ITHEA

International Journal INFORMATION MODELS & ANALYSES

2020 Volume 9 Number 3

International Journal
INFORMATION MODELS AND ANALYSES
Volume 9 / 2020, Number 3

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International Journal "INFORMATION MODELS AND ANALYSES" Volume 9, Number 3, 2020

Edited by the **Institute of Information Theories and Applications FOI ITHEA**, Bulgaria, in collaboration with:
University of Telecommunications and Posts, Bulgaria,
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Publisher: **ITHEA®** Sofia, 1000, P.O.B. 775, Bulgaria. www.ithea.org , e-mail: office@ithea.org
Technical editor: **Ina Markova**

Printed in Bulgaria

ISSN 1314-6416 (printed) **ISSN 1314-6432 (Online)**

The Information Science Paragon: Allow Knowledge to Prevail, from Prehistory to Future – Approaches to Universality, Consistency, and Long-term Sustainability

Claus-Peter Rückemann

Abstract: *Nowerdays we face widely spreaded shisms between disciplines, neologisms, and smattering. Terms are used increasingly loose, for a wide range of reasons. In consequence, we see deficits when solid fundaments dealing with any kind of information are required. In contrary, advanced and inter-disciplinary research, integration, and collaboration tasks, require knowledge and practice of information science fundaments, with a small subset being informatics and technology. Information Science –if understood on a more than 'technical-only' level– can provide a wide and in-depth plethora of methodologies and methods, which essentially contribute to solutions and enable insight, universality, consistency, and long-term sustainability in theory and practice. Information science comprises the fields of collection, documentation, classification, analysis, manipulation, storage, retrieval, movement, dissemination, and protection of information. The key to information science is a solid understanding of knowledge and its context for day-to-day application. The topics provide essential information science fundaments from the base to advanced insight. Presented outcomes result from the last decades of research in information science theory, practice, and scholarly education, spanning advanced application scenarios in management, information and knowledge mining, data processing, information systems, natural sciences, philosophy, prehistory, archaeology, computer science, high end computing, supercomputing, simulation, modeling, and research data reuse.*

Keywords: *Information Science; Information Theory; Epistemology; Logic; Methodologies; Methods; Prehistory; Protohistory; Archaeology; Natural Sciences; Humanities; Informatics; Conceptual Knowledge; Integration Strategies.*

MSC*: 03A10; 03A05; 68Pxx; 62R07; 94A15; 94A17.*

ITHEA Keywords*: Data; Computing Methodologies; Systems and Information Theory; Analysis of Algorithms and Problem Complexity; Mathematical Logic and Formal Languages; Logics and Meanings of Programs.*

Introduction

This article presents the results on knowledge-based approaches to universality, consistency, and long-term sustainability for challenges from multi-disciplinary research and advanced, complex context. The information science methodology and method based implementation solutions address a wide range of knowledge and application scenarios and were triggered to provide solid fundaments contrary to omnipresent deficits seen in many fields dealing with respective challenges.

For quite a time now, public and research have to face smattering and buzz in generalised statements and practice, e.g., "Data, information, knowledge . . . are basically the same . . .", "*Structure* and format are very much alike . . .", "There are *structure* standards . . .", "*Structure* can be created by digitisation . . .", "*Intelligent applications* can deliver solutions . . .". Common views try to imagine that artificial 'intelligence' could deal with structured and unstructured data, that intelligent systems would be based on technical implementations, that solutions would be a matter of informatics realisations, that using ('buzz') technologies could create knowledge from data, that objectives can be seen when looking at present instruments, and that education should focus on training for developing technical apps. Such environments rarely allow to deal with knowledge and contexts in appropriate ways, nevertheless, inspiring the need for more holistic solutions.

With this research, we create methodological approaches and extensive multi-disciplinary, multi-lingual knowledge resources and application scenarios spanning pre- and protohistory and archaeology to natural sciences, computation, and humanities. The knowledge resources are in continuous development for decades and contain millions of knowledge objects and entities, and arbitrary references to distributed resources. The resources can comprise all facets of knowledge, including state-of-the-art conceptual knowledge frameworks. The presented results from information science theory, practice, and scholarly education achieved during the last decades span advanced application scenarios in management, information and knowledge mining, data processing, information systems, natural sciences, philosophy, pre- and protohistoric archaeology, classical archaeology, computer science, high end computing, supercomputing, simulation, modeling, and research data reuse [Rückemann 2017]. This article presents excerpts of the results of a Conceptual Knowledge Reference Implementation (CKRI) created, developed, and used with non-disciplinary context, especially with prehistory, archaeology, natural science, and humanities [Rückemann 2019a]. The goal of this long-term research is to further create knowledge resources, knowledge-based methodologies and methods enabling to handle even challenging scenarios with universality, consistency, and long-term sustainability.

Information Science

Information science fundaments and backgrounds are often neither understood and taught nor practically respected. Society, academy, education are commonly reacting with simplification and training. Our vision, approach, and practice target the insights that we have to start far before getting into technical aspects or implementations of application software, that knowledge is not the output or result of a tool, that the fundament of any Turing machine / computer is formalisation, going along with abstraction and reduction, and that 'information processing' should be addressed via context of knowledge complements. This is also true whenever addressing information, e.g., with gathering, filtering, compression of information but also whenever discussing criteria of hermeticism like quality and optimisation. In consequence, we require education for solid information science fundaments, knowledge complements' integration, information science based methods. A starting point is the fact that data is neither information nor knowledge.

> ***Notion:*** Data are "things given". (Latin) data :: (Latin, singular) "datum", given.
>
> ***Axiom:*** 'Data' cannot be 'turned' into 'information'.
> Trying, consequently leads to arbitrary results and arbitrary organisational states.
>
> Practice: Information can result from information peeling processes, e.g.,
> creating knowledge mining implementations.
>
> ***Notion:*** Information science is the science of information in theory and practice.
>
> The essential fundaments of information science are information and philosophy.
>
> ***Notion:*** Information comprises of aspects of knowledge, e.g., facts or details, supplied to or learned by someone. (Latin) formatio :: formation, construction, arrangement, "creation".

We should always be aware of commonly less holistically consistent descriptions, e.g., in public statements but also in widely known dictionaries. Therefore, we need a more consistent understanding.

Information science investigates the being of information, information related properties, and information processes. Information science focuses on theory and methodologies and their application in practice, understanding information related problems, preserving, developing, and making use of information. Information science primarily tackles systemic problems rather than individual pieces of technology within systems. Information science comprises the fields of collection, documentation, classification, analysis, manipulation, storage, retrieval, movement, dissemination, and protection of information. Information science is associated with psychology, computer science, and technology. Information science is interlinked with cognitive science, archival science, linguistics, museology, management, mathematics, philosophy, commerce, law, public policy, and social sciences. Information science deals with any information and communication, e.g., knowledge in organisations, interaction between people, information systems, understanding information systems, and creating, replacing, improving information systems.

Prehistory to Future: Information, History, and Information Management

There are manifold case scenarios, with content and context from natural sciences, geology, and prehistory to classic, medieval, and modern constellations. Table 1 shows a small range of examples.

Table 1: Example cases of lost knowledge over time.

Prehistory (. . . to protohistory / archaeology)	$\Rightarrow$	"art" to "technology", how to contextualise?
Heron of Alexandria (Greek antique, "Steam Ball")	$\Rightarrow$	"entertainment" but not used as technology.
Isidore of Seville (encyclopaedic, broad documentation)	$\Rightarrow$	end of medieval phase, not further used.
Polyhistor (Martin Fogel)	$\Rightarrow$	broad base, not further used.
Last decades / Internet	$\Rightarrow$	huge amounts of 'knowledge' potential lost.

Summarising, we have to realise that in percentage we nearly know nothing about the past. A reason is that we are widely not practicing universal preservation, documentation, and valorisation. It is necessary, in most cases, to contextualise by consequent multi-disciplinary and sustainable means including appropriate documentation. That means, for pre- and protohistorical, ancient, historical, and near past that realia, their contexts, meaning, and possible documentation are mostly lost and technologies are not fully understood. Therefore, contextualisations of prehistoric to future scenarios require more holistic and basically different approaches than commonly considered when reusing 'known' experiences or written sources. Starting with the experiences on information and management we can consider the following frame of preconditions. Information science can provide fundaments and answers. Prehistory and natural sciences as well as many associated information sources do have and do require special context. Knowledge should be addressable by 'holistic' approaches. Cognition based on prehistoric information is special. Content and context of said research targets require special methodologies. Methodologies need to enable a plethora of implementations, and decision making needs to address knowledge, special content and context.

Information science is based on a fundament of intrinsically tied complements: Episteme (Greek: $\acute{\varepsilon}\pi\iota\sigma\tau\acute{\eta}\mu\eta$) refers to "knowledge", "understanding", and "science". Techne (Greek: $\tau\acute{\varepsilon}\chi\nu\eta$) refers to "craft" and "art". Doxa (Greek: $\delta\acute{o}\xi\alpha$) originates from "to appear", "to seem", "to accept", and "to think".

Knowledge transfer is essential, over generations of objects and subjects. The transfer requires knowledge recognition (expertise), knowledge documentation for any aspect of nature and society (sciences, literature, technical descriptions, tools, cultural heritage, mythology, songs, media, . . .), and long-term means.

The assets we are dealing with in information science are the complements of knowledge (factual, conceptual, procedural, metacognitive, . . .) and the potential of the existing plethora of knowledge and insight.

Background and Fundaments – Information, Sciences, Knowledge

The fundaments of terminology and of understanding knowledge are layed out by Aristotle [Aristotle 2008] [Aristotle 2009], being an essential part of 'Ethics' [Aristotle 2005]. Information science can very much benefit from Aristotle's fundaments and a knowledge-centric approach (Anderson and Krathwohl) [Anderson and Krathwohl 2001] but for building holistic and sustainable solutions they need to go beyond the available technology-based approaches and hypothesis [Plato 2008] as analysed in Platon's Phaidon. In consequence, an updated view on the knowledge complements including the creation of interfaces between methods and applications (e.g., based on the methodology of Superordinate Knowledge [Rückemann 2019b] and the methodology of Knowledge Mapping [Rückemann 2018a]) is addressed in the following excerpts.

Essential factors for application are cognition and contextualisation. The implemented methods and integration modules for result context creation and georeferencing have delivered viable, efficient, and flexible solutions for many case scenarios. Implementations are far from trivial but any discipline being able to ask questions as demonstrated should also be able to deploy the methodology and presented components for creating solid fundaments and own practical solutions for challenging, complex scenarios, e.g., classification and dating of objects [Gleser 2004], geoscientific prospection, surveying [Gleser et al. 2015], and knowledge [Gleser 2015], chorological and chronological context [Gleser 2016], can contribute to the fundaments of cognition in prehistoric and protohistoric archaeology [Gleser 2020] and insight [Gleser 2018] regarding realia and abstract objects, knowledge, and contexts. The presented knowledge-based method and conceptual knowledge framework allow to address context very flexibly, e.g., in order to enable the metacognitive documentation of metacognitive and procedural knowledge of Geoscientific Information Systems or Geographic Information System analysis [Filloramo et al. 2020], filtering contextualised artistic representations [Becker 2018] and managing object collections [Becker 2012]. Knowledge-based approaches can also be beneficial without advanced knowledge resources, e.g., in cases of realia collections, information management and service oriented institutions and research data collection, e.g., The Digital Archaeological Record [tDAR 2020] and Digital Antiquity [DigitalAntiquity 2020]. For example, in focus cases of pre- and protohistory, archaeology, and history context and georeferencing can further be supported by facet creation into more dimensions and also allows the application of a consistent conceptual base for description and fuzziness, beyond common auxiliaries and georeferencing.

Expertise and Skills: Best Practice and Definitions

The 'Best Practice and Definitions Series' introduced with the annual international Symposium on Advanced Computation and Information in Natural and Applied Sciences (SACINAS) so far focussed on the major topics of "Knowledge and Computing", "Data-centric and Big Data – Science, Society, Law, Industry, and Engineering", "Data Sciences – Beyond Statistics", "Data Value", and "Formalisation and Formalism".

Knowledge: "Knowledge is created from a subjective combination of different attainments as there are intuition, experience, information, education, decision, power of persuasion and so on, which are selected, compared and balanced against each other, which are transformed, interpreted, and used in reasoning, also to infer further knowledge. Therefore, not all the knowledge can be explicitly formalised. Knowledge and content are multi- and inter-disciplinary long-term targets and values. In practice, powerful and secure information technology can support knowledge-based works and values." [Rückemann et al. 2015]

Computing: "Computing means methodologies, technological means, and devices applicable for universal automatic manipulation and processing of data and information. Computing is a practical tool and has well defined purposes and goals." [Rückemann et al. 2015]

Data-centric: "The term data-centric refers to a focus, in which data is most relevant in context with a purpose. Data structuring, data shaping, and long-term aspects are important concerns. Data-centricity concentrates on data-based content and is beneficial for information and knowledge and for emphasizing their value. Technical implementations need to consider distributed data, non-distributed data, and data locality and enable advanced data handling and analysis. Implementations should support separating data from technical implementations as far as possible." [Rückemann et al. 2016]

Big Data: "The term Big Data refers to data of size and / or complexity at the upper limit of what is currently feasible to be handled with storage and computing installations. Big Data can be structured and unstructured. Data use with associated application scenarios can be categorised by volume, velocity, variability, vitality, veracity, value, etc. Driving forces in context with Big Data are advanced data analysis and insight. Disciplines have to define their 'currency' when advancing from Big Data to Value Data." [Rückemann et al. 2016]

Data science: "Qualified Data, especially for an enterprise, represents frozen knowledge or in other words frozen value. The abilities to understand and manage these data is what we call data science. Data results from action, hence, data science can be defined secondary to data. The essence of Data Science is to give qualified access to relevant data to owners and users. Hardware and software and their implementation represent the tertiary level of qualified and high level data." [Rückemann et al. 2017]

Data value: "Data value is the primary ranked value in scenarios comprised of data and computing context. In general, processing of data, is the cause for computing. In consequence, data, including algorithms and other factual, procedural, and further knowledge, have to be ranked primary on the scale of values whereas machinery for processing data, including computing, are providing means of secondary ranked value. In addition, further values, including economic values, can be associated with consecutive deployment of data and machinery. (This is unaffected by varying views and attributions, including quality. Nevertheless, different views can scale values.)" [Rückemann et al. 2018]

Formalisation: "Formalisation is the process of creating a defined set of rules, allowing a formal system to infer theorems from axioms. Formal systems may represent well-defined systems of abstract thought. Description and analysis of any detail of any more or less complex system and physical background essentially require a formalisation process. The process includes abstraction and reduction of knowledge, keeping the preconditioned importance of respective context. Consequently, formalisation should be created and context observed by educated experts within the respective discipline. (All mathematical-machine based systems, e.g., computers, are formal systems. Ideologies should be kept outside of formalisation.)" [Rückemann et al. 2019]

Systematical View on Knowledge: FCPM Complements and Instruments

The complements of knowledge (FCPM, factual, conceptual, procedural, metacognitive knowledge) and corresponding example implementations are shown in Table 2.

Table 2: Complements of knowledge and example implementations.

Knowledge Complement		*Example Implementation*
Factual Knowledge	$\Leftrightarrow$	Numerical data, data . . .
Conceptual Knowledge	$\Leftrightarrow$	Classification . . .
Procedural Knowledge	$\Leftrightarrow$	Computing . . .
Metacognitive Knowledge	$\Leftrightarrow$	Experience . . .
. . .	$\Leftrightarrow$	. . .

Obstacles, which should be addressed, otherwise possibly reducing success and efficiency with the processes are, e.g., time consumption (e.g., staff, project timelines), documentation (e.g., low percentage of reusability), classification (e.g., limited views), tools (e.g., changing repeatedly), “standards” (e.g., changing repeatedly), up to different perception of goals, strategies, and completeness.

Examples are methodologies addressing full text and keywords, with various methods available for object groups, regular expressions, search functions, or phonetics, e.g., Soundex. Soundex, for example, provides algorithms for calculating codes from text strings, representing phonetic properties. Originally, Soundex was only used for names, in Englisch. The original algorithm mainly encodes consonants. The goal is to encode homophones with the same representation. Minor spelling differences do result in the same representation. There are various modifications for any language, topics, any kind of words, with support for many programming environments [Russel and Odell 1918; Knuth 1973; Rempel 1998; Rückemann 2013].

Implemented instruments targetting factual, conceptual, procedural, and metacognitive complements include Knowledge Resources, Universal Decimal Classification (UDC), Unified Modeling Language (UML), Documentation, Decision making (in means of information science), Structures (in means of information science), High End Computing (HEC), Open Archives Initiative (OAI), and OAI-Protocol for Metadata Harvesting (OAI-PMH).

Acolytes are always complementary to instruments. Helpers, staff and resources, are needed when dealing with knowledge. The quantity of staff and resources depends. However, the ‘quality’, targetting Quality of Data (QoD) can help to ‘optimise’ requirements for staff and resources.

Structures are relevant for mostly all instruments. We should learn to understand what structures mean in information science and take care that structures are capable to contain and refer to any content and context.

Information Science: Structure and Form

Definition: Structure is an organisation of interrelated entities in a material or non-material object or system. A structured object or system is sometimes called a structure.

Be aware of the etymological background:
structura (Latin) :: fitting together, adjustment, building, a building, edifice, structure.
struere (Latin), structus (past participle) :: pile up, arrange, assemble, build.
Dome / domí, δομή; domes, δομές (pl.) (Greek) :: structure, construction (physical, social, political).
Domus (Latin) :: house.

Information science does require an understanding of structure categories. Examples of basic categories are:

Material structures are *natural objects* such as biological organisms, minerals, and chemicals and *man-made* objects such as architectural buildings and machinery.

Abstract structures are any (knowledge / information / data) structures in information science, used in theory and practice.

In both cases, ***structure types*** are hierarchies or lattices.

From discipline’s views only, information Science and structure may show up heterogeneous. In logic and philosophy, structure is essential, e.g., a structure of arguments. An argument consists of one or more premises from which a conclusion is inferred. Basic inferences are deduction and induction. In problem solving, a data structure is generally an integral part of the respective algorithm. In mathematics, a structure is a set endowed with additional features on the set. Examples of features are topology, operation, relation, metric, especially, algebraic structures (e.g., groups, fields), metric structures (geometries), orders, events, equivalence relations, differential structures, and categories. In chemistry, the term chemical structure refers to both molecular geometry and electronic structure. Structure can be represented by various diagrams, e.g., structural formulas. In informatics, implementations of structures are a) array and index or b) a linked list and pointers. In software architecture, the structure of software system is the way in which interrelated components are partitioned. Associated features and terms are dependencies, modules, robustness, tolerance, and redundancy.

Axiom: Structure can mean features and facilities.

Axiom: Regarding the specification structured / unstructured it is not relevant how much data is compatible with / uniform / part of any of your records, system, accounting, inventory management, order systems etc.

Regarding structure and form, let us take a look on dealing with written text.

The **structure** of a text consists of the particular text units and their context, in order to make the text coherent.

The **form** of a text is the arrangement of the text units, which commonly has to follow predefined rules.

Axiom: There are neither standard structures nor standard forms of data.

Axiom: Information is inherent with form.

Axiom: Form follows function (in best practice).

Especially, if meaning should be expressed by langage, langue, and parole [de Saussure 1916] then the available rules of structure and form should be used. If whatever non linguistic, artistic expression is primary target then different structure an form could be used. Academic use should be aware of the specific academic context. Commercial use should be aware of the specific commercial context. Marketing use should be aware of the specific marketing context. There are rarely reasonable compromises fitting diametrical approaches to form equally well.

Information science fundaments in this context can deal with arbitrary fields of concern and can be applied in any fields and tasks in theory and practice, e.g., knowledge resources (millions of objects, entities, and references), knowledge mining, prehistory, protohistory / archaeology, natural sciences, humanities, creating insight, knowledge-valorisation context, research data reuse, calculation, processing, and computation (simulation, modelling), integration, long-term documentation, algorithms, automation, and autonomous instruments, numerical applications, information and processing systems, and management and governance.

Prehistory to Future

All of the examples, objects, entities, and respective conceptual knowledge references' excerpts are taken from The Prehistory and Archaeology Knowledge Archive (PAKA), DIMF, 2020 [Rückemann 2019a]. Table 3 is showing an illustrative overview of the major divisions and a range of excerpts, from 'prehistory to future'. Conceptual knowledge references are introduced by an 'UDC:' marker.

Table 3: Divisions and example range of consistent conceptual knowledge references, from ‘prehistory to future’.

Code / Sign Ref.	*Verbal Description (EN)*
UDC:0	Science and Knowledge. Organization. Computer Science. Information. Documentation. Librarianship. Institutions. Publications
UDC:1	Philosophy. Psychology
UDC:2	Religion. Theology
UDC:3	Social Sciences
UDC:5	Mathematics. Natural Sciences
UDC:6	Applied Sciences. Medicine, Technology
UDC:7	The Arts. Entertainment. Sport
UDC:8	Linguistics. Literature
UDC:9	Geography. Biography. History
UDC:001	Science and knowledge in general
UDC:113	General laws of nature. Transformation and transience of matter. *Origin of the universe. Creation. Cosmogony*
...	
UDC:903	*Prehistory. Prehistoric remains, artefacts, antiquities*
UDC:902	*Archaeology*
UDC:904	Cultural remains of historical times
UDC:93/94	*History*
...	
UDC:001.18	*Future of knowledge*

Advanced knowledge resources, e.g., collections and containers, built with objects and entities can provide an integration of representations of knowledge complements. Therefore, targetting long-term sustainability, knowledge resources can enable an integration of complexities for universal, multi-disciplinary knowledge in a consistent way.

For example, arbitrary length objects and arbitrary number of objects and entities with multi-line formatting (Listings 1 and 2) can be preserved. Scale and number of referenced objects are not limited by the methodology. A small excerpt of a knowledge resources’ object entry (Listing 1) for the ‘Swimming Reindeer’ [Cook 2015] shows some respective features. The representation of the multi-line formatted object can include excerpts of factual knowledge, conceptual knowledge references, georeferences, and citation references. The conceptual knowledge also carries relevant and characteristic relations for the objects.

```
Swimming Reindeer    [Prehistory]:
                     Prehistoric mammoth ivory scrimshaw work.
                     Location: Montastruc region, France.
                     LATLON: 44:06:07N, 01:17:41E
                     Object:             Swimming Reindeer.
                     Object-Type:        Realia object.
                     Object-Age:         (about) 11,000 B.C.E.
                     Object-Location:    Montastruc, France.
                     Object-Relocation: British Museum, UK.
                     %%IML: UDC-Object:[903+903.2]+(44)+(4)+(23)
                     %%IML: UDC-Relocation:069.51+(41)+(4)+(23)
                     %%IML: cite: YES 20161226 {LXK:mammoth ivory; scrimshaw; Holocene}
                       {UDC:903:"323"} {PAGE:--24..--25} LXCITE://Cook:2015:Reindeer
```

Listing 1: Knowledge resources’ object (‘Swimming Reindeer’): Multi-line formatting, conceptual knowledge, facet, and georeferences (excerpt, LX Knowledge Resources, DIMF).

The following excerpt (Listing 2) shows a an entry, a multi-line object representation of an object, which refers to natural sciences, geology, and archaeology.

```
Nisyros              [Volcanology, Geology, Archaeology]:
```

```
                    Volcano, Type: Strato volcano, Island,
                    Country: Greece, Subregion Name: Dodecanese Islands,
                    Status: Historical, Summit Elevation: 698\UD{m}. ...
                    Craters: ..., VNUM: 0102-05=, ...
                    %%IML: UDC:[911.2+55]:[930.85]:[902]"63"(4+38+23+24)=14
                    %%IML: UDC:[912]
                    %%IML: media:...{UDC:[911.2+55]:"63"(4+38+23)=14}...jpg
                    Stefanos Crater, Nisyros.
                    LATLON: 36.578345,27.1680696
                    %%IML: GoogleMapsLocation: https://www.google.com/...@36
                     .578345,27.1680696,337m/...
                    Little Polyvotis Crater, Nisyros.
                    LATLON: 36.5834105,27.1660736 ...
```

Listing 2: Knowledge resources' object ('Nisyros'): Multi-line formatting, conceptual knowledge, media object entities, and georeferences (excerpt, LX Knowledge Resources, DIMF).

The excerpt shows a representation of a 'Nisyros' multi-line formatted object, including respective conceptual knowledge references, media object entities, and georeferences.

Example Cases and Conditions

There are a number cases of prominent and characteristic content / context challenges and long-term issues.

In ***prehistory and protohistory***, information science content and context differ from disciplines with significant historical documentation, in order to list a few: Information on realia is often lost. Information on context is often lost, isolated, or scattered. Historical sources cannot be used in the vast majority of cases. Language-based context is rarely available / preserved. Contextualisation is difficult to achieve. Cognition and insight require and allow multi-disciplinary approaches. Cognition and insight can foster inter- / trans-disciplinary achievements. Realia objects often require special precautions and long-term documentation – and are beyond classical written sources. All participating disciplines, services, and resources have to be prepared for challenges as big data, critical data, accessibility, longevity, and usability.

