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## ANALYSIS AND OPTIMIZATION OF SYNTHETIC APERTURE ULTRASOUND IMAGING USING THE EFFECTIVE APERTURE APPROACH

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**Abstract:** An effective aperture approach is used as a tool for analysis and parameter optimization of mostly known ultrasound imaging systems - phased array systems, compounding systems and synthetic aperture imaging systems. Both characteristics of an imaging system, the effective aperture function and the corresponding two-way radiation pattern, provide information about two of the most important parameters of images produced by an ultrasound system - lateral resolution and contrast. Therefore, in the design, optimization of the effective aperture function leads to optimal choice of such parameters of an imaging systems that influence on lateral resolution and contrast of images produced by this imaging system. It is shown that the effective aperture approach can be used for optimization of a sparse synthetic transmit aperture (STA) imaging system. A new two-stage algorithm is proposed for optimization of both the positions of the transmitted elements and the weights of the receive elements. The proposed system employs a 64-element array with only four active elements used during transmit. The numerical results show that Hamming apodization gives the best compromise between the contrast of images and the lateral resolution.

**Keywords:** Ultrasound imaging, Synthetic aperture, stochastic optimization.

**ACM Classification Keywords:** J.3 Life and Medical Sciences: Medical information systems; I.5.4 Pattern recognition: Applications --- Signal processing; G.1.6 Numerical Analysis: Optimization --- Simulated annealing

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### 1. Introduction

Medical ultrasound imaging is a technique that has become much more prevalent than other medical imaging techniques since this technique is more accessible, less expensive, safe, simpler to use and produces images in real-time. However, images produced by an ultrasound imaging system, must be of sufficient quality to provide accurate clinical interpretation. The most commonly used image quality measures are spatial resolution, image contrast and frame rate. The first two image quality measures (resolution and contrast) can be determined in terms of beam characteristics of an imaging system beam width and side lobe level. In the design of an imaging system, the optimal set of system parameters is usually found as a tradeoff between the lowest sidelobe peak and the narrowest beam width of an imaging system. In a conventional ultrasound imaging system, the transducer is a phased array with a great number of elements (PA imaging systems). The quality of images produced by a PA system directly depends on the number of active channels used both in transmission and receiving. Thus, the conventional high-resolution PA imaging systems produce images at relatively high cost [1].

Conventional phased array imaging systems employ all elements of the transducer during both transmit and receive during each excitation cycle, while employing delays in order to steer the beam and scan a 2D plane. In receive mode, dynamic (or composite) focus is used, by adjusting the delays of transducer elements as a function of the depth being imaged. In transmit mode, usually the focus point is set in the middle of the region being imaged. At the focus point, the lateral beamwidth is the smallest (and the best lateral resolution is obtained there), while away from the focus point, the lateral beamwidth increases. The spatial resolution of the ultrasound image can be improved by using several transmit beams during the interrogation of each sector, each of which is focused at a different depth. It is done in modern ultrasound imaging systems at the cost of decrease of the frame rate, proportionally to the number of transmit foci [2]. An alternative way to obtain an appropriate spatial resolution, without the decrease of the frame rate, is to use the synthetic aperture technique. This method makes it possible to generate images with dynamic focusing, during both transmit and receive, while maintaining or even drastically decreasing the time of image acquisition.

In a classical Synthetic Aperture Focusing Technique (SAFT), only a single array element transmits and receives at each time. All the elements are excited sequentially one after the other, and the echoes received are recorded

