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MATHEMATICAL MODEL OF RE-STRUCTURING COMPLEX TECHNICAL AND ECONOMIC STRUCTURES

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Abstract: Research and development of mathematical model of optimum distribution of resources (basically financial) for maintenance of the new (raised) quality (reliability) of complex system concerning, which the decision on its re-structuring is accepted, is stated. The final model gives answers (algorithm of calculation) to questions: how many elements of system to allocate on modernization, which elements, up to what level of depth modernization of each of allocated is necessary, and optimum answers are by criterion of minimization of financial charges.

Keywords: system, re-structuring, quality, reliability.

ACM Classification Keywords: I.6.3 Simulation and Modeling: Applications

Introduction

By development of new complex systems, and increase of their efficiency while in service the important factor of increase of adequacy and reliability of mathematical models an estimation of a level of their reliability is ability of the description, formalization and the account in these models of an opportunity of management of reliability [1]. The increment of reliability u due to rational management of reliability is achieved by perfection of algorithm of a system's mode of operation variations, a variation of actions on technical and to preventive maintenance, because that reduction of failure rate after rational procedure of procedural works depends on a level of optimization of this procedure. Reduction of intensity of a refusal's stream, change of its probable structure of limited after action can be achieved also by special modes of external influences. So, for example, separate kinds of integrated circuits at a radioactive irradiation sharply raise accuracy of a presence of parameters in necessary borders [1]. However the time of their life essentially decreases. Realization of such procedure when the system carries out the important and responsible task nevertheless can be quite justified. Value of such task allows neglecting reduction of general time of life of an element due to strict preservation of parameters in the certain limits, though in a smaller time interval. In such situations there is a problem about management of reliability with the purpose of optimization of system by the certain criterion.

Let's investigate such variant of complex system, which is characterized by the linear block diagram of its process of functioning. It can be complex economic structure, or complex technical system, or such system which has both economic and technical properties, that is complex technical-economic structure. Let such complex system contains *n* subsystems, are connected functional consistently. The system has the certain degree of quality of functioning. We for convenience shall interpret this level its level of reliability *P*. At the certain stage of operation of system (for example, by virtue of development of market relations of the country) there comes such moment when the condition of efficiency of system's process of functioning at level *P* becomes insufficient. There is a problem of increase of reliability up to level $P^* = P + + \Delta P$. At this stage of ability to live of many systems there are problems on their reorganization and re-structuring. Thus the powerful part of the general investments should be allocated for an innovation.

Active investments in innovational process of an element can result in an essential gain of reliability of an element or probability of stay of parameters in the necessary limits [2, 3]. This most reliability of system as a whole rises. To increase it is possible would be as much as you want if it was not connected to material inputs. Expansion of quantity of the modernized elements and their parameters, results increase of depth of innovational process of each element in the appropriate growth of charges for realization of all re-structuring. Objectively there is some point (value of a level of completion) after which to increase charges for re-structuring from practical reasons begins inexpedient. There is a task on search of such point, namely: where it is necessary to stop at a choice of a level of depth of completion of a separate element. For all system the problem is, what quantity of elements to subject modernization that economic feasibility of carried out work was not lost. And further, when the question for system as a whole about quantity of the elements chosen for modernization is solved, there will be a following problem: which among *n* elements need to be chosen, and up to which level of depth each of the chosen elements needs to be finished? Clearly, that there is a sense to solve these problems only in view of expenses for completion. Thus the level of completion of an element from the point of view of reliability of system will be determined by a new necessary level of reliability of this element in comparison with reliability *p* which is incorporated during manufacture of an element. Depth of completion, modernization of *i* an element of system quantitatively can be to characterize a gain x_i reliability of this element due to innovations. Thus if *i* the element or unit is not finished it (is not modernized), it is logical to put value $x_i = 0$. Then depth of completion or reconstruction of all system can be characterized *a n*-dimensional vector $\vec{x} = \{x_1, x_2, ..., x_n\}$, where x_i, $i = \overline{1, n}$ - essence the designations entered above. If to designate through p_i reliability of *i* system's an element prior to the beginning of completion after reconstruction reliability of an element will be equal $p_i + x_i$. Reliability of system before realization of modernization was the function $P = P(\vec{p}),$ after modernization it will increase and will get new value:

$$P = P(\vec{p}, \vec{x}).$$

Mathematical Model

It is necessary to investigate dependence of charges for completion of an element and its quality acquired due to completion. That is to investigate dependence of charges and an additional gain of probability x_i, taking into account thus an increment of reliability due to management. Further we shall name this dependence the function of cost of an element's completion [4] and we shall designate it through $K(x)$. As well as at the decision of any other practical problem, here we can not take advantage of real (empirical) dependence of charges for completion and a gain of reliability. For use of this dependence it is necessary to formalize it, construct analytical function which full enough approximates the given empirical dependence that is it is necessary to construct mathematical model of dependence. We shall take the mathematical model of dependence which is investigated and constructed in work [4], and it is proved by economic analogues [5], namely:

$$K(x) = \frac{Ax}{q - u - x},$$

in which q = 1 - p, and the factor A is defined from experimental data for concrete system. As we consider complex system with consecutive connection of subsystems (elements) we suppose, that the system is not a reliable (there comes refusal of system) if even one of n parameters (elements) of system has left for allowable limits. In the assumption of consecutive connection of elements and the account of an increment of reliability due to modernization and managements the equation of reliability of system will have the form:

$$P = \prod_{i=1}^{n} (p_i + u_i + x_i). \tag{1}$$

The total function of charges for modernization of system is equal

$$K(\vec{x}) = \sum_{i=1}^{n} \frac{A_i x_i}{q_i - u_i - x_i}. \tag{2}$$

Let's notice, that function of charges for the modernization (2) and the equation of reliability (1) make sense only with the certain restrictions which are consequence of their physical contents, namely:

$$1.\ K(x) \geq 0; \quad 2.\ K(0) = 0; \quad 3.\ \lim_{x \uparrow 1-p} K(x) = \infty.$$
$$x_i \geq 0, \quad x_i < q_i - u_i, \quad i = \overline{1,n}. \tag{3}$$

Now the problem was reduced to a task of nonlinear mathematical programming. More precisely the problem consists in a finding of such decision of the equation (1) which would provide under conditions (3) minimum of function (2). We already marked, that similar tasks of the theory of nonlinear programming still demand researches. Therefore we are very much limited in a choice of methods for a finding of decisions. For definition of optimum requirements to completion of each element, that is for a finding of a conditional minimum of function (2) in view of restriction on variables (1) and boundary conditions (3) we shall take advantage of the modified [6] method of Lagrange's uncertain multipliers.

Application of the modified method is necessary because direct use of Lagrange's method is impossible because the area of restrictions (3) is open. Therefore we use one substantial equality - restriction, namely (1). This equation information absorbs the others.

We build the function similar to Lagrange's function, we have:

$$f(\vec{x},\lambda) = \sum_{i=1}^{n} \frac{A_i x_i}{q_i - u_i - x_i} + \lambda \left[\prod_{i=1}^{n} (p_i + u_i + x_i) - P^* \right]. \tag{4}$$

From construction of function (4) it is visible, that in it restriction as the equation (1) is used only and boundary conditions as inequalities are not taken into account. Such step, generally speaking, is not absolutely true, but in this case it is justified by information capacity of restriction (1). It will be proved below, that offered use of restriction (1) results in obligatory performance of boundary conditions (3), and procedure of search of the decision provides it. At the same time the offered way releases the decision of a problem in the chosen method from analytical bulkiness of introduction of huge quantity of unknown additional multipliers such as Lagrange's multipliers.

For definition of components of a vector $\vec{x}$, for an optimum set x_i, i = 1, 2, ..., n we shall construct system from n the equations, having calculated individual derivatives from Lagrange's function $f(\vec{x},\lambda)$ on all variable x_i and having equated them to zero. We shall receive:

$$\frac{\partial f}{\partial x_i} = \frac{A_i q_i}{(q_i - u_i - x_i)^2} - \frac{\lambda}{p_i + u_i + x_i} \prod_{j=1}^{n} \left(p_j + u_j + x_j\right) = 0,$$
$$i = 1, 2, \ldots, n. \tag{5}$$

The system of the equations (5) contains (n+1) unknown: n unknown x_i and the unknown λ. If to this system to add the equation (1) we shall receive full system n +1 the equations with n +1 unknown. The received system of the equations (5), (1) has the decision, and besides the only thing that will be proved below.

For convenience we shall transform the equations (5) to a kind:

$$-\lambda \prod_{j=1}^{n}\left(p_j + u_j + x_j\right) = \frac{A_i q_i \left(p_i + u_i + x_i\right)}{\left(q_i - u_i - x_i\right)^2}, \quad i = 1,\ 2,\ ...,\ n. \tag{6}$$

We have allocated to the left the identical parts of the equations (5). As expression which is in the left part of each equation (6) is a constant in relation to an index i also the right parts of the equations should coincide. We admit from some physical or practical conditions it is possible to come to a conclusion, that for k an element of system completion is necessary. It means $x_k > 0$. We shall take k the equation of system (6) and instead of expression in the left part we shall substitute equal to it i expression from the current equation of system. Then the system of the equations (6) will be copied as follows:

$$\frac{A_i q_i \left(p_i + u_i + x_i\right)}{\left(q_i - u_i - x_i\right)^2} = \frac{A_k q_k \left(p_k + u_k + x_k\right)}{\left(q_k - u_k - x_k\right)^2}, \quad i = 1,\ 2,\ ...,\ k-1,\ k+1,\ ...\ n. \tag{7}$$

As we have fixed a choice k an element expression on the right has function which does not depend from i, therefore we shall designate the common right part of parity (7) through B_k, namely:

$$B_k = \frac{A_k q_k \left(p_k + u_k + x_k\right)}{\left(q_k - u_k - x_k\right)^2} \tag{8}$$

