SENSITIVITY AND BIAS WITHIN THE BINARY SIGNAL DETECTION THEORY, BSDT

Petro Gopych

Abstract: Similar to classic Signal Detection Theory (SDT), recent optimal Binary Signal Detection Theory (BSDT) and based on it Neural Network Assembly Memory Model (NNAMM) can successfully reproduce Receiver Operating Characteristic (ROC) curves although BSDT/NNAMM parameters (intensity of cue and neuron threshold) and classic SDT parameters (perception distance and response bias) are essentially different. In present work BSDT/NNAMM optimal likelihood and posterior probabilities are analytically analyzed and used to generate ROCs and modified (posterior) mROCs, optimal overall likelihood and posterior. It is shown that for the description of basic discrimination experiments in psychophysics within the BSDT a 'neural space' can be introduced where sensory stimuli as neural codes are represented and decision processes are defined, the BSDT's isobias curves can simultaneously be interpreted as universal psychometric functions satisfying the Neyman-Pearson objective, the just noticeable difference (jnd) can be defined and interpreted as an atom of experience, and near-neutral values of biases are observers' natural choice. The uniformity or no-priming hypotheses, concerning the 'in-mind' distribution of false-alarm probabilities during ROC or overall probability estimations, is introduced. The BSDT's and classic SDT's sensitivity, bias, their ROC and decision spaces are compared.

Keywords: binary signal detection theory, sensitivity, bias, ROC, mROC, overall likelihood and posterior, neural space, psychometric function, just noticeable difference (jnd), uniformity or no-priming hypotheses.

1. Introduction

Since D.Green & J.Swets' pioneering book [1], classic Signal Detection Theory (SDT) is widely used in psychology for describing different discrimination experiments concerning the study of human/animal sensory and memory abilities. Further developments were summarized by N.A.Macmillan & C.D.Creelman whose monograph [2] reviews the state of the art in this field. Since 1960th the SDT's productivity was successfully demonstrated in numerous experiments performed using different experimental paradigms and for this reason it became extremely popular as a tool for analysis and interpretation of data in sensory and cognitive psychology.

Of course, not all SDT's applications are equally successful and this fact plays the role of an impetus for further SDT development and for designing its new, sometimes technically sophisticated although not always perfect, versions. But, perhaps, the main SDT's disadvantage is conceptual rather than technical: its decision rules act in a so called psychological decision space – the hypothetical space, quite separate from the world of stimuli and having unclear relations to it; decision space is deliberately introduced to define internal (mental) stimulus representations, objects of the SDT. Moreover, it is unclear whether specific relations between stimulus space (world) and the SDT's decision space can be one day discovered even in principle.

In present work using complete numerical examples basic notions and parameters of the optimal Binary (Binomial) SDT (BSDT) [3] are investigated and compared with corresponding notions and parameters of classic SDT [2]. The main two distinctions between BSDT and SDT approaches are emphasized. The first is technical: in contrast to the SDT's continues (Gaussian) probability distributions, BSDT operates with discrete (binomial) probability distributions and for this reason all its predictions are discrete. The second is conceptual: in contrast to the SDT's two separate spaces (the stimulus space and psychological decision space), BSDT defines for stimuli and decisions their common 'neural space' where stimuli are represented as unified neural codes (N-dimensional binary vectors) and decisions as operations over these codes. Furthermore, it may be expected that in the future all objects in the neural space could be related to sensory stimuli using the methods of neuroscience.

2. About the BSDT

As it has already been demonstrated [3], for a binary data coding initially proposed in ref. [4] there exist three data decoding algorithms (neural network, convolutional, and Hamming distance) which have equivalent, and the best in the sense of statistical patterns recognition quality, performance. For such a decoding algorithm that is equivalent to an intact two-layer neural network (NN) operating with a neuron threshold θ, its quality performance were derived [3] as analytical expressions for exact calculation of the probability (likelihood) $L(d,\theta)$ of the best correct decoding N-dimensional binary vectors $x = x(d)$ with components ±1 and a given intensity of cue $q = 1 - d$, $d = m/N$, where m is the number of noise components of x and $N - m$ is the number of intact components of x_0 among the components of x [reference vector x_0 contains the information stored or that should be stored in the NN and, consequently, $L(d,\theta)$ is also the probability of correct recognition of x_0 in $x(d)$] [3]. It is important that BSDT and recent Neural Network Assembly Memory Model (NNAMM) are mathematically similar (NNAMM is in fact a direct implementation of the BSDT for solving the problem of memory storing/retrieval) and some their basic parameters are the same [5]. The similarity between the BSDT and NNAMM (between their mathematical tools) is important as in many cases it allows do not watch for distinctions between patterns' coding/decoding and storing/retrieving and consider these processes using their common, BSDT/NNAMM, point of view.

We refer to ref. 3 and 5 for some BSDT/NNAMM details and pay here the main attention only to those BSDT's parameters which are needed to derive the BSDT's counterparts to sensitivity and bias of the classic SDT. For simplicity, to exclude the consideration of splitting the probability functions $L(d,\theta)$ [3], in this work we shall discuss only the case of odd N, i.e. the case of an odd number of the NN's entrance- or exit-layer neurons. Now we only rewrite the expression for the probability of correct decoding $L(d,\theta) = L(m,N,\theta)$ [3] using a new parameter, the threshold interval index Θ, introduced in Section 3 and compared with other BSDT's parameters in Table 1:

$$L(m,N,\Theta) = \sum_{k=0}^{k\max} C_k^m / 2^m \qquad (1)$$

where if $kmax \le kmax_0$ then $kmax = m$ else $kmax = kmax_0$; for odd N $kmax_0 = (N - \Theta - 1)/2$ where Θ is even, $-(N + 1) \le \Theta \le N - 1$, and $\Delta\Theta = 2$ with a special case if $\Theta = N + 1$ then $L(m,N,\Theta) = L(d, \theta) = 0$.

3. Relations between Some BSDT Parameters

For the case of odd N Table 1 and Figures 1 and 2 illustrate relations between some BSDT's parameters (rather large amount of them is in particular caused by the fact that the decoding algorithm has three different forms).

