THE INVERSE RANKING PROBLEM AND THE ALGORITHM FOR SOLVING IT

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Abstract: *After solving the classical problem of ranking of alternatives for the aggregate of criteria may occur another problem: what the minimum values of changes could improve the ratings of the selected alternative. The desired result of solving this problem is determination of minimum deviation of the initial criterial values which can allow to get the rating which is not below than the pre-set value. In this article the inverse problem of ranking alternatives, which is formalized in the class of discrete programming models, has been considered. The algorithm for solution this problem, based on the ideology of the method of dynamic programming, has been proposed.*

Keywords: *decision-making, ranking alternatives, multi-criteria optimization, discrete programming, dynamic programming*

ACM Classification Keywords: *H. Information Systems*

Introduction

Among tasks of the theory of decision-making, which often occur in practice, tasks of selecting (ranking) alternatives are actual [Ishizaka, 2013; Odu, Charles-Owaba, 2013; Emel'janov, 1985; Larichev, 2003]. Mathematically, these tasks are described by a set of alternatives $x \in X = \{A_1, ..., A_n\}$, for each of which the values of m indicator are given. The solution of this task is considered to be the alternative, which has the best (in aggregate) value of the criteria, which differ in different importance (weighting coefficients) [Emel'janov, 1985; Larichev, 2003]

In practice, there may be a task that is reversed to the specified one, when it is necessary to determine the values of the indicators for the chosen alternative, in which it would receive a given place in the overall rating, and the deviation of the obtained values from the initial ones would be optimal (the minimum of possible). A similar task may be actual when analyzing the results of a rating assessment of the development of certain objects (entities), for example, ranking regions of the country in terms of economic, innovation, industrial activities. In this case, we can talk about developing reasoned recommendations for those ranking participants who failed to occupy the places.

To date, a significant number of methods have been developed for solving the direct task of ranking alternatives in multiple criteria [Ishizaka, 2013; Odu, Charles-Owaba, 2013; Emel'janov, 1985; Larichev, 2003]. The most investigated and commonly used are the analytic hierarchy process (AHP) [Saati, 1993], which is actively used to solve applied problems of decision-making [Baby, 2013; Yadav, Anis, Ali, Tuladhar, 2015; Kutlu et al, 2014].

However, the literature deals with selection problems, which characterized by some specificity: multiple choice alternatives, models with fuzzy relation of advantages, decision-making under uncertainty. [Mustakerov, Borissova, Bantuto, 2012; Borzecka, 2012].

Problem statement

In the general case, the criterion can be considered a certain function $(f_j(x), j \in J = \{1 \dots m\})$, defined on the set of alternatives. The values of this function be-long either to a predefined set or are calculated in accordance with certain mathematical rules. In the first case, variants are possible: the set of values is given by a point scale, linguistic scale [Saati, 1993] or in the form of a numerical interval $[f_j^{min}, f_j^{max}]$, which is formed from all possible values of the function (from minimum to maximum), taking into account the accuracy of its calculation. An example of the second case is the synthesis of local priorities in the AHP [Saati, 1993]. So, we can assume that the value of the j-th criterion is always a countable set, designate it as Q_j. The best one is the result that corresponds to the maximum or minimum value of the function $f_j(x)$, $j \in J$, depending on the direction of optimization of the criterion. Let J_1 and J_2 be the sets of indexes of criteria that are respectively maximized and minimized ($J_1 \cup J_2 = J$). Next, we assume that the values of each function $f_j(x)$, $j \in J$ belong to a common numerical interval $[q_{min}; q_{max}] \subset R$ of the set of real numbers. Otherwise, it is not difficult to construct a corresponding mutually unambiguous trans-formation of the initial values $f_j(x)$, $j \in J$ in a similar interval.

In general, when considering multi-criteria problems, a vector $W = (\omega_1, \omega_2, ..., \omega_m)$ is introduced. Each component of this vector characterizes the importance of the j-th criterion, and $\sum_{j=1}^{m} w_j = 1$, $w_j > 0$ [Emel'janov, 1985; Larichev, 2003; Kini, Rajfa, 1981].

