Conclusions

The presented method of determining the critical points is based on the graph theory and some features of the adjacency matrix, which represents graphs. Searching for critical points in computer networks, as it follows from the above researches is characterized with a large complexity and requires applying of a great computational performance. However, the main advantage of this method is a fact that is uses homogenous structures, and the computations itself are of the same type. Presented experiments were implemented and realized on the multiprocessor cluster and the results of these experiments are presented in the above table. Analyzing these experimental results, the conclusion may be drawn independently.

Bibliography

[Капитонова Ю.В., Кривой С.Л., 1] Капитонова Ю.В., Кривой С.Л., Летичевский А.А., Луцкий Г.М. – *Лекции по дискретной математике*. – Санкт-Петербург: БХВ-Петербург. –2004. 614 стр.

[Свами М., Тхуласираман К., 2] Свами М., Тхуласираман К. Графы, сети и алгоритмы. Москва: Мир. – 1984. 454 стр.

[Hajder M., Loutski G., 3] Hajder M., Loutskij G., Streciwilk W. - *Informatyka*. Rzeszow: WsiZ. - 2002. – 714 s. (на польском языке)

[Hajder M., Dymora P., 4] Hajder M., Dymora P. - *Algorithmical and topological methods of fault tolerace assurance*; Wydawnictwo Uniwersytetu Marii Curie-Skłodowskiej; Annales Informatica, Kazimierz Dolny 2004 – 143-153,

[Hajder M., Dymora P., Bolanowski M., 5] M. Hajder, P. Dymora, M. Bolanowski: *Fault tolerance in multichannel scalability systems;* International Conference of Science and Technology Scalnet'04, Scalable Systems and Computer Networks Design and Aplications, Ministry of Education and Science of Ukraine Kremenchuk State Polytechnical University, Kremenchuk 2004, – pp. 52-57,

[Spohn D.,L., 6] Spohn D. L. - *Data Network Design. New York: McGrew-Hill*. – 1997. – 347 p.

[Harsall F., 7] Harsall F. - *Data Communications, Computer Design Networks and Open Systems*. 4-th Eddition. – Addison-Wesley Company. – Harlow. – 1996. – 412 p.

[Mazurek M., 8] Mazurek M. - *Hyper-deBruijn topology as a result of the Network Product*, International Conference of Science and Technology Scalnet'04, Scalable Systems and Computer Networks Design and Aplications, Ministry of Education and Science of Ukraine Kremenchuk State Polytechnical University, Kremenchuk 2004, pp. 62-65.

[Hajder M., Mazurek M., 9] Hajder M., Mazurek M. - *Embending as a method to improve multichanel bus topology parameters*, Annales Universitatis Mariae Curie-Skłodowska, Sectio Ai Informatica, Lublin, 2004.

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RKHS-METHODS AT SERIES SUMMATION FOR SOFTWARE IMPLEMENTATION

Svetlana Chumachenko, Ludmila Kirichenko

Abstract: Reproducing Kernel Hilbert Space (RKHS) and Reproducing Transformation Methods for Series Summation that allow analytically obtaining alternative representations for series in the finite form are developed.

Keywords: The reproducing transformation method, Hilbert space, reproducing kernel, RKHS, Series Summation Method.

ACM Classification Keywords: G.1.10 Mathematics of Computing: Applications

Introduction

Operating speed of digital logic devices depends on type of silicon: PLD, Gate Array or ASIC. FPGAs are the lowest in risk, low in project budget but have the highest cost per unit. Gate Arrays utilize less custom mask making than standard cell and stand in the middle from all of three and fallen from wide use today. Cell based ASICs have the highest performance and lowest cost per unit in case of mass production, but they also have the longest and most expensive design cycle. Also, digital designs can be divided on CPU based systems on chip (SoC) and non-CPU logic devices. CPU as universal processing unit can solve broad spectrum of various tasks from all areas of human activity. Nevertheless, there exist bottlenecks where CPU can't satisfy required performance. Usually it happens during implementation of mathematical tasks that require big number of iterations and hence big time expenses to obtain desired result with desired accuracy.