Table 1

Relations between some BSDT parameters for the case N = 9, as in our works [3,6]*)

i a)	$\Delta\theta_i$ b)	Q_i c)	D_i d)	ρ_i e)	Θ_i f)	F_i g)	ΔF_i h)
–1	[9, +∞)	9	0	9/9	10	0/512 = 0.00000	–
0	[7, 9)	7, 8	1	7/9, 8/9	8	1/512 = 0.00195	1/512 = 0.00195
1	[5, 7)	5, 6	2	5/9, 6/9	6	10/512 = 0.01953	9/512 = 0.01758
2	[3, 5)	3, 4	3	3/9, 4/9	4	46/512 = 0.08984	36/512 = 0.07031
3	[1, 3)	1, 2	4	1/9, 2/9	2	130/512 = 0.25391	84/512 = 0.16406
4	[–1, 1)	–1, 0	5	–1/9, 0/9	0	256/512 = 0.50000	126/512 = 0.24606
5	[–3, –1)	–3, –2	6	–3/9, –2/9	–2	382/512 = 0.74609	126/512 = 0.24606
6	[–5, –3)	–5, –4	7	–5/9, –4/9	–4	466/512 = 0.91016	84/512 = 0.16406
7	[–7, –5)	–7, –6	8	–7/9, –6/9	–6	502/512 = 0.98047	36/512 = 0.07031
8	[–9, –7)	–9, –8	9	–9/9, –8/9	–8	511/512 = 0.99805	9/512 = 0.01758
9	(–∞, –9)	–	–	–	–10	512/512 = 1.00000	1/512 = 0.00195

*) N is simultaneously the dimension of binary vectors $x = x(d)$, the number of the NN's entrance- and exit-layer neurons, maximal amount of bits of information may be conveyed by vectors x, the NN's information capacity in bits, and the length of convolutional interval for the NN convolutional decoding algorithm [6].

a) The number of a neuron threshold interval $\Delta\theta_i$, false alarm F_i, and etc, $i = -1,0,1,2,\ldots,N$.

b) The ith neuron threshold interval $\Delta\theta_i = [N - 2(i + 1), N - 2(i + 1) + 2)$, $i = 0,1,2,\ldots,N - 1$; $\Delta\theta_{-1} = [N, +\infty)$, $\Delta\theta_N = (-\infty, -N)$; $[\theta_{left}, \theta_{right})$ means $\theta_{left} \leq \theta < \theta_{right}$, $(\theta_{left}, \theta_{right})$ means $\theta_{left} < \theta < \theta_{right}$ (θ_{left} and θ_{right} are left-most and right-most points of an interval $\Delta\theta$); magnitudes of neuron thresholds are continuos, $-\infty < \theta < +\infty$; each $\Delta\theta_i$ is so defined that for all $\theta \in \Delta\theta_i$ probabilities $L(d,\theta)$ are constant, $L(d,\theta) = L(q,\Theta_i)$.

c) The convolution, Q_i, between $x(d)$ and x_0 for the ith neuron threshold interval $\Delta\theta_i$, $i = -1,0,1,2,\ldots,N - 1$, $-N \leq Q_i \leq N$; Q and all other parameters in the table, except θ, are discrete variables; within their common range Q and θ are equivalent, $\theta = Q$.

d) The ith Hamming distance, $D_i = (N - Q_i)/2$; as D_i is integer, for each $\Delta\theta_i$ it may be defined unambiguously.

e) The ith correlation coefficient, $\rho_i = Q_i/N$; in the neuron threshold interval $\Delta\theta_{-1}$ parameters ρ_i, D_i, and Q_i exist only in a single point ($\theta = N = 9$), in $\Delta\theta_N$ they are not defined at all.

f) The ith neuron threshold interval index (for short, threshold interval index), $\Theta_i = N - 2i - 1$, $i = -1,0,1,2,\ldots,N$; the distance between any two neighbor values of Θ_i is $\Delta\Theta_i = 2$; depending on the parity of N and taking into account that for each $\Delta\theta_i$ its Q_i and $Q_i + 1$ values produce the same value of $L(d,Q_i) = L(d,\Theta_i)$, the series of indices Θ_i is defined in such a way that $\Theta_i = 0$ is always among its items; indices Θ_i provide also a possibility to calculate the probability $L(d,\theta)$ in neuron threshold interval $\Delta\theta_{-1}$ where Q, D, and ρ are defined only in one point $\theta = N$, in $\Delta\theta_N$ where they are not defined at all and show $L(d,\theta = 0)$ explicitly.

g) The ith false-alarm probability $F_i = \sum C^N_k/2^N$ where $k = 0,1,\ldots,i$, $C^N_k = N!/(N - k)!/k!$, $i = 0,1,2,\ldots,N$, $F_{-1} = 0$; the value $F_N = 1$ is assigned for $\theta \in \Delta\theta_N$ where Q, D, and ρ are not defined.

h) The ith false-alarm probability interval $\Delta F_i = F_i - F_{i-1} = C^N_i/2^N$, $C^N_i = N!/(N - i)!/i!$, $i = 0,1,2,\ldots,N$.

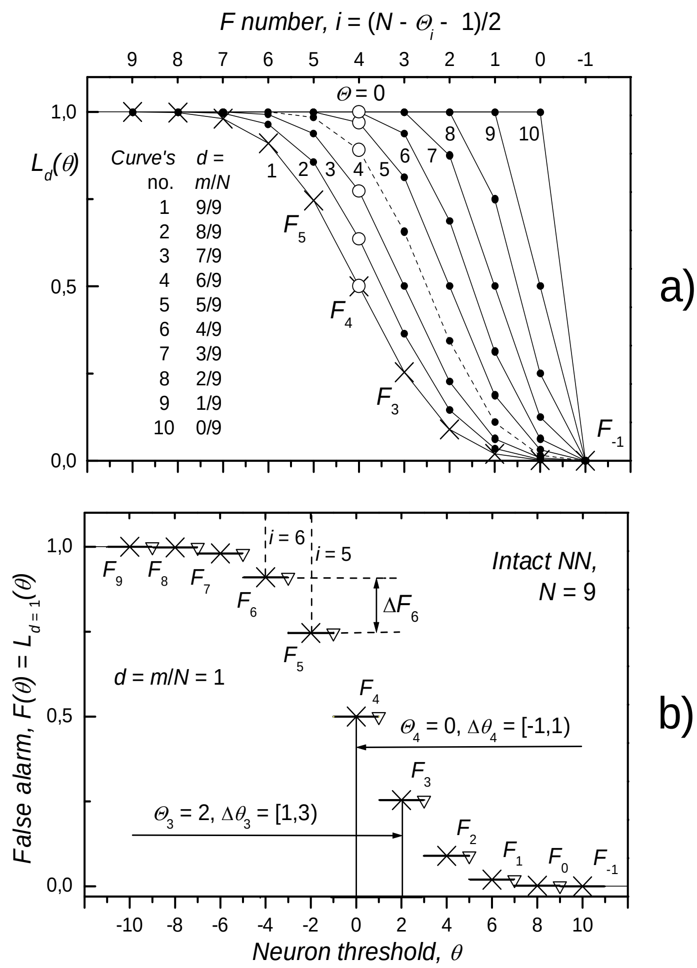

Figure 1. a) Correct decoding probability of vectors $x = x(d)$ (the probability of correct decoding of x under condition that it is x_0 damaged) or the likelihood $L(d,\theta) = P(A|H_1)$ (the probability of the event A under condition that hypotheses H_1 is valid, see Section 4) vs. the neuron threshold θ (lower scale) and index i, the number of F_i, Θ_i, or $\Delta\theta_i$ (upper scale). All values of $L_d(\theta) = L(d,\theta)$ were calculated according to Equation 1: crosses denote false-alarm probabilities (values of F_{-1}, F_3, F_4, and F_5 are marked), open circles denote $L_d(\theta)$ for near-zero neuron thresholds $\theta \in \Delta\theta_4 = [-1,1)$, $\Theta_4 = 0$ [designation $L_d(\theta)$ means that in $L(d,\theta)$ parameter d is fixed]. Probabilities $L_d(\theta)$ specified by a constant value of d are connected in straight lines. Here and in all next Figures values $L_d(\theta)$, $d = 6/9$, are connected in dashed lines. b) The same in more details (see text) but only for the function representing false-alarm probability, $F(\theta)$; crosses denote $F_i = F(\Theta_i)$, all F_i are marked; $\Delta F_6 = F_6 - F_5$ is the interval between two neighbors, F_5 and F_6; vertical lines designate indices $\Theta_3 = 2$ and $\Theta_4 = 0$.