and stored in computer memory. It reduces the system complexity and the frame rate, but requires data memory for all data recordings [3]. The main disadvantage of SAFT is the low signal-to-noise ratio (SNR) and as a result, the poor contrast resolution. In a Multi-element Synthetic Aperture Focusing (MSAF) method, at each time a group of elements transmits and receives signals simultaneously [4]. The transmitted beam is defocused to emulate a spherical wave. The SNR is increased compared to SAFT, in which only a single element is used in transmit and receive. In a Synthetic Transmit Aperture (STA) method, at each time one array element transmits a pulse, and all elements receive the echo signals [5]. Compared to conventional phased array imaging, the advantage of this approach is that a full dynamic focusing can be applied to the transmission and the receiving, producing the highest quality of images at the increased frame rate. The shortcoming is that a huge data memory is required for data recordings. For an  $N$ -element array,  $N$  echo recordings are required to form a conventional phased array image, and, however,  $N \cdot N$  echo recordings are required to synthesize a STA image. This disadvantage can be overcome to some extent, if only a few elements,  $M$ , act as transmitters. In that case  $M \cdot N$  echo recordings are required to synthesize a STA image, where  $M < N$  [6]. This is equivalent to using of a sparse array in transmit. The sparse STA imaging acquires images at higher frame rates, which makes this method very attractive for real-time 3D-ultrasound imaging.

The relation between the employed effective aperture function and the resultant radiation pattern of the imaging system can be used as a strategy for analysis and for optimisation of an imaging system [7]. Since the two-way radiation pattern of a system is the Fourier transform of the effective aperture function, the transmitted and receiving radiation patterns can be optimised by selecting the appropriate transmit and receive aperture functions, to produce the "desired" effective aperture of the imaging system. Thus, when the desired effective aperture of a system is defined, it also provides the two-way radiation pattern that should be used, with the appropriate width of the main-lobe and its sidelobes. In synthetic aperture imaging, the transmit aperture function depends not only on the number of transmit elements, but also on their geometrical locations within the array (sparse synthetic aperture imaging). The received aperture function depends on the length of a physical array and the apodization weights applied to the receiver elements. Thus, the shape of the effective aperture function and, therefore, the shape of the two, one-way radiation patterns of a system, can be optimised depending on the positions of the element in transmit and the weights of the element in receive.

In this paper, it is shown how the effective aperture approach can be used for analysis and parameter optimisation of an ultrasound STA imaging system. Using this approach, the optimal set of system parameters (number of array elements, their configuration within an array) can be determined in result of a compromise between the lowest sidelobe peak and the narrowest beam width of the two-way radiation pattern of an imaging system. The comparison analysis of 3 types of imaging systems is done calculating their effective aperture function and the corresponding two-way radiation pattern using the computational environment of Matlab.

## 2. The Effective Aperture Concept

The effective aperture of an array represents an equivalent aperture that would produce identical two-way radiation pattern if the transmit aperture was a point source. An expression for the effective aperture of an array can be derived from a calculation of the two - way radiation pattern. Consider an uniformly spaced linear array of  $N$  elements with weighting  $w(m)$  ( $m = 0, \dots, N - 1$ ). The one- way far field beam pattern is

$$W(\theta) = \sum_{m=0}^{N-1} w(m) e^{-k^0 m d \sin(\theta)} \quad (1)$$

where  $d$  and  $k^0$  are the inter-element spacing and the wave number, respectively. This equation can also be described as a discrete Fourier transform (DFT) of the aperture function:

$$W(k) = DFT[w(m)] = \sum_{m=0}^{N-1} w(m) e^{-j \frac{2\pi}{N} km}, \quad k=0, 1, \dots, N-1 \quad (2)$$

in which the frequency index  $k$  maps into the beam angle  $\theta$  by  $\sin \theta = k\lambda / (Nd)$  where  $\lambda$  is the wavelength.

Since the round-trip beam pattern is the product of the transmit and receive beams

$$W_{RT}(\theta) = W_R(\theta)W_T(\theta) \tag{3}$$

using the DFT property, we get

$$W_{RT}(k) = DFT[w_R \otimes w_T] \tag{4}$$

where  $\otimes$  denotes convolution and  $w_R$  and  $w_T$  are the apodization functions applied to the array elements in transmit and receive, respectively. Using (4), the effective aperture function of an imaging system is defined as

$$e_{RT} = w_R \otimes w_T \quad \text{and} \quad W_{RT}(\theta) = FFT(e_{RT}) \tag{5}$$

Thus, the round trip beam pattern is determined by the transmitted and the receiving aperture weightings. Every physical beam can be realized by forming the appropriate effective aperture.