To increase efficiency of solving of computational tasks there are used mathematical co-processors, which implement most efficient ways of computing equations, integrals, differential coefficients. It is obvious that after discovering of new methods of increasing computation accuracy and decreasing computation time it is necessary to re-implement mathematical co-processors or use new generation of IP-cores in PLD, Gate Array, ASIC designs. It is presented, easy to implement as IP-core, method of reduction of computation of certain types of series to exact function that is widely used during calculation of parameters of high radio frequency devices. Presented method decrease computation time of such tasks in tens and hundred times and its inaccuracy is equals to zero.

Statement of the Problem

The investigation is based on fundamental works of Aronzajn [1], Razmahnin, Yakovlev [2]. It develops the following research [10-12] on Series Summation in Reproducing Hilbert Space and their approbation [19-22]. Modern papers of Saitoh, Laslo Mate, Daubeshies and others [3-9] are used for revues and staying problem. Classical papers of Tranter, Doetsch, Ango, Titchmarch, Ahiezer [13-18] are used for inter-comparison of results.

Mathematical models based on Reproducing Kernel Hilbert Space methods are used in Wavelets Analysis, namely: at Pattern Recognition, Digital Data Processing, Image Compression, Computer Graphics; and also in Learning theory: for example, at Exact Incremental Learning, in Statistical Learning theory, in Regularization theory and Support Vector Machines. In mentioned arias we have not deal with exact Series Summation because it isn't necessary for considered cases. We use sum and finite summation, not series. But there are areas of scientific study where exact series summation it is necessary.

For such problems Reproducing Transformations Method and its part – Series Summation Method in RKHS – can be useful [20, 22]. We are going to point out these areas.

The purpose of the investigation is to originate a new Series Summation Method based on RKHS-theory and to demonstrate the new results which develop theoretical statements of Series Summation Method in RKHS.

The research problems are:

- Series Summation in RKHS
- Applications of Series Summation in RKHS
- Reproducing Transformations Method as a perspective of this research

Base Theoretical Statements and Investigation Essence

Reproducing Kernel Hilbert Space is a subspace of Hilbert space with Reproducing Kernel (RK). RK is a function Ker of two variables with two properties: 1) $\forall t \in T \quad Ker(s_0,t) \in T$; 2) $\forall f \in H \quad f(t) = < f(s), Ker(s,t) >$, where <…> – inner (scalar) product can be represented as a series on selective values. There is an operator G, which transfers any function from Hilbert space L_2 into function from RKHS and leaves without change function from RKHS H.

Thus, there is an operator G, which transfers any function from Hilbert space L_2 into function from RKHS and doesn't change any function from RKHS H.

For example, the functions with finite spectrum of cosine- and sine-transformations and Hankel-transformation form RKHS. The basic research of expansion problem on selective values was executed by K. Shannon and

V.A. Kotelnikov. There are statements determining particular cases RKHS. Thus, any function from RKHS can be represented as selective value expansion. If there is series where the common summand can be reduced to a standard form, – it means to extract reproducing kernel by equivalent transformations, – then for any series one can put in accordance a function from RKHS. In other words, a series can be summarized by known formulas.

Thus, the main idea of proposed method is to obtain and to use the following relation:

$$f(s) = \sum_k f(t_k) Ker(s, t_k),$$

in right-hand side of this relation we can see a series on selective values of function f(t); left-hand side represents value of function f in point s.

We use four base kinds of Reproducing Kernels, which originate four RKHS accordingly [21]:

1. RKHS H_1 is a space of functions, which have finite Fourier-transformations.
2. Space H_2 contains a class of functions with finite Hankel-transformation.
3. Space H_3 consists of functions with finite sine-transformations.
4. Space H_4 has all functions, those cosine-transformations are finite.

For these spaces there are four Kernel Functions and Series on selective values accordingly [21].

Based RKHS-theory the new approach to definition of series sum is proposed. It is called *Series Summation Method in RKHS.*

It allows analytically obtaining alternative representations for some kinds of series in the finite form [10].

The new formulas for calculating the sum of series (including alternating) have been obtained by proving several theorems [10, 12].

Reproducing Transformations Method are generalization and extension of Series Summation method in RKHS [20, 22]. It can be useful at solving mentioned problems and other important points. It needs further evolution and consideration.