To calculate $F(\theta)$ for different θ, we should posit in Equation 1 $m = N$, i.e. $d = 1$. In Figures 1a and 1b crosses are the same but from the panel *b*) it is seen that they correspond to the middles of neuron threshold intervals $\Delta\theta_i$ (i = 0, 1,…, N – 1) where for all $\theta \in \Delta\theta_i$ Equation 1 gives the same magnitude of the likelihood probability $L_d(\theta)$. As Figure 1b shows, $F(\theta)$ is a stepwise discontinues function where its horizontal line segments denote the constant value of $F(\theta)$ for θ belonging to corresponding threshold interval $\Delta\theta_i$, $\theta \in \Delta\theta_i$. Triangles at the right-most points of all, except $\Delta\theta_{-1} = [9,+\infty)$, segments mean that the values of $F(\theta)$ in these points are not defined. On the number axis, $F(\theta)$ is a discontinuous single-valued total function; its unambiguity is provided by the fact that in each point of discontinuity for each of two neighbor $F(\theta)$ line segments its left frontier point is defined while its right frontier point is not (see also Table 1). Since all $\theta \in \Delta\theta_i$ produce only a single value of the probability $L(d,\theta)$, it is convenient to assign to the *i*th $\Delta\theta_i$ its neuron threshold interval index Θ_i which produces the same $L(d,\theta) = L(d,\Theta_i)$. We define the series of even Θ_i with $\Delta\Theta = 2$ in such a way that without fail it contains its zero-element, e.g., Θ_4 = 0 in Figure 1.

Figure 2 illustrates the way in which all probability values in Figure 1 were calculated under Equation 1. We see that this Equation may be written as a sum of probabilities shown in Figure 2, $L(m,N,\Theta) = \sum p_b(i,m)$, with appropriate summation rules. Summation results obtained equal probabilities shown in Figure 1 (e.g., to calculate $F(\theta)$ the distribution $m = 9$ from Figure 2 should be adopted). We also specially emphasize that the expansion of the standard binomial distribution is needed to calculate $L(d,\theta)$ at $\theta < -9$ and $\theta > 9$ (they provide probabilities 1 and 0).

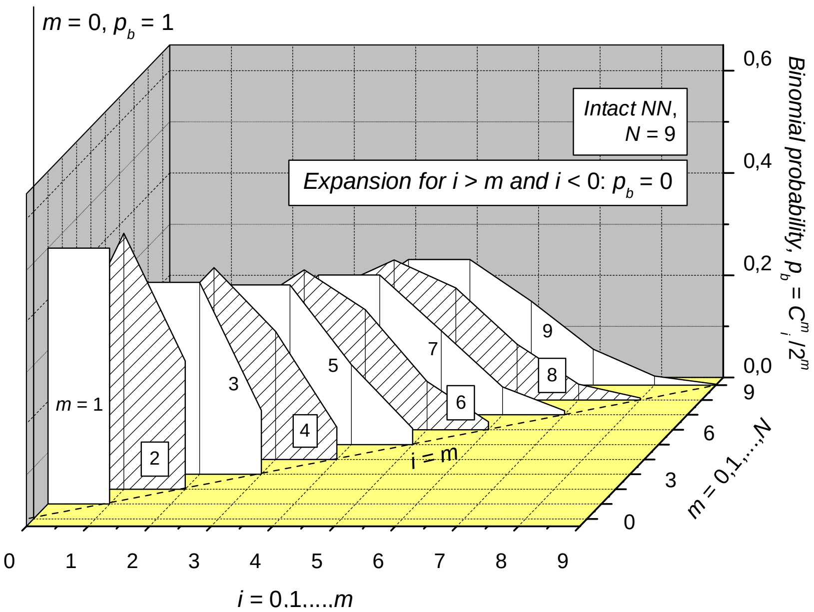

Figure 2. Probability densities $p_b(i,m)$ of Hamming distances between vectors $x(d)$ and x_0 under condition that $x(d)$, $d = m/N$, contains m its noise components ±1 chosen randomly with uniform probability, ½; p_b is a binomial distribution expanded at the ranges $i > m$ and $i < 0$ where we posit that $p_b = 0$. For each m, $0 < m \le N$, exact values of p_b are connected in straight lines bounding corresponding areas, dashed or not. The case $m = 0$ is special as contains only one point $p_b = 1$ which is on the apex of a separate vertical line.

4. Bayes Inferences within the BSDT

Let us define the event A = 'identification of x_0 in $x(d)$, $d = m/N$, by an NN with the neuron threshold θ' (or A = 'decoding vectors $x(d)$, $d = m/N$, by an NN with the neuron threshold θ'') and two alternative hypothesis: H_0 implying that $x(d)$ is a sample of pure binary noise and H_1 implying that $x(d)$ is x_0 damaged to the damage degree d by such a noise. Using these designations and famous Bayes formula we can write

$$P(A)P(H_0|A) = P(H_0)P(A|H_0), \quad P(A)P(H_1|A) = P(H_1)P(A|H_1), \qquad (2)$$

$$P(A) = P(H_0)P(A|H_0) + P(H_1)P(A|H_1) \qquad (3)$$

where $P(A)$, $P(H_0)$, and $P(H_1)$ are prior probabilities of the event A, hypothesis H_0, and H_1; $P(A|H_1) = L(d,\theta)$ and $P(A|H_0) = L(d = 1,\theta) = F(\theta)$ are conditional likelihood probabilities of correct and false decoding; Equation 3 reflects the obvious fact that if the event A occurs then H_0 and H_1 are valid with probabilities $P(H_0)$ and $P(H_1)$, respectively. Combining Equations 2 and 3 we have

$$P_{FD}(d,\theta) = [1 + \kappa(d)L(d,\theta)/F(\theta)]^{-1},\ \ P_{CD}(d,\theta) = \{1 + 1/[\kappa(d)L(d,\theta)/F(\theta)]\}^{-1}, \qquad (4)$$

where $P_{FD}(d,\theta) = P(H_0|A)$ and $P_{CD}(d,\theta) = P(H_1|A)$ are conditional posterior probabilities respectively of false and correct decoding (cf. ref. 3),

