch *-* ***Dr.****, Associate Professor of Information Technology Department, Vasyl Stefanyk Precarpathian National University, IvanoFrankivs'k,Ukraine; Faculty of Mathematics and Informatics; Lessja2501@gmail.com, lesia.khrushch@pu.if.ua*

Research interests*: mathematical economics, optimization techniques, information technology,* web programming