For example, we can see proving the following formula.

Theorem 1. There is the following relation for alternating series

$$\sum_{k=1}^{\infty} (-1)^k \frac{kF(k)}{a^n - k^n} = \begin{cases} \dfrac{\pi a F(a)}{a-k}, \; n = 1; \\ \dfrac{\pi F(a)}{2pa^{2p-2} \sin \pi a}, \; n = 2p, p = 1,2,3,...; \\ \dfrac{\pi F(a)}{(2p-1) a^p \sin \pi a}, \; n = 2p-1, p = 2,3,.... \end{cases} \quad (1)$$

for any $F(x)$ from RKHS, $a \neq 0, \pm 1, \pm 2, \ldots$.

Proof. Let's consider alternating series the common member of which contains the difference of powers in denominator:

$$\sum_{k=1}^{\infty} (-1)^k \frac{kF(k)}{a^n - k^n}.$$

Let's define its sum. For this purpose we would consider the cases, when n is equal to natural number.

1) Let $n = 1$. The common member of series transforms to kind [10] that yields:

$$\sum_{k=1}^{\infty} (-1)^k \frac{kF(k)}{a-k} = \sum_{k=1}^{\infty} F(k) \frac{\cos(k\pi)\sin(\pi a)}{(a+k)\sin \pi a} \frac{a+k}{a-k} \frac{2\pi k}{2\pi} = \frac{\pi}{2 \sin \pi a} \sum_{k=1}^{\infty} \frac{2k}{a+k} \frac{\sin \pi(a-k)}{\pi(a-k)} (a+k) F(k) =$$

$$= \frac{\pi}{2 \sin \pi a} [(a+k)F(k)]\Big|_{k=a} = \frac{2\pi a F(a)}{2 \sin \pi a} = \frac{\pi a F(a)}{\sin \pi a}.$$

2) For $n = 2$ the result obtained in [10]:

$$\sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{a^2 - k^2} = \frac{\pi}{2\sin \pi a} F(a). \tag{2}$$

3) For $n = 3$ we can obtain result by recurrent way with accounting formula (2) and using the decomposition of difference of cubes:

$$\sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{a^3 - k^3} = \sum_{k=1}^{\infty}\frac{(-1)^k k}{(a-k)}\frac{F(k)}{(a^2+ak+k^2)} = \sum_{k=1}^{\infty}\frac{(-1)^k k}{(a^2-k^2)}\frac{(a+k)F(k)}{(a^2+ak+k^2)} =$$

$$= \left[\Phi(k) = \frac{(a+k)F(k)}{(a^2+ak+k^2)}\right] = \frac{\pi}{2\sin \pi a}\Phi(a) = \frac{\pi}{2\sin \pi a}\frac{2aF(a)}{3a^2} = \frac{\pi F(a)}{3a \sin \pi a}.$$

4) For $n = 4$ we can obtain:

$$\sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{a^4 - k^4} = \sum_{k=1}^{\infty}\frac{(-1)^k k}{(a^2-k^2)}\frac{F(k)}{(a^2+k^2)} = \left[\Phi(k) = \frac{F(k)}{(a^2+k^2)}\right] = \frac{\pi}{2\sin \pi a}\Phi(a) =$$

$$= \frac{\pi}{2\sin \pi a}\frac{F(a)}{2a^2} = \frac{\pi F(a)}{4a^2 \sin \pi a}.$$

5) For $n = 5$ we can obtain with decomposition by difference of fifth powers the following result:

$$\sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{a^5 - k^5} = \sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{(a-k)(a^4+ka^3+k^2a^2+k^3a+k^4)} =$$

$$= \sum_{k=1}^{\infty}\frac{(-1)^k k}{(a^2-k^2)}\frac{(a+k)F(k)}{(a^4+ka^3+k^2a^2+k^3a+k^4)} =$$

$$= \left[\Phi(k) = \frac{(a+k)F(k)}{(a^4+ka^3+k^2a^2+k^3a+k^4)}\right] = = \frac{\pi}{2\sin \pi a}\Phi(a) = \frac{\pi}{2\sin \pi a}\frac{2aF(a)}{5a^4} = \frac{\pi F(a)}{5a^3 \sin \pi a}.$$