In ***archaeology***, in context of historical sources, information science content and context show the characteristic: Overall information is widely distributed. Sometimes it can be very difficult and a long lasting challenge not only to create information but even to get access to a few suitable information sources. Digital and realia objects are affected. All participating disciplines, services, and resources have to be prepared for challenges as big data, critical data, accessibility, longevity, and usability.

In both, archaeology of prehistoric and of historic context, the challenges go along with the fact, that even best practice cannot fully preserve realia and data context, context is often destroyed, long-term issues are immanent, and currently there is neither a standard available and used for one discipline nor an international or multi-disciplinary standard. Some essential differences between ***pre- and protohistory*** and ***archaeology in context of historical sources*** may be named, including specialities, goals, and approaches: The search for prehistoric insight can benefit from natural sciences, realia sources, logical / logos-based approaches. Archaeology often benefits from historical sources, realia sources, and further approaches. There is need for an integrated knowledge base for prehistory, archaeology, natural sciences, and humanities. A technical-only approach may target the necessity to collect data from central data centers or registers. Examples of prehistorical, archaeological, and geophysical data [National Park Service 2013] are the North American Database of Archaeological Geophysics (NADAG) [NADAG 2020], the Center for Advanced Spatial Technologies (CAST) [CAST 2020], and the Archaeology Data Service (ADS) [ADS 2020], records as with Center of Digital Antiquity [DigitalAntiquity 2020], and records as with the Digital Archaeological Record (tDAR) [tDAR 2020].

In ***medicine***, information science content and context go along with long-term challenges, e.g., documentation, natural sciences data integration, catalogs (International Classification / Catalog of Diseases, ICD) and universal classification (Universal Decimal Classification, UDC) may require consistent link references, data security, privacy, and anonymity. In ***libraries*** environments, information science content and context go along with long-term challenges, e.g., documentation, catalogues, universality of classification (Universal Decimal Classification, UDC, today about 150,000 libraries are using UDC classification and implementing information systems herewith), referencing, search, and licensing. ***Concordances*** are an important means of bridging implementations when facing heterogeneous realisations and features. For example, this publication / keynote may be associated with various implementations (e.g., Universal Decimal Classification, UDC; American Mathematical Society Mathematics Subject Classification, AMS MSC; Physics and Astronomy Classification Scheme, PACS; ITHEA International Scientific Society), to some extent more or less appropriate:

UDC: 903; 902; 904; 0; 001; 004; 004.01; 004.62; 001.891; 001.92; 021; 165.5; (0.034).

PACS: 02.70.Hm; 89.70.Eg; 93.85.Bc.

MSC: 03A10; 03A05; 68Pxx; 62R07; 94A15; 94A17.

ITHEA: E.0; I.0; H.1.1; F.2; F.4; F.3.

For many aspects, e.g., formalisation, consistency, and meaning, regarding processing, reuse, knowledge complements, locality, and algorithms it is even preferable to deal with knowledge at the most universal level. If that is not achieved, then tackling and breaking consistency is afoot. Avoiding failure means we have to target knowledge and conceptual implementations in practice.

Knowledge and Conceptual Implementations

The Universal Decimal Classification (UDC) [UDCS 2012] is the world's foremost document indexing language in the form of a multi-lingual classification scheme covering all fields of knowledge and constitutes a sophisticated indexing and retrieval tool. The UDC is designed for subject description and indexing of content of information resources irrespective of the carrier, form, format, and language. UDC is an analytico-synthetic and facetted classification. UDC schedules are organised as a coherent system of knowledge with associative relationships and references between concepts and related fields. The Universal Decimal Classification (UDC) is a general plan for the knowledge classification. UDC is a hierarchical decimal classification system that divides the main knowledge fields into 10 main categories (numbered from 0 to 9). Each field is in turn divided into 10 subfields, each subfield is inturn divided into 10 subsubfields, and so on. A more extensive classification code in general describes a more specific subject. UDC-based references in this publication are taken from the multi-lingual UDC summary [UDCS 2012] released by the UDC Consortium (Creative Commons license) [Creative Commons 2012]. The classification deployed for a universal documentation [Rückemann 2012] must be able to describe any object with any relation, structure, and level of detail. Objects include any media, textual documents, illustrations, photos, maps, videos, sound recordings, as well as realia, physical objects such as museum objects. "Facetted" and "multi-disciplinary" is synonym to the Universal Decimal Classification (UDC) [UDCC 2012]. UDC uses a special notation in order to indicate aspect. These descriptions are called facets. In multi-disciplinary object context a facetted classification does provide advantages over enumerative concepts. Facets can be created with any auxiliary tables.

Means to achieve overall efficient realisations, even for complex scenarios: Principles of Superordinate Knowledge, integrating arbitrary knowledge. Core assembly elements of Superordinate Knowledge are methodology, implementation, and realisation [Rückemann 2020b]. Comprehensive focussed subsets of conceptual knowledge provide excellent modular, standardised complements for information systems component implementation, e.g., environmental information management and computation [Rückemann 2018b]. Following references' excerpts are taken from The Prehistory and Archaeology Knowledge Archive (PAKA), DIMF, 2020 [Rückemann 2019a].

As different scenarios (Figure 1), e.g., discovery processes, may require manual and automatable characteristics we will take a look into organisation and procedural deployment with what we call conceptual knowledge reference forks.

Code / Sign Ref.	*Verbal Description (EN)*
UDC:5	Mathematics and natural sciences
UDC:51	Mathematics
UDC:53	Physics
UDC:55	Earth Sciences. Geological sciences
UDC:550	Earth sciences
UDC:550.3	Geophysics
UDC:551.2	Internal geodynamics (endogenous processes)
UDC:551.24	Geotectonics
UDC:9	Geography. Biography. History
UDC:902	Archaeology
UDC:903	Prehistory. Prehistoric remains, artefacts, antiquities
UDC:904	Cultural remains of historical times

Figure 1: Conceptual Knowledge Pattern Matching: Relevant UDC reference implementations [UDCS 2012] regarding the respective matching process, illustrating discovery paths in the decimally organised structure of conceptual knowledge, starting with main tables.

The procedural deployment of conceptual knowledge reference forks [Rückemann 2020a] is illustrated by the Conceptual Knowledge Forks Diagram (CKFD), which can be used to represent a discovery process (Figure 2).

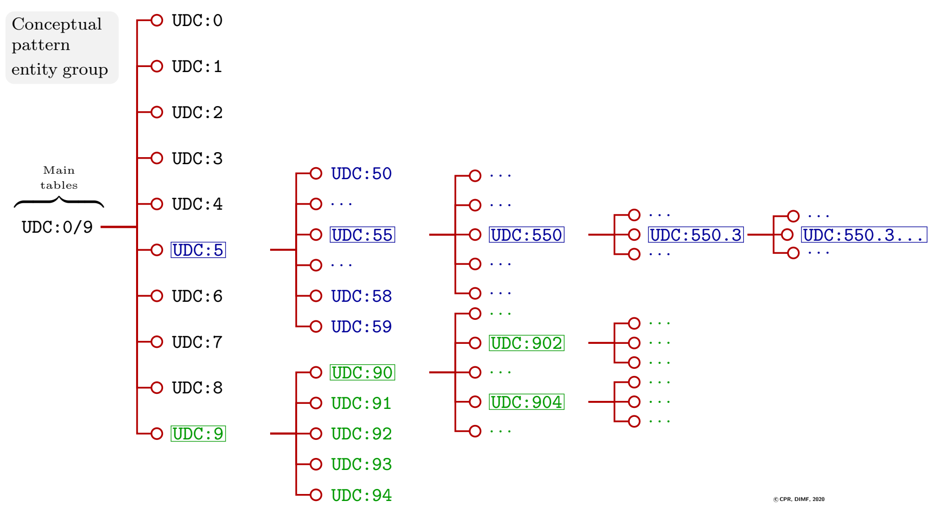

Figure 2: Conceptual Knowledge Pattern Matching: Relevant UDC reference implementations [UDCS 2012] regarding the respective matching process, illustrating discovery paths in the decimally organised structure of conceptual knowledge, starting with main tables.

Resulting Conceptual Knowledge Reference Implementation (CKRI)

The implemented Conceptual Knowledge Reference Implementation (CKRI) as used for pre- and protohistory and arbitrary, universal context from natural sciences to humanities, comprise major divisions of main references. The results are shown in the following excerpts of a core subset. Verbal descriptions are available in 50 languages. The excerpts show verbal descriptions in English language.

CKRI: Universal, main

In these example implementation, universally consistent conceptual knowledge is based on UDC code references for demonstration, spanning the main tables [UDC Consortium 2020h] shown in Table 4.

Table 4: CKRI: Implemented UDC code references, main tables.

Code / Sign Ref.	*Verbal Description (EN)*
UDC:0	Science and Knowledge. Organization. Computer Science. Information. Documentation. Librarianship. Institutions. Publications
UDC:1	Philosophy. Psychology
UDC:2	Religion. Theology
UDC:3	Social Sciences
UDC:5	Mathematics. Natural Sciences
UDC:6	Applied Sciences. Medicine, Technology
UDC:7	The Arts. Entertainment. Sport
UDC:8	Linguistics. Literature
UDC:9	Geography. Biography. History

CKRI: Archaeology, Prehistory, Geography

Major references of geography, biography, history [UDC Consortium 2020e], including prehistory, archaeology, and cultural remains are shown in Table 5.

Table 5: CKRI: Implemented UDC code references of geography, biography, history (excerpt).

Code / Sign Ref.	*Verbal Description (EN)*
UDC:902	Archaeology
UDC:903	Prehistory. Prehistoric remains, artefacts, antiquities
UDC:904	Cultural remains of historical times
UDC:908	Area studies. Study of a locality
UDC:91	Geography. Exploration of the Earth and of individual countries. Travel. Regional geography
UDC:912	Nonliterary, nontextual representations of a region
UDC:92	Biographical studies. Genealogy. Heraldry. Flags
UDC:93/94	History
UDC:94	General history

CKRI: Mathematics and Natural Sciences

Major references of mathematics and natural sciences [UDC Consortium 2020d] are shown in Table 6.

Table 6: CKRI: Implemented UDC code references of mathematics and natural sciences (excerpt).

Code / Sign Ref.	*Verbal Description (EN)*
UDC:51	Mathematics
UDC:52	Astronomy. Astrophysics. Space research. Geodesy
UDC:53	Physics
UDC:54	Chemistry. Crystallography. Mineralogy
UDC:55	Earth Sciences. Geological sciences
UDC:550.3	Geophysics
UDC:551	General geology. Meteorology. Climatology. Historical geology. Stratigraphy. Palaeogeography
UDC:551.21	Vulcanicity. Vulcanism. Volcanoes. Eruptive phenomena. Eruptions
UDC:551.24	Geotectonics
UDC:551.7	Historical geology. Stratigraphy
UDC:551.8	Palaeogeography
UDC:56	Palaeontology
UDC:57	Biological sciences in general
UDC:58	Botany
UDC:59	Zoology

CKRI: Computer Science and Technology, Computing, Data Processing

Major references of computer science and technology [UDC Consortium 2020a] are shown in Table 7.

Table 7: CKRI: Implemented UDC code references of computer science and technology, computing, . . . (excerpt).

Code / Sign Ref.	*Verbal Description (EN)*
UDC:004.2	Computer architecture
UDC:004.22	Data representation
UDC:004.3	Computer hardware
UDC:004.4	Software
UDC:004.41	Software engineering
UDC:004.42	Computer programming. Computer programs
UDC:004.43	Computer languages
UDC:004.45	System software
UDC:004.49	Computer infections
UDC:004.51	Display interface
UDC:004.52	Sound interface
UDC:004.55	Hypermedia. Hypertext
UDC:004.58	User help
UDC:004.62	Data handling
UDC:004.63	Files
UDC:004.65	Databases and their structures
UDC:004.67	Systems for numeric data
UDC:004.7	Computer networks
UDC:004.75	Distributed processing systems
UDC:004.8	Artificial intelligence
UDC:004.91	Document processing and production
UDC:004.92	Computer graphics
UDC:004.93	Pattern information processing
UDC:004.94	Simulation

CKRI: Auxiliaries of Time

Major references from the auxiliaries of time [UDC Consortium 2020g] are shown in Table 8.

Table 8: CKRI: Implemented UDC code references, auxiliaries of time (excerpt).

Code / Sign Ref.	*Verbal Description (EN)*
UDC:“0”	First millennium CE
UDC:“1”	Second millennium CE
UDC:“2”	Third millennium CE
UDC:“3/7”	Time divisions other than dates in Christian (Gregorian) reckoning
UDC:“3”	Conventional time divisions and subdivisions: numbered, named, etc.
UDC:“32”	The year. Seasons and other divisions of the year
UDC:“321/324”	Seasons
UDC:“321”	Spring
UDC:“322”	Summer
UDC:“323”	Autumn (fall)
UDC:“324”	Winter
UDC:“325”	Quarters (quarter years, trimesters)
UDC:“327”	Months
UDC:“328”	Weeks
UDC:“329”	Days
UDC:“34”	Day and night phenomena. Hours or times of day
UDC:“344”	Daytime. Daylight hours
UDC:“345”	Night-time. Hours of darkness or semi-darkness
UDC:“36”	Times of peace, war, danger, emergency, difficulties
UDC:“362”	Peacetime. Time of no danger
UDC:“363”	Time of danger, threat
UDC:“364”	Wartime
UDC:“367”	Times according to volume of use, load, demand
UDC:“37”	Time of work activity, occupation, production, daily routine
UDC:“372”	Working hours. Service hours. Time of occupation
UDC:“377”	Rest and recreation time. Spare time. Free time. Time outside working hours
UDC:“38”	Holidays. Festive and commemorative occasions
UDC:“382”	Religious holidays, festive and commemorative occasions
UDC:“383”	Public, national or regional holidays (other than religious)
UDC:“385”	Personal private holidays, vacation or leave time
UDC:“4”	Duration. Time-span. Period. Term. Ages and age-groups
UDC:“5”	Periodicity. Frequency. Recurrence at specified intervals.
UDC:“6”	Geological, archaeological and cultural time divisions
UDC:“61/62”	Geological (lithological / biological / palaeoecological) time division
UDC:“61”	Precambrian to Mesozoic (from more than 600 to 70 MYBP)
UDC:“62”	Cenozoic (Cainozoic). Neozoic (70 MYBP - present)
UDC:“63”	Archaeological, prehistoric, protohistoric periods and ages
UDC:“67/69”	Time reckonings: universal, secular, non-Christian religious
UDC:“67”	Universal time reckoning. Before Present
UDC:“68”	Secular time reckonings other than universal and the Christian (Gregorian) calendar
UDC:“69”	Dates and time units in non-Christian (non-Gregorian) religious time reckonings
UDC:“7”	Phenomena in time. Phenomenology of time

CKRI: Auxiliaries of Spatial Features and Place

Table 9 shows an excerpt of major auxiliaries of spatial features and place implementation of references [UDC Consortium 2020c].

Table 9: CKRI: Implemented UDC code references of auxiliaries of spatial features and place (excerpt).

Code / Sign Ref.	*Verbal Description (EN)*
UDC:(1)	Place and space in general. Localization. Orientation
UDC:(2)	Physiographic designation
UDC:(3/9)	Individual places of the ancient and modern world
UDC:(3)	Places of the ancient and mediaeval world
UDC:(31)	Ancient China and Japan
UDC:(32)	Ancient Egypt
UDC:(33)	Ancient Roman Province of Judaea. The Holy Land. Region of the Israelites
UDC:(34)	Ancient India
UDC:(35)	Medo-Persia
UDC:(36)	Regions of the so-called barbarians
UDC:(37)	Italia. Ancient Rome and Italy
UDC:(38)	Ancient Greece
UDC:(399)	Other regions. Ancient geographical divisions other than those of classical antiquity
UDC:(4/9)	Countries and places of the modern world
UDC:(4)	Europe
UDC:(5)	Asia
UDC:(6)	Africa
UDC:(7)	North and Central America
UDC:(8)	South America
UDC:(9)	States and regions of the South Pacific and Australia. Arctic. Antarctic

The UDC provides references based on the common auxiliaries of place of the UDC [UDC Consortium 2020c] as excerpted here for facets of place and space, physiographic designation, and places from ancient to modern world.

Tables 10 and 11 provide example excerpts of relevant main conceptual knowledge and details of UDC references used for conceptual mapping. For conceptual knowledge of place and spatial context the implementations requires to provide references to classification codes.

CKRI: Auxiliaries of Boundaries and Spatial Forms

Table 10 shows an excerpt of the major implementation of references of an auxiliary subdivision for boundaries and spatial forms [UDC Consortium 2020c].

Table 10: CKRI: Implemented UDC code references: Auxiliaries of boundaries and spatial forms (excerpt).

Code / Sign Ref.	*Verbal Description (EN)*
UDC:(1-0/-9)	Special auxiliary subdivision for boundaries and spatial forms of various kinds
UDC:(1-0)	Zones
UDC:(1-1)	Orientation. Points of the compass. Relative position
UDC:(1-2)	Lowest administrative units. Localities
UDC:(1-3)	Larger unit within the state
UDC:(1-4)	Units of highest (state) level. Nations. States. Confederations
UDC:(1-5)	Dependent or semi-dependent territories
UDC:(1-6)	States or groupings of states from various points of view
UDC:(1-7)	Places and areas according to privacy, publicness and other special features
UDC:(1-8)	Location. Source. Transit. Destination
UDC:(1-9)	Regionalization according to specialized points of view

CKRI: Auxiliaries of Physiographic Designation

Table 11 shows an excerpt of the major implementation of references of an auxiliary subdivision for physiographic designation [UDC Consortium 2020c].

Table 11: CKRI: Implemented UDC code references of spatial features / place: Auxiliaries of physiographic designation (excerpt).

Code / Sign Ref.	*Verbal Description (EN)*
UDC:(2)	Physiographic designation
UDC:(20)	Ecosphere
UDC:(21)	Surface of the Earth in general. Land areas in particular. Natural zones and regions
UDC:(23)	Above sea level. Surface relief. Above ground generally. Mountains
UDC:(24)	Below sea level. Underground. Subterranean
UDC:(25)	Natural flat ground (at, above or below sea level). The ground in its natural condition, cultivated or inhabited
UDC:(26)	Oceans, seas and interconnections
UDC:(28)	Inland waters
UDC:(29)	The world according to physiographic features

CKRI: Auxiliaries of Form

Table 12 shows an excerpt of the major implementation of references of an auxiliaries of form [UDC Consortium 2020b].

Table 12: CKRI: Implemented UDC code references of auxiliaries of form (excerpt).

Code / Sign Ref.	*Verbal Description (EN)*
UDC:(0.02)	Documents according to physical, external form
UDC:(0.03)	Documents according to method of production
UDC:(0.034)	Machine-readable documents
UDC:(0.04)	Documents according to stage of production
UDC:(0.05)	Documents for particular kinds of user
UDC:(0.06)	Documents according to level of presentation and availability
UDC:(0.07)	Supplementary matter issued with a document
UDC:(0.08)	Separately issued supplements or parts of documents
UDC:(01)	Bibliographies
UDC:(02)	Books in general
UDC:(03)	Reference works
UDC:(04)	Non-serial separates. Separata
UDC:(05)	Serial publications. Periodicals
UDC:(06)	Documents relating to societies, associations, organizations
UDC:(07)	Documents for instruction, teaching, study, training
UDC:(08)	Collected and polygraphic works. Forms. Lists. Illustrations. Business publ.
UDC:(09)	Presentation in historical form. Legal and historical sources
UDC:(091)	Presentation in chronological, historical form. Historical presentation.
UDC:(092)	Biographical presentation
UDC:(093)	Historical sources
UDC:(094)	Legal sources. Legal documents

CKRI: Auxiliaries of Language

Table 13 shows an excerpt of the major implementation of references of an auxiliary subdivision of language [UDC Consortium 2020f].

Table 13: CKRI: Implemented UDC code references of auxiliaries of language (excerpt).

Code / Sign Ref.	*Verbal Description (EN)*
UDC:=1	Indo-European languages of Europe
UDC:=11	Germanic languages
UDC:=12	Italic languages
UDC:=13	Romance languages
UDC:=14	Greek (Hellenic)
UDC:=15	Celtic languages
UDC:=16	Slavic languages
UDC:=17	Baltic languages
UDC:=2	Indo-Iranian, Nuristani (Kafiri) and dead Indo-European languages
UDC:=21	Indic languages
UDC:=29	Dead Indo-European languages (not listed elsewhere)
UDC:=3	Dead languages of unknown affiliation. Caucasian languages
UDC:=35	Caucasian languages
UDC:=4	Afro-Asiatic, Nilo-Saharan, Congo-Kordofanian, Khoisan languages
UDC:=5	Ural-Altaic, Palaeo-Siberian, Eskimo-Aleut, Dravidian and Sino-Tibetan
UDC:=521	Japanese
UDC:=531	Korean
UDC:=541	Ainu
UDC:=6	Austro-Asiatic languages. Austronesian languages
UDC:=7	Indo-Pacific (non-Austronesian) languages. Australian languages
UDC:=8	American indigenous languages
UDC:=81	Indigenous languages of Canada, USA and Northern-Central Mexico
UDC:=82	Indigenous languages of western North American Coast, Mexico and Yucatï£¡n
UDC:=84	Ge-Pano-Carib languages. Macro-Chibchan languages
UDC:=85	Andean languages. Equatorial languages
UDC:=86	Chaco languages. Patagonian and Fuegian languages
UDC:=88	Isolated, unclassified Central and South American indigenous languages
UDC:=9	Artificial languages
UDC:=92	Artificial languages for use among human beings. Int. aux. languages (interlanguages)
UDC:=93	Artificial languages used to instruct machines. Programming/computer languages

CKRI: Operations, Creating Facets, Groups, and References

Table 14 shows common standardised operations with UDC, which can be used with the creation of facets [UDCS 2020]. Facets can be part of the conceptual knowledge of knowledge resources objects as already shown.

Table 14: CKRI: Standard UDC code operations.

Operation	*Symbol*
Addition	+
Consecutive extension	/
Relation	:
Order-fixing	::
Subgrouping	[ ]
Non-UDC notation	∗
Alphabetic extension	A-Z

Table 15 shows some basic examples including facets. Ellipses (". . .") indicate reduced depth of shown references.

Table 15: Examples of conceptual knowledge references.

Example Code / Sign Ref.	*Example Description*
UDC:311:903[622+669](4)	Statistics of prehistoric mining and metallurgy in Europe
UDC:004.38:[550.8+903+902]	(Computer) simulation referring to geophysics, prehistory, and archaeology
UDC:[903]:"323"	Prehistoric remains referring to autumn/fall
UDC:7. . .(36)	Prehistoric art in the regions of the so-called barbarians
UDC:[903]:7. . .	Prehistoric remains referring to art, . . .
UDC:53/55	From physics and chemistry, crystallography, mineralogy to earth sciences, geological sciences

The references, e.g., classification, facets, concordances, and textual description, are usable in all the procedures and steps and allow to consider and implement arbitrary flexibility of fuzziness. Entry points to relevant and associated knowledge may be in any disciplinary context due to the consistent framework of the UDC and the multi-disciplinary and multi-lingual Knowledge Resources. In our practical research projects and implementations, deploying a modular integration of consequent knowledge resources' components and their development with means of conceptual knowledge pattern matching have proven to enable valuable solutions for challenging and complex cases in described disciplines and arbitrary context.

Conclusion

It was shown that conceptual knowledge approaches can enable sustainable long-term knowledge creation, development, and integration. The knowledge resources created and developed are successfully used in many application scenarios for more than three decades, including the resources on pre- and protohistory, archaeology with historical context, geosciences, natural sciences, and humanities.

Special focus was taken on universality (multi-disciplinarity, multi-linguality, attributation, time, space, relocation, . . .), consistency (methodologies, methods, approaches, and practice, especially editions), and long-term sustainability (decades, centuries). When dealing with challenges regarding these issues we must be aware that any formalisation means abstraction and reduction. The more, it is important to implement every step knowing that a 'solid' implementation of information science tasks, e.g., information processing require a 'solid' understanding of information science fundaments. Therefore, any software and hardware supported approaches should be considered tools. Especially, we should always remember that formal systems and algorithms are tools, that knowledge itself cannot be created by formal systems, and that algorithms can neither be intelligent nor create intelligence. A recommended best practice is that procedural implementations should not be done without serious consideration of other knowledge complements, respective methodologies, and structural fundaments.

Among many disciplines, prehistory is unique in a way that in many cases neither common approaches using direct comparisons to documented contexts in presence nor historical sources can be used, not for the creation of new insight, not for documenting knowledge objects. Information science can provide long-term strategies and solutions such as demonstrated. A solution for universality, consistency, and long-term sustainability can be provided by recognising knowledge complements being assets of information science. Based on that we can provide solutions to application scenarios and these can provide fundaments for creating new instruments.

Therefore, further recommendations are to consequently foster scientific research, fundamental, epistemologic background – insight, universality, consistency, and sustainability. We should keep in mind that many disciplines, especially prehistory, provide a higher potential of creating insight from multi-disciplinary scientific research. We should enable contextualisation and cognitive insight and create and provide instruments based on knowledge-based standards. We should ensure to enable continuity while creating knowledge-centric, modular implementations and we should consider knowledge targetting implementation aspects, in this context knowledge-centricity, data-centricity, concordances, structure, processing, and computing. In summary, we should involve experienced, dedicated experts in these fields.