$$\kappa(d) = P(H_1)/P(H_0) = (1 - d)/d \qquad (5)$$

is the ratio of prior probabilities of H_0 and H_1 defined within the BSDT explicitly, $P(H_0) = d$ and $P(H_1) = q = 1 - d$. As at $d = 0$ (the case $x(d) = x_0$) $\kappa(d)$ does not exist and at $d = 1$ (the case of pure noise) $1/\kappa(d)$ does not exist, in these special cases we posit that at $d = 0$ $P_{CD} = 1$, $P_{FD} = 0$ and at $d = 1$ $P_{CD} = 0$, $P_{FD} = 1$ in accordance with our expectations. Hence, now using Equations 1, 4 and 5 $P_{CD}(d,\theta)$ and $P_{FD}(d,\theta)$ can analytically be calculated for any possible values of d and θ. Since $P_{CD}(d,\theta) + P_{FD}(d,\theta) = 1$, it is enough to consider only one of these two posteriors. Below we shall discuss $P_{CD}(d,\theta)$ writing it without its subscripts, $P(d,\theta)$. Also we emphasize that in contrast to ref. 3 within this work likelihood and posterior are always designated respectively as '*L*' and '*P*,' regardless of lists of their subscripts or arguments; such designations directly point to distinctions between conditional probabilities of two types, likelihood and posterior, and are convenient when they are considered together.

In a 3D orthogonal space with axes d, θ, and L (or d, θ, and P) likelihood $L(d,\theta)$ [or posterior $P(d,\theta)$] produces a lattice of discrete points representing a complete set of all possible values of $L(d,\theta)$ [or $P(d,\theta)$]. For short, here we do not display corresponding 3D figures although in Figure 1a one can see a projection of the $L(d,\theta)$-lattice on the coordinate plane (L,θ); projections of $L(q,\Theta)$- and $P(q,\Theta)$-lattices on coordinate planes (L,q) and (P,q) see in Figure 5 ($q = 1 - d$, for relations between θ and Θ see Table 1).

5. ROC, mROC, Overall Likelihood and Posterior

As the values of $L(d,\theta)$, $F(\theta)$, $P(d,\theta)$ and relations between BSDT parameters are known (Equations 1, 4, 5 and Table 1), for different values of d (or more 'physical' parameter $q = 1 - d$ meaning the intensity of cue) it is possible to calculate likelihood, $L_q(F)$, and posterior, $P_q(F)$, as functions of false-alarm probability, F. The dependence $L_q(F)$ is called Receiver Operating Characteristic (ROC) curve; by analogy we refer to the corresponding dependence $P_q(F)$ as modified or posterior ROC curve, mROC; for all q, $0 \le q \le 1$, they are shown in Figures 3a and 4a, respectively. In addition to ROCs and mROCs, we can also define overall, do not depending on F, likelihood and posterior. For this purpose in ref. 3 a simple averaging of probabilities related to particular mROC was used. But taking into account that all ΔF_i are known and constitute a complete binomial probability distribution $\Delta F_i = C^N_i/2^N$, $\Sigma\Delta F_i = 1$, $i = 0,1,\ldots,N$ (see Table 1), it is natural to define overall likelihood, $L_0(q)$, and overall posterior, $P_0(q)$, as binomial averaging of corresponding sets of likelihoods, $L_q(F_i)$, and posteriors, $P_q(F_i)$:

$$L_0(q) = \sum L_q(F_i)\Delta F_i = \sum L_q(F_i)C^N_i/2^N, \qquad (6)$$

$$P_0(q) = \sum P_q(F_i)\Delta F_i = \sum P_q(F_i)C^N_i/2^N \qquad (7)$$

where all summations are made over $i = 0,1,\ldots,N$ [above $L_q(F_i)\Delta F_i$ and $P_q(F_i)\Delta F_i$ are areas of rectangles with the base ΔF_i and heights $L_q(F_i)$ and $P_q(F_i)$; $L_0(q)$ and $P_0(q)$ are areas under stepwise curves connected discrete ROC and mROC values, respectively]. The choice of values of ΔF_i as weights in Equations 6 and 7 means that if any value of F, e.g. F_x, is randomly chosen with uniform probability within the range $0 \le F_x \le 1$ then the probability (fre-

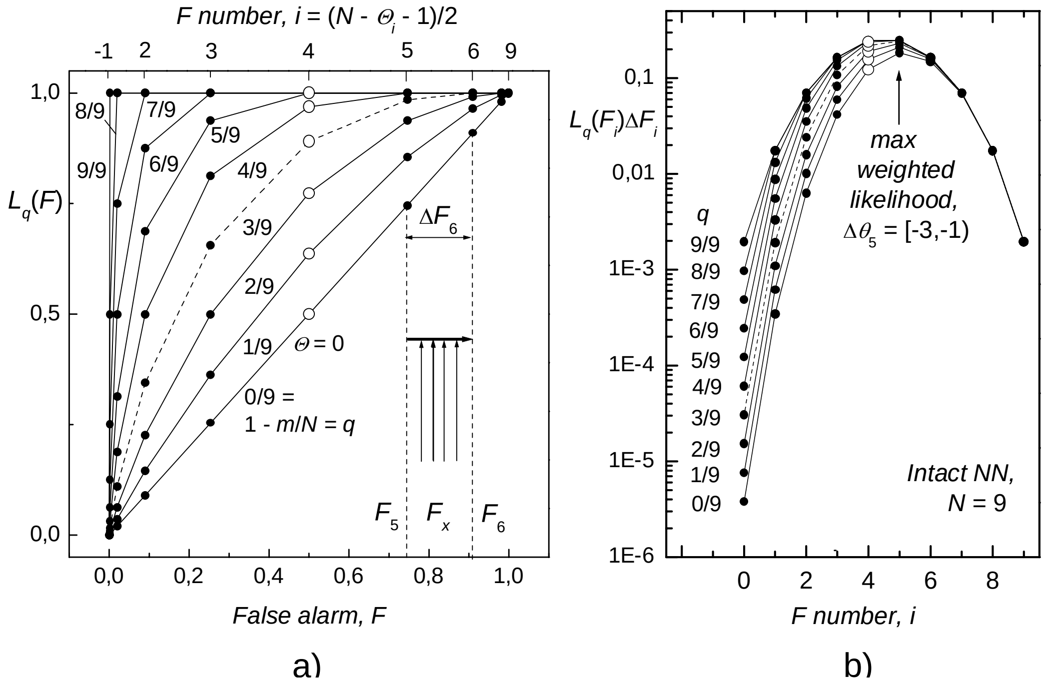

Figure 3. a) ROCs, $L_q(F_i)$, and b) weighted likelihoods, $L_q(F_i)\Delta F_i$; values $L_q(F_i)$ were calculated under Equation 1. In panel a) straight lines connect signs related to a specific value of the intensity of cue, $q = 1 - d$, and constitute a specific ROC; dashed lines designate ROC with $q = 3/9$ ($d = 6/9$); open circles reflect values of $L_q(F_4) = L_q(1/2)$ corresponding to the threshold interval index $\Theta = \Theta_4 = 0$; ΔF_6 is the interval between two neighbors, F_5 and F_6; vertical arrows represent schematically a fraction of values of F belonging to ΔF_6, $F_x \in \Delta F_6$. In panel b) vertical arrow points to maximum items of sums $\sum L_q(F_i)\Delta F_i$, they correspond to $i = 5$ and $\theta \in \Delta\theta_5$.

quency) of the event $F_x \in \Delta F_i$ equals ΔF_i and the probability of the emerging values of $L_q(F_i)$ and $P_q(F_i)$ also equals ΔF_i [the same may relate to $L_q(F_{i-1})$ and $P_q(F_{i-1})$ although this case will not be considered in this work]. Hence, if our assumption that values of F are randomly chosen within the range $0 \le F \le 1$ with uniform probability is valid (we shall call this assumption the *uniformity* or *no-priming hypotheses*) then overall probabilities $L_0(q)$ and $P_0(q)$ are optimal, i.e. the best among other ones calculated according to other possible averaging rules.

