ANALYTICAL MODEL OF A QUEUING SYSTEM IN A TELECOMMUNICATION NETWORK

Velin Andonov, Stoyan Poryazov and Emiliya Saranova

Abstract: The paper represents shortly one analytical model of a queuing system as a part of overall telecommunication network. The analytical expressions for the parameters of the queuing system are derived in the papers [Andonov et al, 2019c; Poryazov et al, 2020b] as a part of an analytical model of overall telecommunication system.

Keywords: *Analytical modelling, Service systems, Queueing systems.*

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ITHEA Keywords: *C.4 Performance of Systems, H.1.2 User/Machine Systems, I.6 Simulation and Modelling, I.6.0 General, H.1.2 User/Machine Systems, K.6 Management of Computing and Information Systems*.

Introduction

The paper summarizes our recent results in the analytical modelling of queuing systems in telecommunication networks published in the papers [Andonov et al, 2019; Andonov et al, 2019c; Poryazov et al, 2020b]. The problem for conceptual and analytical modeling of queuing systems consisting of buffer and server as a part of overall telecommu-nication system arose when we focused on extending the conceptual model of overall telecommunication system (see [Poryazov & Saranova, 2012]) by inclusion of a queuing system in the switching stage. The two approaches to the conceptual modelling of queuing systems which we use are Service Systems Theory and the Generalized Nets (GNs, see [Atanassov, 2007]). First, in the papers [Andonov et al, 2019; Andonov et al, 2020] the means for constructing of GNs conceptual models of service systems are described. In the series of papers [Tomov et al, 2018; Tomov et al, 2019; Andonov et al, 2018; Poryazov et al, 2018a; Andonov et al, 2019b], different conceptual models of queuing systems are proposed and compared. The most suitable of these conceptual models for the purpose of the analytical modelling are chosen and included in the conceptual models of overall telecommunication system with queuing [Andonov et al, 2019; Andonov et al, 2019b]. Based on these conceptual models, an analytical model of overall telecommunication system with queuing is derived in [Andonov et al, 2019b]. Analytical expressions for the important parameters of the queuing systems such as mean service time in the buffer, mean service time in the server, etc., are also obtained.

In Section 2, the basic concepts from Service Systems Theory which are used in the conceptual modelling are presented. In Section 3, a conceptual model of overall telecommunication system

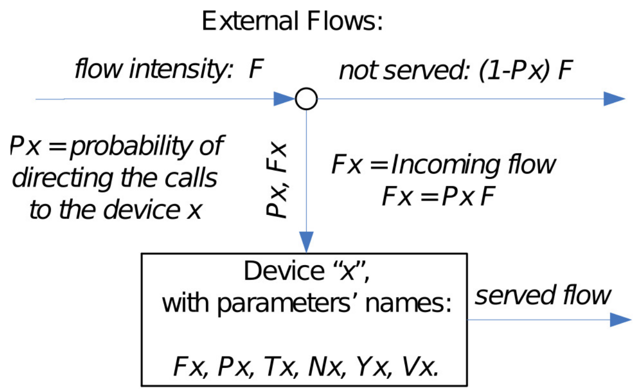

Figure 1: Graphical representation of a base virtual device x (see [Andonov et al, 2019c]).

including a queuing system in the switching stage is shortly described. A conceptual model of a queuing system which is used in the construction of the analytical model is described in Section 4. In Section 5, an analytical model of a queuing system as part of overall telecommunication system is presented.

Base virtual devices and their parameters

For the purpose of the analytical modelling, after a comparison of the different conceptual models of queuing systems, proposed in [Tomov et al, 2018; Tomov et al, 2019; Andonov et al, 2018; Poryazov et al, 2018a; Andonov et al, 2019b] the Service Systems Theory approach is chosen. Here, we present the basic concepts used in the conceptual models and the conceptual models of queuing systems which are used in the analytical modelling.

The basic building blocks of the conceptual models are the base virtual devices. They do not contain any other virtual devices. A general graphical representation of a base virtual device is shown in Fig. 1.

Every such base virtual device x has the following parameters (see [ITU-T E.600 ,1993] for terms definition):

- Fx - intensity or incoming rate (frequency) of the flow of requests (i.e. the number of requests per time unit) to device x;
- Px - probability of directing the requests towards device x;
- Tx - service time (duration of servicing of a request) in device x;
- Yx - traffic intensity [Erlang];
- Vx - traffic volume [Erlang - time unit];
- Nx - number of lines (service resources, positions, capacity) of device x.

