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TRAINING A LINEAR NEURAL NETWORK WITH A STABLE LSP SOLUTION FOR JAMMING CANCELLATION

Elena Revunova, Dmitri Rachkovskij

Abstract: *Two jamming cancellation algorithms are developed based on a stable solution of least squares problem (LSP) provided by regularization. They are based on filtered singular value decomposition (SVD) and modifications of the Greville formula. Both algorithms allow an efficient hardware implementation. Testing results on artificial data modeling difficult real-world situations are also provided*

Keywords: *jamming cancellation, approximation, least squares problem, stable solution, recurrent solution, neural networks, incremental training, filtered SVD, Greville formula*

ACM Classification Keywords: *I.5.4 Signal processing, G.1.2 Least squares approximation, I.5.1 Neural nets*

Introduction

Jamming cancellation problem appears in many application areas such as radio communication, navigation, radar, etc. [Shirman, 1998], [Ma, 1997]. Though a number of approaches to its solution were proposed [McWhirter, 1989], [Ma et al., 1997], no universal solution exists for all kinds of jamming and types of antenna systems, stimulating further active research to advance existing methods, algorithms and implementations.

Consider an antenna system with a single primary channel and *n* auxiliary channels. Signal in each channel is, generally, a mixture of three components: a valid signal, jamming, and channel's inherent noise. The problem consists in maximal jamming cancellation at the output while maximally preserving valid signal.

Within the framework of weighting approach [Shirman, 1998], the output is obtained by subtraction of the weighted sum of signals provided by the auxiliary channels from the primary channel signal. The possibility of determining a weight vector $\mathbf{w}^*$ that minimizes noise at the output while preserving the valid signal to a maximum degree is, in general case, provided by the following. The same jamming components are present both in primary and auxiliary channels, however, with different mixing factors. Valid signal has small duration and amplitude and is almost absent in auxiliary channels. Characteristics of jamming, channel's inherent noise, and their mixing parameters are stable within the sliding "working window".

These considerations allow us to formulate the problem of obtaining the proper $\mathbf{w}^*$ as a linear approximation of a real-valued function $y=f(x)$:

$$F(x) = w_1h_1(x) + w_2h_2(x)+\ldots+ w_n h_n(x) = \sum_{i=1,\, n} w_i\, h_i\, (x), \qquad (1)$$

where $h_1(x),\ldots,h_n(x)$ is a system of real-valued basis functions; $w_1,\ldots,w_n$ are the real-valued weighting parameters, $F(x)$ is a function approximating $f(x)$.

In our case, $h_1(x),\ldots,h_n(x)$ are signals provided by the *n* auxiliary channels. Information about $y=f(x)$ at the output of the primary channel is given at discrete set of (time) points $k=1,\ldots,m$ (*m* is the width of the working window) by the set of pairs $(\mathbf{h}^k,y^k)$. It is necessary to find $\mathbf{w}^*$ approximating $f(x)$ by $F(x)$ using linear least squares solution:

$$\mathbf{w}^* = argmin_w\, ||\mathbf{H}\, \mathbf{w}-\mathbf{y}||_2, \qquad (2)$$

where $\mathbf{H}$ is the so-called $m\times n$ "design matrix" containing the values provided by the n auxiliary channels for all $k=1,\ldots,m$; and $\mathbf{y} = [y_1,\ldots,y_m]^T$ is the vector of corresponding *y* values provided by the primary channel.

After estimating $\mathbf{w}^*$, the algorithm's output $\mathbf{s}$ is the residual discrepancy:

$$\mathbf{s} = \mathbf{H}\, \mathbf{w}^* - \mathbf{y}. \qquad (3)$$

Such a system may be represented as a linear neural network with a single layer of modifiable connections, $n+1$ input and single output linear neurons connected by a weight vector $\mathbf{w}$ (Fig. 1). In the case of successful training $\mathbf{w}^*$ provides an increased signal-jamming ratio at the output *s* compared to the input of the primary channel *y*, at least, for the training set $\mathbf{H}$.