### 3. Synthetic Aperture Imaging

First, the concept of synthetic aperture was originally used in radar for highly resolution imaging terrain, but it can be successfully used in ultrasound imaging systems as well. In this case, the benefit of the synthetic aperture is the reduction of system complexity and cost. Several methods were proposed to form a synthetic aperture for ultrasonic imaging. In SAFT imaging, at each time only a single array element transmits a pulse and receives the echo signal. (Fig. 1). The system complexity is reduced, because only a single set of circuit for transmit and receive is needed. In this case the effective aperture can be calculated by

$$e_N = \sum_{m=1}^N w_R(m) \otimes w_T(m), \text{ where } w_R(m) = w_T(m) = [0, 0, \dots, i_m, \dots, 0] \text{ and } i_m = 1 \tag{6}$$

In MSAF imaging, a group of elements transmit and receive signals simultaneously, and transmit beam is defocused to emulate a single element response (Fig. 2). The acoustic power and the signal-to-noise ratio are increased compared to SAFT where a single element is used. This method requires also memory for data recordings. In MSAF, a  $K_t$ -element transmit subaperture sends an ultrasound pulse and echo signals are recorded at a  $K_r$ -element receive subaperture. At the next step, one element is dropped and a new element is included to the transmitted and receiving subaperture, repeating the transmission and receiving process. Usually  $K_t=K_r=k$ . The effective aperture is:

$$e_N = \sum_{m=1}^{N-k+1} w_R(m) \otimes w_T(m), \text{ where} \tag{7}$$

$$w_R(m) = w_T(m) = [0, 0, \dots, i_m, i_{m+1}, \dots, i_{m+k-1}, 0, \dots, 0] \text{ and } i_m = i_{m+1} = \dots = i_{m+k-1} = 1$$

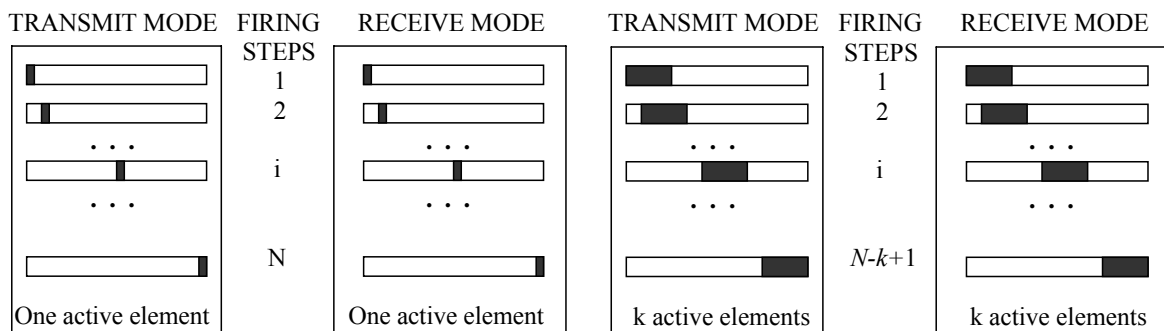


Fig.1: SAFT imaging method

Fig.2: MSAF imaging method

In STA imaging, at each time one array element transmits a pulse and all elements receive the echo signals (Fig. 3). The advantage of this approach is that a full dynamic focusing can be applied to the transmission and

the receiving, giving the highest quality of image. The disadvantage is that a huge data memory is required and motion artifacts may occur. The effective aperture is calculated as:

$$e_N = \sum_{m=1}^N w_R \otimes w_T(m) \text{ where } w_R = [1,1,\dots,1], w_T(m) = [0,0,\dots,i_m,\dots,0] \text{ and } i_m = 1 \quad (8)$$