The task of ranking alternatives $x \in X = \{A_1, ..., A_n\}$ in multiple criteria $f(x) = (f_1(x), \dots, f_m(x))$ is to establish a certain order

$$A_{i_1} \succ A_{i_2} \succ \dots \succ A_{i_n} \qquad (1)$$

on the basis of computing the values of a generalized significance $G(X)$ for each element of the set:

$$G(x) = G(f(x), W) = G((f_1(x), \dots, f_m(x)), (\omega_1, ..., \omega_m)),\ x \in X \qquad (2)$$

where the values $G(A_i)$ are calculated according to a certain rule (algorithm), which is determined by the mathematical method, which used in each particular case, and

$$G(A_{i_1}) \geq G(A_{i_2}) \geq \ldots G(A_{i_n}). \tag{3}$$

Thus, in decision-making theory, the most known and widespread is the method of an ideal point, a linear convolution, a power-additive convolution, and some others [Lotov, Pospelova, 2008; Makarov et al., 1982; Chernoruckij, 2005; Shtojer, 1992].

In the task of ranking alternatives, the alternative A_{i_1} considered best, which in order (1) takes the first place, respectively, the worst alternative is A_{i_n}. Next we will say that the alternative in the order (1) is at k-th place. Hence, the alternative A_{i_k}, $k = \overline{1,n}$ in the order (1) is at *k*-th place.

However, in practice, after solving the problem (1) - (3) for another alternative A' might be necessary to analyze the place k' that it occupied in the order (1). Such type of analysis can be a research: "At which deviations from the existing values $f_j(A')$, $j \in J$ of the alternative A' would it take another, predetermined and different place?" A concrete illustrative example may be the problem of developing an individual plan for training a reserve sports team player in order to he would be able to get into the first team in the near future. It is about what exactly the indicators that are analyzed in the process of training should be given the most attention. Another example is to argue and to point out the shortcomings of a particular project (any sphere of human activity) that participated in a competition for a grant or funding allocation, etc.

Suppose, after the solving of problem (1) - (3), it has become necessary that the alternative A' occupies a certain place in order (1), not lower than p $(p < k')$, and the decision maker is empowered to determine the subset of the criteria $J' \in J$ for which the values $f_j(A')$, $j \in J$ are allowed to change. The set of vector-parameters $\Theta = (\Theta_1 \times \ldots \times \Theta_m,\ \Theta \subset R^m)$ is defined in the following way:

$$\Theta_j = \bigcup_{q_j \in Q_j} (q_j - f_j(A')), j \in J', \quad \Theta_j = \{0\}, j \in J \setminus J'$$

where each set $\Theta_j, j \in J'$ represents all possible deviations of values $q_j \in Q_j$ from $f_j(A')$. The mathematical model of the problem under consideration will look like this:

$$\begin{gathered} H(A', \theta, p, W) \to \min \\ H(A', \theta, p, W) = G(f(A', \theta), W) - G(f(A_{i_p}), W) \end{gathered} \tag{4}$$

$$G(f(A', \theta), W) > G(f(A_{i_p}), W), \tag{5}$$

$$\theta = (\theta_1 \times \ldots \times \theta_m) \in \Theta = (\Theta_1 \times \ldots \times \Theta_m,\ \Theta \subset R^m), \tag{6}$$

where $f(A', \theta) = (f_1(A') + \theta_1, f_2(A') + \theta_2, \ldots, f_m(A') + \theta_m),\ \theta \in \Theta.$

If for solving of problems (1) - (3), (4) - (6) uses a linear convolution as a generalized criterion, then the function $G(\cdot)$ in (2) will have the form:

$$G(x) = G(f(x), W) = \sum_{j \in J_1} w_j(f_j(x)) - \sum_{j \in J_2} w_j(f_j(x)),$$