6) For $n = 6$ we can analogically obtain:

$$\sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{a^6 - k^6} = \frac{\pi F(a)}{6a^4 \sin \pi a}.$$

Thus, based on mentioned transformations we can conclude:

$$\sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{a - k} = \frac{\pi a F(a)}{a - k},\ n = 1;$$

$$\sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{a^{2p} - k^{2p}} = \frac{\pi F(a)}{2pa^{2p-2} \sin \pi a},\ \ n = 2p, p = 1,2,3,...;$$

$$\sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{a^{2p-1} - k^{2p-1}} = \frac{\pi F(a)}{(2p-1)a^p \sin \pi a},\ n = 2p-1, p = 2,3,....$$

Thus, the theorem 1 has been proof.

The following examples illustrate application of the theorem 1.

Example 1. To proof of the following formula truth:

$$\sum_{k=1}^{\infty}(-1)^k \frac{k \sin kx}{a^2 - k^2} = \frac{\pi}{2}\frac{\sin ax}{\sin \pi a},\ -\pi < x < \pi,\ a > 0, a \neq 1,2,...,$$

the residues theory is used in [23]. However, application of the theorem 1 gives the same result:

$$\sum_{k=1}^{\infty}(-1)^k \frac{k \sin kx}{a^2 - k^2} = \frac{\pi}{2\sin \pi a}\sin kx\big|_{k=a} = \frac{\pi \sin ax}{2\sin \pi a},\ a > 0, a \neq 1,2,....$$

Example 2. To proof of the following identity truth Laplace transformation is used [24]. However application of the theorem 1 reduces to the same result but it is simpler solution:

$$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{k^2-a^2}J_{2n+1}(kx)=\sum_{k=1}^{\infty}\frac{(-1)^{k}k}{a^2-k^2}J_{2n+1}(kx)=\frac{\pi}{2}J_{2n+1}(ax)cosec(a\pi),\ -\pi<x<\pi.$$

Example 3. To proof of the following identity truth

$$\sum_{k=1}^{\infty}(-1)^k\frac{k\cos kx}{a^4-k^4}=\frac{\pi\cos ax}{4a^2\sin\pi a},\qquad (3)$$

we can apply the theorem 1. Also we can show numerically this result (see fig. 1, 2). On Fig. 1 there are two diagrams in the equal co-ordinates for parameter $a=2{,}5$. Graphs of function from right-hand side of (3) (the bold curve) and left-hand side (thin curve) of (3). Fig. 2 demonstrates the absolute uncertainty.

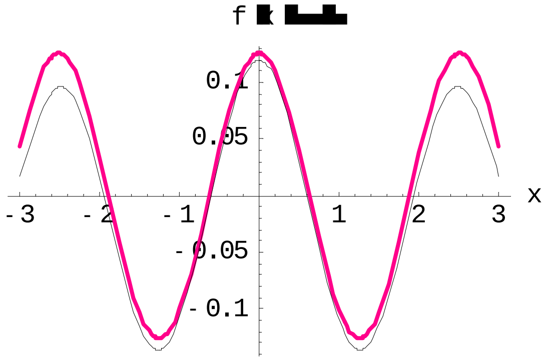

Fig. 1. Results comparison

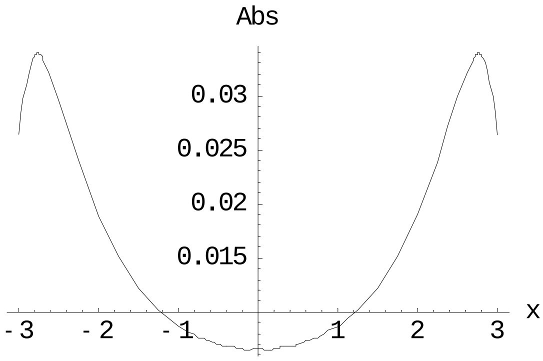

Fig. 2. Absolute error

Conclusion

Thus, we can direct the following *areas* for applications of new Series Summation Method:

– Exact summation of series;

– solving summatory equations and its systems;

– solving integral equations and its systems;

– solving integral-summatory equations and systems of complex form;

– proving integral identities.