When dealing with knowledge and information, participated parties should learn how to decide on multi-dimensional aspects of knowledge complements, how to decide on intrinsic and extrinsic (information) properties, how to deploy logic / logos, how to deploy knowledge complements and conceptual knowledge references, about the hermeticism of 'quality'.

Concrete future research with the presented long-term projects will focus on prehistory knowledge resources, contextualisation, multi-disciplinary perspectives, and documentation.

Acknowledgements

We are grateful to the “Knowledge in Motion” (KiM) long-term project, Unabhängiges Deutsches Institut für Multi-disziplinäre Forschung (DIMF), for partially funding this research, implementation, case studies, and publication under grants D2018F1P04962, D2018F6P04938, D2019F1P04998, and D2020F1P05228 and to its senior scientific members and members of the permanent commission of the science council, especially to Dr. Friedrich Hülsmann, Gottfried Wilhelm Leibniz Bibliothek (GWLB) Hannover, to Dipl.-Biol. Birgit Gersbeck-Schierholz, Leibniz Universität Hannover, to Dipl.-Ing. Martin Hofmeister, Hannover, and to Olaf Lau, Hannover, Germany, for fruitful discussion, inspiration, practical multi-disciplinary case studies, and the analysis of advanced concepts. We are grateful to Dipl.-Ing. Hans-Günther Müller, HPE, Germany, for his excellent contributions and assistance providing practical private cloud and storage solutions. We are grateful to the members of the Eastern Mediterranean research and studies campaign 2018–2020, DIMF, and all national and international partners in the Geo Exploration and Information cooperations for their constructive and trans-disciplinary support. Many thanks go to the scientific colleagues at the Westfälische Wilhelms-Universität (WWU), as well as to my postgraduate students in the European Legal Informatics Study Programme (EULISP), Leibniz Universität Hannover, for prolific discussion, participating in case studies on knowledge resources, and sharing scientific, multi-disciplinary, technical, and computational experiences over the last years. We are grateful to The Science and High Performance Supercomputing Centre (SHPSC) for long-term support. / DIMF-PIID-DF98_007.

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Highly Efficient Monte Carlo Approaches for Multidimensional Integrals for Options Evaluation

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Abstract: *In the last few years new approaches have been developed that outperform standard Monte Carlo in terms of numerical efficiency. It has been found that there can be efficiency gains in using deterministic sequences rather than the random sequences which are a feature of standard Monte Carlo. These deterministic sequences are carefully selected so that they are well dispersed throughout the region of integration. Sequences with this property are known as low discrepancy sequences. These sequences are often more efficient than standard Monte Carlo in evaluating high dimensional integrals if the integrand is sufficiently regular and for many finance applications this is the case. In the present paper we evaluate European style options with an exponential payoff function with a lattice rule based on Fibonacci generalized vectors of the corresponding dimensionality.*

Keywords: *Multidimensional integration, Monte Carlo methods, Option Pricing.*

MSC: *65C05, 65F10, 91G20*

ITHEA Keywords: *NUMERICAL ANALYSIS: Applications, SIMULATION AND MODELLING: Applications.*

Introduction

The pricing of options is a very important problem encountered in financial markets today. The famous Black-Scholes model provides explicit closed form solutions for the values of European style call and put options. Options have been widely traded since the creation of an organized exchange in 1973 Boyle [1977]. Much of the focus to-date has been on high-dimensional problems since these are more challenging from a computational viewpoint.

Nowadays Monte Carlo method has become a popular computational device for problems in finance Dimov [2010]. The finance discipline has become more sophisticated and more quantitative in the last two decades. New approaches have been developed that outperform standard Monte Carlo in terms of numerical efficiency. It has been found that there can be efficiency gains in using deterministic sequences rather than the random sequences which are a feature of standard Monte Carlo. These deterministic sequences are carefully selected so that they are well dispersed throughout the region of integration. Sequences with this property are known as low discrepancy sequences. These sequences are often more efficient than standard Monte Carlo in evaluating high dimensional integrals if the integrand is sufficiently regular and for many finance applications this is the case. The Monte Carlo method has proven to a very useful tool for numerical analysis, particularly when the number of dimensions ranging from medium to large Wilmott et al. [1995].

Much of the focus to-date has been on high-dimensional problems since these are more challenging from a computational viewpoint. However, it is also of interest to examine low to medium dimension problems. Low to medium sized problems are of practical interest since there are popular contracts whose value depends on a small to medium number of variables. Here are a few examples Boyle et al. [2001]:

Options whose payoff depends on the relative performance of two underlying assets. A particular version of this option known as a spread option popular in the energy industry.

Basket options where the payoff depends on the ending values of a number of assets such as different common stocks or stock market indices. The payoff could be based on the average, the maximum or the minimum of the asset prices.

Path dependent options where the payoff is a function of the asset price at a number of discrete monitoring points along the path. In the case of Asian options the payoff is based on the average of these points. In the case of lookback options the payoff is based on the largest (or smallest) value recorded at one of these monitoring points. The dimensions of the problem are directly related to the number of discrete points in the path.

Monte Carlo and/or quasi-Monte Carlo methods can be directly applied to finance problems involving multidimensional integrals Dimov [2003]. For example, Paskov uses a quasi-Monte Carlo sequence the Sobol sequence - to find the present value of securities which involve up to 360 dimensional integrals; see Paskov [1994].

The basic definitions are taken form Lai and Spanier [1998].

A European call option is a contract such that the owner may (without obligation) buy some prescribed asset (called the underlying) S at a prescribed time (expiry date) T at a prescribed price (exercise or strike price) E.

A European put option is the same as a call option, except that "buy" is replaced by "sell" .

A look-back option is an option whose payoff depends not only on the asset price at expiry, but also on the maximum or minimum of the asset price over some time prior to expiry. Risk neutrality is the characteristic ascribed to an investor who is indifferent with respect to risk. A rigorous definition may be given (see Broadie and Glasserman [1997] or Duffie [1988]) based on an order relationship on a space of random variables on an appropriately defined probability space. However, the most important use of this notion for us is in its application to the risk-neutral evaluation formula.

The risk-free interest rate r is an idealized interest rate, usually taken to be that of an appropriate Treasury Bond.

The Wiener process (also called Brownian motion) dX is a special type of Markov stochastic process with the following properties: $dX \sim N(0, \sqrt{(dt)})$, where $N(\mu, \sigma)$ is the normal distribution with mean μ and variance σ^2.

Multidimensional integrals related to evaluation of European style options

One of the basic problems in option pricing is: given the current price of an asset S, the strike price E, the time to expiry T, the risk-free interest rate r, and the equation that is assumed to control the behavior of S as a function of time t:

$$dS = \mu S dt + \sigma S dX, \tag{1}$$

where dX is a Wiener process , μ (a measure of the average rate of growth of the asset price) is the drift rate and σ is the volatility of the asset (characterizing fluctuations in the price S), how may one determine a "fair" current value $V(S,t)$ of the option? The well-known Black-Scholes model for a European call option can be described (**?** or Wilmott et al. [1995]) by the following (diffusion-type) partial differential equation for this value:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0, \tag{2}$$

with final condition

$$V(S,T) = max(S-E,0),$$

and boundary conditions

$$V(0,t) = 0, V(S,t) \sim S, S \to \infty.$$

The European put option satisfies the same equation as (2), but with final condition

$$V(S,T) = max(E-S,0),$$

and boundary conditions

$$V(0,t) = Ee^{-r(T-t)}, V(S,t) \sim 0, S \to \infty.$$

In both cases, there are explicit closed form solutions. For the call option, the solution is

$$V(S,t) = C(S,t) = SN(d1) - Ee^{-r(T-t)}N(d2),$$

with

$$d_1 = \frac{ln(\frac{S}{E}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt(T-t)}$$

and

$$d_2 = \frac{ln(\frac{S}{E}) + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt(T-t)}$$

and $N(z)$ is the cumulative distribution function of the standard normal distribution. For the put option,

$$V(S,t) = P(S,t) = Ee^{-r(T-t)}N(-d2) - SN(-d1),$$

with the same $d1, d2$, and $N(z)$.

Monte Carlo methods can be very useful in such cases if the solution (i.e., the value, V) can be expressed as the expectation of some random variable(s). This is made possible by the risk-neutral valuation formula for the European option Broadie and Glasserman [1997]:

$$V(S,t) = E(e^{r(Tt)}h(S(T)) \mid S(t) = S, \mu = r) \tag{3}$$

where $E(.)$ is the expectation, $h(S)$ is the payoff function, $h(S) = max(S-E,0)$ for a call option and $h(S) = max(E-S,0)$ for a put option.

We follow the idea of Lai and Spanier in Lai and Spanier [1998]. Consider a European option whose payoff depends on $k > 1$ assets with prices $S_i, i = 1, ..., k$. Each asset follows the random walk

$$dS_i = \mu_i S_i dt + \sigma_i S_i dX_i$$

where σ_i is the annualized standard deviation for the i-th asset and dX_i is Brownian motion. Suppose at expiry time T that the payoff is given by $h(S_1^{'}, \ldots, S_k^{'})$ (where $S^{'}$ denotes the value of the i-th asset at expiry). Then the current value, V, of the option (assuming risk neutrality) will be

$$V = e^{-r(T-t)}(2\pi(T-t))^{-k/2}(det\Sigma)^{-1/2}(\sigma_1, \ldots, \sigma_k)^{-1} \tag{4}$$

$$\int_0^\infty \cdots \int_0^\infty \frac{h(S_1^{'}, \ldots, S_k^{'})}{S_1^{'}, \ldots, S_k^{'}} exp(-0.5\alpha^T\Sigma^{-1}\alpha)dS_1^{'}, \ldots, dS_k^{'}, \tag{5}$$

where

$$\alpha_i = (\sigma_i(T-t)^{1/2})^{-1}(log\frac{S_i^{'}}{S_i} - (r - \frac{\sigma^2}{2})(T-t)),$$

r is the risk-free interest rate and Σ is the covariance matrix,where the (i,j) entry is the covariance of dX_i and dX_j for the k assets. The infinite domain of integration can be mapped into the k-dimensional unit hypercube in a variety of ways. For example, $\frac{2}{\pi}\arctan(x)$ maps $(0,\infty)$ to $(0,1)$. Such a mapping transforms the problem to one in which an integral $\int_0^1 \ldots \int_0^1 g(x_1,\ldots,x_k)dx_1\ldots dx_k$ over the hypercube is sought. When g is the exponent function, with appropriate choices of the constants involved in the equation for the value of the option (4) according to Lai and Spanier [1998] we obtain k dimensional integral $\int\limits_{[0,1]^k} exp(x_1,\ldots,x_k)dx_1\ldots dx_k$. We compare four very effective and highly accurate techniques for numerical integration for solving the last integral.

Monte Carlo algorithms for numerical integration

Lattice rules are based on the use of deterministic sequences rather than random sequences. They are a special type of so-called low discrepancy sequences. It is known that as long as the integral is sufficiently regular, lattice rules generally outperform not only basic Monte Carlo, but also other types of low discrepancy sequences. It is well known that Sobol algorithm has some advantageous over the other low discrepancy sequences such as Halton as it is shown its superiority for high dimensional integrals up to 360 dimensions by Paskov Paskov [1994]. The monograph of Sloan and Joe [1994] provide comprehensive expositions of the theory of integration lattices. We implemented a specific lattice rule and compared its performance with an implementation of Sobol, Crude and the Adaptive method over integrals of smooth functions.

First we will demonstrate the power of the plain Monte Carlo over the deterministic methods Dimov [2003]. Suppose $f(x)$ is a continuous function and let a quadrature formula of Newton or Gauss type be used for calculating the integrals. Consider an example with $d = 20$. We generate a grid in the 20-dimensional domain and take the sum of the function values at the grid points. Let a grid be chosen with 20 nodes on the each of the coordinate axes in the 20-dimensional cube $G = [0,1]^{20}$. In this case we have to compute about 10^{20} values of the function $f(x)$. Suppose a time of $10^{-7}s$ is necessary for calculating one value of the function. Therefore, a time of order 10^{13}s will be necessary for evaluating the integral (remember that 1 year = 31536×10^3s, and that there has been less than 9×10^{10}s since the birth of Pythagoras). If the formula of rectangles is applied then the error in the approximate integral calculation is

$$\epsilon \leq cMh^3,$$

where $h = 0.1$ is the mesh size, c is constant independent of h and M is the maximal value of the second derivative.

Consider a plain Monte Carlo algorithm for this problem with a probable error of the same order. The algorithm itself consists of generating N pseudo random values (points) (PRV) in G; in calculating the values of $f(x)$ at these points; and averaging the computed values of the function. For each uniformly distributed random point we have to generate 20 random numbers uniformly distributed in $[0,1]$. The probable error is:

$$\epsilon \leq 0.6745\sigma(\theta)\frac{1}{\sqrt{N}}, \tag{6}$$

where $\sigma(\theta)$ is the standard deviation of random variable θ for which $E\theta = \int_G f(x)p(x)dx$ and N is the number of realizations of the random variable. The probable error is estimated:

$$\epsilon \leq 0.6745||f||_{L_2}\frac{1}{\sqrt{N}}. \tag{7}$$

From above equations we conclude that

$$N \approx (\frac{0.6745||f||_{L_2}}{cM})^2 \times h^{-6}. \tag{8}$$

Suppose that the expression in front of h^{-6} is of order 1. Since $h = 0.1$, we have $N \approx 10^6$; hence, it will be necessary to generate $20 \times 10^6 = 2 \times 10^7$ PRV. Usually, two operations are sufficient to generate a single PRV. Suppose that the time required to generate one PRV is the same as that for calculating the value of the function at one point in the domain. Therefore, in order to solve the problem with the same accuracy, a time of

$$2 \times 10^7 \times 2 \times 10^{-7} \approx 4s$$

will be necessary. The advantage of the Monte Carlo algorithms to solve such problems is obvious, in the case of 20 dimensional integral it is 2.5×10^{12} times faster than the deterministic one.

The Crude Monte Carlo method has rate of convergence $O(N^{-1/2})$ which is independent of the dimension of the integral, and that is why Monte Carlo integration is the only practical method for many high-dimensional problems. Much of the efforts to improve Monte Carlo are in construction of variance reduction methods which speed up the computation or to use quasi-random sequences. A quasi-random or low discrepancy sequence, such as the Faure, Halton, Hammersley, Niederreiter or Sobol sequences, is "less random" than a pseudo-random number sequence, but more useful for such tasks as approximation of integrals in higher dimensions, and in global optimization. This is because low discrepancy sequences tend to sample space "more uniformly" than random numbers. We use a specific implementation of Sobol sequence that is an adaptation of the INSOBL and GOSOBL routines Bratley and Fox [1988] in ACM TOMS Algorithm 647 and ACM TOMS Algorithm 659. The routine adapts the ideas of Antonov and Saleev Antonov and Saleev [1980]. The original code can only compute the "next" element of the sequence. The revised code allows the user to specify the index of the desired element. The algorithm has a maximum spatial dimension of 40 since MATLAB doesn't support 64 bit integers. A remark by Joe and Kuo shows how to extend the algorithm from the original maximum spatial dimension of 40 up to a maximum spatial dimension of 1111. The FORTRAN90 and C++ versions of the code has been updated in this way, but updating the MATLAB code has not been simple, since MATLAB doesn't support 64 bit integers. We generate a new quasi-random Sobol vector with each call. The parameters of the algorithm are an integer DIM_{NUM}, the number of spatial dimensions. The algorithm starts with integer $SEED$, the "seed" for the sequence. This is essentially the index in the sequence of the quasi-random value to be generated. On output, $SEED$ has been set to the appropriate next value, usually simply $SEED + 1$. Output is the real $QUASI(DIM_{NUM})$, the next quasi-random vector.

The Adaptive Monte Carlo algorithm that we developed is based on the ideas and results of the importance separation Georgieva [2010], a method that combines the idea of separation of the domain into uniformly small subdomains with the importance sampling approach. The Adaptive method does not use any a priori information about the smoothness of the integrand, but it uses a posteriori information about the variance. The idea of the method consists of the following (see for instance Dimov [2003]; Georgieva [2010]): the domain of integration G is separated into subdomains with identical volume. The interval [0,1] on every dimension coordinate is partitioned into M subintervals, i.e.

$$G = \sum_j G_j, \; j = 1, M^d.$$

Denote by p_j and I_{G_j} the following expressions:

$$p_j = \int_{G_j} p(x)dx, \; I_{G_j} = \int_{G_j} f(x)p(x)dx.$$

Consider now a random point $\xi_i^{(j)} \in G_j$ with a density function $p(x)/p_j$ and in this case

$$I_{G_j} = \mathrm{E}\left[\frac{p_j}{N}\sum_{i=1}^{N} f(\xi_i^{(j)}))\right] = \mathrm{E}\theta_N.$$

We start with a relatively small number M which is given as input data. For every subdomain the integral I_{G_j} and the variance are evaluated. After that the variance is compared with a preliminary given value ε. If the calculated variance in any region is greater than some constant ε, then this region is divided to new M^d subregions, again by partitioning the segment of the region on every coordinate to M subintervals. The algorithm is described below.

Algorithm

1. **Input data**: *number of points* N*, constant* ε *(estimation for the variance), constant* δ *(stop criterion; estimation for the length of subintervals on every coordinate).*

2. **For** $j = 1,\ M^d$:

   2.1. Calculate *the approximation of* I_{Ω_j} *and the variance* $\mathbf{D}_{\Omega_j}$ *in subdomain* Ω_j *based on* N *independent realizations of random variable* θ_N;

   2.2. If $(\mathbf{D}_{\Omega_j} \geq \varepsilon)$ **then**

   2.2.1. Choose *the axis direction on which the partition will perform*,

   2.2.2. Divide *the current domain into two* (G_{j_1}, G_{j_2}) *along the chosen direction*,

   2.2.3. If *the length of obtained subinterval is less than* δ **then go to** *step 2.2.1*
   else $j = j_1\ (G_{j_1}\ is\ the\ current\ domain)$ *and* **go to** *step 2.1*;

   2.3. Else if $(\mathbf{D}_{\Omega_j} < \varepsilon)$ *but an approximation of* $I_{G_{j_2}}$ *has not been calculated yet*,
   then $j = j_2\ (G_{j_2}\ is\ the\ current\ domain\ along\ the\ corresponding\ direction)$ *and* **go to** *step 2.1*;

   2.4. Else if $(\mathbf{D}_{\Omega_j} < \varepsilon)$ *but there are subdomains along the other axis directions*, **then go to** *step 2.1*;

   2.5. Else *Accumulation in the approximation* I_N *of* I.

More detailed information about **Lattice sets** can be found in the work of Wang and Hickernell Wang and Hickernel [2002]. Let G_s denote the unit cube in s-dimensional space, i.e.,

$$G_s = [0,1)^s = \{x = (x_1, \ldots, x_s)\} \mid 0 \leq x_j < 1, j = 1, \ldots, s\}. \tag{9}$$

Let $n_1 < n_2 < \ldots$ be a sequence of positive integers, and let P_{n_l} be any set of n_l points in G_s. (Here a set may have multiple copies of the same point.) For any $r = (r_1, \ldots, r_s) \in G_s$ note that $r_1, \ldots, r_s$ is the volume of the box $[0, r)$. Let $N_{n_l}(r)$ denote the number of points in P_{n_l} lying inside the box $[0, r)$. The discrepancy of the set P_{n_l} is defined as the largest difference between the proportion of points in the box and the volume of the box:

$$D(n_l) := \sup_{r \in G_s} |\frac{N_{n_l}(r)}{n_l} - r_1 \ldots r_s| \tag{10}$$

This notion was introduced by Weyl (1916). If $D(n_l) = o(1)$ as $n_l \to \infty$, then the sequence of sets P_{n_l} , $n_1 < n_2 < \ldots$ is said to be uniformly distributed on G_s with discrepancy $D(n_l)$. The subscript l is often omitted for simplicity. Not only is the discrepancy a geometric method for measuring uniformity of a set, the discrepancy of a set measures its quality for use in numerical quadrature. The error of this approximation is bounded by the Koksma-Hlawka inequality:

$$|\int_{G_s} f(x)dx - \frac{1}{n}\sum_{k=1}^{n} f(x_k)| \leq D(n).V(f), \tag{11}$$

where $D(n)$ is the discrepancy of the set $P(n) = \{x_1, \ldots, x_n\}$, and $V(f)$ is the bounded variation of f in the sense of Hardy and Krause. If the integrand is smoother and also periodic, then better error bounds may be obtained, in particular for quadrature rules using lattice point sets.

The lattice S is an infinite set of points in R^s with the following three properties:

1. If x and x' belong to S, then $x+x'$ and $x-x'$ also belongs to S.
2. S contains s linearly independent points.
3. There exists a sphere centered at 0 that contains only 0 itself.

A multiple-integration lattice is a lattice that contains $\mathbb{Z}^s$ as a sub-lattice. By a "lattice rule" then, we shall mean a rule of the form

$$I_N(f) = \frac{1}{N}\sum_{j=0}^{N-1} f(x_j),$$

in which $x_0, \dots, x_{N-1}$ are all the points of a multiple-integration lattice that lie in $[0,1]^s$. The cubic lattice is

$$\left\{\left(\frac{j_1}{n}, \dots, \frac{j_s}{n} : j_i \in \mathbb{Z}, 1 \leqq i \leqq s\right)\right\},$$

where n is a positive integer. The corresponding lattice rule is the "rectangle rule"

$$I_N(f) = \frac{1}{N}\sum_{j_1=0}^{n-1} \cdots \sum_{j_s=0}^{n-1} f\left(\frac{j_1}{n}, \dots, \frac{j_s}{n}\right),$$

where $N = n^s$. Because N rises very rapidly with s, the rectangle rule suffers in a very obvious way from the "curse of dimensionality." Note that this rule is equivalent, because of the assumed periodicity, to a product-trapezoidal rule.

Let n be an integer, $n \geq 2$ and $a = (a_1, a_2, \dots a_s)$ be an integer vector modulo n. A set of the form

$$P_n = \left\{\left\{\frac{ak}{n}\right\} = \left(\left\{\frac{a_1 k}{n}\right\}, \dots, \left\{\frac{a_s k}{n}\right\}\right) \mid k = 1, \dots, n\right\}$$

is called a lattice point set, where $\{x\}$ denotes the fractional part of x. The vector a is called called a lattice point or generator of the set. As one can see, the formula for the lattice point set is simple to program. The difficulty lies in finding a good value of a, such that the points in the set are evenly spread over the unit cube. The choice of good generating vector, which leads to small errors, is not trivial. Complicated methods from theory of numbers are widely used, for example Zaremba's index or error of the worst function. Korabov consider the following vectors:

$$a = (1, a, a^2, \dots, a^{s-1}) \, mod \, N, 1 \leq a \leq N-1, gcd(a, N) = 1.$$

The method can be applied only for for number of points $n_l = F_l^{(s)}$, i.e. only for generalized Fibonacci number of points. This set used the generating vector

$$a = (1, F_{l+1}^{(s)}, ..., F_{l+s-1}^{(s)})), \quad n_l = F_l^{(s)},$$

where $F^{(s)}$ is the corresponding generalized Fibonacci number of dimensionality s:

$$F_{l+s}^{(s)} = F_l^{(s)} + F_{l+1}^{(s)} + ... + F_{l+s-1}^{(s)}, l = 0, 1, \dots$$

with initial conditions:

$$F_0^{(s)} = F_1^{(s)} = \dots = F_{s-2}^{(s)} = 0, F_{s-1}^{(s)} = 1,$$

for $l = 0, 1, \dots$.

The discrepancy of the set obtained by using the vector described above is asymptotically estimated in Wang and Hickernel [2002]. We have the following estimation:

$$D(n_l) = \mathcal{O}(n_l^{-\frac{1}{2}-\frac{1}{2^{s+1}.\log 2}-\frac{1}{2^{2s+3}}}).$$

The advantage of the Lattice method is the linear computational complexity and reduced time for calculating the multidimensional integrals. The number of calculation required to obtain the generating vector is asymptotically less than $\mathcal{O}(N_l)$. The generation of a new point requires constant number of operations thus to obtain a lattice set of the described kind consisting of N_l points, $\mathcal{O}(N_l)$ number of operations are necessary.

Numerical examples and results

Our experimental results include the evaluation of the following 3 and 20 dimensional integrals:

$$\int_{[0,1]^3} exp(x_1x_2x_3) \approx 1.14649907. \quad (12)$$

$$\int_{[0,1]^{20}} exp(\prod_{i=1}^{20} x_i) \approx 1.00000949634. \quad (13)$$

The results are given in the tables below. Each table contains information about the MC approach (Crude for the plain Monte Carlo, Adaptive for the Adaptive approach, Sobol for the Sobolalgorithm and Lattice for the Lattice method developed in this study), the obtained relative error, the needed CP-time and the number of points. Note that when the lattice method is tested, all of these numbers are Generalized Fibonacci numbers of the corresponding dimensionality. We have used CPU Intel Core i5-2410M @ 2.30GHz and MATLAB. The advantage of the Lattice method for low dimensions is superior. For higher dimensions Sobol sequence has advantages for fixed number of points and it is slightly worse than Lattice for a fixed computational time. The Adaptive Monte Carlo is better than Crude Monte Carlo after some seconds, and it strength is when the integrand have some pecularities.