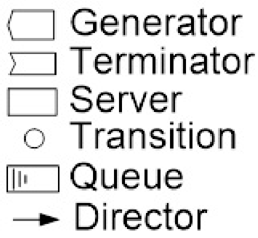

Figure 2: Types of base virtual devices and their graphical representation (see [Poryazov & Saranova, 2012]).

For the better understanding of the models and for a more convenient description of the intensity of the flow, a special notation including qualifiers (see [ITU-T E.600 ,1993]) is used. For example $dem.F$ for demand flow; $inc.Y$ stands for incoming traffic; $ofr.Y$ for offered traffic; $rep.Y$ for repeated traffic, etc.

Different types of base virtual devices are used in the conceptual models. Some of them together with their graphical representations are shown in Fig. 2. Each type of base virtual device has a specific function (see [Poryazov & Saranova, 2012]):

- Generator – generates call attempts (requests, transactions);
- Terminator – eliminates each request which enters it;
- Server – models traffic and time characteristics of the model, the delay (service time, holding time) of the requests;
- Transition – selects one of its possible exits for every request which has entered it;
- Queue – buffer device of the queuing system;
- Director – points to the next device to which the request is transferred without delay.

Conceptual model of an overall telecommunication system including a queuing system in the switching stage

In the conceptual model each virtual device has a unique name. The names of the devices are constructed according to their position in the model. The model is partitioned into service stages (dialing, switching, ringing and communication). Every service stage has branches (enter, abandoned, blocked, interrupted, not available, carried), corresponding to the modeled possible cases of ends of the calls' service in the branch considered. Every branch has two exits (repeated, terminated) which show what happens with the calls after they leave the telecommunication system. Users may make a new bid (repeated call), or stop the attempts (terminated call). In the names of