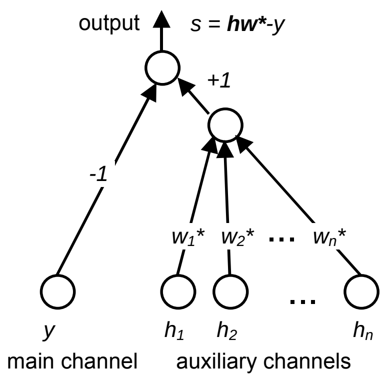

Fig 1. A neural network representation of a jamming canceller

A peculiarity of jamming cancellation problem in such a formulation consists in contamination of both $\mathbf{y}$ and $\mathbf{H}$ by the inherent noise of channels. Existing algorithms for jamming cancellation in the framework of weighting processing (2)-(3) [Shirman, 1998] do not take into account inherent noise contamination of $\mathbf{y}$ and $\mathbf{H}$. This results in instability of $\mathbf{w}$ estimation, leading, in turn, to a deterioration of cancellation characteristics, and often even to amplification of noise instead of its suppression. Therefore, methods for obtaining $\mathbf{w}^*$ should be stable to inherent noise contamination of $\mathbf{y}$ and $\mathbf{H}$. Other necessary requirements are real-time operation and simplicity of hardware implementation.

Least Squares Solution and Regularization

Generally, the solution of the least squares problem (LSP) (2) is given by

$$\mathbf{w}^* = \mathbf{H}^+ \mathbf{y}; \tag{4}$$

where $\mathbf{H}^+$ is pseudo-inverse matrix. If $\mathbf{H}$ is non-perturbed (noise is absent), then:

$$\text{for } m = n,\ rank\ (\mathbf{H}) = n = m \Rightarrow \det \mathbf{H} \neq 0.\ \mathbf{H}^+ = \mathbf{H}^{-1}; \tag{5}$$

$$\text{for } m > n,\ rank\ (\mathbf{H}) = n, \Rightarrow \det \mathbf{H} \neq 0:\ \mathbf{H}^+ = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T; \tag{6}$$

$$\text{for } m = n,\ m > n,\ rank(H) < n \Rightarrow \det H = 0,\ \mathbf{H}^+ = \lim_{v\to 0} (\mathbf{H}^T\mathbf{H} + v^2\mathbf{I})^{-1}\mathbf{H}^T. \tag{7}$$

$\mathbf{H}^+$ for (7) can be obtained numerically using SVD [Demmel, 1997] or the Greville formula [Greville, 1960].

The case when $\mathbf{y}$ and elements of matrix $\mathbf{H}$ are known precisely is very rare in practice. Let us consider a case that is more typical for jamming cancellation, i.e. when $\mathbf{y}$ and $\mathbf{H}$ are measured approximately: $\mathbf{y} = \mathbf{y} + \varsigma$; $\mathbf{H} = \mathbf{H} + \Xi$; where ς is noise vector, Ξ is noise matrix. In such a case, solutions (5)-(7) may be unstable, i.e. small changes of $\mathbf{y}$ and $\mathbf{H}$ cause large changes of $\mathbf{w}^*$ resulting in instable operation of application systems based on (5)-(7).

To obtain a stable LSP solution, it is fruitful to use approaches for solution of "discrete ill-posed problems" [Hansen, 1998], [Jacobsen et al., 2003], [Reginska, 2002], [Wu, 2003], [Kilmer, 2003]. Such a class of LSPs is characterized by $\mathbf{H}$ with singular values gradually decaying to zero and large ratio between the largest and the smallest nonzero singular values. This corresponds to approximately known and near rank-deficient $\mathbf{H}$.