And the function $H(\cdot)$ in (4) –

$$H(A', \theta, p, W) = \left(\sum_{j \in J_1} w_j(f_j(A') + \theta_j) - \sum_{j \in J_2} w_j(f_j(A') + \theta_j) \right) - \left(\sum_{j \in J_1} w_j(f_j(A_{i_p})) - \sum_{j \in J_2} w_j(f_j(A_{i_p})) \right) =$$

$$= \sum_{j \in J_1} w_j(f_j(A') + \theta_j - f_j(A_{i_p})) - \sum_{j \in J_2} w_j(f_j(A') + \theta_j - f_j(A_{i_p})) \qquad (7)$$

For power-additive convolution:

$$G(x) = G(f(x), W) = \sum_{j \in J_1} (f_j(x))^{w_j} - \sum_{j \in J_2} (f_j(x))^{w_j},$$

$$H(A', \theta, p, W) = \left(\sum_{j \in J_1} (f_j(A') + \theta_j)^{w_j} - \sum_{j \in J_2} (f_j(A') + \theta_j)^{w_j} \right) - \left(\sum_{j \in J_1} (f_j(A_{i_p}))^{w_j} - \sum_{j \in J_2} (f_j(A_{i_p}))^{w_j} \right) =$$

$$= \sum_{j \in J_1} w_j((f_j(A') + \theta_j)^{w_j} - (f_j(A_{i_p}))^{w_j}) - \sum_{j \in J_2} ((f_j(A') + \theta_j)_{w_j} - (f_j(A_{i_p}))^{w_j})$$

If the task (1) - (3) is solved by the method of the ideal point, and A^* – the ideal alternative (point), then

$$G(x) = G(f(x), W) = \left(\sum_{j \in J} w_j(f_j(x) - f_j(A^*))^2 \right)^{\frac{1}{2}},$$

However, in this form, the function $G(x)$ does not meet the requirements (3), since for this case the best will be not bigger but smaller values $G(x)$ (the distance to the ideal point is minimized), therefore, for correct use of the model (1) - (3) $G(x)$ could be modify in following way:

$$G(x) = \tilde{G} - \left(\sum_{j \in J} w_j(f_j(x) - f_j(A^*))^2 \right)^{\frac{1}{2}},$$

where $\tilde{G} = \left(\sum_{j \in J_1} w_j(\min_{x \in X} f_j(x) - f_j(A^*))^2 + \sum_{j \in J_2} w_j(\max_{x \in X} f_j(x) - f_j(A^*))^2 \right)^{\frac{1}{2}},$

$\tilde{G}$ – the distance between the ideal worst and ideal best alternatives. Then

$$H(A', \theta, p, W) = G(f(A', \theta), W) - G(f(A_{i_p}), W) = \left(\sum_{j \in J} w_j(f_j(A_{i_p}) - f_j(A^*))^2 \right)^{\frac{1}{2}} - \left(\sum_{j \in J} w_j(f_j(A') - f_j(A^*))^2 \right)^{\frac{1}{2}}$$

Algorithm

The algorithm for solving the problem (4) - (6) for the case of application of the linear convolution of the criteria in (2) is consider in detail. The cost function (7) can be rewrited int the form:

$$H(A',\theta,p,W)=\sum_{j\in J_1}w_j\theta_j-\sum_{j\in J_2}w_j\theta_j+\sum_{j\in J_1}w_j(f_j(A')-f_j(A_{i_p}))-\sum_{j\in J_2}w_j(f_j(A')-f_j(A_{i_p}))=\sum_{j\in J_1}w_j\theta_j-\sum_{j\in J_2}w_j\theta_j-$$

$$-\left(\sum_{j\in J_1}w_j(f_j(A_{i_p}))-\sum_{j\in J_2}w_j(f_j(A_{i_p}))\right)+$$

$$+\left(\sum_{j\in J_1}w_j(f_j(A'))-\sum_{j\in J_2}w_j(f_j(A'))\right)=\sum_{j\in J_1}w_j\theta_j-\sum_{j\in J_2}w_j\theta_j-\left(G(A_{i_p})-G(A')\right)=$$

$$=h(\theta_1,\theta_2,\dots,\theta_m)-G^* \tag{8}$$

In the resulting formula

$$h(\theta_1,\theta_2,\dots,\theta_m)=\sum_{j\in J_1}w_j\theta_j-\sum_{j\in J_2}w_j\theta_j\ ,$$

and G^* is the constant value, which sets the initial advantage of the alternative A_{i_p} over A', so $G^*=G(A_{i_p})-G(A')$. According to (3) G^* is not a negative number.