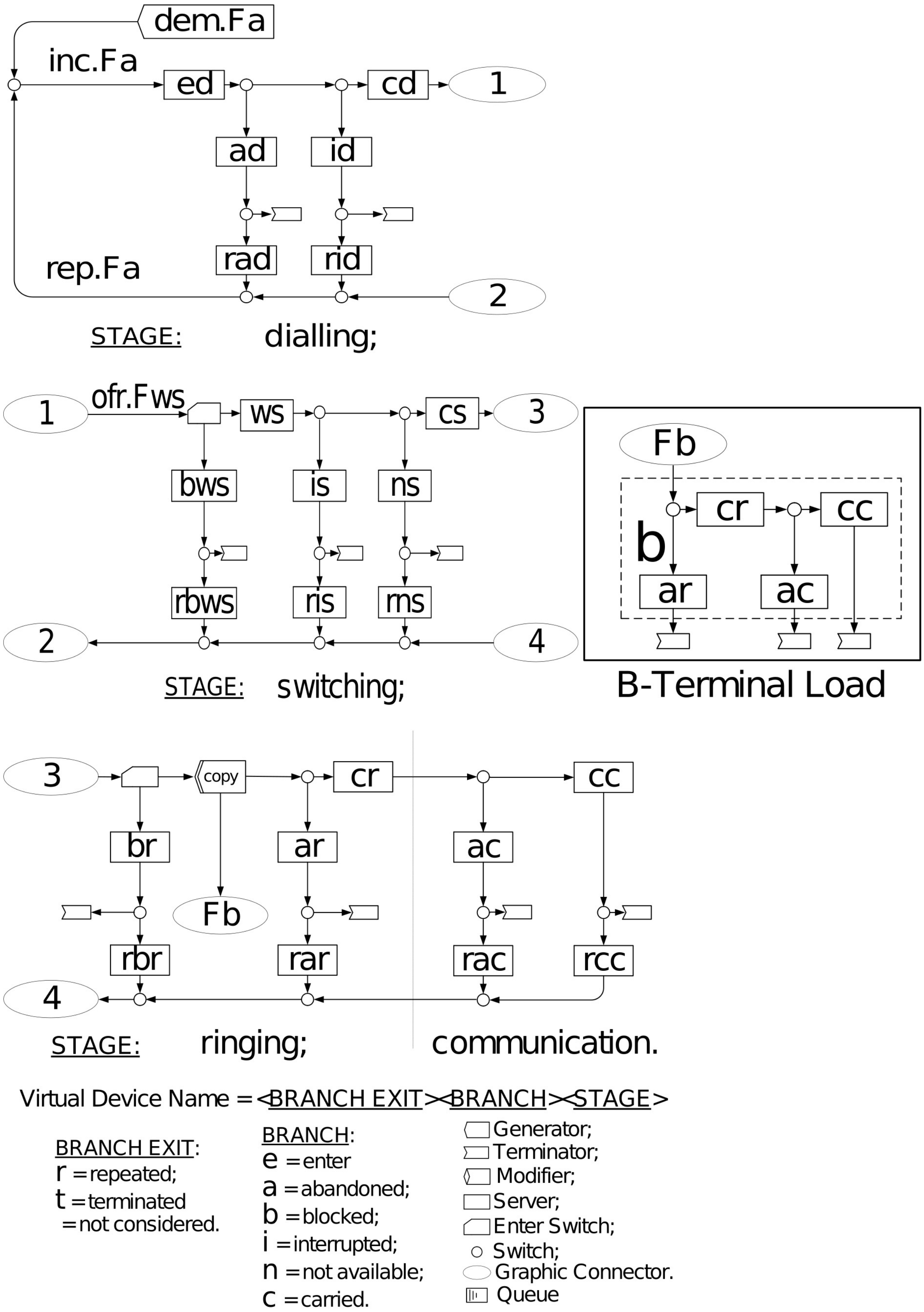

Figure 3: Conceptual model of an overall telecommunication system including a queueing system in the switching stage (see [Andonov et al, 2019]).

the virtual devices the corresponding bold first letters of the names of stages, branches end exits are used in the following way:

Virtual Device Name = <BRANCH EXIT><BRANCH><STAGE>

The names of the parameters of a virtual device are concatenations of the letter denoting the parameter and the name of the virtual device. For example, “Yid” means “traffic intensity in interrupted dialing case”; “Fid” – “flow (calls) intensity in interrupted dialing case”; “Pid” – “probability for interrupted dialing”; Tid – “mean duration of the interrupted dialing”; “Frid” – “intensity of repeated flow calls, caused by (after) interrupted dialing”.

Apart from base virtual devices, the following comprise virtual devices denoted by **b** (shown in dash line box in Fig. 3) and **a, ab, s** (not shown in Fig. 3) are also included in the model.

- **a** comprises all calling terminals (A-terminals) in the system. It is not shown in Fig. 3;
- **b** comprises all called terminals (B-terminals) in the system (dashed line box in Fig. 3);
- **ab** comprises all the terminals (calling and called) in the system. It is not shown in Fig. 3;
- **s** virtual device corresponding to the switching system. It is not shown in Fig. 3.

The flow of calls (B-calls), with intensity Fb, occupying the B-terminals, is coming from the Copy device. This corresponds to the fact that at the beginning of the ringing a second (B) terminal in the system becomes busy. The second reason for this conceptual modelling trick is that the paths of the A and B-calls are different in the telecommunication system’s environment, after releasing the terminals. There are two virtual devices of type Enter Switch (see Fig. 3) – before Blocked Waiting for Switch (**bws**) and Blocked Ringing (**br**) devices. These devices deflect calls if the buffer (**bw**) has reached its capacity or the intent B-terminal is busy, respectively. The corresponding transition probabilities depend on the macrostate of the system (Yab). The macrostate of a (virtual) device (including the overall network, considered as a device) is defined as the mean number of simultaneously served calls in this device, in the observed time interval (similar to “mean traffic intensity” in (see [ITU-T E.600 ,1993]).

An important remark regarding the analytical modeling of the system should be made. The mean service time of the call attempts in the **s** device depends, among other parameters, on the mean service time of the carried requests by the switching system. The mean service time of the carried requests by the switching system depends on Pbr, Tbr, Tb and on the mean service time of the requests in the **cs** device. Therefore, to avoid confusion in the analytical modeling of the system, we denote by T^*cs the mean service time of the requests in the **cs** device and by Tcs the mean service time of the carried requests by the switching system. This notation allows to avoid the inclusion of a comprise virtual device representing the service of the carried requests by the switching system.

Detailed description of the conceptual model can be found in the papers [Andonov et al, 2019; Andonov et al, 2019c].

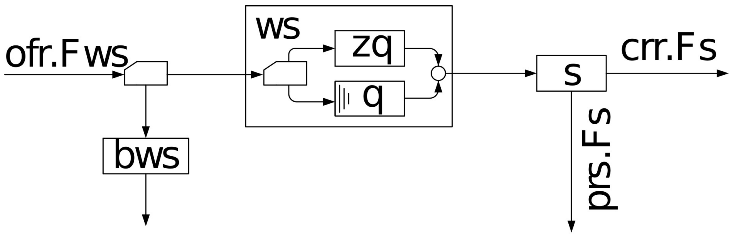

Figure 4: Conceptual model of a queuing system in the switching stage of an overall telecommunication system.