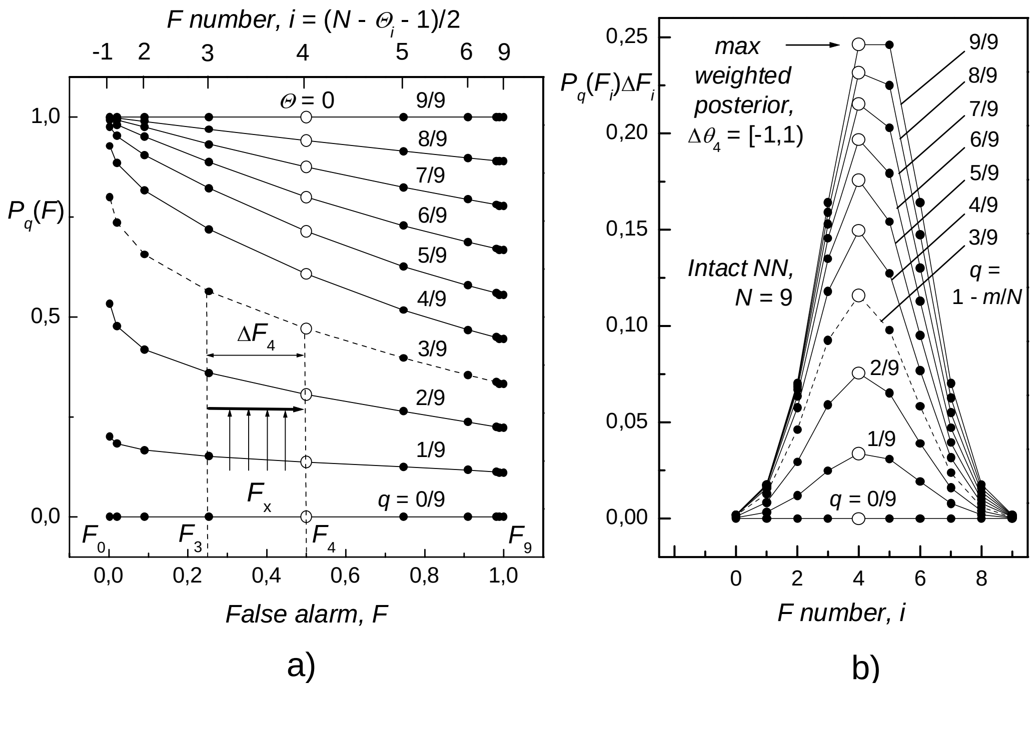

Figure 4. a) mROCs, $P_q(F_i)$, and b) weighted posteriors, $P_q(F_i)\Delta F_i$; posteriors $P_q(F_i)$ were calculated under Equations 1, 4, and 5 [$P_q(F_{-1})$ in panel a) and $P_q(F_{-1})\Delta F_{-1}$ in panel b) are not shown]. In panel a) straight lines connect signs related to a specific value of the intensity of cue, $q = 1 - d$, and constitute a specific mROC; dashed lines designate mROC with $q = 3/9$ ($d = 6/9$); open circles reflect values of $P_q(F_4) = P_q(1/2)$ corresponding to the threshold interval index $\Theta = \Theta_4 = 0$; ΔF_4 is the interval between two neighbors, F_3 and F_4; F_0 and F_9 are also marked; vertical arrows represent schematically a fraction of values of F belonging to ΔF_4, $F_x \in \Delta F_4$. In panel b) vertical arrow points to maximum items of sums $\sum P_q(F_i)\Delta F_i$, they correspond to $i = 4$ and $\theta \in \Delta\theta_4$.

For all q, $0 \le q \le 1$, in Figures 3b and 4b components of sums $L_0(q) = \sum L_q(F_i)\Delta F_i$ and $P_0(q) = \sum P_q(F_i)\Delta F_i$ are shown as functions of their summation index, i. It is remarkable that all corresponding curves have a common maximum, for weighted likelihoods at $i = 5$ and for weighted posteriors at $i = 4$. Hence, if our uniformity hypotheses concerning the choice of F for ROC and mROC values estimation is valid then an observer/computer code, who/that does not use any prior information about probabilities of hypothesis H_0 and H_1, naturally (most probably) choices values of the neuron threshold θ which are slightly smaller than zero, $\theta \in \Delta\theta_5 = [-3, -1)$; another observer, who in contrast uses completely the prior information mentioned, naturally choices for θ its near-zero values, $\theta \in \Delta\theta_4 = [-1,1)$. The same Figures demonstrate also that during the estimation of overall probabilities, $L_0(q)$ and $P_0(q)$, in right-hand sums of Equations 6 and 7 their items with their numbers i near to 0 and near to N (them correspond 'small' ΔF_i) are not so essential as their 'central' items (them correspond 'large' ΔF_i). Moreover, $L_0(q)$ and $P_0(q)$ are defined without the use of probabilities $L_q(F_{-1}) = 0$ and $P_q(F_{-1}) = 1$ in corresponding sums of Equations 6 and 7 and, consequently, these probabilities are at all not requested for estimating particular values of overall probabilities (in other words, left-most points of ROCs and mROCs may in practice be not claimed).

6. BDPs, mBDPs, and Psychometric Functions

Each of probability functions $L(d,\theta) = L(d,\Theta) = L(d,F)$ and $P(d,\theta) = P(d,\Theta) = P(d,F)$ has two arguments (for relations between θ, Θ, F, and etc see Table 1; $d = 1 - q$). If for the likelihood $L(d,F)$ one of them, e.g. d, is fixed then we obtain ROC curves, $L_d(F)$ or $L_q(F)$; if for the posterior $P(d,F)$ the same parameter is fixed then we obtain mROC curves, $P_d(F)$ or $P_q(F)$. Similarly, if in $L(d,F)$ or $L(q,F)$ the argument F is fixed then we obtain the function $L_F(q)$ or $L_\Theta(q)$ which we shall call Basic Decoding Performance (BDP) curve; if in $P(d,F)$ or $P(q,F)$ the same argument is fixed then we obtain a modified (posterior) BDP or mBDP curve, $P_F(q)$ or $P_\Theta(q)$. Examples of BDP and mBDP curves are shown in Figures 5a and 5b, respectively.