Reducing of an ill-posed LSP to a well-posed one by introduction of the appropriate constraints to the LSP formulation is known as regularization [Hansen, 1998]. Let us consider a problem known as standard form of the Tikhonov regularization [Tikhonov, 1977]:

$$\operatorname{argmin}_w \{||\mathbf{y} - \mathbf{H}\,\mathbf{w}||_2 + \lambda\ ||\mathbf{w}||_2\}. \tag{8}$$

Its solution $\mathbf{w}_\lambda$ may be obtained in terms of SVD of $\mathbf{H}$ [Hansen, 1998]:

$$\mathbf{w}_\lambda = \Sigma_{i=1,\,n}\ f_i\ \mathbf{u}_i^T\mathbf{y} / \sigma_i\ \mathbf{v}_i; \tag{9}$$

$$f_i = \sigma^2_i / (\sigma^2_i + \lambda^2), \tag{10}$$

where σ_i are singular values, $\mathbf{u}_1 \ldots \mathbf{u}_n$, $\mathbf{v}_1 \ldots \mathbf{v}_n$ are left and right singular vectors of $\mathbf{H}$, f_i are filter factors.

Note that solution of (8) using truncated SVD method [Demmel, 1997] is a special case of (9), (10) with $f_i \in \{0,1\}$.

Algorithmic Implementation of Solutions of Discrete ill-posed LSPs

Requirements of an efficient hardware implementation of jamming cancellation pose severe restrictions on the allowable spectrum of methods and algorithms. In particular, methods of $\mathbf{w}^*$ estimation are required to allow for parallelization or recursion. Taking this into account, let us consider some algorithmic implementations of (8).

SVD-based solution. The implementation of SVD as a systolic architecture with paralleled calculations is considered in [Brent, 1985]. We have developed a systolic architecture that uses effective calculation of $\mathbf{u}_i$, σ_i, and $\mathbf{v}_i$ for obtaining the regularized solution $\mathbf{w}_\lambda^*$ (9)-(10). The architecture implements two regularization techniques: truncated and filtered SVD [Hansen, 1998].

Advantages of SVD-based solution are accuracy and parallelism. Drawbacks are connected with the hardware expenses for calculation of trigonometric functions for diagonalization of sub-matrices and implementation of systolic architecture itself.

Solution based on the Greville formula and its modifications. Let us consider another stable method for $\mathbf{w}^*$ estimation based on the Greville formula, which can be readily implemented in hardware because of its recursive character. A recursive procedure for the LSP solution [Plackett, 1950] for a full-rank $\mathbf{H}$ is as follows:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mathbf{b}_{k+1}(y_{k+1} - \mathbf{h}^T_{k+1}\mathbf{w}_k); \quad k = 0, 1,\dots, \tag{11}$$

$$\mathbf{b}_{k+1} = \mathbf{P}_k \mathbf{h}_{k+1}/(1 + \mathbf{h}^T_{k+1}\mathbf{P}_k \mathbf{h}_{k+1}); \tag{12}$$

$$\mathbf{P}_{k+1} = (\mathbf{H}_{k+1}{}^T \mathbf{H}_{k+1})^{-1} = (\mathbf{I} - \mathbf{b}_{k+1}\mathbf{h}^T_{k+1})\mathbf{P}_k; \tag{13}$$

where $\mathbf{h}_k$ is the kth row (sample) of $\mathbf{H}$; $\mathbf{P}_0 = 0$; $\mathbf{w}_0 = 0$. Note that this provides an iterative version of training algorithm for a neural network interpretation of Fig. 1.

The Greville formula [Greville, 1960] allows $\mathbf{b}_{k+1}$ calculation for (11) without $(\mathbf{H}_{k+1}{}^T \mathbf{H}_{k+1})^{-1}$ calculation, thus overcoming the problem of rank-deficiency of $\mathbf{H}$:

$$\mathbf{b}_{k+1} = (\mathbf{I} - \mathbf{H}^+_k \mathbf{H}_k)\mathbf{h}_{k+1} / \mathbf{h}^T_{k+1}(\mathbf{I} - \mathbf{H}^+_k \mathbf{H}_k)\mathbf{h}_{k+1}; \text{ if } \mathbf{h}^T_{k+1}(\mathbf{I} - \mathbf{H}^+_k \mathbf{H}_k)\mathbf{h}_{k+1} \neq 0; \tag{14}$$