Mentioned areas can be used at solving some problems of: antenna theory; diffraction theory; electrodynamics and can be useful at Software/Hardware implementations (See Fig. 3).

The obtained results allow making the following *conclusions*:

1) RKHS-theory can be used for summation of selected series. For this purpose, *Series Summation Method in RKHS* has been proposed.

2) *Advantages of this method* consist of:
   - application of equivalent transformations to the common member of a series, that enables to obtain the *analytical solution* for smaller quantity of steps;
   - in absence of necessity to use the tables of integral transformations and to use the integration in complex area.

3) The application of obtained results of RKHS-theory for *solving the boundary electrodynamics problems* gives possibility *to simplify known methods* and to receive on their basis the *analytical solutions*, that is represented as essential for the further numerical experiment;

4) New mathematical results for solving the summatory and integral equations are obtained by proving some theorems.

5) The obtained results can be included into the reference mathematical library and implemented into Mathematics program products, MathCAD, Math Lab means. It can be useful for scientists, engineers, mathematics at solving the different problems.

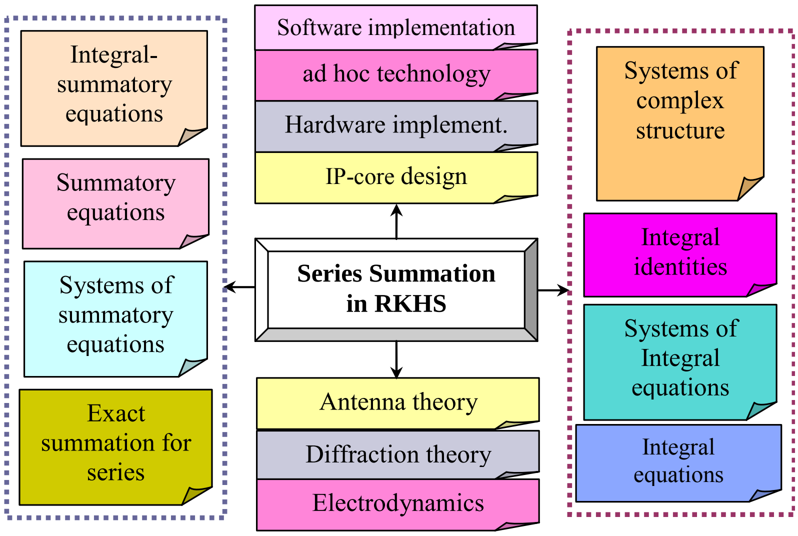

Fig. 3. Application arias of Series Summation in RKHS

Bibliography

[1] *Aronzajn A.* Theory of reproducing kernels // Trans. American Math. Soc., 68:337-404, 1950.

[2] *Functions* with double orthogonality in radio electronics and optician / By edition M.K. Razmahnin and B.P. Yakovlev. M.: Sov. radio, 1971. 256p. (in Russian).

[3] *Saitoh S.* Integral Transforms, Reproducing Kernels and Their Applications, Pitman Res. Notes in Math. Series, 369 (1997), Addison Wesley Longman, UK.

[4] *Laslo Mate.* Hilbert Space Methods in Science and Engineering, 1990. 200 p.

[5] *Daubeshies I.* Ten lectures on wavelets. CBMS-NSF. Regional conference series in applied mathematics. SIAM, Philadelphia, 1992.

[6] *Sethu Vijayakumar, Hidemitsu Ogawa.* RKHS based Functional Analysis for Exact Incremental Learning / Neurocomputing: Special Issue on Theoretical analysis of real valued function classes, Vol.29, No.1-3, pp.85-113, Elsevier Science (1999). http://www-clmc.edu/sethu/research_detail.1html.

[7] *Saitoh S.* New Norm Type Inequalities for linear mappings // Jornal of Inequalities in Pure and Applied Mathemathics. Victoria Univ., 2003. Volume 4. Issue 3. Article 57. http: // jipam.vu.edu.au.

[8] *Girosy F.* An equivalence between sparse approximation and support vector machines // Neural Computation, 10(6):155-1480, 1998.