Table 1: The relative error for 3 dimensional integral

N	Crude	time	Adaptive	time	Lattice	time	Sobol	time
19513	8.93e-4	0.01	3.21e-4	2.21	4.69e-4	0.02	4.98e-5	0.56
35890	2.18e-3	0.04	6.55e-5	6.41	5.46e-6	0.06	1.56e-5	1.45
66012	5.65e-4	0.07	5.12e-5	9.86	5.34e-6	0.11	8.11e-6	2.31
121415	6.46e-4	0.12	5.11e-5	15.4	5.34e-6	0.12	3.08e-6	3.80
223317	4.15e-4	0.20	9.34e-5	24.2	1.73e-6	0.22	2.05e-6	6.13

The advantage of the lattice method for the (12) integral is superior, it gives $1.32e-6$ for 0.1s and $3.22e-7$ for 1s in Table 1, while Sobol needs 20s to achieve the same accuracy. While for more computational time Sobol improves a lot - see Table 2, Adaptive and Crude algorithms performs in similar way and the results are worse and as can be seen Adaptive Monte Carlo is better than Crude Monte Carlo after $5s$. It can be concluded that the best approach for lowest dimensions is the Lattice method, which completely outperforms the other methods - see Figure 1.

We expect that for 20 dimension Sobol is the best but again Lattice method has minor advantage over the Sobol for a fixed time for the (13) integral. In Table 4, for 1s it produces error $1.48e-5$, which is the same result as Sobol

Table 2: Times for 3 dimensional integral

time,s	Crude	Adaptive	Lattice	Sobol
0.1	6.56e-4	8.67e-4	1.32e-6	3.21e-4
1	1.37e-4	2.96e-5	3.22e-7	8.21e-5
2	5.29e-5	5.45e-4	2.06e-7	2.96e-5
5	1.84e-4	1.14e-4	1.47e-7	5.00e-6
10	7.79e-5	6.56e-5	3.89e-7	2.71e-6
20	4.57e-5	2.04e-5	1.53e-8	1.88e-6

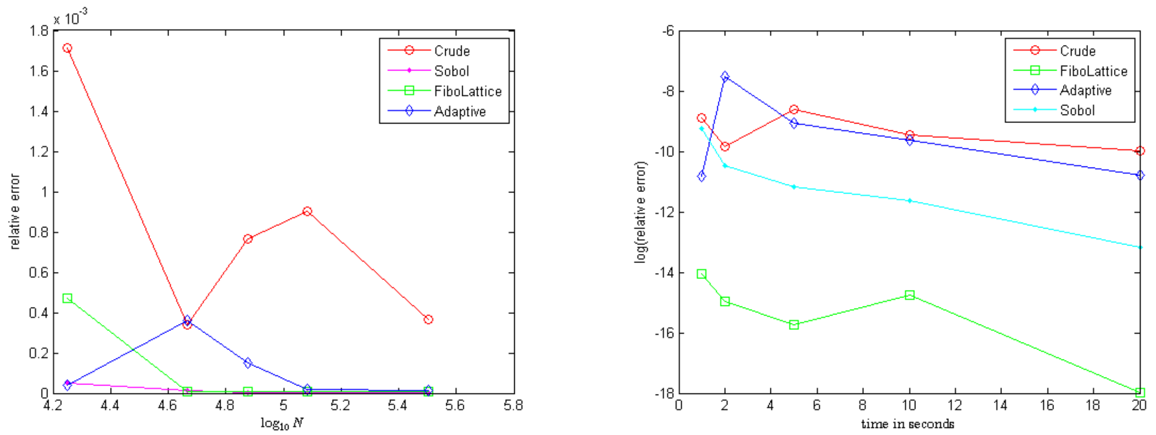

Figure 1: Relative error and computational time for 3 dimensional integral with Monte Carlo and quasi Monte Carlo methods.

Table 3: The relative error for 20 dimensional integral

N	Crude	time	Adaptive	time	Lattice	time	Sobol	time
2048	2.84e-2	0.02	1.14e-2	8.6	8.22e-5	0.03	8.44e-4	0.13
16384	8.23e-4	0.12	4.96e-4	60.3	3.12e-5	0.13	6.82e-5	1.68
65536	8.61e-3	0.91	9.75e-4	474.2	1.36e-5	1.17	8.34e-6	8.69
131072	4.13e-4	2.13	1.25e-5	888.3	8.85e-6	2.34	3.77e-6	14.36
524288	1.22e-4	8.13	1.96e-6	2356	2.15e-6	8.34	1.91e-7	57

Table 4: Times for the 20 dimensional integral

time,s	Crude	Adaptive	Lattice	Sobol
1	9.14e-3	1.58e-3	1.48e-5	3.25e-5
2	3.68e-3	1.028e-3	9.17e-6	3.97e-5
5	2.67e-3	8.58e-4	5.19e-6	1.45e-5
10	3.34e-4	4.02e-4	1.73e-6	2.71e-6
20	1.53e-4	1.13e-4	1.38e-7	1.76e-6

for 5s. However, for the same number of points Sobol gives better accuracy - see Table 3. For the 20 dimensional integral Adaptive algorithm performs better than Crude algorithm and has better accuracy for the same number of points closer to Lattice method. The Lattice algorithm is the fastest method and it is definitely the choice when one needs to have a very good accuracy for less than a minute on a laptop - see Figure 2.

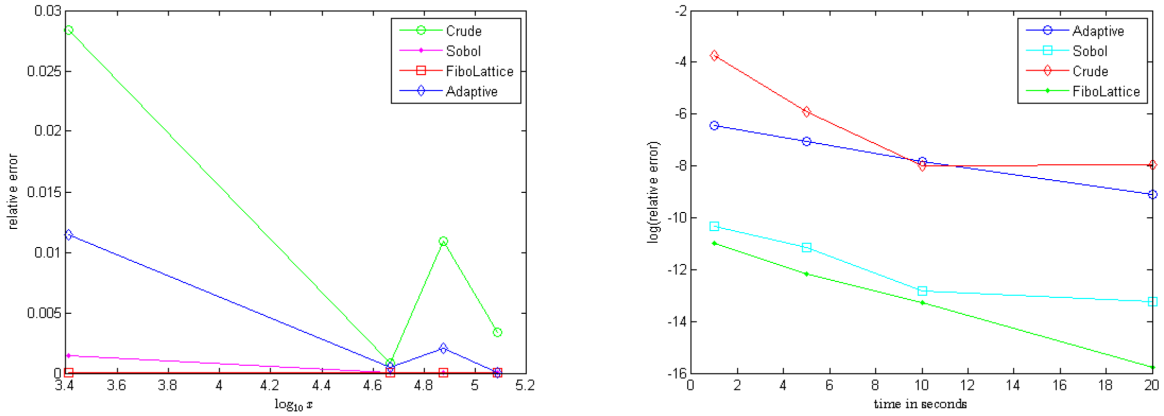

Figure 2: Relative error and computational time for 20 dimensional integral with Monte Carlo and quasi Monte Carlo methods.

Conclusion

This paper shows that a particular type of low discrepancy sequence known as lattice rules have strong advantages in the case of low to medium dimension problems as long as the integrands are sufficiently regular. The quasi-random Sobol sequence is slower but gives closer accuracy, being better for a fixed number of points for higher dimensions of the integral. The Adaptive algorithm requires more computational time to achieve better accuracy, gives good results regardless of the dimensions and it is the best method when the integrand functions are not smooth. The plain(Crude) Monte Carlo algorithm gives worst, but fast results closer to this obtained by the lattice rule. The four approaches are completely different thus it is a question of interest to know which one of them outperforms the other. The main advantage of the presented Lattice algorithm is its linear computational complexity for very high dimensions, where the deterministic methods suffer from the so-called curse of dimensionality and become impractical. The experimental results from Tables show that the Lattice algorithm has the best performance with respect to relative error and computational time. The progress on the problem of option pricing and the computational finance area is closely related to the development of reliable algorithms for multidimensional numerical integration.

Acknowledgements

Venelin Todorov is supported by the Bulgarian National Science Fund under Project KP-06-M32/2-17.12.2019 "Advanced Stochastic and Deterministic Approaches for Large-Scale Problems of Computational Mathematics" and by the National Scientific Program "Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICT in SES)", contract No DO1-205/23.11.2018, financed by the Ministry of Education and Science in Bulgaria.

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PRACTICAL APPROACH TO SPEECH IDENTIFICATION

Serhii Zybin, Yana Bielozorova

***Abstract**: The rules for building a speaker identification system was described. The approaches to creation of such system was analyzed. The ways of creating such system were proposed.*

***Keywords**: speaker identification system, wavelet, coding and information theory, pattern analysis.*

Introduction

Parameters of involvant's individual voice characteristics are the basis of every voice search system. Parameters of individual voice characteristics in modern automatic systems of speaker identification are usually determined on the basis of two main factors - the main tone frequency F0 and spectral characteristics [Solovyov, 2014]. Efficiency of such systems mainly depends on the methods for determination of these factors and stability of spectral characteristics and the main tone frequency F0.

Speaker Recognition Evaluation (SRE) carried out by the National Institute of Standards and Technologies (NIST) since 1996 is the most objective and competent source of information as to capabilities of the modern systems and methods of automatic speaker identification.

SRE allows to obtain data on real capabilities of identification methods and systems including in comparison with others and to choose the most perspective trends of development.

For the period of the last ten years the test results show considerable progress in this field. However, modern systems of automatic speaker identification are essentially behind the effectiveness of the speaker identification implemented by human acoustical apparatus.

Traditionally NIST publishes impersonal test results by means of which it is impossible to determine what identification method or system is found to be the best one. Absence (with few exceptions) of the test results data at the test participants' websites usually points out the poor results or restricted developments which are often performed for authorities ensuring the state security or for commercial purposes.

At the same time achieved level of developments and progress in outlined perspective directions in this field allow to proceed with the stage of collective developments of such systems within the frames of the EU countries.

Complex long-term investigations of scientific and research groups in this area in Ukraine allowed creating an experimental model of modern system for search of involvants in the voice database [Rubalsky et al, 2014]. Further, there is presented a physical and mathematic model based on which the investigations were performed, and experimental model was developed.

Model of voice characteristics identification

We will consider fragments of speech in audio data as discrete time series of the amplitude of a sound wave. Let us consider the problem of determining the characteristics of self-similar structures in a time series. These can be various geometrically similar structures, visually observed when examining the graphs of changes in the amplitude of the sound wave. We will consider self-similarity as a geometric similarity associated with transformations of compression, extension, both along the time axis and along the amplitude coordinate. To reveal approximately similar structures, we use the methods of wavelet analysis. For this purpose, we will use the complex Morlet wavelet [Bielozorova, 2019], [Solovyov et al, 2014], [Solovyov, 2013].

Figure 1 shows an illustration that combines fragments of an audio recording of speech and a scalograms built on the basis of the model under consideration. The features of the scalograms construction were considered in the work [Bielozorova, 2019], [Solovyov et al, 2014], [Solovyov, 2013].

The analysis shows that the location of the tops of the scalograms in terms of the time parameter in Fig. 1 strictly corresponds to the local maxima of the amplitude of the sound wave in the time domain. In this case, the local maxima correspond to bursts of the amplitude of the sound wave due to the frequency F0.

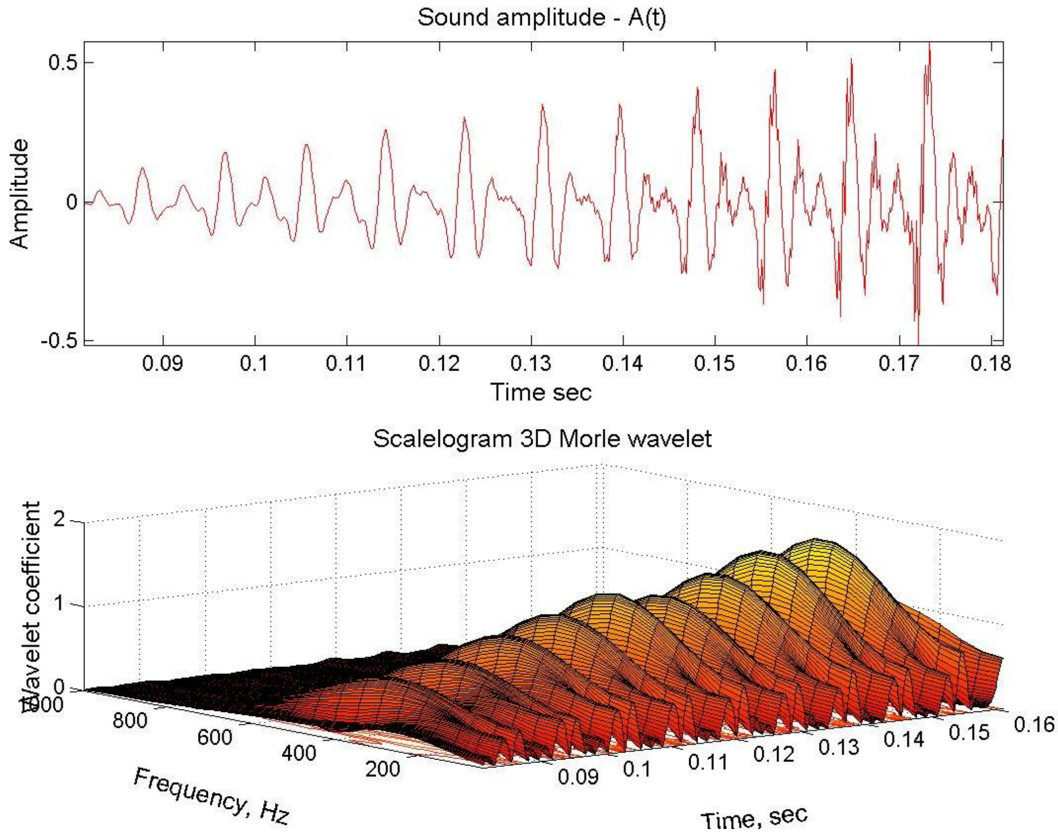

Fig.1. Scalogram of a voice signal using the complex Morlet wavelet

An essential feature of the characteristics of the ridges is the shape of the ridge. Studies show that after rational normalization of the description function, the value of the modulus of the wavelet transform coefficient at fixed parameters: the position of the time window in time and the width of the wavelet at the peak, these functions have a high degree of geometric similarity. At the same time, the shape of the normalized peaks is individually different when the characteristics of the voice differ.

The reason for the similarity of structures of the type of peaks in the frequency domain on the scalogram for the same characteristics of the voice is the specificity of the affinity of the Morlet wavelet to the temporal structure of the amplitude of the sound wave in the regions of local maxima. The Morlet wavelet transform effectively separates these structures with this approach. The similarity under consideration has a very transparent physical interpretation. The structures of the amplitude of the sound wave in the region of local maxima corresponding to the frequency F0 have a fairly pronounced geometric symmetry with respect to the amplitude of the local maximum. In this case, the Morlet wavelet, due to its affinity, makes it possible to reveal this symmetry in the form of pronounced extrema of the scalograms.

In fig. 2 and fig. 3 two-dimensional slices of the spatial scalogram in frequency and in time are presented. The smoothness of these dependencies illustrates the thesis about the reduction of the mathematical complexity of identifying self-similar structures in the time-frequency domain with the considered approach.

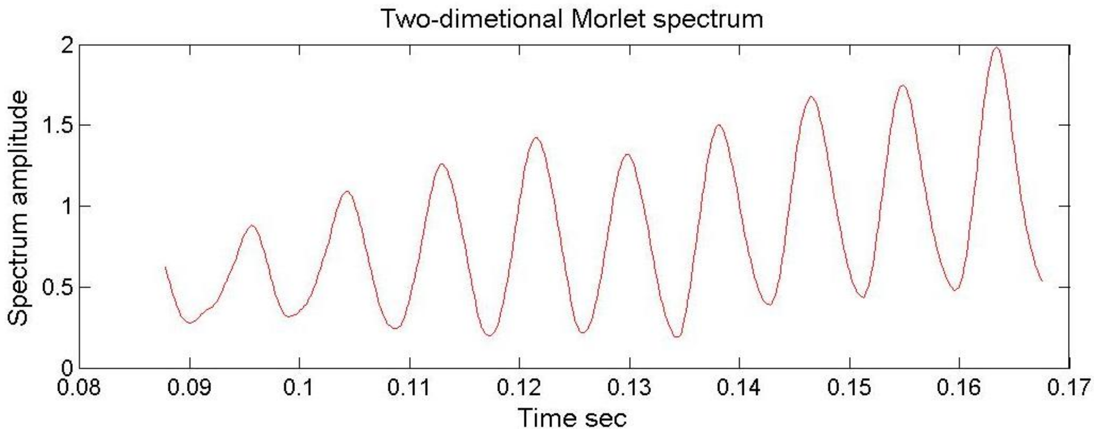

Fig.2. Two-dimensional time scale chart

To extract speech characteristics, the following approach was used:

1. Divide signal into short overlapping frames (frame-20msec, overlapping-10msec)

2. Calculate spectrogram for each frame by using complex wavelet Morle (as fig.2). A feature of the applied method for calculating the spectrum is that the calculation of the spectral characteristics is carried out with a resolution of 1 Hz (The wavelet frames were used in the work, which allow obtaining a higher frequency resolution of signal, instead of the standard wavelet decomposition with a frequency resolution of 50 Hz).

3. Extracts two-dimensional frequency scale chart (as fig.3) from 7 local maxima of spectrogram for each frame (A feature of this stage is the selection of two-dimensional slices only for the positions of the time maxima of the two-dimensional slices of the scalograms in time. That is, the frequency dependences of the wavelet coefficients are considered only at the peaks).

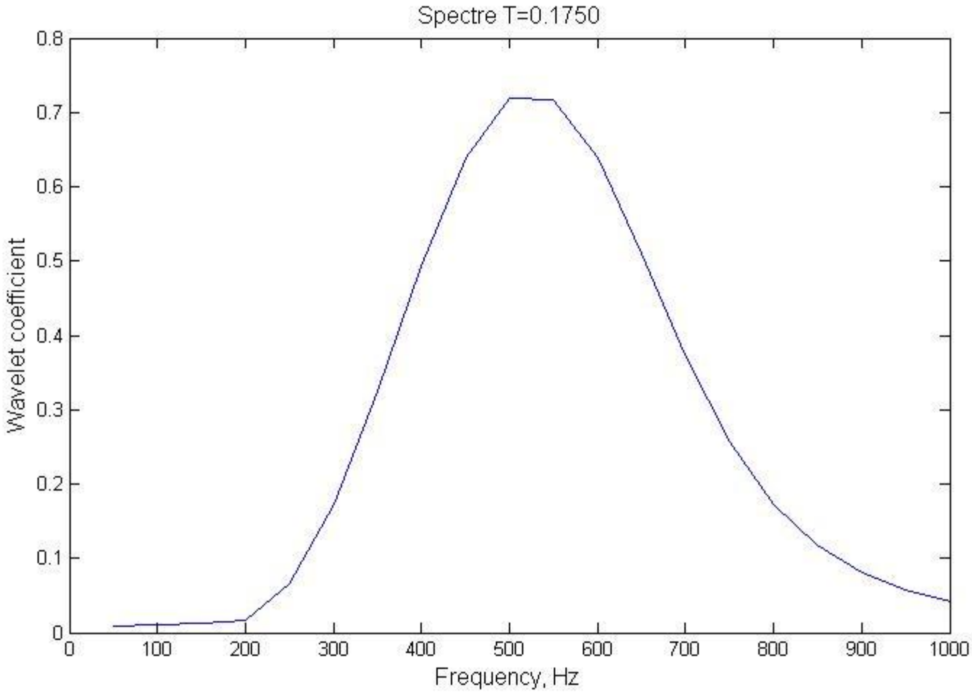

Fig.3. Two-dimensional frequency scale chart

The frequency range of the frequency F0 is divided into intervals in the range from 100 to 1500 Hz. For all fragments of speech, the probability density is determined in terms of frequency F0 and the function of voice characteristics Fc (see Fig. 1). Thus, the system is based on a combined estimation method based on the frequency F0 and spectral characteristics. Proximity of the two characteristics of the voice are determined by the absolute differences between two-dimensional probability densities. Of course, the projection of two-dimensional probability densities on each of the one-dimensional coordinate axes gives the frequency distribution F0 and the distribution of the function of voice characteristics Fc, which can also be represented as separate dependencies. This allows you to calculate the proximity of the curves of the probability density functions for each of these features separately.

The analysis of these dependences showed that at different values of the magnitudes of the maxima in frequency, when combining the graphs of frequency dependences in the position of the maxima, they have a high degree of similarity after the corresponding normalizations for the same characteristics of the voice. The final acceptance of the hypothesis about the identity of the characteristics of the voice of two speech fragments is made after testing the hypothesis about the identity of the distributions of frequency dependences.

In the identification process, a comparison of the parameters of the distributions selected in the preparatory stage will be performed.

There are 2 main approaches to building systems based on the method under consideration:

1.The correlation of the normalized temporal spectrograms (point 2 of the preparatory stage for the selection of characteristics) of each frame of one file with the frames of another file is estimated (by searching for the Pearson's correlation coefficient or calculating the maximum of the correlation function). In the case of high correlation characteristics of the normalized time spectrograms, the characteristics of the frequency scalograms of these frames are compared (point 3 of the preparatory stage) and a decision is made on the similarity of the speakers in 2 files.

2.Evaluation of the similarity of speakers is carried out on the basis of a separate consideration of the probability density distribution of the pitch frequency and the function of voice characteristics as described above

The Experimental studies of speech files and speech fragments based on the developed model have shown the effectiveness of the approach in solving problems of identifying voice characteristics.

Scientific novelty of investigations and development

In view of importance and commercial nature of investigations and developments of leading designers in the field of voice characteristics identification, there is not enough information about detailed principles of modern system operation in open access publications. However, based on the analysis of the many years investigations, it is possible to suppose that the methodology of investigations and developments, described in above sections, is a complex new scientific approach to tasks of voice identification.

Scientific novelty of the approach under consideration is an application of complex of the following mathematical methods and technologies of digital processing of acoustic audio information:

1 complex discrete two-parameter Morlet transformation;
2 non-orthogonal discrete transformations with discrete frequency step of 1 Hz;
3 application of the wavelet transforms maxima approach for determination of the features of frequency characteristics;
4 determination of connection of frequency characteristics features with the frequency F0 and other parameters of voice characteristics;
5 analysis of voice information in small time intervals – 10-30 ms.

Essence of the innovative nature of the project

Confidential nature of the modern most advanced developments in the field of identification of the voice characteristics creates essential problems both for accelerated evolution of these systems and for wide application in the intergovernmental voice search databases.

The present development, if the practical efficiency is proved, can be distributed in any EU country with appropriate version of language localization.

Innovative nature of the proposed project consists of posing the task for creation of prototypes of more open modern systems for identification of voice characteristics within the frame of voice databases for specialized structures.

The second important moment ensuring innovative nature of the project from our point of view is new technologies and models for processing of discrete voice information.

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ANALYTICAL FOUNDATION OF MODEL TO CODE TRANSFORMATION ACTIVITIES

Anton Shyrokykh

Abstract: *The Codegeneration problem is not new in AGILE approach. Many wide spread used tools and formal approaches and papers are devoted to this problem.*

From the other hand codegenetation tools, that are used in development practices, have some drawbacks that not allow to transform structure of class diagram to code without mistakes (Shyrokykh, 2020).

It is explained by peculiarities of human cognitive comprehension. When a developer "reads" structure of software represented in graphical notation of UML class diagrams some details are convenient for visual representation, for example interrelations between classes. Proposed transformation rules of model to code transformation languages must consider structure of class diagrams elements in more detailed way.

Paper is devoted to designing of a new codegeneration approach based on idea of preliminary refinement of class diagrams before model to code transformation and further transformation using newly proposed transformation rules. Formal foundation of approach is grounded on model to model transformation language. Aim of this approach is to design intermedia analytical representation of class diagram using algebra describing static software models (Chebanyuk. 2013).

***Keywords**: Codeneration, Class Diagram, Transformation Rules, Model to Model Transformation.*

Introduction

Model to code transformation operation is one of the activities reducing development efforts. Software models, represented as UML diagrams, easily comprehend by specialists in comparison with skeletons of code, represented as texts.

Challenges to use modeling notations in real software development companies are the next:

- it must be flexible to represent future software system from different points of view with different levels of details.
- it must be supported by variety of application life cycle management tools

- it must be grounded on professional standard to be easily implement of different specialists

There is no modeling notation that fully answers to the challenges listed above. From the other point of view UML is more close standard to formulated challenges.

Implementing chain of operations supported sequence of transformations from high-level behavioral software models, (represented as UML Use Case or Communication diagrams) to the code allow reduce development efforts to design a chain of software development artifacts that correspond to requirement specification (architectural solutions, code modules, test cases, etc.).