Intensity of cue $q = 1 - d$ defines a fraction of undamaged signal components among m noise components of N-dimensional vectors $x(d)$, $d = m/N$, or the quality of data analyzed: the more the q the better the quality is. Functions describing the signal's detection probability against the quality of data analyzed (e.g., the signal's intensity, amplitude, or area) are called *psychometric functions* [2, chapter 8], PMFs. Consequently, BDF and mBDP curves may respectively be interpreted as PMFs (Figure 5a) and modified or posterior PMFs, mPMFs (Figure 5b).

In the classic SDT arguments of PMFs are continuos and ranged from zero to positive infinity [2] while within the BSDT q is discrete, with the discreteness degree $\Delta q = 1/N$, and changes in the limited range, $0 \le q \le 1$. As in practice magnitudes of all variables are always limited and measurement results are usually discrete, the finiteness and discreteness of q are not its disadvantages as the PMF's or mPMF's argument. Indeed, if discrete values of a variable V are from the range $0 \le V \le V_{max}$ then $V = (k/N)V_{max}$ ($k = 0,1, \ldots,N$) and by changing N arbitrary small discreteness of V, $\Delta V = V_{max}/N$, may be achieved. Hence, for any V_{max} such signal detection experiment may be designed that the psychometric function (PMF) measured [6] will have the form as one of curves shown in Figure 5a. To confirm this claim it is simply enough to transform the variable V into a new dimensionless variable $V/V_{max} = k/N$ and assume that $k/N = q$, i.e. $V/V_{max} = q = (N - m)/N$ where $N - m = k$ is the number of undamaged signal components of a vector $x(d)$. Consequently, discrete PMFs as in Figure 5a may be considered as a universal (dimensionless) PMFs, UPMFs, matching to any positive variable V with its arbitrary large maximum value V_{max} (the number of points along particular UPMF defines its fit parameter N may be chosen arbitrary large). Finally, let us pay an attention to a corollary arising: all points along a UPMF or PMF are equidistant ($\Delta q = 1/N$, $\Delta V = V_{max}/N$) and ΔV may be considered as a *just noticeable difference* (*jnd*) [2, p.25], the minimal difference between two stimuli, $V_k = kV_{max}/N$ and $V_{k+1} = (k + 1)V_{max}/N$, that leads to a change in experience, $\Delta L_0(V_{k+1}) = L_0(V_{k+1}) - L_0(V_k)$, $k = 0,1,\ldots, N - 1$. For likelihood and posterior PMFs their jnd's values are the same although the values of $\Delta L_0(V_k)$ depend essentially on V_k and for this reason in different experiments different 'seeming' values of the jnd can be observed. The jnd (i.e. ΔV) could be an atom of experience and this atom-of-experience hypotheses still proposed by Gustav Fechner [2, p. 26] is consistent with our model of an atom of consciousness [5].

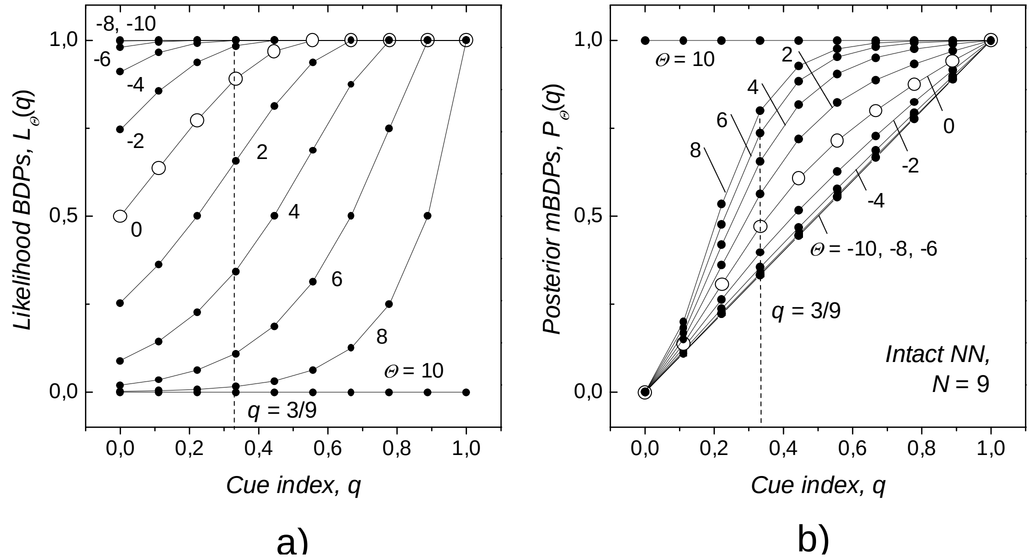

Figure 5. a) BDPs or universal psychometric functions, UPMFs, and b) mBDPs or modified (posterior) UPMFs, mUPMFs. $F_\Theta = L_\Theta(q = 0)$ is the value of F for a UPMF specified by the parameter Θ. Consequently, for each UPMF the Neyman-Pearson objective is achieved; the same concerns to mUPMFs in spite of the fact that for them $P_\Theta(q = 0) = 0$ at all Θ. For the special case $L_{10}(q) = F_{10} = 0$ posteriors $P_{10}(q)$ cannot be calculated (Equations 4) and for this reason for all q it is needed to posit: $P_{10}(q) = 1$. Other designations were explained previously.

7. Sensitivity and Bias, a Comparison between the SDT and BSDT

For psychophysics experiments, where likelihoods are measured, within the SDT sensitivity and bias are defined using ROCs [2]. Within the BSDT not only ROCs, $L_q(F)$, but also mROCs, $P_q(F)$, exist and, consequently, using

Table 2

Discrimination experiment descriptions within the SDT and BSDT, a comparison

Notion	Classic SDT approach [2]		BSDT approach [3,5]	
	Definition	Comments	Definition	Comments
1	2	3	4	5
Decision space	One-dimensional[a)], its psychological decision variable is a single stimulus attribute called familiarity x, $-\infty < x < +\infty$; its objects are underlying familiarity distributions or probability densities $f(x\|S_i)$[b)] related to samples of trials S_i; M_i is the mean for the ith $f(x\|S_i)$, $-\infty < M_i < +\infty$.	Familiarity x is continuous, values of x define criterion locations or biases c, differences of x define perception distances or sensitivities d'; the world of stimuli and psychological decision space are separate with unclear relations between them.	Two-dimensional, its variables are cue index (intensity of cue) $q = 1 - m/N$ and neuron threshold θ, $0 \le q \le 1$, $-\infty < \theta < +\infty$; its objects are N-dimensional binary (±1) vectors $x = x(d)$ with m uniformly distributed noise components, $0 \le m \le N$, $d = m/N$.	q is discrete, θ is continuous and both are statistically independent; a 'neural space' where stimuli are represented as neural codes (binary vectors x) [c)] is also the decision space.