[9] *Vapnik V.* The nature of Statistical Learning Theory. Springer-Verlag. New York, 1995.

[10] *Chumachenko S.V.* Summation method of selected series for IP-core design // Radioelectronics and Informatics. 2003. N3. C. 197-203 (in Russian).

[11] *Chumachenko S.V.* Theorem about some integral identities based on Series Summation Method in RKHS // Radioelectronics and Informatics. 2004. №1. C. 113-115 (in Russian).

[12] *Chumachenko S.V.* Reproducing kernel Hilbert spaces and some theirs applications // Radioelectronics and Informatics. 2003. № 4. C. 141-144 (in Russian).

[13] *Tranter C.J.* Integral Transforms in Mathematical Physics. London: Methuen & Co. Ltd., New York: John Wiley & Sons, Inc. 1951.

[14] *Doetsch G.* Anleitung Zum Praktischen Gebrauch Der Laplace-transformation und Der Z-transformation. R. Oldenbourg. Munhen, Wien, 1967.

[15] *Angot A.* Complements de Mathematiques. A L'usage Des ingenieurs de L'elektrotechnique et Des Telecommunications. Paris, 1957.

[16] *Watson G.N.* A Treatise on the Theory of Bessel Functions. Cambridge: Cambridge University Press. 1952.

[17] *Titchmarch E.C.* The theory of functions. Oxford Univ. Press, 1939.

[18] *Ahiezer N.I.* The lectures of the approximating theory. Moscow: Nauka, 1965. 408p. (in Russian).

[19] *Chumachenko S.V., Govhar Malik, Imran Saif Chattha.* Series Summation Method in HSRK // Proc. of the international Conference TCSET'2004 "Modern Problems of Radio Engineering Telecommunications and Computer Science". February 24-28. 2004. Lviv-Slavsko, Ukraine. P.248-250.

[20] *Chumachenko S.V.* Solving Electrodynamics Problems by Reproducing Transformations Method // Proc. BEC 2004. Tallinn. October 3-6, 2004. PP. 319-322.

[21] *Chumachenko S.V., Gowher Malik, Khawar Parvez*. Reproducing Kernel Hilbert Space Methods FOR CAD Tools // Proc. EWDTW, 2004. Yalta-Alushta, September 23-26. PP. 247-250.

[22] *Svetlana Chumachenko, Vladimir Hahanov.* Reproducing Transformations method for IP-core of summatory and integral equations solving // Proc. of DSD 2004 Euromicro Symposium on Digital System Design: Architectures, Methods and Tools. August 31 - September 3, 2004, Rennes – France (Work in progress).

[23] *Titchmarch E.C.* The theory of functions. Oxford Univ. Press, 1939.

[24] *Prudnikov A.P., Bryichkov Yu.A., Marichev O.I.* Integrals and series. Moscow: Nauka, 1981. 800p. (in Russian).

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FUZZY SETS: ABSTRACTION AXIOM, STATISTICAL INTERPRETATION, OBSERVATIONS OF FUZZY SETS

Volodymyr Donchenko

Abstract: The issues relating fuzzy sets definition are under consideration including the analogue for separation axiom, statistical interpretation and membership function representation by the conditional Probabilities.

Keywords: fuzzy sets, membership function, conditional distribution

ACM Classification Keywords: I.5.1. Pattern Recognition: Models - Fuzzy set

Introduction

Fussy sets introduced by Lotfi Zadeh [Zadeh, 1965] (see also [Kaufmann, 1982]) were considered on the one hand as the modeling method alternative to statistical ones which ought to realized the idea of uncertainty, vagueness in the situation under consideration, on the other – as the theory which should to generalize the classical Set theory.

Such generalized pretensions syringed up on the fuzzy logic, which considered as the algebra with two binary operations namely max and min on the real numbers from the interval [0,1] for which all the properties of classical Boolean operations are valid excepting the low of excluded middle. Nevertheless nothing out of this frame in the fuzzy logic was considered. Particularly, such axiom of paramount importance known as abstraction [Stoll, 1960] or separation [Kuratovski K, Mostowski A.6 1967] principle was out of consideration. As is well known this