Further we pass from system (7) to a classical kind of the square-law equation:

$$B_k\left(x_i + u_i - q_i\right)^2 - A_i q_i\left(x_i + u_i - q_i\right) - A_i q_i = 0, \quad i = 1,\ 2,\ ...,\ k-1,\ k+1,\ ...\ n. \tag{9}$$

The system of the equations of the second degree (9) is received. It is equivalent to system of the equations (5). We research it on existence and uniqueness of the decision. With this purpose we shall enter functions

$$\varphi_i(x_i) = A_i q_i(p_i + u_i + x_i),$$
$$\psi_i(x_i) = B_k(x_i + u_i - q_i)^2,$$
$$i = 1,\ 2,\ ...,\ k-1,\ k+1,\ ...\ n.$$

These are functions according to the right and left parts of the equation $\varphi(x_i) = \psi(x_i)$ which is the system equivalent (9).

As for i the equations of system at value $x_i = q_i - u_i$ functions get values $\varphi_i(q_i - u_i) = A_i q_i > 0$ and $\psi_i(q_i - u_i) = 0$ the parity takes place $\varphi_i(q_i - u_i) > \psi_i(q_i - u_i)$. Graphically it means, that the straight line $\varphi_i(x_i)$ will cross a parabola $\psi_i(x_i)$ in two points x_i. One of the points x_i is located more to the left $q_i - u_i$, that is $x_i < q_i - u_i$. Other point is located to the right of $q_i - u_i$, that is $x_i > q_i - u_i$. Thus, the second roots of system of the equations (9) have no sense. They do not represent for us interest as they lie outside of area of physically legal values x_i, outside of area of existence of the decision of a task in view and do not satisfy to boundary conditions (3). From properties of monotony and a continuity of functions $\varphi_i(x_i)$ and $\psi_i(x_i)$ follows, that the point of crossing of these curves more to the left of value $x_i = q_i - u_i$ is unique. Then the unique left roots of system of quadratics (9) look like:

$$x_i = q_i - u_i + \frac{1 - \sqrt{1 + 2\alpha_i}}{\alpha_i}, \quad i = 1,\ 2,\ ...,\ k-1,\ k+1,\ ...,\ n, \tag{10}$$

in which $\alpha_i = 2B_k / A_i q_i$.

Let's calculate values of the entered functions in the left ends of an interval $[0;\ q_i - u_i)$ - ranges of definition of variables x_i as in the right ends we have already compared boundary their values. At $x_i = 0$ function $\varphi_i(x_i)$ accepts value $\varphi_i(0) = A_i q_i\left(p_i + u_i\right)$, and function $\psi_i(x_i)$ turns to number $\psi_i(0) = B_k\left(q_i - u_i\right)^2$.

For those i elements, for which $\varphi_i(0) > \psi_i(0)$, the point of crossing of curves $\varphi_i(x_i)$ and $\psi_i(x_i)$ lays more to the left of zero. Values of a root turn out $x_i < 0$. It contradicts a physical nature of a variable x_i and means what to modernize such element it is not necessary. It is necessary to put $x_i = 0$, thus the condition (3) will be provided. If for determined i an element of value of the entered functions will be razed to the ground $\varphi_i(0) = \psi_i(0)$, that is $\frac{A_i q_i(p_i + u_i)}{(q_i - u_i)^2} = B_k$, that the point of crossing of diagrams $\varphi_i(x_i)$ and $\psi_i(x_i)$ will get on an axis of ordinates. Thus the root $x_i = 0$ will turn out. It also means what to modernize an element it is not necessary. Thus the boundary condition (3) was provided automatically, as proves the statement stated above.

As a result we have: on the chosen value of a level of a gain of reliability x_k for k an element of system unequivocally we determine a level of completion i (i = 1, 2, ..., k – 1, k + 1, ..., n) an element x_i by means of parities:

$$x_i = \begin{cases} 0, & \frac{A_i q_i(p_i + u_i)}{(q_i - u_i)^2} \geq B_k, \\ q_i - u_i + \frac{1 - \sqrt{1 + 2\alpha_i}}{\alpha_i}, & \frac{A_i q_i(p_i + u_i)}{(q_i - u_i)^2} < B_k, \end{cases} \quad i = 1, 2, ..., k-1, k+1, ..., n. \tag{11}$$

Constructed mathematical model (11) solves a task in view of distribution of resources (financial) for optimum re-structuring complex system with the purpose of increase of its quality (reliability) in view of management of reliability from level P up to level $P^* = P + \Delta P$.

Conclusion

Using the initial information on system (reliability, cost of elements and the units, the available unsatisfactory level of reliability P new P^* required degree of quality of system in which achievement consists sense of re-structuring), mathematical model (11) is constructed. It gives answers to all questions of achievement of concrete decisions which are formulated in statement of a problem and are textually formulated in the summary in the beginning of work. We shall note simplicity of calculations, applied and their transparent interpretation at practical use of the developed model.

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