ROC space	Two-dimensional, its variables (measured or calculated) are hit rate $H = \Phi(d'/2 - c)$ and false-alarm rate $F = \Phi(-d'/2 - c)$, $0 \le H \le 1$, $0 \le F \le 1$[d]; for transformed ROC space its variables, $z(H)$ and $z(F)$, are H and F transformed into a z-score (z is the inverse of the normal distribution function), $-\infty < z(H) < +\infty$, $-\infty < z(F) < +\infty$.	H and F are continuous and statistically independent [e]; to generate an ROC, d' should be stable while c changeable; in H,F-coordinates ROCs are curvilinear with changing shape across d' values; in z-scores ROCs have a straight-line form.	Two-dimensional, its variables (measured or calculated) are hit rate $H = L(q,\theta)$ and false-alarm rate $F = L(q = 0,\theta)$, $0 \le H \le 1$, $0 \le F \le 1$, $F \le H$ [d]; as each stimulus is represented by its vector x whose binomial probability density is specific for each m, a transformed common z-ROC space cannot be defined.	F and H are discrete and functionally related; to generate an ROC, probabilities $H = L(q,\theta) = L_q(F)$ are calculated at a fixed q with changing θ (or with changing F as θ, Θ, F, Q, ρ, and D are related, see Table 1).
Sensitivity	Perception distance, $d' = z(H) - z(F)$ or $d' = M_2 - M_1$, $0 \le d' < +\infty$ [f]; proportion correct, $p(c) = p(S_2)H + p(S_1)(1 - F)$ where $p(S_i)$ is the probability that S_i is presented; area under the ROC, A', A_g, or A_z [g].	d' is continuous; a fixed d' and changing c define an isosensitivity or ROC curve, if $d' = 0$ then ROC provides chance-level performance; for unbiased ($c = 0$) observers $p(c)$ is a nonparametric overall sensitivity.	Cue index q; proportion correct, $p(q,\theta)$, may also be defined[h] and its definition is valid for any possible values of q and θ; overall likelihood, $L_0(q)$, an 'area' estimation under the discrete ROC.	q is discrete; a fixed q and changing θ define an isosensitivity or ROC curve, if $q = 0$ then ROC provides chance level performance.
Bias	Criterion location, $c = -[z(H) + z(F)]/2$, $-\infty < c < +\infty$; relative criterion location, $c' = c/d'$; likelihood ratio or the slope of transformed ROC for a given c, $\beta_G = f(c\|S_2)/f(c\|S_1) = H(c)/F(c) = \exp(cd')$; $\log(\beta_G) = [z(H)^2 - z(F)^2]/2$[i].	d' and c are independent and constitute a pair of variables alternative to the pair H and F; a fixed c (c' or β_G) and changing d' define an isobias curve; on isobias curves ($c = 0$) as F increases H must decrease [j]; observers do not naturally use a neutral value of the bias (confidence level).	Neuron threshold θ, $-\infty < \theta < +\infty$ (or convolution Q, threshold interval index Θ, correlation coefficient ρ, false-alarm probability F, or Hamming distance D as θ, Q, Θ, ρ, F, and D are related; see Table 1).	θ is continuous; a fixed θ (F, Q, Θ, ρ, or D) and changing q define an isobais curve, $H = L_\Theta(q)$, which is a UPMF (see Figure 5a); observes naturally use near-neutral values of the bias, θ.

[a] There exist experimental paradigms for which two- and many-dimensional versions of the SDT were developed [2, chapter 10]; recently, a new original two-dimensional version of the SDT has also been proposed [7] but here it is not discussed as only classic SDT [1,2] is the comparison subject in this work.

[b] Here and below only normal densities are considered and for this reason $f(x|S_i)$ is always a Gaussian.

[c] Within the BSDT a set of neural codes $x(d)$ representing particular stimuli (a 'neural space') is simultaneously a decision space where operations over these codes are defined. The world of stimuli and psychological space within the BSDT are directly not requested but it is supposed that rules for transformation of external/internal sensory stimuli (the world of stimuli or stimuli space) into their corresponding neural representations (the neural space) may be discovered by methods of neuroscience.

[d] If an ROC curve passes through the points $(F,H) = (0, 0)$ and $(1,1)$ then it is called a regular ROC curve.

[e] Formally, H and F are defined as statistically independent but while $d' \ge 0$ they can take independently only values $F \le H$ which are on and upper the ROC's main diagonal, $H = F$. If additionally values $d' < 0$ are admitted then H and F may be independent in all ROC space but in this case events $H < F$ become possible.

[f] d' is defined under condition that underlying distributions for samples of trials S_1 and S_2 have common standard deviation; if that is not the case then for S_1 and S_2 their distinct standard deviations d'_1 and d'_2 are introduced as well as their root-mean-square average, d_a.

[g] $A' = 0.5 + (H - F)(1 + H - F)/[4H(1 - F)]$ gives an area estimation under the one-point ROC; $A_g = 0.5\sum(F_{i+1} - F_i)(H_{i+1} + H_i)$ provides area under the multipoint ROC (the summation is made over all ROC points numbered

from the left lower corner), within the BSDT the counterpart of A_g is overall likelihood $L_0(q)$; $A_z = \Phi(D_{YN}) = \Phi(d_a/\sqrt{2})$ gives area under the non-regular ROC curve (D_{YN} is the distance between the origin and the non-unit-slope *z*-ROC, $\Phi(x)$ is a cumulative distribution function or the integral of a Gaussian $f(x)$ taken from $-\infty$ to x).

h) By analogy to proportion correct, $p(c)$, $p(q,\theta) = p(S_2)\, L(q,\theta) + p(S_1)[1 - L(q = 0,\theta)]$ where $L(q,\theta) = H(q,\theta) = H$, $L(q = 0,\theta) = F(\theta) = F$ and $p(S_i)$ is the probability that S_i is presented; for intact NN, odd N, and $\theta = 0$ $p(q,0) = p(S_2)L(q,0) + p(S_1)½$ as here $F = ½$ [3].

i) Within the singe high-threshold theory [2, chapter 4], an SDT's alternative, false alarm *F* is introduced as natural bias (confidence level) index but such bias definition cannot be accepted as it leads to isobias curves with constant value of *F*, in contradiction to the SDT's so called monotonically condition demanding that along isobias curves as *F* increases *H* must decrease and vice versa [2, p.93].

j) Such definition of isobias curves does not satisfy to the Neyman-Pearson objective as *F* and *H* are changing simultaneously (due to the monotonically condition); within the SDT to generate an isobias curve satisfying this objective, *c* and M_1 should be constant while M_2 changeable.

mROCs a possibility arises to define similar parameters as well for posteriors, $P_q(F)$. In Table 2 sensitivity, bias, their decision and ROC spaces defined using the ROCs within the SDT and BSDT are compared. For examples of experiments where posterior probabilities could be measured and mROCs derived see ref. 3, corresponding posterior sensitivity and bias should be discussed separately as till now they have no the SDT's counterparts.