Let $\theta_j\in\{\theta_j^1,\theta_j^2,\dots,\theta_j^{r_j}\}\subseteq\Theta_j$, where r_j – Number of deviations given by decision-maker, and $\theta_j^1,\theta_j^2,\dots,\theta_j^{r_j}$ is a numerical sequence, and for $j\in J_1$ $0<\theta_j^1<\theta_j^2<\dots<\theta_j^{r_j}$; $j\in J_2$ $0>\theta_j^1>\theta_j^2>\dots>\theta_j^{r_j}$

.

The purpose of the algorithm is to find such a vector $\theta=(\theta_1\times\dots\times\theta_m)$, which minimizes $h(\theta_1,\theta_2,\dots,\theta_m)$ among all possible sets θ for which (8) the positive value. For the minimized criteria, the elements of the set $\{\theta_j^1,\theta_j^2,\dots,\theta_j^{r_j}\}$ are redefined as follows: $\theta_j^1:=-\theta_j^1,\theta_j^2:=-\theta_j^2,\dots,\theta_j^{r_j}:=-\theta_j^{r_j}$, where "$:=$" is considered as a operation of the reassignment of the values. Then (8) can be rewritten in the form

$$H(A',\theta,p,W)=h(\theta_1,\theta_2,\dots,\theta_m)-G^*=H(A',\theta,p,W)=\sum_{j=1}^{m}w_j\theta_j-G^* \tag{9}$$

The algorithm is based on the ideology of the method of dynamic programming [Bellman, 1960]. In accordance with it, a recurrence relation is determined, according to which the initial problem is reduced to problems of smaller dimension. Hence, $\theta^k=(\theta_1\times\dots\times\theta_k)$, the vector of deviations for the first *k* criteria, $k\in\{1,2,\dots,m\}$. Consequently

$$H(A',\theta^m,p,W)=\sum_{j=1}^{m}w_j\theta_j-\left(G(A_{i_p})-G(A')\right)$$

$$H(A',\theta^m,p,W)=w_m\theta_m+H(A',\theta^m,p,W)-G^*,$$

$$H(A',\theta^{m-1},p,W)=w_{m-1}\theta_{m-1}+H(A',\theta^{m-2},p,W)-(G^*-w_m\theta_m),$$

$$\dots$$

$$H(A',\theta^k,p,W)=w_k\theta_k+H(A',\theta^{k-1},p,W)-(G^*-\sum_{j=k+1}^{m}w_j\theta_j), \quad (10)$$

$$\dots$$

$$H(A',\theta^1,p,W)=w_1\theta_1-(G^*-\sum_{j=2}^{m}w_j\theta_j).$$

Initially, G^* is a value as far as the alternative A' is "worse" from A_{i_p} when taking into account all *m* criteria.

The parts of the formula (10) for the dimension problem *k* are considered. The expression in brackets $G^*-\sum_{j=k+1}^{m}w_j\theta_j$ means how much the "lag" alternative A' from A_{i_p} is reduced by obtaining the value $\theta_{k+1},\theta_{k+2},\dots,\theta_m$ at the initial *m-k* steps of the algorithm; $H(A',\theta^{k-1},p,W)$ is the minimum positive value of the desired advantage of the alternative A' over A_{i_p}, taking into account only *k* -1 of the first criteria.

In the part $w_k\theta_k$ the value of θ_k is selected from the set $\{\theta_k^1,\theta_k^2,\dots,\theta_k^{r_k}\}$.