$$\mathbf{b}_{k+1} = \mathbf{H}^+_k(\mathbf{H}^+_k)^T \mathbf{h}_{k+1} / (1 + \mathbf{h}^T_{k+1}\mathbf{H}^+_k(\mathbf{H}^+_k)^T \mathbf{h}_{k+1}); \text{ if } \mathbf{h}^T_{k+1}(\mathbf{I} - \mathbf{H}^+_k \mathbf{H}_k)\mathbf{h}_{k+1} = 0; \tag{15}$$

$$\mathbf{H}^+_{k+1} = (\mathbf{H}^+_k - (\mathbf{b}_{k+1}\mathbf{h}^T_{k+1}\mathbf{H}^+_k \mid \mathbf{b}_{k+1})). \tag{16}$$

$\mathbf{w}^*$ obtained by (11)-(13) using (14)-(16) is equivalent to $\mathbf{w}^*$ obtained by (7) for precisely specified $\mathbf{H}$. Presence of $\mathbf{H}^+_k$ and $\mathbf{H}_k$ in (14)-(16) makes recursion more resource- and computation-expensive than (12)-(13). As a new sample arrives, it is necessary to calculate $\mathbf{H}^+_k\mathbf{H}_k$ or $\mathbf{H}^+_k(\mathbf{H}^+_k)^T$ that requires calculation of $\mathbf{H}^+_k$ and storage of $\mathbf{H}^+_k$ and $\mathbf{H}_k$. These drawbacks are overcome by an improvement of the Greville formula proposed recently in [Zhou, 2002].

For $\mathbf{h}^T_{k+1}\mathbf{Q}_k = 0$ calculations of $\mathbf{b}_{k+1}$ and $\mathbf{P}_{k+1}$ are made by (12)-(13). If $\mathbf{h}^T_{k+1}\mathbf{Q}_k \neq 0$

$$\mathbf{b}_{k+1} = \mathbf{Q}_k\mathbf{h}_{k+1}/(\mathbf{h}^T_{k+1}\mathbf{Q}_k\mathbf{h}_{k+1}); \tag{17}$$

$$\mathbf{P}_{k+1} = (\mathbf{I} - \mathbf{b}_{k+1}\mathbf{h}^T_{k+1})\mathbf{P}_k(\mathbf{I} - \mathbf{b}_{k+1}\mathbf{h}^T_{k+1})^T + \mathbf{b}_{k+1}\mathbf{b}^T_{k+1}; \tag{18}$$

$$\mathbf{Q}_{k+1} = (\mathbf{I} - \mathbf{b}_{k+1}\mathbf{h}^T_{k+1})\mathbf{Q}_k. \tag{19}$$

Here $\mathbf{P}_k = \mathbf{H}_k{}^+(\mathbf{H}_k{}^+)^T$ is Hermitian $n \times n$ matrix; $\mathbf{P}_0 = 0$, $\mathbf{Q}_k = \mathbf{I} - \mathbf{H}_k{}^+\mathbf{H}_k$; $\mathbf{Q}_0 = \mathbf{I}$.

We further modified the Greville formula so that $\mathbf{w}_{k+1}$ is equivalent to the regularized solution $\mathbf{w}_\lambda^*$ (9). This is achieved by comparison of vector norm $\mathbf{h}^T_{k+1}\mathbf{Q}_k$ not with 0, but with some threshold value 0_{eff} calculated from noise matrix $\boldsymbol{\Xi}$. We name such an algorithm "pseudo-regularized modification of the Greville formula" (PRMGF).

The algorithm (11)-(13), (17)-(19) calculates $\mathbf{w}^*$ using all previous samples. However, for a non-stationary case it is necessary to process only a part of the incoming samples inside a sliding working window. Full recalculation of $\mathbf{H}_{k+1}{}^+$ for estimation of $\mathbf{w}_{k+1}$ as each new sample arrives can be avoided by using inverse recurrent representation of [Kirichenko, 1997]. For the purpose of removing the row $\mathbf{h}^T_1$ from $\mathbf{H}_{k+1}$, $\mathbf{H}_k{}^+$ is represented through $\mathbf{H}^+_{k+1} = (\mathbf{b}_1|\mathbf{B}_{k+1})$ as follows.