Architectural solutions, represented as UML Components or Class Diagrams, are initial sources of codegeneration procedures. Today in the market different codegeneration tools are represented as separate software tools and as plug-ins to IDE (Shyrokykh A, 2020).

Many codegeneration tools have drawbacks that are sources of loosing some parts of UML class diagrams' or incorrect transformation of structure when model to code transformation is performed (Shyrokykh A, 2020). Different tools needs different efforts after generating skeleton of code. That why correctness of final code structure depends upon qualification of designer (developer) and his efforts to avoid refine mistakes.

Review of papers

Problems of codegenaration approach in different types of software development considered in different papers.

One of the fundamental papers in codegeneration approach in domain specific modeling area (Midingoyi, C et. al, 2020) explaines difficulties of implementing codegeneration focuses by several reasons: pure coding concepts are, in most cases, too far from the requirements and from the actual problem domain. Models are used to raise the level of abstraction and hide the implementation details. In a traditional development process, models are, however, kept totally separate from the code as there is no automated transformation available from those models to code (Midingoyi, C et. al, 2020).

In paper (Midingoyi, C et. al, 2020) several approaches of collaborating models and code are considered. Approaches are represented on figure 1.

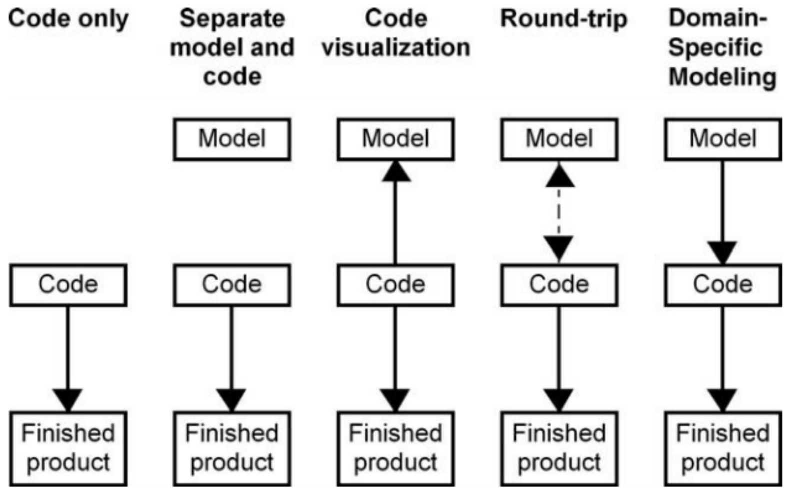

Figure 1 Code and software models collaboration (Midingoyi, C et. al, 2020).

Approach, proposed this in paper is focused on forward engineering activities, namely on round-trip engineering and domain-specific engineering. Also such approaches can be used in other forward engineering software development process.

In order to perform codegeneration operations successfully authors (Midingoyi, C et. al, 2020) propose two languages, namely Crop2ML and CyML.

Language Crop2ML provides a model component specification based on XML meta-language. It consists of unified concepts. A Crop2ML model is an abstract model that may be either a unit model with fine granularity or a composite model represented as a graph of unit models connected by their inputs and outputs to manage model complexity. A model specification contains formal descriptions of the model, the inputs, outputs, state variable initializations, auxiliary functions and a set of parameters and unit tests. Thus, it allows for checking that a model reproduces the expected output values with a given precision. In order to be adopted to model to code transformation tasks CROP needs additional plug-ins and clearly documentation. Abstract model can’t consider some platform specific details. For example, there is no multiple inheritance of classes, in C++ vise versa and so on.

Authors performed a great step into development of serious analytical foundation and proposed codegeneration framework, but task to read abstract model is more complex in comparison with UML duagrams. Due to this fact wide using of such languages is limited.

Formal foundation of designing model to model transformation language is proposed in paper (Chebanyuk, 2018). As our codegeneration approach generates object-oriented code that is a type of model, consider application of the proposed formalization to this approach.

Paper also represents in clear structured way challenges to the abstract syntax of model to model transformation language, to the metamodel of language, to the concrete syntax, and to the transformational rules.

Then author describes elements of mode to model transformation language and proposes a metamodel of Model to Model Transformation language (M2MTL).It is represented in the figure 2.

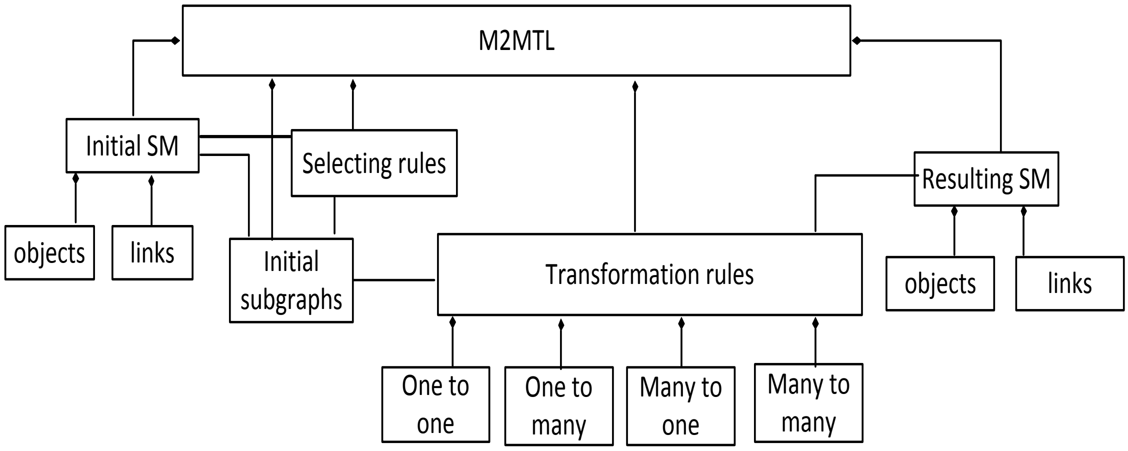

Figure 2 Metamodel of model to model transformation languages (Chebanyuk O., 2018)

Task and research questions

Task: propose analytical foundations of model to code transformation approach. As initial model class diagram is used. As resulting model object model of c# language is used.

Research questions (RQ)

RQ1: Consider architecture of Model to Code Transformation Approach in connection with specific frameworks and tools.

RQ2: Choose flexible analytical apparatus allowing represent structure both of static software model and skeleton of code.

RQ3: Perform an experiment proving correctness of proposed analytical foundation of software model to code transformation framework.

Frameworks and Tools and for proposed Model to code Transformation approach

Distribution of proposed frameworks and tools is represented according to classical schema of Model to Model transformation approach (Figure 3).

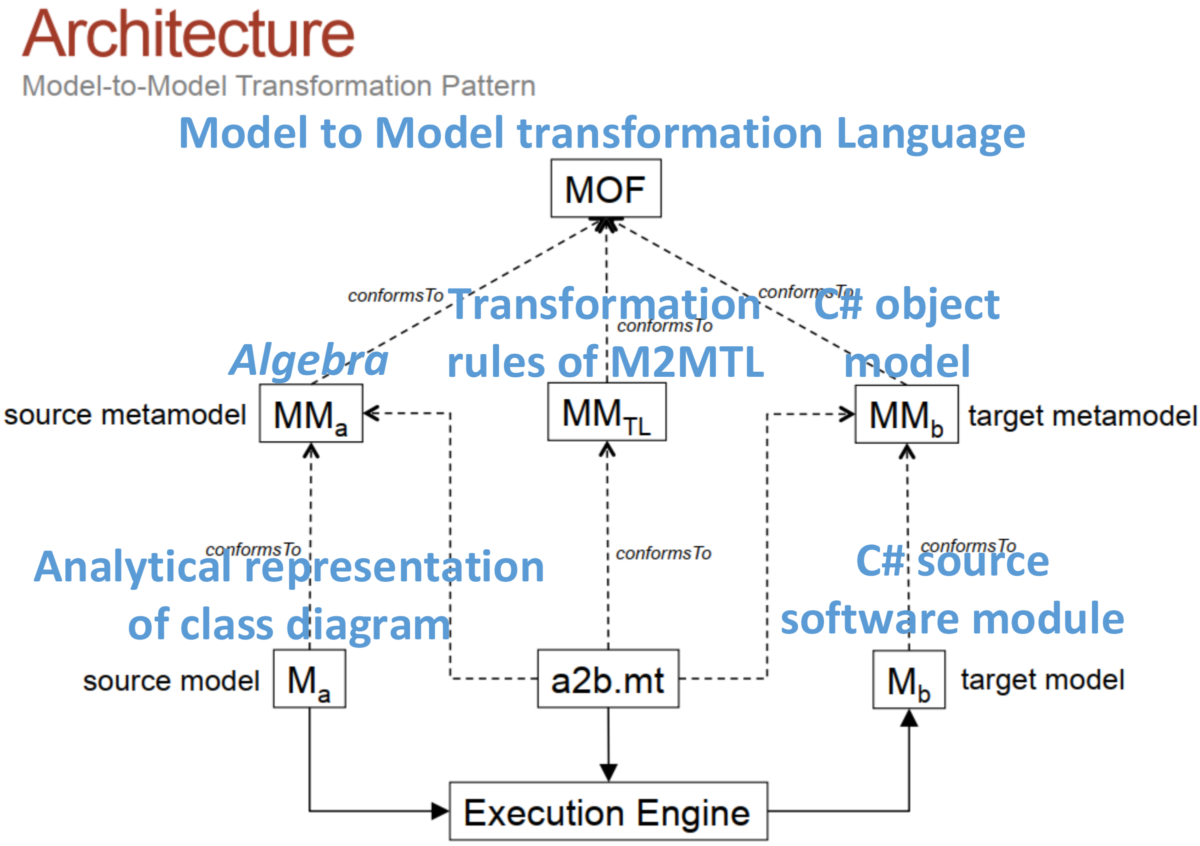

Figure 3 Model to code transformation approach (Cabot J., 2015)

In the Table 1 detailed explanation about used tools for codegeneration approach is represented.

Table 1. Description of Model-to-code transformation approaches architecture

Part of codegeneration architecture	Explanation
Metametalevel	
Metamodel of model to model transformation language (Chebanyuk, 2018).	This metamodel is flexible and contains all necessary elements to describe transformation process with necessary level of details.
Source and target Metamodels	

Source Metamodel Algebra, describing static software models (Chebanyuk, 2013)	Algebra that allows describing structure of class diagram considering all variants of class diagram elements composition.
Target Metamodel	C# object model (Microsoft, 2017). Also can be XMI standard (XMI, 2015)
Model level Source and target models	
Source model	Analytical representation of class diagram in terms of algebra describing software static models
Target models	Modules of C# source code. XMI representation of class diagrams (Chebanyuk E. & Povalyaev D., 2017). Many Modeling environments store UML diagrams in this format
Transformation rules	Rules to represent how to transform initial analytical representation of class diagram to its intermedia analytical representation. Rules how to transform intermedia analytical representation to C# or to XMI representation.
Execution engine	Visual Studio compiler

Analytical representation of transformation rules

Transformation rules must allow to influence on changing structural characteristics of class diagram elements. In order to represent transformation results corresponding analytical approach must be involved. After review of different analytical approaches aimed to reflect information about static diagrams algebra, describing software static model is chosen (Chebanyuk, 2013). Let's describe transformation rules allowing to precise class diagram structure before codegeneration operation.

The first transformation rule aimed to improve structure of class diagram before codegeneration. It is formulated as follows:

If in the class diagram there is class with composition relationship with other classes (denote is as C) these classes must be included to the list of properties of C.

According to (Chebanyuk, 2018) the first step of realization of this rule is to find on class diagram all classes that are connected with other ones with composition relationship.

General selection riles are denoted as follows:

$$select\, S\ from(SMI_{type}) = SMI_{sub} \tag{1}$$

Forming SMI_{sub} , allowing to form a set of classes that have composition links with other classes, (namely *COMP*) according to this rule is performed by the following:

$$select\, F^{comp}(C)\ from ClassDiagram = COMP \tag{2}$$

Transformation rule is consisted from two steps. The first step is to refine class structure, namely for every class from $F^{comp}(C) \in COMP$ add attributes that matches with types of the connected classes. This transformation rule is aimed to refine class diagram structure and it is denoted by the following:

$$\begin{aligned} &F^{comp}(class) \longrightarrow F^{comp}(A \cup \bigcup_{i=1}^{n} \alpha(class_i(name)) \times X \times B), i = 1,...,n \\ &F^{comp}(class) \in COMP \end{aligned} \tag{3}$$

The second step is to fill class attributes according to the specific template of some programming language. According to C# language template is looking by the following:

$$\begin{aligned} &class\, www\{ \\ &\quad public\, type1\, attribute1\{get; set;\} \\ &\quad ...... \\ &\quad public\ \ class_1\ \ \alpha(class_1(name)\{get; set;\} \\ &\quad ...... \\ &\quad public\ \ class_n\ \ \alpha(class_n(name)\{get; set;\} \\ &\} \end{aligned} \tag{4}$$

Second transformation step is represented by the following:

$$\begin{aligned} &F^{comp}(A \cup \bigcup_{i=1}^{n} \alpha(class_i(name)) \times X \times B) \longrightarrow class\, www\{ \\ &\qquad public\, type1\, attribute1\{get; set;\} \\ &\qquad ...... \\ &\qquad public\ \ class_1\ \ \alpha(class_1(name)\{get; set;\} \\ &\qquad ...... \\ &\qquad public\ \ class_n\ \ \alpha(class_n(name)\{get; set;\} \\ &\} \end{aligned} \tag{5}$$

Let’s describe other transformation rule, namely adding a list of methods to skeleton of class when class inherits an interface. The rule selecting all classes that inherit interfaces (set *INTERF*) is represented by the following:

$$select\ F^{inh}(C^{public})\ from\ ClassDiagram = INTERF \qquad (6)$$

Transformation rule allowing to complete classes from the set *INTERF* by a set of methods from interfaces is written by the following:

$$F^{inh}(C^{public}) \longrightarrow F^{inh}(A \times B \times X \bigcup_{i=1}^{m} B^{j}),\ j = 1,...,m$$
$$F^{inh}(C^{public}) \in INTERF \qquad (7)$$

The second step is to fill class attributes according to the specific template of some programming language. According to C# language template is looking by the following

$$\begin{array}{l} class\ www\{ \\ \quad public\ \beta_{1,1}; \\ \quad public\ \beta_{1,2}; \\ \quad .... \\ \quad public\ \beta_{1,m_1}; \\ \quad public\ \beta_{2,1}; \\ \quad .... \\ \quad public\ \beta_{n,k}; \\ \} \end{array} \qquad (8)$$

Where $\beta_{1,1}$ - is a signature of the first method of the first interface, and generally $\beta_{i,j}$ public signature of method *i* of interface *j*.

Second transformation step is represented by the following:

$$F^{inh}(A \times B \times X \bigcup_{i=1}^{m} B^{j}) \longrightarrow \begin{array}{l} class\ www\{ \\ \quad public\ \beta_{1,1}; \\ \quad public\ \beta_{1,2}; \\ \quad .... \\ \quad public\ \beta_{1,m_1}; \\ \quad .... \\ \quad public\ \beta_{n,k}; \\ \} \end{array} \qquad (9)$$

Proposed approach

1. Parse input XMI of class diagram to obtain analytical representation of class diagram according to algebra describing software static models. Parse rules are represented in paper (Chebanyuk E. & Povalyaev D., 2017).
2. On analytical representation of class diagram search fragments that satisfy patterns of transformation rules using (2) and (6).
3. Obtain skeletons of source codes according to transformation rules represented in (5) and (9).
4. Using visual studio codegeneration environment obtain *.cs files with source codes in C# language.
5. Merge textual representation of class diagram fragments, obtained after performing of previous point and skeleton of source code obtained in point 3.
6. Optional point for testing – compile obtained source module by visual studio compiler.

Case study

Let’s consider proposed codegeneration approach investigating Visual Studio class designer plug-in (Microsoft, 2018). As it was mentioned in (Shyrokikh, 2020) Visual Studio does not contain composition relationship only association one (figure 4).

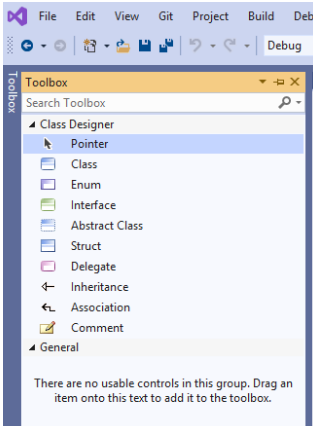

Figure 4. Visual Studio Class designer components

Codegeneration rules of Visual Studio plug-in add datatypes of classes to the central (composition) class. From the other hand it is important to mention that when designer establish association relationship between two classes property pointing that one class becomes a field of other is added automatically. Then developer must think about semantic of generated code and spend additional time for software module analysis and editing.

For example – class diagram that is represented on the figure 5 contains three classes. Screen is a part of SmartPhone, and classes Screen and Smartphone are connected by composition relationship. Classes User and Smartphone must be connected through association relationship.

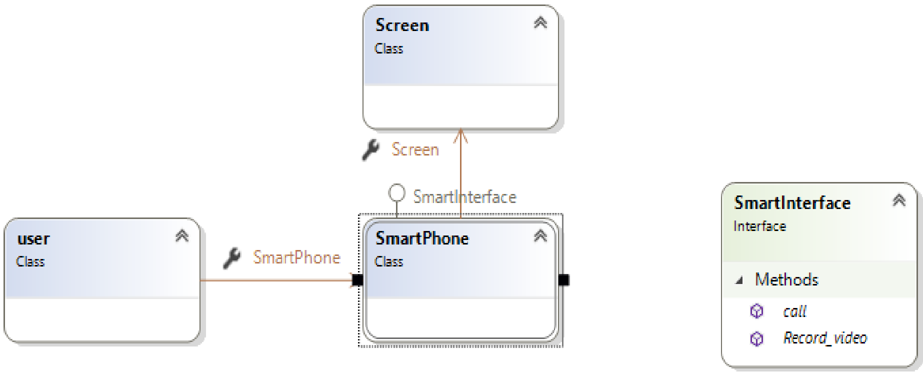

Figure 5. Example of class diagram

Codegeneration plug-in assumes that these two association links are the same. Result of codegenaration plug-in is represented below.

```
 public class user
  {
Extra fragment needed to be deleted
      public SmartPhone SmartPhone
      {
         get => default;
         set
         {
         }
      }
  }
public class SmartPhone : SmartInterface
  {
      public Screen Screen
      {
         get => default;
         set
         {
         }
```

```
    }
Absent fragment – needed to be added
    void call();
    void Record_video();

  }
 public class Screen
  {
  }
public interface SmartInterface
  {
    void call();
    void Record_video();
  }
```

Representation of transformation rules is given in the Table 2.

Table 2. Representation of transformation rules on metalevel and model level

Analytical representation of class diagram initial fragment	Analytical representation of class diagram resulting fragment
Metalevel	
$P(Classes) =$ $\exists class \in Classes$ $where\ F^{comp}(class) \neq \varnothing$	$class^{comp} = A \times X \times B$ $A = A \bigcup_{i=1}^{n} name_i$
Model level	
$F(SmartPhone)^{comp} =$ $= F(SmartPhone) \cup F(Screen)$ $SmartPhone \subseteq A \times X \times B$	$F(SmartPhone)^{comp} =$ $= F(SmartPhone) \cup F(Screen)$ $SmartPhone \subseteq A \times X \times B$ $A^* = A \cup Screen$
Metalevel	
$P(Classes, Ic) =$ $\exists class \in Classes\ \forall I = \{i_i, ..., i_k\}, i \in Ic$ $where\ F^{inh}(class) \bigcup_{i=1}^{n} I_i \neq \varnothing$	$class^{inh} = A \times X \times B$ $B^* = B \bigcup_{i=1}^{n} \bigcup_{j=1}^{m} \beta_{i,j}$
Model level	

$F(SmartPhone)^{inh} = F(SmartPhone) \cup$ $\cup F(SmartInterface)$ $SmartPhone \subseteq A \times X \times B$	$F(SmartPhone)^{inh} = F(SmartPhone) \cup$ $\cup F(SmartInterface)$ $SmartPhone \subseteq A \times X \times B$ $B^{*} = B \cup B(SmartInterface)$

Conclusion

In this paper Codegeneration approach is proposed. Advantages of the proposed approach are the next:

- It uses flexible analytical apparatus for representation of class diagram with given level of details;
- such a representation allows to set transformation rules to improve drawbacks of codegeneration of different designing environments;
- remain for codegenetation environment possibility to design class diagram convenient for human cognitive perception (for example represent relationship between classes graphically).
- transforming analytical representation to XMI and vise versa (Chebanyuk E. & Povalyaev D., 2017) proposed codpgeneration approach can be used to improve round trip engineering activities.

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ANALYTICAL MODEL OF A QUEUING SYSTEM IN A TELECOMMUNICATION NETWORK

Velin Andonov, Stoyan Poryazov and Emiliya Saranova

Abstract: The paper represents shortly one analytical model of a queuing system as a part of overall telecommunication network. The analytical expressions for the parameters of the queuing system are derived in the papers [Andonov et al, 2019c; Poryazov et al, 2020b] as a part of an analytical model of overall telecommunication system.

Keywords: *Analytical modelling, Service systems, Queueing systems.*

MSC: *68Q85, 68M10, 90B22*.

ITHEA Keywords: *C.4 Performance of Systems, H.1.2 User/Machine Systems, I.6 Simulation and Modelling, I.6.0 General, H.1.2 User/Machine Systems, K.6 Management of Computing and Information Systems*.

Introduction

The paper summarizes our recent results in the analytical modelling of queuing systems in telecommunication networks published in the papers [Andonov et al, 2019; Andonov et al, 2019c; Poryazov et al, 2020b]. The problem for conceptual and analytical modeling of queuing systems consisting of buffer and server as a part of overall telecommu-nication system arose when we focused on extending the conceptual model of overall telecommunication system (see [Poryazov & Saranova, 2012]) by inclusion of a queuing system in the switching stage. The two approaches to the conceptual modelling of queuing systems which we use are Service Systems Theory and the Generalized Nets (GNs, see [Atanassov, 2007]). First, in the papers [Andonov et al, 2019; Andonov et al, 2020] the means for constructing of GNs conceptual models of service systems are described. In the series of papers [Tomov et al, 2018; Tomov et al, 2019; Andonov et al, 2018; Poryazov et al, 2018a; Andonov et al, 2019b], different conceptual models of queuing systems are proposed and compared. The most suitable of these conceptual models for the purpose of the analytical modelling are chosen and included in the conceptual models of overall telecommunication system with queuing [Andonov et al, 2019; Andonov et al, 2019b]. Based on these conceptual models, an analytical model of overall telecommunication system with queuing is derived in [Andonov et al, 2019b]. Analytical expressions for the important parameters of the queuing systems such as mean service time in the buffer, mean service time in the server, etc., are also obtained.

In Section 2, the basic concepts from Service Systems Theory which are used in the conceptual modelling are presented. In Section 3, a conceptual model of overall telecommunication system

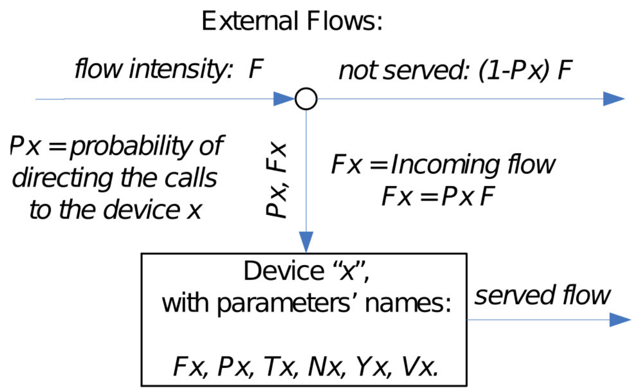

Figure 1: Graphical representation of a base virtual device x (see [Andonov et al, 2019c]).

including a queuing system in the switching stage is shortly described. A conceptual model of a queuing system which is used in the construction of the analytical model is described in Section 4. In Section 5, an analytical model of a queuing system as part of overall telecommunication system is presented.

Base virtual devices and their parameters

For the purpose of the analytical modelling, after a comparison of the different conceptual models of queuing systems, proposed in [Tomov et al, 2018; Tomov et al, 2019; Andonov et al, 2018; Poryazov et al, 2018a; Andonov et al, 2019b] the Service Systems Theory approach is chosen. Here, we present the basic concepts used in the conceptual models and the conceptual models of queuing systems which are used in the analytical modelling.

The basic building blocks of the conceptual models are the base virtual devices. They do not contain any other virtual devices. A general graphical representation of a base virtual device is shown in Fig. 1.

Every such base virtual device x has the following parameters (see [ITU-T E.600 ,1993] for terms definition):

- Fx - intensity or incoming rate (frequency) of the flow of requests (i.e. the number of requests per time unit) to device x;
- Px - probability of directing the requests towards device x;
- Tx - service time (duration of servicing of a request) in device x;
- Yx - traffic intensity [Erlang];
- Vx - traffic volume [Erlang - time unit];
- Nx - number of lines (service resources, positions, capacity) of device x.