- For effectively work of the algorithm of problem solving (4) - (6), it is necessary to perform several procedures..
- For each of the m criteria, the arithmetic mean values of the weighted deviations are determined:

$$\tilde{\theta}_k=\frac{\sum_{i=1}^{r_k}w_k\theta_k^i}{r_k},\ k=\overline{1,m}.$$

- The criteria of the base task are sorted by the growth of the obtained values $\tilde{\theta}_k$, $k=\overline{1,m}$. In order to prevent the reassignment of the index of the criteria it is assumed that inequality $\tilde{\theta}_1\le\tilde{\theta}_2\le\dots\le\tilde{\theta}_m$ is initially carried out. This does not reduce the generalization of the task.
- A sequence of hash sums S_k is created according to the following rule:

$$S_1=0,S_2=S_1+\theta_1^{r_1},\dots,S_k=S_{k-1}+\theta_{k-1}^{r_{k-1}},\dots,$$

$$S_m=S_{m-1}+\theta_{m-1}^{r_{m-1}}$$

Next, a table is created that will consist of m column of values (bottom-up) $0,w_k\theta_k^1,w_k\theta_k^2,\dots,w_k\theta_k^{r_{jk}}$ for each criterion and have the row of the received hash amounts:

In table 1 $r_2 = \max(r_j), j \in \{1,...,m\}$, $r_k > r_{m-1} > r_1$, Therefore, the columns have different heights, depending on the number of possible deviations of one or another criterion.

The algorithm allows to generate permissible solutions of the problem (4) - (6) without a complete overview of all possible variants and is verbally described as follows.

0. If sum $S_m + \theta_{m,r_m}$ * w_m lower than G^*, then the problem (4)-(6) have no solution.

1. Initialization. Initial values are determined $k = m, g_k = G^*, l_k = 0, \theta = (0,\ldots,0), \theta^{min} = \theta$. Here and further in the algorithm *k* is the number of the analyzed criterion, g_k is the "not overcome" advantage A_{i_p} over A', which remains at the moment of consideration of the subtask with *k*-th dimension. l_k is index of weighted deviation of the criterion under the number *k* $(0 \le l_k \le r_k)$, $\theta = (\theta_1, \theta_2, \ldots, \theta_k, \ldots, \theta_m)$ is variant of problem solution. Initially at (*k* = *m*) is laid $\theta_1 = 0, \theta_2 = 0, \ldots, \theta_m = 0$. At any time of the algorithm's work, the current resulting minimal function value $H(A', \theta, p, W)$ corresponds to the solution θ^{min} .

2. For a sub-task of *k*-th dimension, the column of weighted deviations of the *k*-th criterion is considered. The index l_k increases as long as the weighted deviation $w_k\theta_k^{l_k}$ with the hashed amount S_k does not become larger than g_k . It should be noted that S_k the maximum possible value, which may be reduced to advantage A_{i_p} over A' with already fixed deviations $\theta_{k+1} = \theta_{k+1}^{l_{k+1}}, \theta_{k+2} = \theta_{k+2}^{l_{k+2}}, \ldots, \theta_m = \theta_m^{l_m}$ in previous steps. There are three possible cases.
   - 2.1. The sum of the maximum possible weighted deviation $w_k\theta_k^{r_k} + S_k$ is less than g_k . Go to Step 3.
   - 2.2. Weighted deviation $w_k\theta_k^{l_k}$ without hash amount S_k is more than g_k . This means that the solution is obtained $\theta = (0,...,0, \theta_k^{l_k}, \theta_{k+1}^{l_{k+1}},...,\theta_{m-1}^{l_{m-1}}, \theta_m^{l_m})$. Go to Step 4.
   - 2.3. Weighted deviation $w_k\theta_k^{l_k}$ with the hash amount S_k is more than g_k . Go to Step 5.
3. If k = m, go to step 6, otherwise return to (k + 1) -th criterion; k = k + 1. Go to step 2.

4. If the value $H(A', \theta, p, W)$ for the received solution is less than $H(A', \theta^{min}, p, W)$, then $\theta^{min} = (0,...,0, \theta_k^{l_k}, \theta_{k+1}^{l_{k+1}},...,\theta_{m-1}^{l_{m-1}}, \theta_m^{l_m})$. Go to step 3.

5. The dimensionality of the task decreases; k = k - 1, $l_k = 0$. Go to step 2.

6. θ^{min} is optimal solution of the problem (1)-(6).