For a linear independent row:

$$(\mathbf{H} + \mathbf{h}\mathbf{e}^T)^+ = \mathbf{H}^+ - \mathbf{H}^+\mathbf{h}\mathbf{h}^T\mathbf{Q}/(\mathbf{h}^T\mathbf{Q}\mathbf{h}) - \mathbf{Q}\mathbf{e}\mathbf{e}^T\mathbf{H}^+/\mathbf{e}^T\mathbf{Q}\mathbf{e} + \mathbf{Q}\,\mathbf{e}\mathbf{h}^T\mathbf{Q}(\mathbf{H}^T)(1+\mathbf{e}^T\mathbf{H}^+\mathbf{h})/\mathbf{h}^T\mathbf{Q}(\mathbf{H}^T)\mathbf{h}\,\mathbf{e}^T\mathbf{Q}\mathbf{e}; \tag{20}$$

and for a linear dependent row:

$$(\mathbf{H} + \mathbf{h}\mathbf{e}^T)^+ = (\mathbf{I} - \mathbf{z}\mathbf{z}^T/\|\mathbf{z}\|^2)\mathbf{H}^+; \; \mathbf{z} = \mathbf{H}^+\mathbf{h} - \mathbf{e}/\|\mathbf{e}\|^2. \tag{21}$$

Thus, we propose to use PRMGF with a sliding window for the case, when it is required to obtain $\mathbf{w}^*$ not for the whole training set, but for its subset of a fixed size. For initial $k < m$ samples $\mathbf{h}_k$, $\mathbf{w}^*_k$ is recursively calculated by PRMGF. For $k > m$, (20)-(21) are used for updating $\mathbf{H}^+$ by removing the sample that has left a working window, and the incoming sample s is taken into account using PRMGF as earlier.

Advantages of PRMGF with a sliding window include:

- natural embedding into recursive algorithm for $\mathbf{w}^*$;

- increase of calculation speed due to using $\mathbf{h}^T_{k+1}$ instead of $\mathbf{H}_k$, also resulting in reduction of required memory;
- additional memory reduction since $\mathbf{P}_k$, $\mathbf{K}_k$ and $\mathbf{Q}_k$ have fixed $n \times n$ dimension for any k;
- further increase of calculation speed when sliding window is used due to the Greville formula inversion;
- considerably smaller hardware expenses in comparison with SVD;
- $\mathbf{w}^*$ close to the Tikhonov regularized solution for noisy, near rank-deficient $\mathbf{H}$ (at least, for small matrices);
- natural interpretation as an incrementally trained neural network.

Example of Modeling a Jamming Cancellation System

Let's compare the following jamming cancellation algorithms: ordinary LS-based (6); non-truncated SVD-based [Demmel, 1997]; truncated SVD-based (9) with $f_i = \{0,1\}$; PRMGF-based (section 3.2). We use near rank-deficient $\mathbf{H}$, which is critical for traditional jamming cancellation algorithms – e.g., for ordinary LS-based ones.

Testing scheme and cancellation quality characteristics. In a real situation, all antenna channels receive jamming signals weighted by the gain factor that is determined by the antenna directivity diagram in the direction of particular jamming. We simulated signals in antenna channels as follows:

$$\mathbf{X} = \mathbf{S}\,\mathbf{M} + \mathbf{\Xi}; \qquad (22)$$

where $\mathbf{X}$ is $L \times (n+1)$ matrix of signals in antenna channels ($\mathbf{H}$ is sub-matrix of $\mathbf{X}$); L is the number of samples; n is the number of auxiliary channels; $\mathbf{S}$ is jamming signals' matrix; $\mathbf{\Xi}$ is channel inherent noise matrix; $\mathbf{M}$ is mixing matrix.