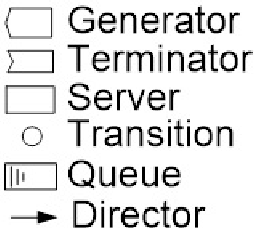

Figure 2: Types of base virtual devices and their graphical representation (see [Poryazov & Saranova, 2012]).

For the better understanding of the models and for a more convenient description of the intensity of the flow, a special notation including qualifiers (see [ITU-T E.600 ,1993]) is used. For example $dem.F$ for demand flow; $inc.Y$ stands for incoming traffic; $ofr.Y$ for offered traffic; $rep.Y$ for repeated traffic, etc.

Different types of base virtual devices are used in the conceptual models. Some of them together with their graphical representations are shown in Fig. 2. Each type of base virtual device has a specific function (see [Poryazov & Saranova, 2012]):

- Generator – generates call attempts (requests, transactions);
- Terminator – eliminates each request which enters it;
- Server – models traffic and time characteristics of the model, the delay (service time, holding time) of the requests;
- Transition – selects one of its possible exits for every request which has entered it;
- Queue – buffer device of the queuing system;
- Director – points to the next device to which the request is transferred without delay.

Conceptual model of an overall telecommunication system including a queuing system in the switching stage

In the conceptual model each virtual device has a unique name. The names of the devices are constructed according to their position in the model. The model is partitioned into service stages (dialing, switching, ringing and communication). Every service stage has branches (enter, abandoned, blocked, interrupted, not available, carried), corresponding to the modeled possible cases of ends of the calls' service in the branch considered. Every branch has two exits (repeated, terminated) which show what happens with the calls after they leave the telecommunication system. Users may make a new bid (repeated call), or stop the attempts (terminated call). In the names of

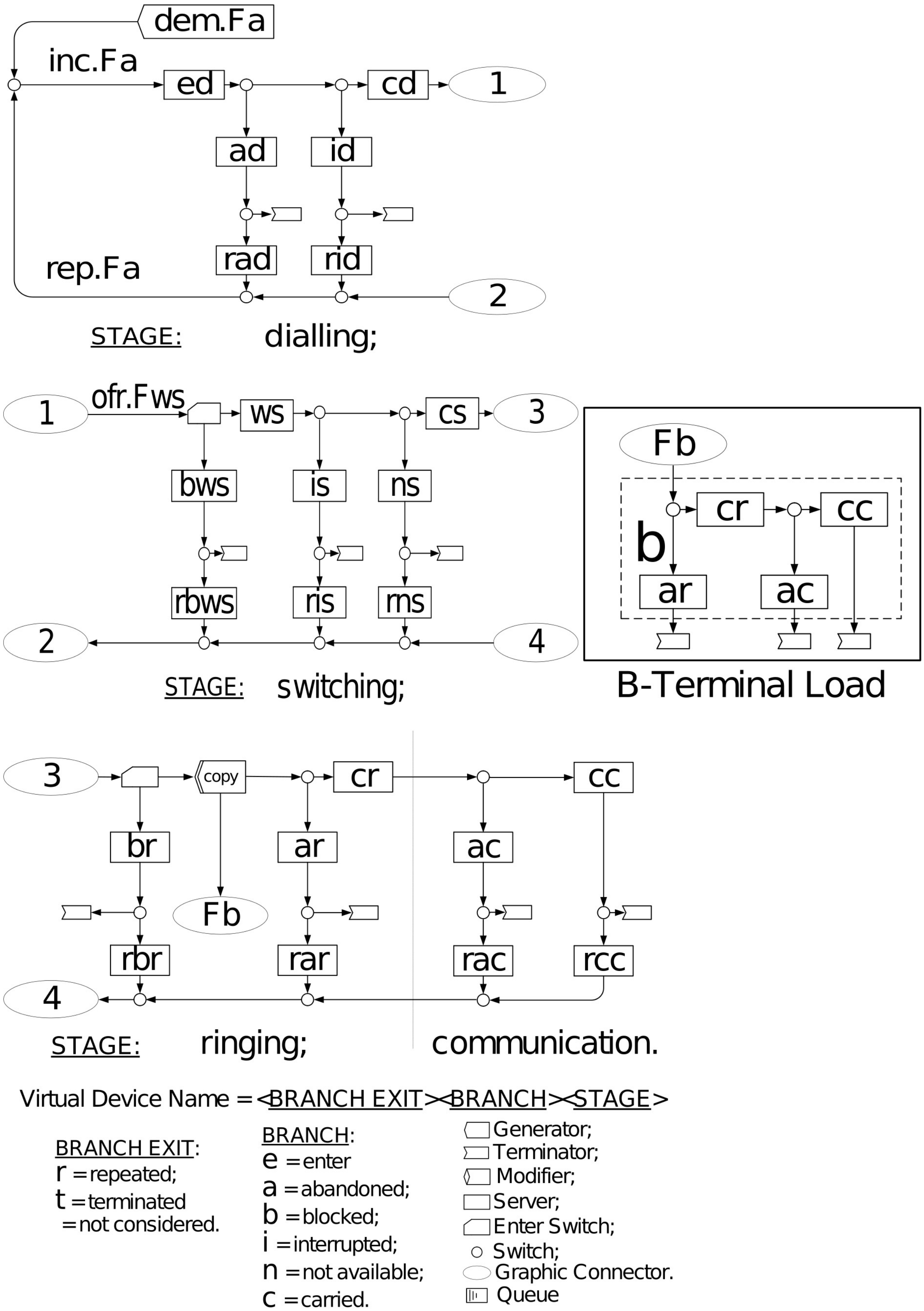

Figure 3: Conceptual model of an overall telecommunication system including a queueing system in the switching stage (see [Andonov et al, 2019]).

the virtual devices the corresponding bold first letters of the names of stages, branches end exits are used in the following way:

Virtual Device Name = <BRANCH EXIT><BRANCH><STAGE>

The names of the parameters of a virtual device are concatenations of the letter denoting the parameter and the name of the virtual device. For example, “Yid” means “traffic intensity in interrupted dialing case”; “Fid” – “flow (calls) intensity in interrupted dialing case”; “Pid” – “probability for interrupted dialing”; Tid – “mean duration of the interrupted dialing”; “Frid” – “intensity of repeated flow calls, caused by (after) interrupted dialing”.

Apart from base virtual devices, the following comprise virtual devices denoted by **b** (shown in dash line box in Fig. 3) and **a, ab, s** (not shown in Fig. 3) are also included in the model.

- **a** comprises all calling terminals (A-terminals) in the system. It is not shown in Fig. 3;
- **b** comprises all called terminals (B-terminals) in the system (dashed line box in Fig. 3);
- **ab** comprises all the terminals (calling and called) in the system. It is not shown in Fig. 3;
- **s** virtual device corresponding to the switching system. It is not shown in Fig. 3.

The flow of calls (B-calls), with intensity Fb, occupying the B-terminals, is coming from the Copy device. This corresponds to the fact that at the beginning of the ringing a second (B) terminal in the system becomes busy. The second reason for this conceptual modelling trick is that the paths of the A and B-calls are different in the telecommunication system's environment, after releasing the terminals. There are two virtual devices of type Enter Switch (see Fig. 3) – before Blocked Waiting for Switch (**bws**) and Blocked Ringing (**br**) devices. These devices deflect calls if the buffer (**bw**) has reached its capacity or the intent B-terminal is busy, respectively. The corresponding transition probabilities depend on the macrostate of the system (Yab). The macrostate of a (virtual) device (including the overall network, considered as a device) is defined as the mean number of simultaneously served calls in this device, in the observed time interval (similar to “mean traffic intensity” in (see [ITU-T E.600 ,1993]).

An important remark regarding the analytical modeling of the system should be made. The mean service time of the call attempts in the **s** device depends, among other parameters, on the mean service time of the carried requests by the switching system. The mean service time of the carried requests by the switching system depends on Pbr, Tbr, Tb and on the mean service time of the requests in the **cs** device. Therefore, to avoid confusion in the analytical modeling of the system, we denote by T^*cs the mean service time of the requests in the **cs** device and by Tcs the mean service time of the carried requests by the switching system. This notation allows to avoid the inclusion of a comprise virtual device representing the service of the carried requests by the switching system.

Detailed description of the conceptual model can be found in the papers [Andonov et al, 2019; Andonov et al, 2019c].

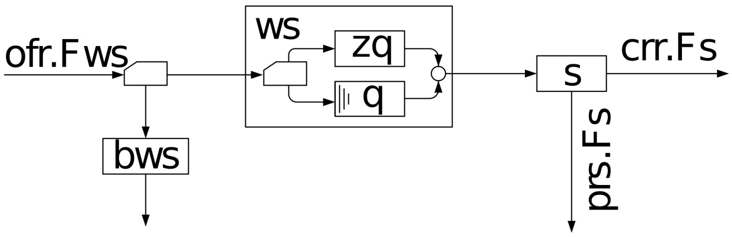

Figure 4: Conceptual model of a queuing system in the switching stage of an overall telecommunication system.

Conceptual model of a queuing system

For the derivation of analytical model of the queuing system as part of the overall telecommunication system, a detailed conceptual model of the queuing system should be used. Its graphical representation is shown in Fig. 4.

This detailed representation allows for the two different ways of service of the request in the queuing system to be distinguished: service with waiting and service without waiting. When the switching system (**s**) has not reached its capacity (Ns), the requests enter the zero queuing (zq) device from where they are sent to the switching system without delay. If the switching system has reached its capacity but there are free places in the buffer, the requests enter the buffer device q where they wait to be serviced, depending on the discipline of service in consideration. In the present paper, we consider FIFO discipline of service of the requests. The mean service time of the requests in the buffer (Tws), for both the waiting and the non-waiting requests, is given by:

$$Tws = Pq\,Tq + (1 - Pq)\,Tzq\,, \tag{1}$$

where Pq is the probability that the request is serviced with waiting and Tq is mean service time in the buffer for the waiting requests. We shall consider that the mean service time of the non-waiting requests is 0, i.e., $Tzq = 0$. In this way the mean service time in the buffer of both the waiting and the non-waiting requests becomes:

$$Tws = Pq\,Tq\,. \tag{2}$$

From $Tzq = 0$ if follows $Yzq = 0$. Therefore, the capacity of the buffer (Nws) is equal to the capacity of the q device Nq. The intensity of the offered flow of requests to the switching system is denoted by $ofr.Fws$ (see Fig. 4). The qualifier “parasitic” (prs) is defined in [Poryazov et al, 2018b]. In Fig. 4 the outgoing flow intensity of the switching system ($out.Fs$) is given by:

$$out.Fs = crr.Fs + prs.Fs\,. \tag{3}$$

For $crr.Fs$ and $prs.Fs$ we have:

$$crr.Fs = Fcc\,. \tag{4}$$

The parasitic flow ($prs.Fs$) represented in Fig. 4 according to its definition and the conceptual model shown in Fig. 3 can be expressed through the equation:

$$prs.Fs = Fis + Fns + Fbr + Far + Fac\,. \tag{5}$$

Problem statement. The problem that we are solving can be stated as deriving equations for the

- output parameters: $Pbws$, Yws, Tws, related to the **ws** device

given the

- input parameters: $Ns, Nws, ofr.Fws, Ts$ which can be measured.

In order to compactly describe single queuing stations in an unambiguous way, the so called Kendall notation is often used (see [Haverkort, 1998]). A queuing system is described by 6 identifiers separated by vertical bars in the following way:

Arrivals | Services | Servers | Buffersize | Population | Scheduling

where “Arrivals” characterizes the arrival process (arrival distribution), “Services” characterizes the service process (service distribution), “Servers” – the number of servers, “Buffersize” – the total capacity, which includes the customers possibly in the server (infinite if not specified), “Population” – the size of the customer population (infinite if not specified), and finally, “Scheduling” – the employed service discipline.

In our model, the queuing system in the Switching stage of the telecommunication network in Kendall notation is represented as $M|M|Ns|Ns + Nws|Nab|FIFO$, where M stands for exponential distribution, Ns is the capacity of the Switching system (number of equivalent internal switching lines) and Nab is the total number of active terminals which can be calling and called. This is related to the derivation of the analytical model of the system.

Analytical model of the queuing system

The queueing system in the switching stage differs from other queuing systems such as the ones studied in [Schneps, 1979; Vishnevskiy, 2003] in that it has more exits. The exits are represented in the conceptual model in Fig. 3 with the branches is, ns, br, ac, cc. In [Andonov et al, 2019], we have derived analytical expressions for the parameters of the queuing system, starting with the simplest queuing system $M|M|1|FIFO$ and gradually advancing to the most complicated system with finite buffer and finite capacity of the server. Here we shall use the results from [Andonov et al, 2019] but adapted to the more detailed conceptual model presented here.

The density functions of the arrival and service times are respectively:

$$a(t) = \lambda e^{-\lambda t}\,, \tag{6}$$
$$b(t) = \mu e^{-\mu t}\,, \tag{7}$$

where $1/\lambda$ is the mean value of time between two arrivals (interrarival time) and $1/\mu$ is the mean time of service. For our queuing system, they are given by:

$$\lambda = ofr.Fws\,, \tag{8}$$
$$\mu = \frac{1}{Ts}\,. \tag{9}$$

They are assumed to be statistically independent which results in a birth-death process. Let us denote with p_n the probability that the queuing system is in state n that is:

$$p_n = Pr\{\text{there are n requests in the queuing system}\}.$$

There are different ways to solve the birth-death equations. The solution is well-known and can be found for example in [Schneps, 1979]. First, we notice that the arrival rate λ_n is equal to 0 when $n \geq Ns + Nws$. The probability for the system to be in state n is now given by:

$$p_n = \begin{cases} \frac{\lambda^n}{n!\mu^n} p_0 & \text{for } 1 \leq n < Ns. \\ \frac{\lambda^n}{Ns^{n-Ns} Ns! \mu^n} p_0 & \text{for } Ns \leq n \leq Ns + Nws. \end{cases} \tag{10}$$

Again, the condition that the sum of the probabilities p_n should be equal to 1, gives us the following expression for p_0:

$$p_0 = \left(\sum_{n=0}^{Ns-1} \frac{\lambda^n}{n!\mu^n} + \sum_{n=Ns}^{Ns+Nws} \frac{\lambda^n}{Ns^{n-Ns} Ns! \mu^n} \right)^{-1}\,. \tag{11}$$

In order to simplify the expression we set $r = \lambda/\mu$ and $\rho = r/Ns$. After elementary operations, the above expression for p_0 becomes

$$p_0^{-1} = \begin{cases} \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \frac{r^{Ns}}{Ns!} \frac{1-\rho^{Nws+1}}{1-\rho} & \text{for } \rho \neq 1\,. \\ \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \frac{r^{Ns}}{Ns!}(Nws+1) & \text{for } \rho = 1\,. \end{cases} \tag{12}$$

Using (10), we can confirm the validity of the following theorem.

Theorem 1. *The probability of blocked waiting for switch ($Pbws$) is equal to the probability that the system is in state $Ns + Nws$, i.e.,*

$$Pbws = \frac{\lambda^{Ns+Nws}}{Ns^{Nws} Ns! \mu^{Ns+Nws}} p_0\,. \tag{13}$$

Theorem 2. *The expected length of the queue is given by the following expression:*

$$Yws = \sum_{n=Ns+1}^{Ns+Nws} (n - Ns)p_n = \frac{p_0 r^{Ns} \rho}{Ns!(1-\rho)^2} [(\rho - 1)\rho^{Nws}(Nws + 1) + 1 - \rho^{Nws+1}] \,. \quad (14)$$

The proof of the above theorem is given in [Andonov et al, 2019].

Theorem 3. *The mean service time of the requests in the **ws** device for both, the waiting and non-waiting requests, is given by:*

$$Tws = \frac{p_0^2 (Ns\rho r)^{Ns} \rho (1 - \rho^{Nws})}{(Ns!)^2 (1-\rho)^3} . \frac{[(\rho - 1)\rho^{Nws}(Nws + 1) + 1 - \rho^{Nws+1}]}{\lambda(1 - Pbws)} \,. \quad (15)$$

Proof: The mean service time of the requests in **ws** device for both the waiting and non-waiting requests, given the condition $Tzq = 0$, is

$$Tws = PqTq + (1 - Pq)Tzq = PqTq \,. \quad (16)$$

The mean service time of the waiting requests in the **q** device (Tq) is given by:

$$Tq = \frac{p_0 r^{Ns} \rho}{Ns!(1-\rho)^2} \frac{[(\rho - 1)\rho^{Nq}(Nq + 1) + 1 - \rho^{Nq+1}]}{\lambda(1 - Pbws)} \,. \quad (17)$$

The probability Pq is the probability that the system is in any of the states $Ns, Ns + 1, ..., Ns + Nws - 1$, i.e.,

$$Pq = \sum_{k=Ns}^{Ns+Nws-1} p_k = \sum_{k=Ns}^{Ns+Nws-1} \frac{\lambda^k}{Ns^{k-Ns} Ns! \mu^k} p_0 \,. \quad (18)$$

After simpification we obtain:

$$Pq = \frac{p_0 Ns^{Ns} \rho^{Ns} (1 - \rho^{Nws})}{Ns!(1-\rho)} \,. \quad (19)$$

After substitution of (19) and (17) in (16), the theorem is proved. □

Acknowledgements

The work of Velin Andonov is supported by the 2020 National Scientific Program for post doctoral and young scientists of the Ministry of Education and Science of Bulgaria.

The work of Stoyan Poryazov is partially supported by the joint research project “Symbolic-Numerical Decision Methods for Algebraic Systems of Equations in Perspective Telecommunication Tasks” of IMI-BAS, Bulgaria and JINR, Dubna, Russia.

The work of Emiliya Saranova was supported by the Task 1.2.5. “Prediction and Guaranteeing of the Quality of Servicein Human-Cyber-Physical Systems” of the National Scientific Program “Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICT in SES)” financed by the Bulgarian Ministry of Education and Science.

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AN OVERVIEW OF SOME CONCEPTUAL MODELS OF QUEUING SYSTEMS IN SERVICE NETWORKS

Stoyan Poryazov, Velin Andonov and Emiliya Saranova

Abstract: The paper summarizes the authors' research in recent years on the conceptual modelling of queueing systems in service networks. The two approaches used are the Service Systems Theory and the Generalized nets. Some of the proposed conceptual models of queuing systems are included in the analytical models of overall telecommunication system developed by the authors. The models are developed with the aim of deriving analytical models. An important direction of research is the conceptual modelling of the causal structure of a queuing system. The proposed causal conceptual models are used in the study of the Quality of Service (QoS) composition in service networks.

Keywords: *Conceptual modelling, Generalized nets, Queueing systems.*

MSC: *68Q85, 68M10, 90B22*.

ITHEA Keywords: *C.4 Performance of Systems, H.1.2 User/Machine Systems, I.6 Simulation and Modelling, I.6.0 General, H.1.2 User/Machine Systems, K.6 Management of Computing and Information Systems*.

Introduction

The paper summarizes our recent results in the area of conceptual modelling of queuing systems in service networks. The problem for conceptual modelling of queuing systems appeared when we studied the possibilities to extend the conceptual model of overall telecommunication system described in [Poryazov & Saranova, 2012] with the inclusion of a queuing system at the switching stage. Despite the fact that queuing systems are a part of virtually all service networks, their conceptual models are not well studied in the literature. This is true to an even greater degree about the graphical representation of the models. The model of overall telecommunication system in [Poryazov & Saranova, 2012] is, to our knowledge, the most detailed such model in the literature. In order for a queuing system to be included in this model, we need to construct a conceptual model of a queuing system which has detailed graphical representation and, also, is suitable for analytical modelling.

The choice of a suitable conceptual model is of high importance. With a view to achieve maximum conceptual clarity and simplicity of the derivation of mathematical models from the conceptual ones, it is necessary to compare various approaches such as the methods for conceptual modelling [Robinson et al, 2011], teletraffic theory [Iversen, 2010], the modern methods for network planning [Larsson, 2014], the apparatus of the Generalized Nets (GNs) theory [Atanassov, 2007]. The conceptual models of telecommunication system and its environ-ment make use of notions from the Service Systems Theory. However, the latest results of the theory of the GNs, which can have certain advantages in some cases, are rarely used. With a view to compare GNs models with models based on Service Systems Theory, it is necessary to construct GNs representations of the base and often used elements of Service Systems Theory [Andonov et al, 2019a]. If necessary, new extensions of the ordinary GNs can be proposed, similar to the already existing ones in the conceptual modelling of telecommunication networks with Quality of Service (QoS) guarantees [Andonov et al, 2018] - Generalized Nets with Characteristics of the Places (GNCP) [Andonov & Atanassov, 2013].

The duration of staying of the requests in the queue has an impact on the Quality of Experience (QoE). This is another reason why the inclusion of queuing systems in the models of overall telecommunication systems is important. An overview of the literature shows that in the models of overall telecommunication systems with QoS guarantees, queuing systems are practically not considered because their inclusion leads to increase of the complexity of the model. This makes the construction and comparison of various conceptual models of queuing systems a necessary step. This, in turn, would allow for the most suitable model for the purpose of the analytical modelling to be determined.

The first GN models of queuing systems are described in the papers [Tomov et al, 2018; Andonov et al, 2018], while in [Poryazov et al, 2018a] four conceptual models of queuing systems are compared. Some of these models are extended in [Andonov et al, 2019b].

The problem for the presentation of the traffic quality in queuing systems, as a composition of the quality of the components of the queuing systems, is studied in [Poryazov et al,2020a]. The queuing system is considered a part of information service system. The causal structure of the queuing system is extended with not-served traffic devices and consists of 5 causal virtual devices. The naming system for the virtual devices is also extended. The concepts of time for partial service and “pie" intensity of the traffic are considered.

Section 2 summarizes the approaches to the conceptual modelling of queuing systems which are used in the models. In Section 3, the conceptual models of queuing systems based on Service Systems Theory are presented. In Section 4, a summary of the generalized net models of queuing systems is made.

Two approaches to the conceptual modelling of queuing systems

The two approaches which we use in the conceptual modelling of queuing systems are the Service Systems Theory and the theory of the Generalized Nets. In this section, we present the basic concepts of both of these approaches and compare them.

01 Service Systems Theory

In the conceptual models of service networks, base virtual devices are used (see [Poryazov & Saranova, 2012]). A general representation of a base virtual device is shown in Fig. 1. Every such device named x has the following parameters:

- Fx – Intensity or incoming rate (frequency) of the flow of requests (i.e., the number of requests per time unit) to device x;

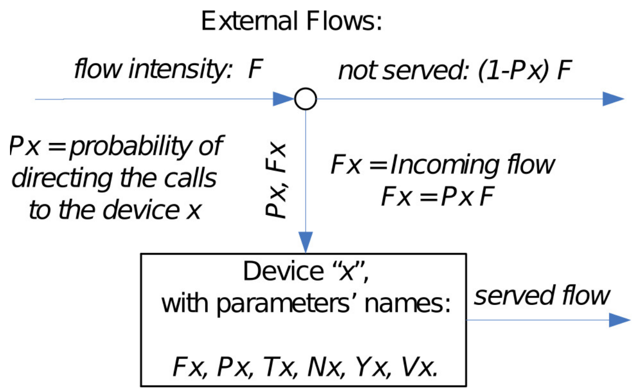

Figure 1: Graphical representation of a base virtual device.

- Px – Probability of directing the requests towards device x;
- Tx – Service time (duration of servicing of a request) in device x;
- Yx – Traffic intensity [Erlang];
- Vx – Traffic volume [Erlang - time unit];
- Nx – Number of lines (service resources, positions, capacity) of device x.

Different types of base virtual devices are used in the models. Some of them together with their graphical representations are shown in Fig. 2.
The devices of each type have specific functions (see [Poryazov & Saranova, 2012]):

- Generator – generates call attempts (requests, transactions);
- Terminator – eliminates each request which enters it;
- Server – models traffic and time characteristics of the model, the delay (service time, holding time) of the requests;
- Transition – selects one of its possible exits for every request which has entered it;
- Queue – buffer device of the queuing system;
- Director – points to the next device to which the request is transferred without delay.

02 On the concept of a generalized net

The GN is a relatively complex object. Detailed definition of a *transition of GN*, *GN* and the algorithms for transition and net functioning can be found in [Atanassov, 2007]. The concepts of a GN model can be divided into:

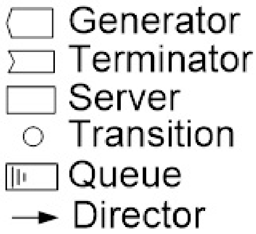

Figure 2: Types of base virtual devices and their graphical representation.

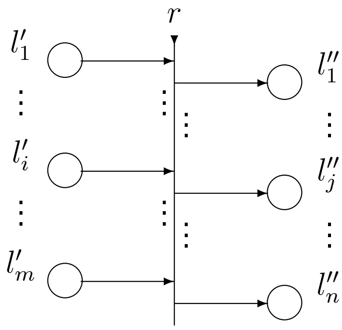

Figure 3: Graphical representation of a transition of a GN.

- model description concepts;
- graphical representation concepts.