Synthetic Receive Aperture (SRA) method of imaging was proposed to improve lateral resolution (Fig. 4). It is known that the lateral resolution can be improved by increasing array length. In practice, it is not very expensive to build a large transmit aperture, but is very complex to form a large receive aperture. This method uses a large transmit aperture and enables an imaging system to address a large number of transducer receive elements without the same number of parallel receive channels. In the receive mode the aperture is split into two or more subapertures. In order to form each line of image data in the SRA system, the transmitters must be fired once for each receive subaperture. For a single transmit pulse (from all transmit elements), the RF sum for one receive subaperture is formed and stored in memory. Then a second identical pulse is transmitted in the same direction and the RF sum for another subaperture is formed and stored. After the RF signals have been acquired from all receive subapertures, the total RF sum is formed by coherently adding together the sums from various subapertures. For an  $N$ -element linear array, receive aperture is split into  $N_S = N/K_R$  subapertures, and each subaperture contains  $K_R$  elements. The effective aperture is:

$$e_N = \sum_{m=0}^{N_S-1} w_R(m) \otimes w_T, \text{ where} \quad (9)$$

$$w_T = [1,1,\dots,1] \quad , \quad w_R(m) = [0,0,\dots,i_n,i_{n+1},\dots,i_{n+K_R-1},0,\dots,0] \quad , \quad n = m * K_R + 1$$

$$i_n = i_{n+1} = \dots = i_{n+K_R-1} = 1$$

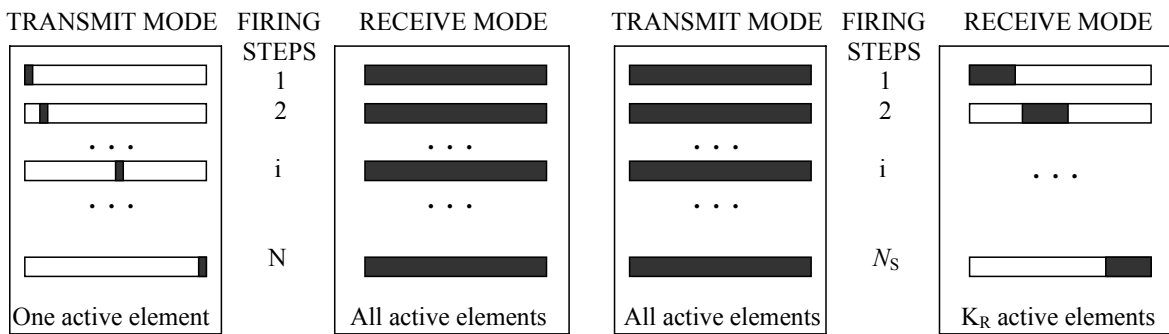


Fig.3: STA imaging method

Fig.4: SRA imaging method

A sparse STA imaging method is proposed to increase system frame rate (Fig. 5). Only a small number of elements are used to transmit a pulse but all array elements receive the echo signals. For an  $N$ -element aperture,  $M \times N$  data recordings are needed for image reconstruction., where  $M \ll N$ . All data recordings must then be combined with dynamic focusing. The effective aperture is:

$$e_N = \sum_{m=1}^M w_T(m) \otimes w_R \quad (10)$$

where  $w_T(m) = [0,0,\dots,i_{K_m},\dots,0]$ ,  $w_R = [1,1,\dots,1]$  and  $i_{K_m} = 1$

The two-way radiation pattern of a synthetic aperture imaging system is calculated using the Fourier transform of the corresponding effective aperture function defined by the expressions (6,7,8,9 and 10).

### 4. Optimization of a Sparse Array

For a sparse STA imaging system with an array with  $N$ -elements, the two-way radiation pattern is evaluated as the Fourier Transform of the effective aperture function  $e_N$ , defined as:

$$e_N = \sum_{m=1}^M a_m \otimes B, \text{ and } a_m = [0, 0, \dots, i_m, \dots, 0], \text{ where } i_m = 1 \tag{11}$$