Conceptual model of a queuing system

For the derivation of analytical model of the queuing system as part of the overall telecommunication system, a detailed conceptual model of the queuing system should be used. Its graphical representation is shown in Fig. 4.

This detailed representation allows for the two different ways of service of the request in the queuing system to be distinguished: service with waiting and service without waiting. When the switching system (**s**) has not reached its capacity (Ns), the requests enter the zero queuing (zq) device from where they are sent to the switching system without delay. If the switching system has reached its capacity but there are free places in the buffer, the requests enter the buffer device q where they wait to be serviced, depending on the discipline of service in consideration. In the present paper, we consider FIFO discipline of service of the requests. The mean service time of the requests in the buffer (Tws), for both the waiting and the non-waiting requests, is given by:

$$Tws = Pq\,Tq + (1 - Pq)\,Tzq\,, \tag{1}$$

where Pq is the probability that the request is serviced with waiting and Tq is mean service time in the buffer for the waiting requests. We shall consider that the mean service time of the non-waiting requests is 0, i.e., $Tzq = 0$. In this way the mean service time in the buffer of both the waiting and the non-waiting requests becomes:

$$Tws = Pq\,Tq\,. \tag{2}$$

From $Tzq = 0$ if follows $Yzq = 0$. Therefore, the capacity of the buffer (Nws) is equal to the capacity of the q device Nq. The intensity of the offered flow of requests to the switching system is denoted by $ofr.Fws$ (see Fig. 4). The qualifier “parasitic” (prs) is defined in [Poryazov et al, 2018b]. In Fig. 4 the outgoing flow intensity of the switching system ($out.Fs$) is given by:

$$out.Fs = crr.Fs + prs.Fs\,. \tag{3}$$

For $crr.Fs$ and $prs.Fs$ we have:

$$crr.Fs = Fcc\,. \tag{4}$$

The parasitic flow ($prs.Fs$) represented in Fig. 4 according to its definition and the conceptual model shown in Fig. 3 can be expressed through the equation:

$$prs.Fs = Fis + Fns + Fbr + Far + Fac\,. \tag{5}$$

Problem statement. The problem that we are solving can be stated as deriving equations for the

- output parameters: $Pbws$, Yws, Tws, related to the **ws** device

given the

- input parameters: $Ns, Nws, ofr.Fws, Ts$ which can be measured.

In order to compactly describe single queuing stations in an unambiguous way, the so called Kendall notation is often used (see [Haverkort, 1998]). A queuing system is described by 6 identifiers separated by vertical bars in the following way:

Arrivals | Services | Servers | Buffersize | Population | Scheduling

where “Arrivals” characterizes the arrival process (arrival distribution), “Services” characterizes the service process (service distribution), “Servers” – the number of servers, “Buffersize” – the total capacity, which includes the customers possibly in the server (infinite if not specified), “Population” – the size of the customer population (infinite if not specified), and finally, “Scheduling” – the employed service discipline.

In our model, the queuing system in the Switching stage of the telecommunication network in Kendall notation is represented as $M|M|Ns|Ns + Nws|Nab|FIFO$, where M stands for exponential distribution, Ns is the capacity of the Switching system (number of equivalent internal switching lines) and Nab is the total number of active terminals which can be calling and called. This is related to the derivation of the analytical model of the system.

Analytical model of the queuing system

The queueing system in the switching stage differs from other queuing systems such as the ones studied in [Schneps, 1979; Vishnevskiy, 2003] in that it has more exits. The exits are represented in the conceptual model in Fig. 3 with the branches is, ns, br, ac, cc. In [Andonov et al, 2019], we have derived analytical expressions for the parameters of the queuing system, starting with the simplest queuing system $M|M|1|FIFO$ and gradually advancing to the most complicated system with finite buffer and finite capacity of the server. Here we shall use the results from [Andonov et al, 2019] but adapted to the more detailed conceptual model presented here.