First, we shall describe non-formally the elements used in the graphical representation of a GN. GN's *places* are represented by ◯. A GN's *transition* is a part of the net whose graphical representation looks like the object shown in Fig. 3. Every transition contains *transition's conditions* which are graphically represented by .

Like Petri nets, GNs contain tokens which are transferred from place to place through the *arcs* of the net. The arcs are denoted by arrows in Fig. 3. They begin at a place and end at the transition's condition or begin at a transition's condition and end at a place.

The names of the transitions and the places are also included in the graphical representation of the GN model. They can be very important for the understanding of the model by non-specialists in the area of GNs and for the users in general.

To summarize, the concepts of a GN model which are represented graphically are: *transition, place, arc* and the names of the transitions and the places.

Definition 1. Transition of a GN is the following ordered seven-tuple:

$$Z = \langle\langle L', L'', t_1, t_2, r, M, \square \rangle\rangle\,, \tag{1}$$

where

1. L' and L'' are finite non-empty sets of the transition's input and output places, respectively. For the transition in Fig. 3 these sets are:

$$L' = \{l'_1, l'_2, \ldots, l'_m\},$$
$$L'' = \{l''_1, l''_2, \ldots, l''_n\}.$$

2. t_1 is the current time moment at which the transition can be activated.

3. t_2 is the duration of the active state of the transition.

4. r is the Index Matrix (IM, see [Atanassov, 2007]) of the transition's conditions which determine the output places to which the tokens in the input places can be transferred. It has the form:

$$r = \begin{array}{c|c} & l''_1 \ \ldots \ l''_j \ \ldots \ l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} \ - \ \text{predicate}) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array} ;$$

where $r_{i,j}$ is the predicate which expresses the condition for transfer from the i-th input place to the j-th output place.

When $r_{i,j}$ has truth-value "$true$", then a token from the i-th input place can be transferred to the j-th output place. Otherwise, this is impossible.

5. M is an IM of the arcs' capacities. It has the form:

$$M = \begin{array}{c|c} & l''_1 \ \ldots \ l''_j \ \ldots \ l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \ge 0 - \ \text{natural number or } \infty) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array} ;$$

6. $\square$ is the transition type. It is an object having a form similar to a Boolean expression. It has as variables the same symbols that serve as labels for the transition's input places. It is constructed of these variables and the Boolean connectives $\wedge$ and $\vee$. When the value of its type (calculated as a Boolean expression) is "$true$", the transition can become active. Otherwise, it cannot.

Definition 2. *Generalized net E* is the ordered four-tuple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, b\rangle\rangle, \quad (2)$$

where

1. A is the set of GN-transitions.
2. π_A is a function which gives the priorities of the transitions, i.e., $\pi_A : A \to \mathcal{N}$.
3. π_L is a function which gives the priorities of the places, i.e., $\pi_L : L \to \mathcal{N}$, where

$$L = pr_1 A \cup pr_2 A$$

is the set of all GN-places.
4. c is a function which gives the capacities of the places, i.e., $c : L \to \mathcal{N}$.
5. f is a function which calculates the truth values of the predicates of the transition's conditions. It obtains the values "$false$" or "$true$", or values from the set $\{0, 1\}$. if $\mathcal{P}$ is the set of the predicates used in a given GN-model, then f can be defined as $f : \mathcal{P} \to \{0, 1\}$.
6. θ_1 is a function which gives the next time-moment when a given transition Z can be activated, i.e., $\theta_1(t) = t'$, where $pr_3 Z = t, t' \in [T, T + t^*]$ and $t \leq t'$. The value of this function is calculated at the moment when the transition terminates its functioning.
7. θ_2 is a function which gives the duration of the active state of a given transition Z, i.e., $\theta_2(t) = t'$, where $pr_4 Z = t \in [T, T + t^*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition starts functioning.
8. K is the set of the GN's tokens. In some cases, it is convenient to consider this set in the form

$$K = \bigcup_{l \in Q^I} K_l,$$

where K_l is the set of tokens which enter the net from place l, and Q^I is the set of all input places of the net.
9. π_K is a function which gives the priorities of the tokens, i.e., $\pi_K : K \to \mathcal{N}$.
10. θ_K is a function which gives the time moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$.
11. T is the time-moment when the GN starts functioning; this moment is determined with respect to a fixed (global) time-scale.
12. t^0 is an elementary time-step, related to the fixed (global) time-scale.
13. t^* is the duration of the GN's functioning;
14. X is a function which assigns initial characteristics to each token when it enters an input place of the net.
15. Φ is a characteristic function which assigns new characteristics to each token when it makes a transfer from an input to an output place of a given transition;

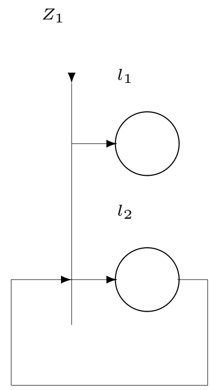

Figure 4: Generalized net representation of Generator.

16. b is a function which gives the maximum number of characteristics a given token can receive, i.e., $b : K \to N$.

The input and output places of the transitions, the IMs of the capacities of the arcs and the types of the transitions determine the static structure of a GN. The dynamic nature of a GN is expressed in the tokens and the transitions' condition. The temporal properties of a GN are presented through the time components T, t^0, t^* and the elements of the set $pr_{3,4}A$, i.e., the functions θ_1 and θ_2 of the transitions. The functions Φ, X and b serve as a GN's memory. The functions π_A, π_L, c are related to the GN's static structure, f and π_K are related to the GN's dynamic elements and θ_1, θ_2 and θ_K are related to the time components of a GN.

03 GN representations of the basic elements of Service Systems Theory

Despite the fact that GNs have been used as a tool for modelling of service systems, and telecommunication systems in particular, starting with the paper [Poryazov & Atanassov, 1997], the connection between the GN conceptual models and the models based on Service Systems Theory had not been studied. In [Andonov et al, 2019a], for the first time GN representations of the basic elements of Service Systems Theory were proposed. The work in this direction continues with the paper [Andonov et al, 2020a] where GN representations of more complex elements of Service Systems Theory are proposed, such as information feedback, information feedback and feedforward and requests feedback. The proposed representations allow for the easier construction of GNs conceptual models of service systems using the already existing models based on Service Systems Theory.

The graphical representations of the proposed GN representations of the basic elements of Service Systems Theory are shown below. The description of the transitions of the GNs can be found in [Andonov et al, 2019a].

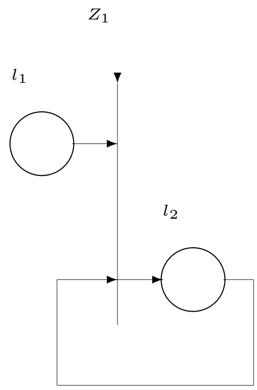

Figure 5: Generalized net analogue of Terminator.

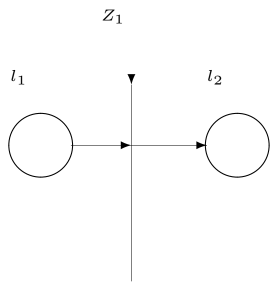

Figure 6: Generalized net representation of Transportation.

031 Generator

The function of the Generator is to create requests, for instance call attempts. Its execution does not increase the model time. The device has characteristics: capacity and time interval between two consecutive requests – usually a pseudo random variable. The requests generated belong to a certain type. A graphical representation of Generator is shown in Fig. 4.

032 Terminator

The Terminator removes the requests from the model. It has capacity. The duration of its execution is 0, i.e. it does not change the model time.

A GN representation of the Terminator is shown in Fig. 5. The requests that should be terminated are represented by tokens in place l_1.

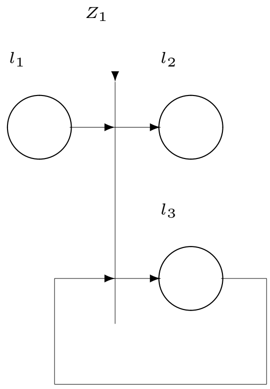

Figure 7: Generalized net representation of Delay.

033 Transportation

The function of the Transportation is to represent the movement of objects from one part of the model to another. It reflects the dynamics of the modeled process. A GN representation of Transportation is shown in Fig. 6. The places of the transition represent two different parts of the modeled system for which there is a flow of information in one direction – from the input place l_1 to the output place l_2.

034 Delay

The Delay is used to represent situation in the modelled process when requests must wait for a certain period of time until the condition for their transfer is satisfied. The GN representation of the Delay is shown in Fig. 7. The tokens in place l_1 representing requests that must wait enter place l_3. They stay there until the condition for transfer is satisfied. The evaluation of the truth value of the predicate corresponding to the transfer of the tokens from input to output places does not change the model time, i.e. it is performed outside of it.

The GN representation can be used not only when requests must wait a given number of steps but also when their further transfer depends on some logical condition and the delay must be determined at any time step. For example, the delay could be as a result of accumulation of many events within the model.

035 Server

The Server represents control or comparison of results with their standard or expected values. This includes checking of requests' quality or quantity parameters, control of the results of experiments, reading documents before taking a decision etc. A GN representation of Server is shown in Fig. 8. The change of the of the characteristics of the requests is modelled through the characteristic function of the places of the GN.

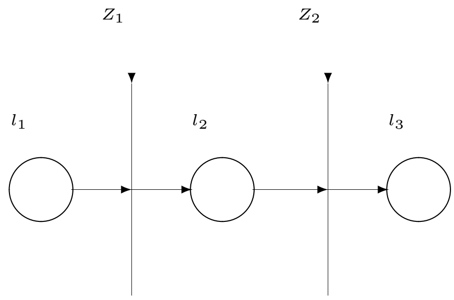

Figure 8: Generalized net representation of Server.

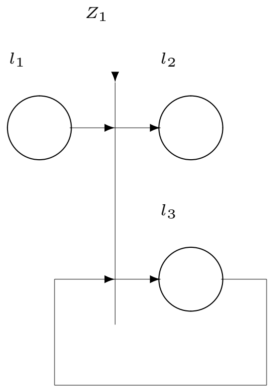

Figure 9: Generalized net representation of Information gathering.

036 Information gathering

The Information gathering denotes the accumulation and storage of data obtained within the model. In particular it describes the passive storage of data. The GN representation of the Information gathering is shown in Fig. 9. The information that must be stored is preserved in the form of characteristics of a token which stays permanently in place l_3.

037 Unifying Transition

The Unifying Transition denotes two channels that merge to form a single channel. Its GN representation is shown in Fig. 10. The tokens entering place l_3 merge to form a single token which preserves their characteristics.

038 Distributive Transition

The Distributive Transition represents one channel that splits into two. Its GN representation is shown in Fig. 11.

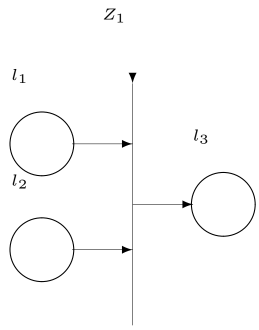

Figure 10: Generalized net representation of Unifying Transition.

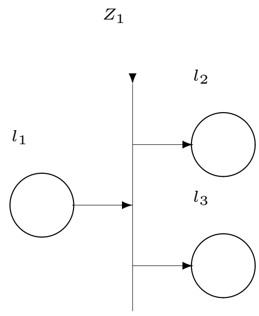

Figure 11: Generalized net representation of Distributive Transition.

039 Queue

The Queue represents waiting lines. It is important to know where the information comes from and what is the management of the queue, i.e. FIFO, LIFO or some other rule. A GN representation of a Queue is shown in Fig. 12. The management of the Queue can be described in terms of GNs in different ways. For example, through the priorities of the places and the predicates of the transition's condition.

Conceptual models of queuing systems based on Service Systems Theory

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For the queuing systems containing buffer and server, the first quantitative models are created by Erlang [Erlang, 1917]. After that newer methods have been used [Kleinrock, 1975]. These systems are used in practically all service networks, including telecommunication, computing, logistic, etc [Otsetova, 2016]. Despite this, in the broad scientific literature their conceptual models are being represented graphically in a very simple way [Gupta & Aurora, 2018]. The existing conceptual models are not suitable for inclusion in a conceptual model of overall telecommunication network such as the one described in [Poryazov & Saranova, 2012]. Therefore, different conceptual models of queuing systems based on Service Systems Theory have been proposed and compared in the papers [Poryazov et al, 2018a; Andonov et al, 2019b; Poryazov et al, 2020b]. Later, in the paper [Poryazov et al,2020a] a conceptual

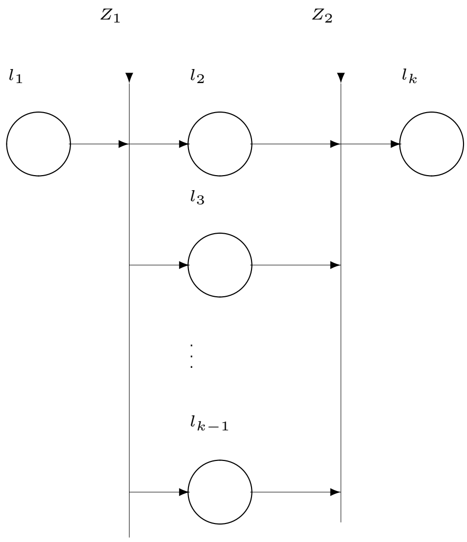

Figure 12: Generalized net representation of a Queue.

model of the causal decomposition of the traffic in a queuing system is proposed. Here, we summarize briefly these models.

04 Classical conceptual model of a queuing system

The classical representation of a queuing system is shown in Fig. 13. The buffer device is denoted by **ws** (abbreviation of “waiting for server”). The server is denoted by **s**. A generator generates requests with flow intensity $ofr.Fws$ (intensity of the offered flow of requests to the **ws** device). If both the server and the buffer have reached their capacities, the requests enter the **bws** device (abbreviation for “blocked waiting for switch") with probability $Pbws$. The “blocked queuing” device is outside the queuing system. It corresponds to the duration of specific signalization, e.g. listening of the busy tone in telephone systems. From there the requests leave the system through the terminator device after the **bws** device. If the buffer hasn’t reached its capacity, then the requests enter the **ws** device with probability $1 - Pbws$. The requests wait in the buffer to be serviced if the server has reached its capacity. Otherwise, they are sent without delay to the server. In both cases, the flow intensity of the requests entering the server is denoted by $inc.Fs$.

05 Detailed conceptual model of a queuing system

An extension of the classical conceptual model of a queuing system is shown in Figure. 14. As above, a generator generates offered requests to the server with flow intensity $ofr.Fws$. If the queuing system has reached its capacity, i.e., both the server (**s**) and the buffer (**ws**) have reached their capacities, the requests are sent to the **bws** device with probability $Pbws$ and from there they leave the system through the terminator. Otherwise, with probability $1 - Pbws$ the requests enter the **ws** device. Here,

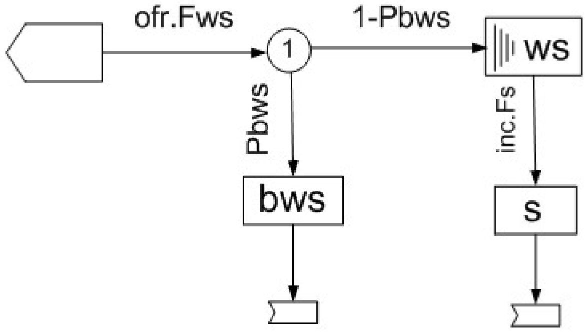

Figure 13: Classical conceptual model of a queuing system.

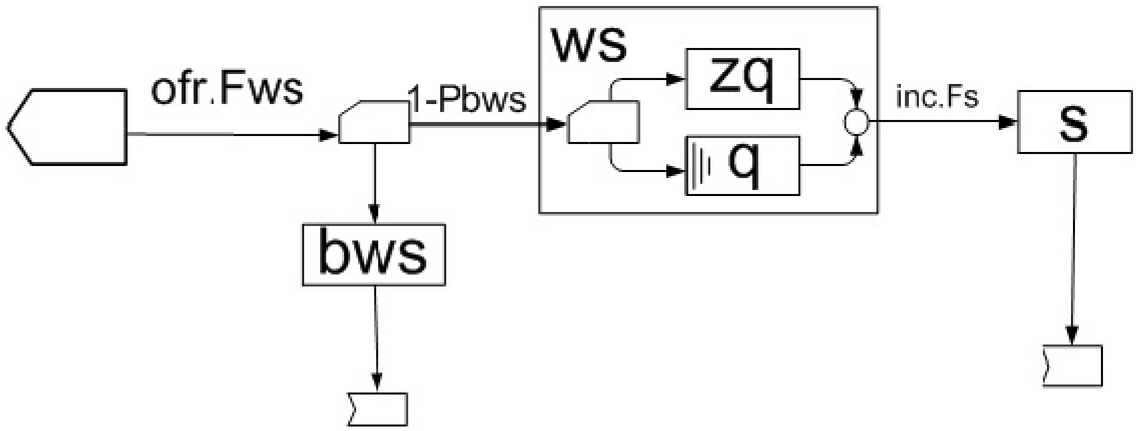

Figure 14: Extension of the classical conceptual model of a queuing system.

the **ws** device is a comprise virtual device, i.e. it contains other devices. The enter switch device inside the **ws** device sends the requests to the zero queuing (**zq**) device when the server has not reached its capacity. From there without delay, they enter the server. This corresponds to the service of the requests by the server without delay. If the server has reached its capacity, the requests are sent to the **q** device. This corresponds to the service of the requests with waiting.

This extension of the classical model allows for the two ways of service of the request in the queuing system to be represented graphically: service with waiting and service without waiting. The mean service time of the requests in the buffer (Tws), for both the waiting and the non-waiting requests, is given by

$$Tws = Pq\ Tq + (1 - Pq)\,Tzq\,, \tag{3}$$

where Pq is the probability that the request is serviced with waiting and Tq is the mean service time in the buffer for the waiting requests. Tzq is the duration of service in the buffer without waiting.

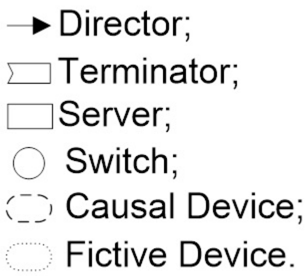

Figure 15: Base virtual device types and their graphical representations.

06 Causal structure of a qeueuing system

The importance of the Quality of Service (QoS) indicators grows with the usage of the informational service networks and it became a commodity in 2015 [Varela et al, 2015]. The QoS and Quality of Experience (QoE) are defined in different ways, but we follow the definition in standardization documents such as the ITU-T [ITU-T, 2017]. The prediction of the overall network quality, as a function of qualities of composed services, is a foremost question in service networks design and maintenance. There are two main approaches to QoS aggregation - analytical (e.g. [Zheng et al, 2011]) and simulational (e.g. [Gatnau et al, 2013]).

In order to study the problem for the QoS composition in queuing systems, first in the paper [Poryazov et al,2020a] conceptual models of the causal structure of virtual devices with limited and unlimited capacities are proposed. A special notation of the parameters is proposed and three types of quality indicators are defined: Traffic Quality Indicators, Flow Quality Indicators and Time Quality Indicators. The proposed approach is applied to a queuing system with limited capacity of the buffer and server.

07 Causal structure of a device with limited capacity

The base virtual devices used in the causal decomposition of the traffic together with their graphical representations are shown in Fig. 15.

The devices of each type have specific functions (see [Poryazov & Saranova, 2012]):

- Director – this device points unconditionally to the next device, which the request shall enter but without transferring, changing or delaying it.
- Terminator – this device eliminates every request entered (so it leaves the model without any traces).
- Server – this device models the delay (service time, holding time) of requests in the corresponding device without their generation or elimination. It models also traffic and time characteristics of the requests processing.
- Switch (Transition) – this device selects one of its possible exits for each request entered, thus determining the next device where this request shall go to.

- Causal device – virtual device defined for presentation of causes of service ending, e.g., successful (carried) or not (interrupted, abandoned, etc.).
- Fictive device – device presenting fictive traffic which is necessary for engineering. For example, not carried traffic is fictive, but it is used for calculating of the offered traffic, which is necessary for device dimensioning.

We group causes of service ending, and corresponding causal devices, in generalized comprising causal devices. If device has unlimited capacity, three causal generalizations are enough: “parasitic”, “carried” and “served” (see [Poryazov et al, 2018b]).

The *Parasitic Traffic* in a pool of resources is the traffic, which was unsuccessfully served in the pool. Parasitic traffic occupies real resources but not for a useful service execution.

The *Carried Traffic* in a pool of resources is the traffic, which was successfully served in the pool (and carried to the next service device). We distinguish two types of carried traffic:

- 'zero service' e.g. zero waiting in a buffer if the buffer is empty and there is free requested place for the service in the following device. The requests are receiving zero service in the causal 'zero service device' and may be served without delay;
- 'genuine service' - successfully and real served requests in the pool. The service time is noticeable.

The *Served Traffic* in a pool of resources is any traffic, occupying (using) resources in the pool. The Served Traffic is a sum of carried and parasitic traffic.

Every Causal Device Parameter's Name is a concatenation:

Causal name =<qualifier><qualifier>.<Parameter's Symbol>.<Device Name>.

The qualifiers used in the conceptual models of the causal decomposition are:

- crr. = carried;
- gen. = genuine;
- nsr. = not served;
- ofr. = offered;
- prs. = parasitic;
- srv. = served;
- zer. = zero.

'Parameter's symbol' is one of letters P, F, T, Y, V, N, as they are described in Section 2. Qualifiers are used to characterize the parameters of the devices [Poryazov et al, 2018b]. Used qualifiers may be two, one or none. If parameter's symbol is omitted, the causal name is a name of a device (Fig. 16). Device name may be in small or subscript letters. For instance, $crr.Fx$ is the intensity of the carried

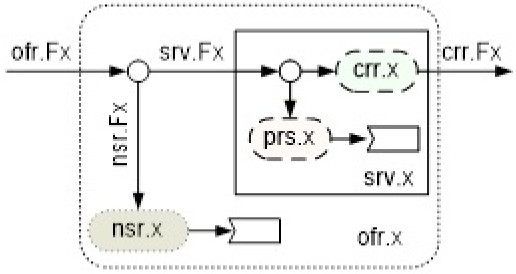

Figure 16: A causal structure of a network portion represented by device x with a limited capacity.

flow of requests of the device x, $prs.Fx$ is the intensity of the parasitic flow of requests of the device x (see Fig. 16).

In the figures, only the names of causal devices may be presented. The names of device parameters are implicit. In Fig. 16 there are three generalized conceptual devices: x, $prs.x$, and $crr.x$.

A more general and real presentation of a service network portion "x" is shown in Fig. 16. It contains a virtual device srv.x with limited capacity. This may cause requests rejection due to lack of service place (call attempts blocking).

The fictive virtual device $nsr.x$ corresponds to the not served traffic, due to blocking or other reasons that would be served, with the same time duration:

$$nsr.Tx = srv.Tx \; . \tag{4}$$

The not served traffic intensity, following the theorem of Little, is given by:

$$nsr.Yx = nsr.Fx \; srv.Tx \; . \tag{5}$$

Therefore, the equivalent traffic offered [ITU-T E.501,] to device x is:

$$ofr.Yx = nsr.Yx + srv.Yx \; . \tag{6}$$

The probability of not service (blocking) $nsr.Px$ may be predicted using a blocking formula, e.g., the B-formula of Erlang. The offered traffic concept leads to necessity of definition of a conceptual device called $ofr.x$ with its parameters P, F, T, Y, and V (the capacity of the offered traffic device is not considered usually). Hence, in Fig. 16 the network portion x is presented by 5 generalized conceptual devices: $prs.x$, $crr.x$, $srv.x$, $nsr.x$ and $ofr.x$ and 2 terminators, 2 transitions and 8 directors.

071 Conceptual model of the causal structure of a queuing system

We propose the following causal conceptual model of a network portion, consisting of queuing system named 'qs' (Fig. 17).

The buffer and the server have limited capacities. The requests to the queuing system with flow intensity $ofr.Fqs$ try to enter the server (s). If there is a free place in the server, the requests pass through the buffer without waiting. If in the server there is no free place for service, the requests may

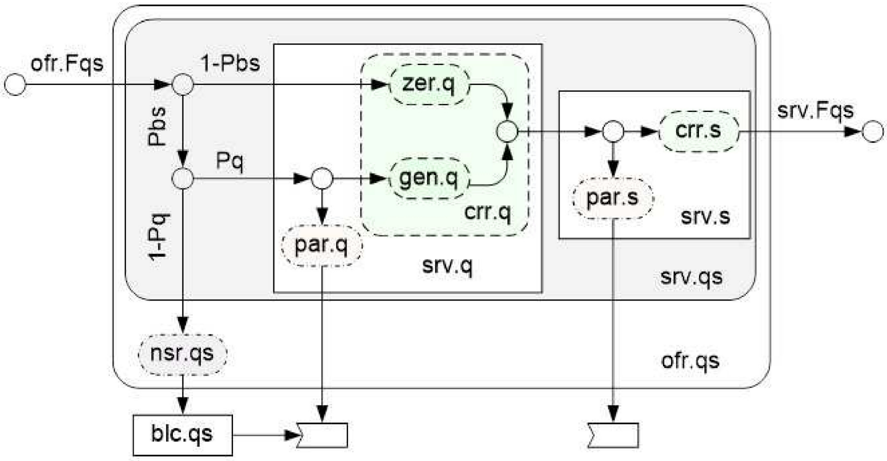

Figure 17: A causal structure of a network portion with queuing system qs with a limited capacity of the buffer and the server.

wait for service in the buffer device. So, the buffer service may be with or without queuing. In Fig. 17 the buffer is denoted by 'queuing device' (q). We consider the probability 'blocked server' (Pbs), i.e. the server to be full and the requests' service to be blocked. The blocked server probability is different from the 'not served in the server' probability because some of the blocked requests, due to the busy server, may be served after waiting in the buffer (the name '$nsr.Ps$' is not shown in Fig. 17).