where  $a_m$  is the transmit aperture during the  $m$ 'th firing,  $B$  is the apodization function applied to the receiver elements, and  $\otimes$  is the convolution operator. The speed of the image acquisition is determined by the number of transmit elements ( $M$ ),  $M < N$ . Since the geometrical locations of the transmit elements in a sparse array system impact the two-way radiation pattern of that system, the image quality parameters, the lateral resolution and contrast, all depend on the locations of the transmit elements within the sparse array ( $i_1, i_2, \dots, i_M$ ). Since the weighting applied to each receiver element also impacts the radiation pattern of the system, the image quality also depends on the type of the apodization function ( $B$ ). Therefore, the optimization of a sparse STA imaging system can be formulated as an optimization problem of both the location of the elements of the sparse array in transmit and the weights assigned to the elements of the full array during receive [8]. Different algorithms have been proposed for optimization of the locations of the transmitted elements in a sparse array – genetic, linear programming and simulated annealing algorithms [8]. For most cases the optimization criterion is minimal sidelobe peak of the radiation pattern.

In this paper, another optimization criterion is proposed. It is the minimal width of the mainlobe ( $W$ ) combined with a condition on the maximum sidelobe level ( $SL < Q$ ). It is suggested here to divide the optimization process into two stages. In the first stage, the optimal positions of transmit elements ( $i_1, i_2, \dots, i_M$ ) are found, for a set of known apodization functions  $\{B_k\}$ ,  $k=1, 2, \dots, K$ . Such a set of apodization functions may include several well-known window-functions (Hamming, Hann, Kaiser, Chebyshev and etc). At this stage, the optimization criterion can be written as follows:

$$\text{Given } M, N \text{ and } \{B_k\}_K, \text{ choose } (i_1, i_2, \dots, i_M)_K \text{ to minimize } W \text{ subject to } SL < Q \tag{12}$$

where  $Q$  is the threshold of acceptable level of the sidelobe peak. In the second stage, the final layout of transmit elements is chosen, which is a layout that corresponds to the most appropriate apodization function  $B = (b_1, b_2, \dots, b_N)$ . This choice is a compromise between minimal width of the mainlobe and the acceptable level of the peak of the sidelobes. Mathematically, it can be written as follows:

$$\text{Given } M, N, \{B_k\}_K \text{ and } \{i_1, i_2, \dots, i_M\}_K, \text{ choose } (b_1, b_2, \dots, b_N) \text{ to minimize } W \text{ subject to } SL < Q \tag{13}$$

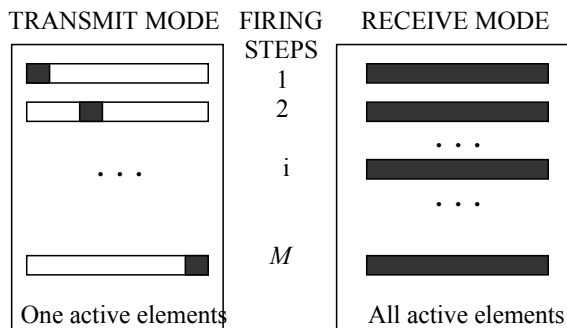


Fig.5: Sparse STA imaging method

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begin
  Initialize (I0, T0)
  for k=1 to number_iterations
    Tk = Tk-1 · α
    for j=1 to number_perturbations
      Ip = perturbate(Ij-1)
      ΔW = W(Ip) - W(Ij-1)
      ΔSL = SL(Ip) - Q
      if {ΔW < 0 or exp(-ΔW/Tk) < rand(0,1)}
        &
        {ΔSL < 0 or exp(-ΔSL/Tk) < rand(0,1)}
          Ij = Ip
        else
          Ij = Ij-1
        endif
      endfor
    endfor
  end
end
    