The density functions of the arrival and service times are respectively:

$$a(t) = \lambda e^{-\lambda t}, \tag{6}$$

$$b(t) = \mu e^{-\mu t}, \tag{7}$$

where $1/\lambda$ is the mean value of time between two arrivals (interrarival time) and $1/\mu$ is the mean time of service. For our queuing system, they are given by:

$$\lambda = ofr.Fws\,, \tag{8}$$

$$\mu = \frac{1}{Ts}\,. \tag{9}$$

They are assumed to be statistically independent which results in a birth-death process. Let us denote with p_n the probability that the queuing system is in state n that is:

$$p_n = Pr\{\text{there are n requests in the queuing system}\}.$$

There are different ways to solve the birth-death equations. The solution is well-known and can be found for example in [Schneps, 1979]. First, we notice that the arrival rate λ_n is equal to 0 when $n \geq Ns + Nws$. The probability for the system to be in state n is now given by:

$$p_n = \begin{cases} \frac{\lambda^n}{n!\mu^n} p_0 & \text{for } 1 \leq n < Ns. \\ \frac{\lambda^n}{Ns^{n-Ns} Ns! \mu^n} p_0 & \text{for } Ns \leq n \leq Ns + Nws. \end{cases} \tag{10}$$

Again, the condition that the sum of the probabilities p_n should be equal to 1, gives us the following expression for p_0:

$$p_0 = \left(\sum_{n=0}^{Ns-1} \frac{\lambda^n}{n!\mu^n} + \sum_{n=Ns}^{Ns+Nws} \frac{\lambda^n}{Ns^{n-Ns} Ns! \mu^n} \right)^{-1}. \tag{11}$$

In order to simplify the expression we set $r = \lambda/\mu$ and $\rho = r/Ns$. After elementary operations, the above expression for p_0 becomes

$$p_0^{-1} = \begin{cases} \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \frac{r^{Ns}}{Ns!} \frac{1-\rho^{Nws+1}}{1-\rho} & \text{for } \rho \neq 1\,. \\ \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \frac{r^{Ns}}{Ns!}(Nws + 1) & \text{for } \rho = 1\,. \end{cases} \tag{12}$$

Using (10), we can confirm the validity of the following theorem.

Theorem 1. *The probability of blocked waiting for switch ($Pbws$) is equal to the probability that the system is in state $Ns + Nws$, i.e.,*

$$Pbws = \frac{\lambda^{Ns+Nws}}{Ns^{Nws} Ns! \mu^{Ns+Nws}} p_0\,. \tag{13}$$

Theorem 2. *The expected length of the queue is given by the following expression:*

$$Yws = \sum_{n=Ns+1}^{Ns+Nws} (n - Ns)p_n = \frac{p_0 r^{Ns} \rho}{Ns!(1-\rho)^2}[(\rho-1)\rho^{Nws}(Nws+1)+1-\rho^{Nws+1}]\,. \quad (14)$$

The proof of the above theorem is given in [Andonov et al, 2019].

Theorem 3. *The mean service time of the requests in the* ***ws*** *device for both, the waiting and non-waiting requests, is given by:*

$$Tws = \frac{p_0^2(Ns\rho r)^{Ns}\rho(1-\rho^{Nws})}{(Ns!)^2(1-\rho)^3}.\frac{[(\rho-1)\rho^{Nws}(Nws+1)+1-\rho^{Nws+1}]}{\lambda(1-Pbws)}\,. \quad (15)$$

Proof: The mean service time of the requests in **ws** device for both the waiting and non-waiting requests, given the condition $Tzq = 0$, is

$$Tws = PqTq + (1 - Pq)Tzq = PqTq\,. \quad (16)$$

The mean service time of the waiting requests in the **q** device (Tq) is given by:

$$Tq = \frac{p_0 r^{Ns}\rho}{Ns!(1-\rho)^2}\frac{[(\rho-1)\rho^{Nq}(Nq+1)+1-\rho^{Nq+1}]}{\lambda(1-Pbws)}\,. \quad (17)$$

The probability Pq is the probability that the system is in any of the states $Ns, Ns+1, ..., Ns+Nws-1$, i.e.,

$$Pq = \sum_{k=Ns}^{Ns+Nws-1} p_k = \sum_{k=Ns}^{Ns+Nws-1} \frac{\lambda^k}{Ns^{k-Ns}Ns!\mu^k}p_0\,. \quad (18)$$

After simpification we obtain:

$$Pq = \frac{p_0 Ns^{Ns}\rho^{Ns}(1-\rho^{Nws})}{Ns!(1-\rho)}\,. \quad (19)$$

After substitution of (19) and (17) in (16), the theorem is proved. □

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