If the server is not full (the probability of blocked server is less than one), with a probability of $1 - Pbs$ the requests pass through the buffer without queuing, in the zero queuing ($zer.q$) device.

With a probability of Pbs (blocked server) the server is busy and the requests try to enter the queue in the buffer, with a probability of Pq. If they enter the queue, the waiting may be successful in the genuine queuing ($gen.g$) device, or unsuccessful in the parasitic queuing ($prs.q$) causal device.

The following assumptions have been stated.

Assumption 1: All considered processes are in a stationary state. The values of all parameters are random and we consider their means (or mathematical expectations).

Assumption 2: In case of zero queuing, there is no parasitic service in the buffer, due to the little service duration. The causal device 'carried queuing' ($crr.q$) comprises devices 'zero queuing' ($zer.q$) and 'genuine queuing' ($gen.g$). With a probability of $1 - Pq$ the buffer is busy and the requests are not served in the queuing system – they enter the fictive device 'not served in the queuing system' ($nsr.qs$).

After carried queuing, the requests enter the server device ($srv.s$) with a flow intensity of $srv.Fs$ (the name is not shown in Fig. 17) and they are served: successfully in the carried service ($crr.s$) device or unsuccessfully in the parasitic service ($prs.s$) device. Obviously:

$$srv.Fs = crr.Fq\,. \tag{7}$$

The flow intensity of the not served in the queuing system requests ($nsr.Fqs$) is equal to that of the queuing device ($nsr.q$):

$$nsr.Fqs = nsr.Fq,. \tag{8}$$

Assumption 3: Following the definition of equivalent traffic offered, the fictive service times in the 'not serve' devices are:

$$nsr.Tqs = srv.Tqs\,. \tag{9}$$

$$nsr.Tq = srv.Tq\,. \tag{10}$$

$$nsr.Ts = srv.Ts\,. \tag{11}$$

Conceptual modelling of queuing systems with generalized nets

The first GN model of a queuing system is described in [Andonov et al, 2018] as a part of a GN model of the Switching stage of an overall telecommunication network. A GN model corresponding to the classical model of a queuing system with FIFO discipline of service of the requests is described in [Poryazov et al, 2018a]. In [Tomov et al, 2018], GN models of queuing systems with various disciplines of service of the requests are described. Some of the models of queuing systems proposed in the previous publications are extended and modified in [Tomov et al, 2019] with the inclusion of Intuitionistic Fuzzy Pairs (IFPs, see [Atanassov, 2013]) and Interval-Valued Intuitionistic Fuzzy Pairs (IVIFPs), which determine the way (discipline) of service of the requests by the queuing system. The buffer has limited capacity and is represented by two GN transitions. The places of the buffer are represented by places of the GN. Simple disciplines of service of the requests are considered (FIFO, LIFO), as well as more general models with IFPs (IVIFPs), in which the requests can change their parameters and places within the buffer.

A GN model of the causal structure of a queuing system is constructed in [Andonov et al, 2020]. Also there, the problem for representation of comprise virtual device through GNs is discussed.

In this section, we shall briefly decribe some of these models.

08 Generalized net models of queuing systems

081 First generalized net model of a queuing system

A GN model corresponding to the classical conceptual model of a queuing system is shown in Fig. 18. In comparison to the similar model proposed in [Poryazov et al, 2018a], here only 4 transitions and 10 places are used. This is achieved through the representation of the terminator devices as a places of the GN. This possibility is mentioned in [Andonov et al, 2019a].

Each of the four transitions represents some function of the corresponding base virtual devices.

- Z_1 represents the function of the Generator from Figure 13.
- Z_2 represents the function of Transition 1 from Figure 13.
- Z_3 represents the function of the **ws** device from Figure 13.
- Z_4 represents the function of the **s** (the Server device) from Figure 13.

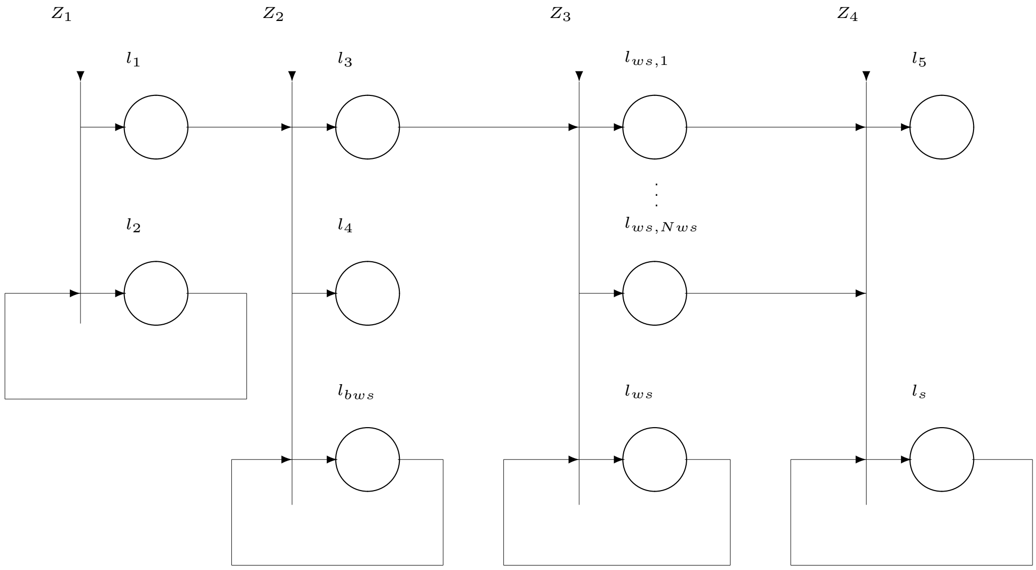

Figure 18: First generalized net model of a queuing system.

The places of the GN represent virtual devices in the following way:

- l_1 and l_2 represent the Generator before Transition 1 in Figure 13.
- l_3 has no analogue in Figure 13.
- l_4 represents the terminator after the **bws** device.
- l_{bws} represents the **bws** device.
- $l_{ws,1}, \dots, l_{ws,Nws}$ represent the waiting places of the buffer device **ws**.
- l_{ws} represents the buffer device **ws**.
- l_5 represents the terminator after the **s** device.
- l_s represents the server.

Four types of tokens are used:

- Tokens of type α represent the requests. In the initial time moment of the GN functioning, token α stays in place l_2 with initial characteristic "*formula for generating the flow of requests*".
- Token of type β stays in place l_{bws} in the initial time moment with initial characteristic "initial values of $Y_{bws}, P_{bws}, F_{bws}, T_{bws}, N_{bws}$". It is used to accumulate data about the **bws** device.
- Token of type γ stays in place l_{ws} in the initial time moment with initial characteristic "initial values of $Y_{ws}, P_{ws}, F_{ws}, T_{ws}, N_{ws}$". It is used to accumulate data about the **ws** device. The discipline of service of the requests can be specified in this initial characteristic. In the present paper, we consider only FIFO discipline of service of the requests.

- Token of type δ stays in place l_s in the initial time moment with initial characteristic “initial values of Y_s, P_s, F_s, T_s, N_s”. It is used to accumulate data about the **s** device.

The formal description of the transitions of the GN can be found in [Andonov et al, 2019b].

082 Second generalized net model of a queuing system

The second GN model of a queuing system, which we consider worth mentioning, corresponds to the extended classical model (see Fig. 14). The graphical representation of the GN is shown in Fig. 19. The GN consists of 7 transitions and $14 + Nq$ places where Nq is the capacity of the buffer device. The transitions represent functions of the corresponding virtual devices as follows:

- Z_1 represents the function of the Generator from Figure 14.
- Z_2 represents the function of the Enter switch device before the **ws** device from Figure 14.
- Z_3 represents the function of the Enter switch device inside the comprise virtual device **ws** from Figure 14.
- Z_4 represents the function of the **zq** device inside the comprise virtual device **ws** from Figure 14.
- Z_5 represents the function of the **q** device inside the comprise virtual device **ws** from Figure 14.
- Z_6 represents the function of the Transition device inside the comprise virtual device **ws** from Figure 14.
- Z_7 represents the function of the **s** from Figure 14.

The places of the GN correspond to virtual device in the following way:

- l_1 and l_2 represent the Generator before Enter switch device in Figure 14.
- l_3 has no analogue in Figure 14.
- l_4 represents the terminator after the **bws** device.
- l_{bws} represents the **bws** device.
- l_5 and l_6 have no analogue in Figure 14.
- l_7 and l_{zq} represent the **zq** device in Figure 14.
- $l_{q,1}, ..., l_{q,Nq}, l_q$ represent the waiting places of the buffer device **q**.
- l_8 and l_{ws} represent formally the comprise buffer device **ws** but transitions Z_4 and Z_5 are also part of the comprise virtual device **ws**.

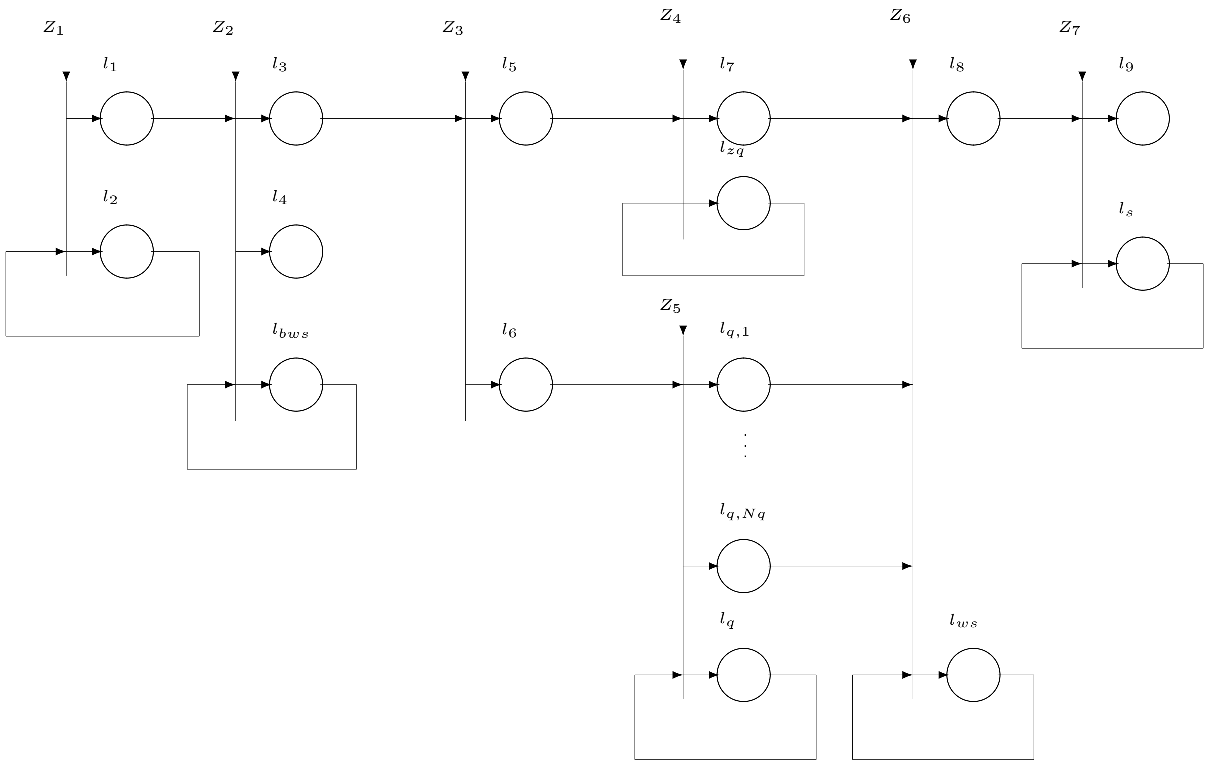

Figure 19: Second generalized net model of a queuing system.

- l_9 represents the terminator after the **s** device.
- l_s represents the server.

Six types of tokens are used in the model:

- Tokens of type α represent the requests. In the initial time moment of the GN functioning, token α stays in place l_2 with initial characteristic "formula for generating the flow of requests".
- Token of type β stays in place l_{bws} in the initial time moment with initial characteristic "initial values of $Y_{bws}, P_{bws}, F_{bws}, T_{bws}, N_{bws}$ ". It is used to accumulate data about the **bws** device.
- Token of type γ stays in place l_{zq} in the initial time moment with initial characteristic "initial values of $Y_{zq}, P_{zq}, F_{zq}, T_{zq}, N_{zq}$". It is used to accumulate data about the **zq** device.
- Token of type δ stays in place l_q in the initial time moment with initial characteristic "initial values of Y_q, P_q, F_q, T_q, N_q". It is used to accumulate data about the **q** device. The discipline of service of the requests can be specified in this initial characteristic. We consider only FIFO discipline of service of the requests.
- Token of type ϵ stays in place l_{ws} in the initial time moment with initial characteristic "initial values of $Y_{ws}, P_{ws}, F_{ws}, T_{ws}, N_{ws}$". It is used to accumulate data about the **ws** device.

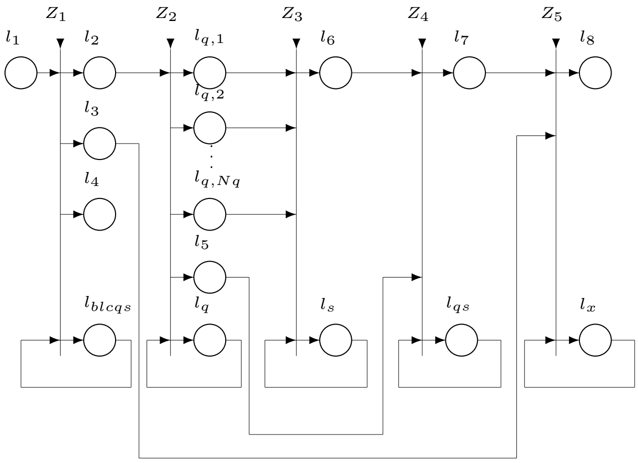

Figure 20: GN model of the simple causal structure representation of a queuing system.

- Token of type ζ stays in place l_s in the initial time moment with initial characteristic "initial values of Y_s, P_s, F_s, T_s, N_s". It is used to accumulate data about the **s** device.

The formal description of the transitions can be found in [Andonov et al, 2019b].

09 Generalized net model of the causal structure of a queuing system

The first two GN representations of the causal structure of a queuing system are described in [Andonov et al, 2020a].

091 First generalized net model of the causal structure of a queuing system

We propose a GN representation of the simple causal structure of a queuing system in which the comprise devices are included in the gaphical representation. The GN model is shown in Fig. 20.

The GN model consists of 5 transitions and $13 + Nq$ places where Nq is the capacity of the buffer. The labels of those places which represent virtual devices are in the form l_y where "y" is the name of the corresponding virtual device. The transitions represent functions of the corresponding virtual devices as follows:

- Z_1 represents the function of the Switch before the **qs** device.
- Z_2 represents the function of the Director pointing to the **q** device.
- Z_3 represents the function of the Director between the **q** device and the **s** device.
- Z_4 represents the function of the comprise virtual device **qs**.

- Z_5 represents the function of the comprise virtual device **x**.

The places of the GN correspond to virtual devices in the following way:

- l_{blcqs} represents the **blc.qs** device.
- $l_{q,1}, ..., l_{q,Nq}, l_q$ represent the waiting places of the buffer device **q**.
- l_s represents the server device **s**.
- l_{qs} represents the comprise virtual device **qs**.
- l_x represents the comprise virtual device **x**.

Six types of tokens are used in the model:

- Tokens of type α represent the requests. In the initial time moment of the GN functioning, token α stays in place l_1 with initial characteristic "formula for generating the offered flow of requests to the queuing system".
- Token of type β stays in place l_{blcqs} in the initial time moment with initial characteristic "initial values of $Y_{blc.qs}, P_{blc.qs}, F_{blc.qs}, T_{blc.qs}, N_{blc.qs}$". It is used to accumulate data about the **blc.qs** device.
- Token of type γ stays in place l_q in the initial time moment with initial characteristic "initial values of Y_q, P_q, F_q, T_q, N_q". It is used to accumulate data about the **q** device. The discipline of service of the requests can be specified in this initial characteristic. Here, we consider only FIFO (First-In, First-Out) discipline of service of the requests.
- Token of type δ stays in place l_s in the initial time moment with initial characteristic "initial values of Y_s, P_s, F_s, T_s, N_s". It is used to accumulate data about the **s** device.
- Token of type ϵ stays in place l_{qs} in the initial time moment with initial characteristic "initial values of $Y_{qs}, P_{qs}, F_{qs}, T_{qs}, N_{qs}$". It is used to accumulate data about the **s** device.
- Token of type ζ stays in place l_x in the initial time moment with initial characteristic "initial values of Y_x, P_x, F_x, T_x, N_x". It is used to accumulate data about the **x** device.

092 Second generalized net model of the causal structure of a queuing system

A GN model corresponding to the detailed conceptual model of the causal structure of a queuing system (see Fig. 17) is shown in Fig. 21.

The GN consists of 9 transitions and $23 + Nq$ places, where Nq is the capacity of the buffer. The labels of those places which represent virtual devices are in the form l_y where "y" is the name of the corresponding virtual device but the "." symbol, if present in the name of the device, has been omitted. The transitions represent functions of the corresponding virtual devices as follows:

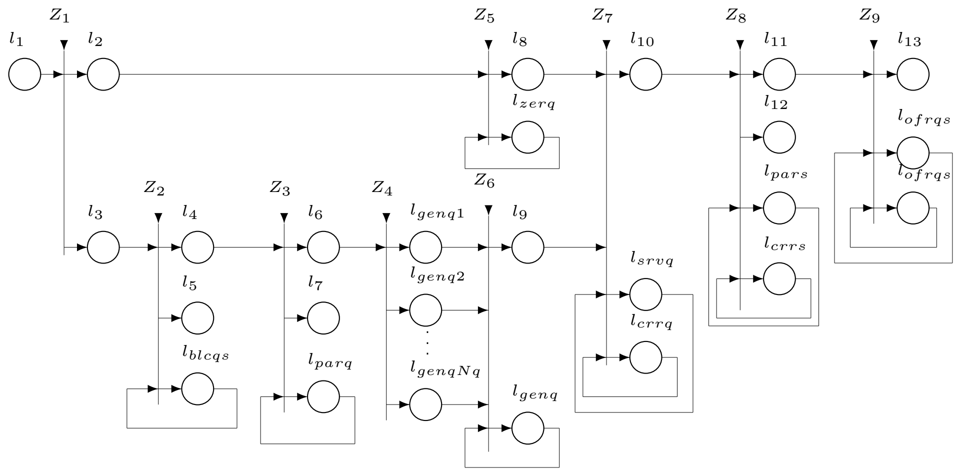

Figure 21: GN model of a detailed representation of the causal structure of a queuing system.

- Z_1 represents the function of the Director outgoing of the first Switch device in Fig. 17.
- Z_2 represents the function of the Director pointing to the **blc.qs** device from Fig. 17.
- Z_3 represents the function of the Switch before the **crr.q** causal device inside the **srv.q** causal device in Fig. 17.
- Z_4 represents the function of the Director entering the causal device **gen.q** in Fig. 17.
- Z_5 represents the function of the causal device **zer.q** from Fig. 17.
- Z_6 represents the function of the causal device **gen.q** in Fig.17.
- Z_7 represents the function of the Switch device to which the requests are sent from the causal devices **zer.q** and **gen.q** inside the causal device **crr.q** in Fig.17.
- Z_8 represents the function of the Switch device inside the **srv.s** device in Fig.17.
- Z_9 represents the function of the comprise devices **srv.qs** and **ofr.qs** in Fig.17.

Eleven types of tokens are used in the model:

- Tokens of type α represent the requests. In the initial time moment of the GN functioning, token α stays in place l_1 with initial characteristic "formula for generating the offered flow of requests to the queuing system".
- Token of type β stays in place l_{blcqs} in the initial time moment with initial characteristic "initial values of $Y_{blc.qs}, P_{blc.qs}, F_{blc.qs}, T_{blc.qs}, N_{blc.qs}$". It is used to accumulate data about the **blc.qs** device.

- Token of type γ stays in place l_{parq} in the initial time moment with initial characteristic "initial values of $Y_{par.q}, P_{par.q}, F_{par.q}, T_{par.q}, N_{par.q}$". It is used to accumulate data about the **par.q** device. The discipline of service of the requests can be specified in this initial characteristic. Here, we consider only FIFO discipline of service of the requests.

- Token of type δ stays in place l_{zerq} in the initial time moment with initial characteristic "initial values of $Y_{zer.q}, P_{zer.}, F_{zer.q}, T_{zer.q}, N_{zer.q}$". It is used to accumulate data about the **zer.q** device.

- Token of type ϵ stays in place l_{genq} in the initial time moment with initial characteristic "initial values of $Y_{gen.q}, P_{gen.q}, F_{gen.q}, T_{gen.q}, N_{gen.q}$". It is used to accumulate data about the **gen.q** device.

- Token of type ζ stays in place l_{srvq} in the initial time moment with initial characteristic "initial values of $Y_{srv.q}, P_{srv.q}, F_{srv.q}, T_{srv.q}, N_{srv.q}$". It is used to accumulate data about the **srv.q** device.

- Token of type η stays in place l_{crrq} in the initial time moment with initial characteristic "initial values of $Y_{crr.q}, P_{crr.q}, F_{crr.q}, T_{crr.q}, N_{crr.q}$". It is used to accumulate data about the **crr.q** device.

- Token of type θ stays in place l_{pars} in the initial time moment with initial characteristic "initial values of $Y_{par.s}, P_{par.s}, F_{par.s}, T_{par.s}, N_{par.s}$". It is used to accumulate data about the **par.s** device.

- Token of type κ stays in place l_{crrs} in the initial time moment with initial characteristic "initial values of $Y_{crr.s}, P_{crr.s}, F_{crr.s}, T_{crr.s}, N_{crr.s}$". It is used to accumulate data about the **crr.s** device.

- Token of type λ stays in place l_{ofrqs} in the initial time moment with initial characteristic "initial values of $Y_{ofr.qs}, P_{ofr.qs}, F_{ofr.qs}, T_{ofr.qs}, N_{ofr.qs}$". It is used to accumulate data about the **ofr.qs** device.

- Token of type μ stays in place l_{srvqs} in the initial time moment with initial characteristic "initial values of $Y_{srv.qs}, P_{srv.qs}, F_{srv.qs}, T_{srv.qs}, N_{srv.qs}$". It is used to accumulate data about the **srv.qs** device.

The formal description of the GN transitions can be found in [Andonov et al, 2020].

Conclusion

As a result of the presented research on the conceptual modelling of queuing systems in service networks, two approaches have been developed – Service Systems Theory approach and GNs approach. The comparison of the two approaches shows that depending on the purpose of the modelling one or the other may give better results. Furthemore, the proposed GNs representation of the elements of Service Systems Theory allows easier construction of GNs conceptual models based on Service Systems Theory and vice versa.

The large number of conceptual models of queuing systems that have been developed as a result of our research has lead to the rise of another problem, i.e., which one of the models to choose. This problem is partially addressed in the paper [Andonov et al, 2020b], where a general scheme for conceptual optimization of GN models is proposed based on the indicators for complexity of GNs. Also there, the conceptual optmization scheme is applied to a GN model of a queuing system.

In [Andonov et al, 2019], a conceptual model of a queuing system based on Service Systems Theory is included in a conceptual model of overall telecommunication system. Analytical expressions for the important parameters of the queuing system are derived.

In [Andonov et al, 2019c], an analytical model of overall telecommunication system with queuing is constructed using the conceptual model based on Service Systems Theory.

In [Andonov et al, 2020c], a GN model of overall telecommunication system with queuing is described. One of the GN models of a queuing system(corresponfing to the detailed representation) is included in the model.

Acknowledgements

The work of Velin Andonov is supported by the 2020 National Scientific Program for post doctoral and young scientists of the Ministry of Education and Science of Bulgaria.

The work of Stoyan Poryazov is partially supported by the joint research project “Symbolic-Numerical Decision Methods for Algebraic Systems of Equations in Perspective Telecommunication Tasks” of IMI-BAS, Bulgaria and JINR, Dubna, Russia.

The work of Emiliya Saranova was supported by the Task 1.2.5. “Prediction and Guaranteeing of the Quality of Servicein Human-Cyber-Physical Systems” of the National Scientific Program “Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICT in SES)” financed by the Bulgarian Ministry of Education and Science.

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