```

Fig.6: The simulated annealing algorithm

where  $\{i_1, i_2, \dots, i_M\}_K$  are the selected positions of transmit elements, as found at the first stage of the optimization.

One way of selecting the positions  $\{i_1, i_2, \dots, i_M\}_K$  is by using a modification of the simulated annealing algorithm based on a Monte Carlo simulation. This approach was suggested initially for combinatorial optimization by Kirkpatrick et al. [9]. The simulated annealing algorithm realizes an iterative procedure that is determined by simulation of the arrays with variable transmit element positions. In order to speed up the simulation process it is assumed that two of the  $M$  transmit elements are always the two outer elements of the physical array; their positions are not changed and are assigned numbers 1 and  $M$ . The positions of the other transmit elements are shifted randomly, where a shift in position to the left or to the right has equal probability (of 0.5). Once the process is initiated, with an initial layout of transmit elements  $I_0=(i_1, i_2, \dots, i_M)$ , a neighbour layout  $I_1=(i_1, i_2, \dots, i_M)$  is generated, and the algorithm accepts or rejects this layout according to a certain criterion. The acceptance is decided stochastically and may be described in terms of probability as:

$$P = \begin{cases} 1 & \text{if } \Delta W < 0 \ \& \ \Delta SL < 0 \\ \exp(-\Delta W / T_k) & \text{if } \Delta W > 0 \ \& \ \Delta SL < 0 \\ \exp(-\Delta SL / T_k) & \text{if } \Delta W < 0 \ \& \ \Delta SL > 0 \\ \exp(-\Delta W / T_k) \times \exp(-\Delta SL / T_k) & \text{if } \Delta W > 0 \ \& \ \Delta SL > 0 \end{cases} \quad (14)$$

where  $P$  is the probability of acceptance,  $\Delta W$  is the difference of width of the mainlobe,  $\Delta SL$  is the difference of the height of the peak of the sidelobe between the current configuration of transmit elements and the best one obtained at preceding steps.  $T_k$  is the current value of ‘temperature’, where the current ‘temperature’ is evaluated as  $T_k=0.95 T_{k-1}$ , and the algorithm proceeds until the number of iterations reaches the final value. A pseudo-code of the proposed simulated annealing algorithm is given in Fig.6.

### 5. Computations and Comparison Analysis

The effective aperture function and the corresponding two-way radiation pattern of several ultrasound imaging systems were calculated using Eqns (1-14), in order to compare the quality of images produced by the questioned systems.

**Synthetic aperture.** Three more perspective types of synthetic aperture imaging systems are investigated.

The investigated MSAF imaging system employs a linear array with 64 elements and active sub-apertures with 4, 8 (Fig. 7), 16 (Fig. 8) or 32 elements, respectively. In the study no apodization is used. The best results for the lateral resolution and  $SL$  are obtained when the active transmit sub-aperture consists of only 4 elements (Table 1). The main disadvantage of the small number of active transmit elements is that the transmitted power is less, hence the SNR is low.

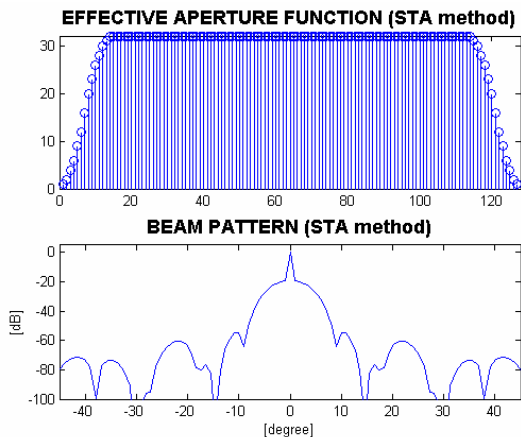


Fig. 7: MSAF method with 8 active elements

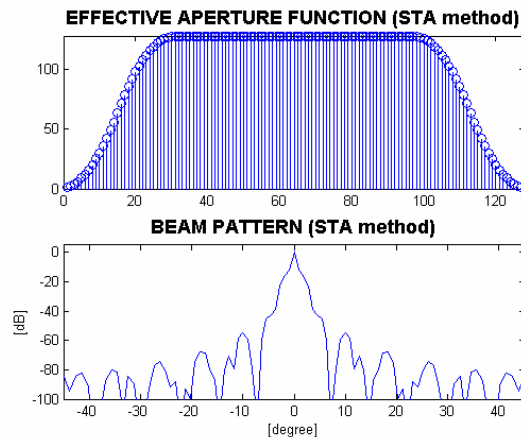


Fig. 8: MSAF method with 16 active elements

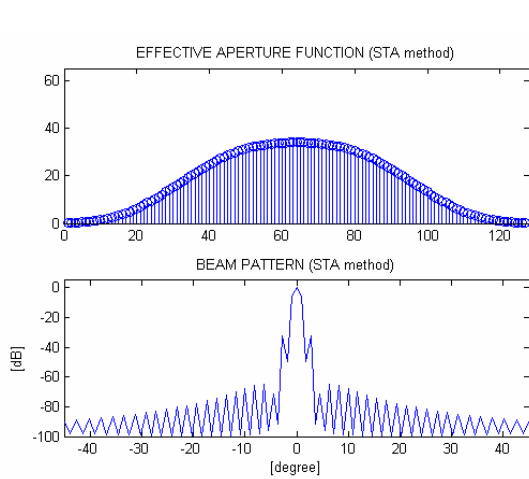


Fig.9 Conventional STA method (64-elements, Hamming apodization)

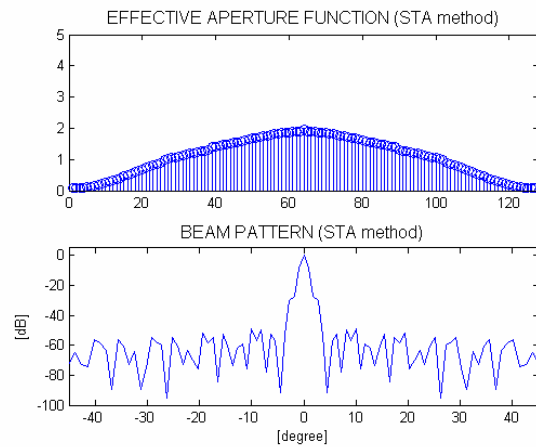


Fig. 10: Sparse STA method (4 transmit elements, 64 receive elements, Hamming apodization)

Number of transmit elements	4	8	16	32
$\Delta\theta$ °	0.3896	0.5664	0.9188	1.7685
SL, dB	-59.6213	-54.3839	-54.574	-35.2764

Table 1: Mainlobe and sidelobe peak level of a MSAF imaging system