Table 1. Weighted deviations of values of the criteria of the problem (4)-(6).

	K₁	**K₂**	**…**	**Kₖ**	**…**	**Kₘ₋₁**	**Kₘ**
$\max\limits_{j\in\{1,\dots,m\}}(r_j)$		$w_2\theta_2^{r_2}$		$\vdots$		$\vdots$	
$\vdots$	$\vdots$			$w_k\theta_k^{r_k}$			$\vdots$
		$\vdots$	…		…	$w_{m-1}\theta_{m-1}^{r_{m-1}}$	
	$w_1\theta_1^{r_1}$			$\vdots$		$\vdots$	$w_m\theta_m^{r_m}$
	$\vdots$						$\vdots$
2	$w_1\theta_1^2$	$w_2\theta_2^2$	…	$w_k\theta_k^2$	…	$w_{m-1}\theta_{m-1}^2$	$w_m\theta_m^2$
1	$w_1\theta_1^1$	$w_2\theta_2^1$	…	$w_k\theta_k^1$	…	$w_{m-1}\theta_{m-1}^1$	$w_m\theta_m^1$
0	0	0	…	0	…	0	0
Hash	S_1	S_2	…	S_k	…	S_{m-1}	S_m

The formally described algorithm can be represented as follows. (Objects used in the algorithm are completely in line with the data structures defined above).

/* *k,m,i,j* : integer positive values,

Rez – value of the cost function: real;

G^* – Initial deviation A_{i_p} from A': real,

$g, w, S, \theta^{\min}$ – array of *m*-th dimension: real numbers,

l, r – array of *m*-th: integer positive values,

θ – two-dimensional array of m columns of different heights: real numbers,

*/

$S_1 = 0$; // Calculation of hash sums $S_k,\ k = \overline{1,m}$

For $(i = 2; i \le m; i{+}{+})$ $S_i = S_{i-1} + \theta_{i-1,r_{i-1}} * w_{i-1}$

EndFor

$\mathrm{Rez} := S_m = \theta_{m,r_m} * w_m - G^*$; // Initial value of the cost function

If $(\mathrm{Rez} < 0)$ **Then Exit EndIf** // The problem has no solutions.

// Initialization

$k := m; g_k := G^*$;

For $(i = 1; i \le m; i{+}{+})$ $l_i := 0; \theta_i^{min} := 0;$ **EndFor**

Do while $(k \le m)$

$j := r_i + 1$ **;**

For $(i = l_k; i \le r_k, i{+}{+})$

If $S_k + w_k\theta_{k,i} - g_k > 0$ **Then** $j := i$ **;**

Break **EndIf**

EndFor

$l_k := j$;

If $(l_k \le r_k)$ **Then**

If $((w_k\theta_{k,l_k} - g_k > 0)$ **Then**

If $(w_k\theta_{k,l_k} - g_k < \operatorname{Re} z))$ **Then**

For $(i = 1; i \le m; i{+}{+})$ $\theta_i^{min} := \theta_{i,l_i}$ **EndFor**

$\operatorname{Re} z := w_k\theta_{k,l_k} - g_k$;

EndIf

$l_k := 0; k := k + 1; l_k := l_k + 1;$

Else $g_{k-1} := g_{k-1} - \theta_{k,l_k}; k := k - 1; l_k := 0;$

EndIf

Else $l_k := 0; k := k + 1; l_k := l_k + 1;$

Endif

EndDo

Conclusion

For solving the problem (1) - (3) a decision support system (DSP) Verum EST [Gorborukov, 2011] was created. The system is intended for the solution of applied problems arising in the administrative activity of collegial bodies of management, enterprises, organizations, institutions, etc... for making responsible and scientifically grounded decisions. The considered algorithm is implemented and included in the library of mathematical methods of this system.

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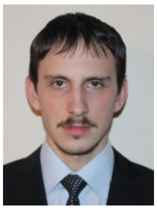

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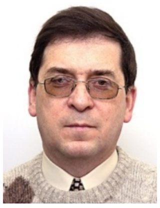

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