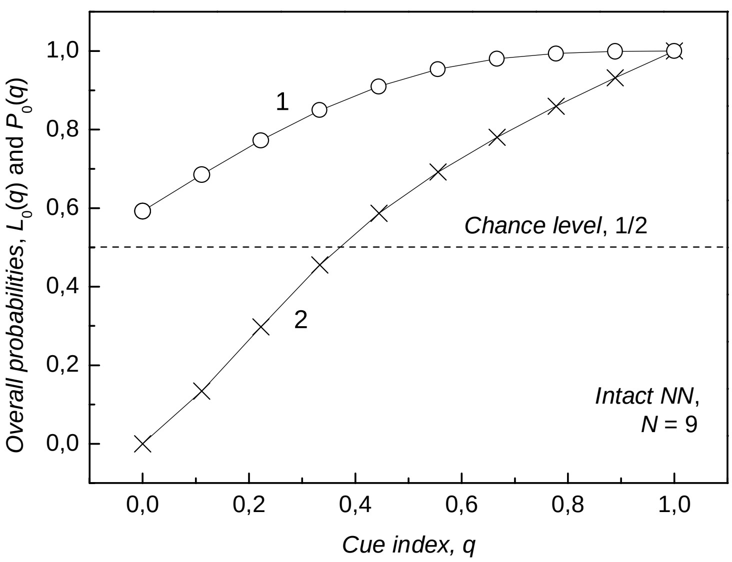

Figure 6. The overall likelihood $L_0(q)$ (open circles, curve 1) and overall posterior $P_0(q)$ (crosses, curve 2) calculated under Equations 6 and 7 as functions of the intensity of cue, *q*. To each circle on curve 1 corresponds a curve in Figure 3b specified by the same *q*, to each cross on curve 2 corresponds a curve in Figure 4b specified by the same *q*.

Finally, Figure 6 demonstrates overall probabilities $L_0(q)$ and $P_0(q)$ for subjects (computer codes) do not using prior information about probabilities of hypothesis H_0 and H_1 and for subjects (computer codes) having and completely using this prior information (in both cases it is supposed that our no-priming hypotheses about 'in-mind' distribution of *F* is valid). We see that curve 1 begins in a point above the chance-level line while curve 2 in the origin; both curves end in the point (1,1). Overall likelihoods $L_0(q)$ (circles on curve 1) may be compared with areas $A(d')$ ranged from ½ at $d' = 0$ to 1 at $d' \to \infty$ [$A(d')$ is area under the SDT's particular regular ROCs]. Hence, $L_0(q = 0) > ½$ while $A(d' = 0) = ½$ and $L_0(q = 1) = A(d' \to \infty) = 1$ [d' and q are sensitivities of the SDT and BSDT, respectively; $A_g(d') \to A(d')$ if the number of points measured on the SDT's regular ROC goes to infinity, see Table 2 and its footnote g)].

8. Conclusions

Above for a simple example, the BSDT's decoding algorithm has been studied using its exact quality performance functions analytically calculated. In this way an attempt was made to reveal some similarities and distinctions between the classic SDT [1,2] and recent BSDT [3,5]. We saw that the main similarity consists in the fact that SDT and BSDT can produce the same functions for the description their decoding algorithms' quality performance, ROCs and psychometric functions (PMFs), and the same basic parameters, sensitivity and bias.

Hence, using the BSDT those measurement results can in principle be described that the SDT describes. The source of distinctions between them is, in our opinion, in the BSDT's original binary data coding [3,4] and its corresponding decoding algorithm existing simultaneously in NN, convolutional, and Hamming distance forms having equivalent and the best (in the sense of statistical patterns recognition quality) performance. For this reason in contrast to the classic SDT, BSDT is based on discrete final (binomial) probability distributions and all its predictions and parameters (except the neuron threshold θ) are discrete. In summary: essential features and inferences of the BSDT follow in fact directly from analyzes of the original mathematical form of its performance functions. For example, the notion of a neural space introduced is a direct consequence of the BSDT/NNAMM's binary coding/decoding approach though here it is also implied implicitly that vectors $x(d)$ represent neural codes of sensory stimuli in the brain; psychometric functions satisfying the Neyman-Pearson objective are simply projections of likelihood probability function $L(q,\theta)$ on the coordinate plane (L,q); the just noticeable difference (jnd) is a counterpart to the discreteness, Δq, of the cue index, q (and due to its discreteness Δq can be naturally related to an atom of experience); our new uniformity or no-priming hypotheses is simply a requirement needed to define optimally ROCs, mROCs, and overall probabilities; and, finally, our conclusion that subjects (computer codes) naturally (most probably) choice near-zero thresholds follows from the existence of a maximum among items of weighted sums of particular likelihoods and posteriors.

Our computations of likelihood, $L(q,\theta)$, posterior, $P(q,\theta)$, overall likelihood, $L_0(q)$, and overall posterior, $P_0(q)$, probabilities of correct decoding confirm that the BSDT's two basic parameters (sensitivity or intensity of cue, q, and bias or neuron threshold, θ) are sufficient to parametrize the decoding algorithm's quality performance functions, traditional (ROCs and PMFs) as well newly introduced [mROCs, mPMFs, $L_0(q)$, and $P_0(q)$]. The only limitation consists in use of the decoding algorithm in the form of an intact NN and that is why only regular ROCs and their related functions were discussed so far. This limitation can be evaded yet even now it does not hinder to begin to reinterpret some psychophysics results in terms and notions of the BSDT which, we believe, in many cases are more natural and attractive than terms and notions of the classic SDT.

I am grateful to my family and my friends for their help and support.

Bibliography

1. D.Green & J.Swets. Signal Detection Theory and Psychophysics. New York, Wiley, 1966.
2. N.A.Macmillan & C.D.Creelman. Detection Theory: A User's Guide. New York, Cambridge University Press, 1991.
3. P.M.Gopych. ROC Curves within the Framework of Neural Network Assembly Memory Model: Some Analytic Results. International Journal Information Theories & Applications. 2003, 10(2), 189-197; http://arXiv.org/abs/cs.AI/0309007.
4. P.M.Gopych. Determination of Memory Performance. JINR Rapid Communications, 1999, 4[96]-99, 61-68 (in Russian).
5. P.M.Gopych. A Neural Network Assembly Memory Model Based on an Optimal Binary Signal Detection Theory. Programming Problems, 2004, no.2-3, 473-479; http://arXiv.org/abs/cs.AI/0309036.
6. P.M.Gopych. Identification of Peaks in Line Spectra Using the Algorithm Imitating the Neural Network Operation. Instruments & Experimental Techniques, 1998, 41(3), 341-346.
7. C.M.Rotello, N.A.Macmillan, J.A.Reeder. Sum-Difference Theory of Remembering and Knowing: A Two-Dimensional Signal Detection Model. Psychological Review, 2004, 111, 588–616.

Author Information

Petro Mykhaylovych Gopych – V.N.Karazin Kharkiv National University; Svoboda Sq., 4, Kharkiv, 61077, Ukraine; e-mail: pmg@kharkov.com.