In a conventional STA imaging method, the transducer is of 64 elements. In the receive mode all elements are active. In the transmit mode, the linear array is split into 64 sub-apertures, each of them has only one active element. The effective aperture function and the corresponding two-way radiation pattern of such a STA imaging system is shown in Fig.9. The  $-6\text{dB}$  beamwidth obtained for the STA imaging system is  $1.82^\circ$ . However, the sidelobe peak level of the STA system is only  $-32\text{ dB}$ .

**Sparse array optimization.** Computer simulations were performed in order to optimize the design and performance of a sparse array probe, to be used for synthetic transmit aperture imaging. The example given here is of a 64 elements sparse array, where 64 active elements are used in receive and only 4 elements are used in transmit. The properties of the system are optimized using the two-stage algorithm described in section 4. First, the optimal positions of transmit elements are found for three apodization functions – Boxcar (i.e. no apodization of the receiver elements), Hamming and the Blackman-Harris.

Optimized positions of transmit elements	Receiver Apodization	Mainlobe width ( $\Delta\theta$ °)			SL (dB)
		-6 dB	-20 dB	-40 dB	
1, 2, 63, 64	-	0.33	1.11	3.2	-33
1, 26, 39, 64	Hamming	1.34	2.92	6.2	-50
1, 21, 44, 64	Blackman-Harris	1.32	4.33	11.16	-100

Table 2: Numerical results obtained after employing the two stages of the optimization.

For each apodization function, the positions of transmit elements are shifted until optimal performance is obtained, as described earlier, using the simulated annealing algorithm presented in Fig.6. In order to obtain a radiation pattern with a sharper mainlobe, the optimization criterion was formulated as the minimal width of

the mainlobe at  $-20\text{dB}$  ( instead of at  $-6\text{ dB}$ ) below the maximum where the condition that the maximal level of the sidelobe peak is below  $-50\text{ dB}$ . The positions of transmit elements that were found to optimize the performance of the system, studied for a physical array with  $\lambda/2$  element spacing, together with the achieved widths of the mainlobe (at  $-6\text{ dB}$ ,  $-20\text{ dB}$  and  $-40\text{ dB}$ ) and the levels of the peaks of the sidelobe, are all presented in Table 2. Both optimized functions, the effective aperture function and the corresponding two-way radiation pattern, are plotted for Hamming apodization function (Fig. 10). It may be seen that the apodization reduces the levels of the peaks of the sidelobes from  $-33\text{ dB}$  to  $-100\text{ dB}$ , but at the cost of widening the mainlobe of the radiation pattern. Since the dynamic range of a computer monitor is limited to about  $50\text{ dB}$ , the sparse array is chosen with the Hamming apodization and the locations of the transmit elements are set to be at positions 1, 26, 39 and 64. Comparison analysis of numerical results (Table 2, Fig.9) shows that a sparse STA imaging system improves significantly lateral resolution of images because  $\Delta\theta=1.82^\circ$  - for a conventional STA imaging system and  $\Delta\theta=1.34^\circ$  - for a sparse STA imaging system.

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## 6. Conclusions

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It is shown that the effective aperture approach can be successfully used as a tool for analysis and parameter optimization of the synthetic aperture imaging systems. The effective aperture function and the corresponding two-way radiation function provide information about two of the most important parameters of images produced by an ultrasound system - lateral resolution and contrast. Therefore, in the design, optimization of the effective aperture function leads to optimal choice of such parameters of a STA imaging system that influence on lateral resolution and contrast of images produced by this imaging system.