Jamming signals and channels' inherent noise are modeled by normalized centered random variables with normal and uniform distribution correspondingly. $\mathbf{M}$ is constructed manually, values of its elements are about units, rank deficiency was achieved by entering identical or linearly dependent rows. For ideal channels without inherent noise, rank deficiency of $\mathbf{M}$ gives rise to strict rank deficiency of $\mathbf{H}$. Inherent noise results in near rank-deficient $\mathbf{H}$. Tests were carried out for n=8 auxiliary channels.

The main characteristics of jamming cancellers are: jamming cancellation ratio (K^c) and jamming cancellation ratio vs inherent noise level in auxiliary channels K^{naux}: $K^c = f(K^{naux})$ [Bondarenko, 1987] at fixed inherent noise at the primary channel K^{n0}.

$$K^c = P^{in}/P^{out}, \qquad (23)$$

where P^{in} and P^{out} is power of jamming in the primary channel and in the output of jamming canceller, respectively. In all tests, the valid signal with amplitude not exceeding amplitude of primary channel jamming was present at the input for 5 nearby samples. L=1000; m=16; K^{n0} = {0.1, 0.2, 0.3}, $K^{n0} >> K^{naux}$ to complicate the algorithm's operation.

Testing results. A family of jamming cancellation characteristics $K^c = f(K^{naux})$ for rank-deficient $\mathbf{M}$ and near rank-deficient $\mathbf{H}$ is shown in Fig.2. K^{naux} varied from 1.6 10^{-9} up to 6.4 10^{-6}. K^c for the ordinary LS did not exceed 1 at K^{naux} < 2.5 10^{-8}. For truncated SVD and PRMGF $K^c \approx 10$ (K^{n0} = 0.1) are nearly constant over the whole range of K^{naux} and close to each other. Note that for a full-rank matrix $\mathbf{H}$, K^n for all algorithms was approximately the same and large in the considered range of K^{naux}.

It may seem from the analysis of the shown results that one may use the ordinary LS algorithm at increased level of K^{naux}. However, roll-of of cancellation characteristic is also observed when jamming intensity in auxiliary channels is much more than the inherent noise level. To show that, let us consider $K^c(P^{in})$ for near rank-deficient $\mathbf{H}$ (Fig.3). Jamming power P^{in} changed from $9 \cdot 10^3$ to $1.2 \cdot 10^7$ by step $2 \cdot 10^2$, K^{naux} = 0.1. For $P^{in} > 10^4$, K^c for ordinary LS and non-truncated SVD decreases. For truncated SVD and PRMGF, $K^c \approx 9$ (K^{n0} = 0.1) are constant and close to each other. In this case, we cannot artificially increase inherent noise level because it will completely mask the valid signal.