The numerical results show that each system has its own advantages and disadvantages. The choice of imaging system should depend on the task, which it will be used for. It is shown that Hamming apodization gives the best compromise between the contrast of images and the lateral resolution.

A MSAF system has better lateral resolution and SL level with less active elements, but in that case the SNR is lowered.

The sparse synthetic transmit aperture imaging systems can be proposed as an alternative and superior approach to the conventional STA systems. Yet, the sparse STA imaging systems suffer from some deficiencies. With proper design, these deficiencies can be overcome and the sparse STA imaging system can perform extremely well for specific applications. To do so, an effective aperture approach is used for optimization of the sparse STA imaging system. A two-stage algorithm is proposed for optimizing both the locations of transmit elements within the ultrasound probe and the weights of the receive element. The first stage of the optimization procedure employs a simulated annealing algorithm that optimizes the locations of the transmit elements for a set of apodization functions. At the second stage, an appropriate apodization function is selected.

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## A MATHEMATICAL APPARATUS FOR ONTOLOGY SIMULATION. SPECIALIZED EXTENSIONS OF THE EXTENDABLE LANGUAGE OF APPLIED LOGIC<sup>1</sup>

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**Abstract:** *A mathematical apparatus for domain ontology simulation is described in the series of articles. This article is the second one of the series. It describes a few specialized extensions of the extendable languages of applied logic that was described in the first article of the series. A few examples of some ideas related to domain ontologies and formalization of these ideas using the language are presented.*

**Keywords:** *Extendable language of applied logic, ontology language specification, specialized extensions of the extendable language of applied logic.*

**ACM Classification Keywords:** *I.2.4 Knowledge Representation Formalisms and Methods, F4.1. Mathematical Logic*

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### Introduction

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The definition of the extendable language of applied logic was given in [Kleshchev et al, 2005]. This definition consists of the kernel of the language and of its standard extension only. When the semantic basis is extended for particular applications the following two classes of elements are possible. The elements of the first class can be impossible or undesirable to be defined by means of the kernel of the language and by extensions built. On the contrary, the elements of the second class can be naturally defined by means of the kernel and extensions built. The elements of the first class are described in specialized extensions in the same form that is used in the description of the kernel of the language and of its standard extension. A specialized extension of the language defines elements of the semantic basis that are necessary for a comparatively narrow class of applications. Because the same specialized extensions can be used in different applications such extensions have names. Every particular language of applied logic contains the kernel and usually the standard extension and possibly some specialized extensions. By this means, every particular language of applied logic is characterized by a set of extension names rather than a signature. A signature is introduced by a particular logical theory represented by

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