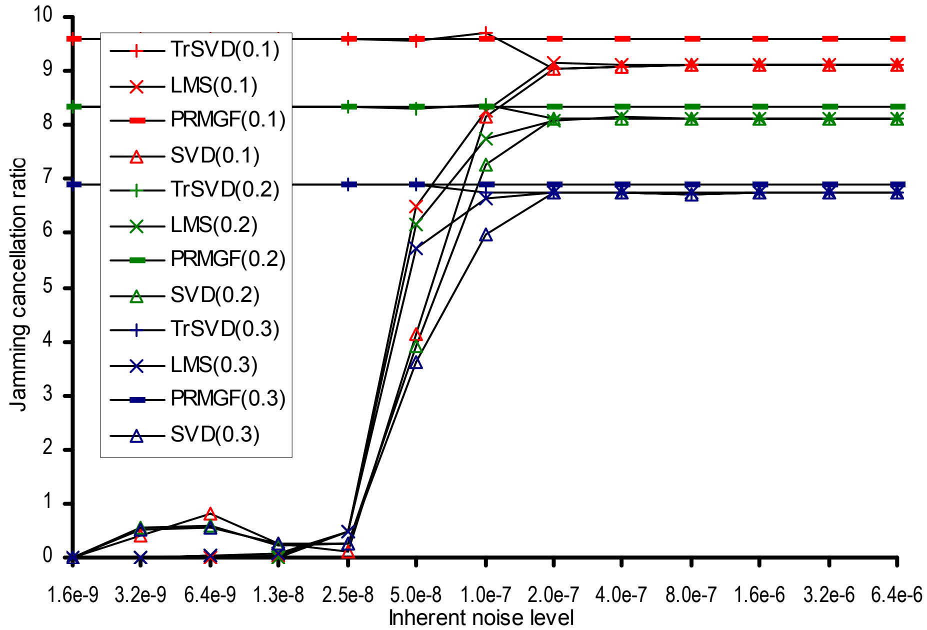

Fig. 2. Kc= f(Knaux) for near rank-deficient H

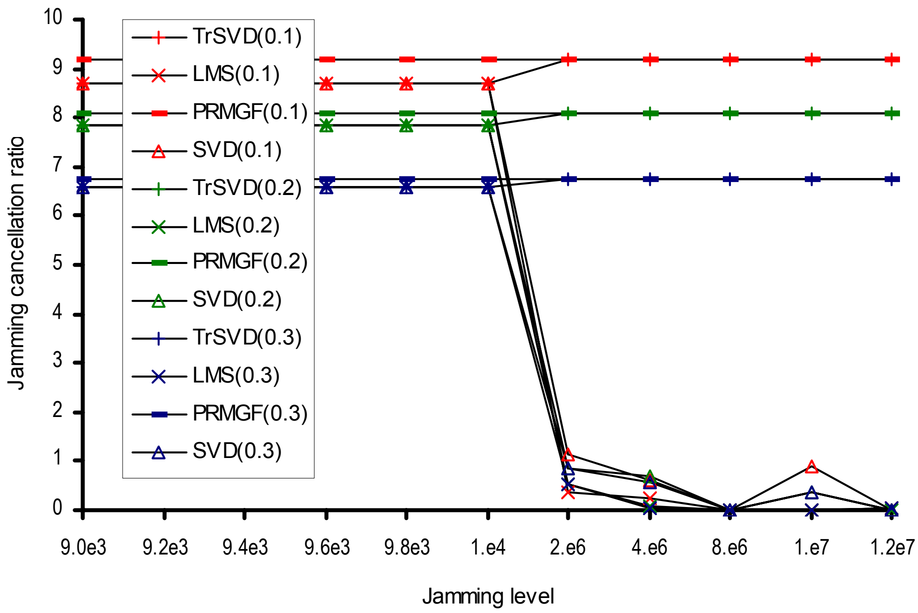

Fig. 3. Kc(Pin) for near rank-deficient H

Conclusions

In the framework of this work, two new jamming cancellation algorithms have been developed based on the so-called weighting approach. Special requirements to the problem have resulted in its classification as a discrete ill-posed problem. That has allowed us to apply an arsenal of the regularization-based methods for its stable solution - estimation of weight vector ***w****.

The standard form of Tikhonov regularization based on SVD has been transformed to efficient hardware systolic architecture. Besides, pseudo-regularized modification of the Greville formula allowed us to get weight vector estimations very close to estimations for a truncated SVD based regularization - at least for ***H*** of about tens of columns. Testing on near rank-deficient ***H*** has shown that distinctions in ***w**** obtained by both algorithms are of the order 10^{-5}. A combined processing technique based on a regularized modification of the Greville formula and inverse recurrent representation of Kirichenko permits a more efficient processing of data for a sliding working window.

Testing on artificial data that model real-world jamming cancellation problem has shown an efficient cancellation for near rank-deficient ***H***. For the developed PRMGF-based algorithm the jamming cancellation ratio is near constant and considerably higher than 1 in the whole range of variation of auxiliary channels' inherent noise and jamming amplitude. On the contrary, for the non-regularized LS method the ratio roll-offs to less than 1, meaning jamming amplification.

A straightforward neural network interpretation of such a system is provided. The developed algorithms and computer architectures for their implementation can be applied to solution of other discrete ill-posed LS problems and systems of linear algebraic equations.

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