THE LOCALIZATION PROCEDURES OF THE VECTOR OF WEIGHTING COEFFICIENTS ON THE SET OF TEACHING EXPERTS IN THE TASKS OF CONSUMING

Elena Drobot

Abstract: In terms of binary relations the author analyses the task of an individual consumers' choice on the teaching excerpts set. It is suggested to analyse the function of consumer's value as additive reduction. For localization of the vector of weighting coefficients of additive reduction the procedures based on metrics of object distance towards the ideal point are suggested.

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Introduction

The typical task in the theory of mathematical economy consuming is traditionally considered [Пономаренко, 1994] as the task of the construction of the function of consumer's value. It defines their preferences considering the definite set of goods. In this case the so-called "teaching excerpts" are under consideration: the vector sets of goods $X = (x_1, x_2, ..., x_n)$ each component of them is the quantity of corresponding goods unities. The prices of goods and budget limits are considered to be set.

The consumer's choice is characterized by the attitude of preference R. The sense is the following: the consumer can point either availability of preference or the fact they are equal (about each 2 sets of goods). A priori is considered that the consumer's choice is made in accordance with his own function of value $U(X)$. Its meaning on the teaching excerpt $X = (x_1, x_2, ..., x_n)$ corresponds to the individual consumer's appreciation for this set. The task of consumer's choice is supposed to be in the choice of such a consumer's set which maximizes its function of value within the defined budget limit.

The classical methods used for the definition of the function of value on binary set of preferences R relations in general are pretty rigid. The basis for their usage, in particular, is a sufficient condition of its existence which are set, for example, by Debre theorem [Пономаренко, 1994]. The preference relation must be complete, reflexive, transitive and continuous, the set of the decisions – connected. If Debre's conditions are not completed (subjective attitude of preference can be, firstly, intransitive), and the function of value which introduces the attitude and does not exist, it's impossible to use the classical methods of the consuming theory.

The alternative approach to defining the function of value is suggested. It's considered that the expert (the consumer) while evaluating the object means its vector value. The procedure of problem formalization is suggested in the way of transition of "vector value" into the additive reduction. Then the task is in specifying weighting coefficients of additive reduction.

The Task Set

Let's consider the final set of consumer's goods, the teaching excerpt on the endless set of goods. The prices of goods are set, the budget limit on summary value of goods unities in the excerpt is set as well. Let's take X as the set of teaching excerpts x^j, $j \in J$, when J is the set of excerpts indexes, which is formed within the budget limit frames.

Each excerpt $x^j \in X$, $j \in J$, is characterized by its unities distribution for each goods $x^j = (x^j{}_1,...,x^j{}_i,...,x^j{}_n)$. Let's mark the set of indexes of goods of excerpts I, $I = \{1,...,n\}$. We suppose that each excerpt, $x^j \in X$, $j \in J$, vectors mark in the dimension of goods Ω^n.

The corresponding set $\omega(x^j{}_i)$, $i \in I$, $j \in J$, where ω – some monotonous reorganization which defines the degree of quantitative characteristics declination from the optimum meanings for each goods $x^j{}_i$, $i \in I$, $j \in J$, and reorganizes all the meanings of goods quantitative characteristics towards the normal type in interval [0,1].

Let the consumer [expert] consequently define his preferences on the set X as binary relations of preference R.

The following approach to the task solution is suggested: we suppose that evaluating the object (in our case – the teaching excerpt) the expert (consciously or subconsciously) means its vector value. If we consider "the vector" function of value as additive reduction, the task is considered as defining of weighting coefficients' reduction (1)-(2):

$$x^1 R x^2 \Leftrightarrow \sum_{i \in I} \rho_i \omega_i(x^1) \le \sum_{i \in I} \rho_i \omega_i(x^2),\ x^1, x^2 \in X, \tag{1}$$

$$\rho = (\rho_1,...,\rho_n),\ i \in I,\ \rho_i > 0,\ \sum_{i \in I} \rho_i = 1, \tag{2}$$

where (2) – normal vector of object's relative importance parameters for the experts statement about the preference relation between the objects.

So, the task is in localization of weighting coefficients of additive reduction (1) – (2).

The Localization Procedures of the Vector of Weighting Coefficients

The procedures of localization the of vector's component of weighting coefficients in the way of successive intervals specifying of changing of corresponding vector ρ components (hyperparallelepiped of weighting coefficients in the sphere of preferences):

$$\rho \in \Pi = \prod_{i \in I} [\rho_i^H, \rho_i^B],\ \rho = (\rho_i,\ i \in I),\ 0 < \rho_i^H \le \rho_i^B < 1, \tag{3}$$

$$\sum_{i \in I} \rho_i = 1,\ \rho_i > 0,\ i \in I.$$

The ideological procedures' basis is the hypothesis about the "ideal point" which reflects the "ideal excerpt" in the dimension of goods (vector of preferences in the preferences sphere). The expert possesses the excerpt's complex "image" of this. It is supposed that while comparing teaching excerpts the expert compares their closeness degrees within some metrics limits of to the "ideal" set with the optimum good units' distribution.

For reorganization of all the meanings of goods units x… to the normalized kind in the interval [0,1] the formula [Волкович, Волошин, Заславский, 1993] in particular is used.

$$\omega(x^j{}_i) = \frac{x^{opt}{}_i - x^j{}_i}{x^{opt}{}_i - x^0{}_i}, \tag{4}$$

where $x^j{}_i \in X$, $i \in I$, $j \in J$; $x^{opt}{}_i \in X$, $i \in I$ - the most desirable quantity of the units of i-goods on the set of possible excerpts: $x^0{}_i \in X$, $i \in I$ - the least desirable quantity of the units of i-goods on the set of possible excerpts. Let's consider that x^{opt} and x^0 can be set directly by the expert on the set of admitted teaching excerpts.

Taking into consideration (4), the generalized criteria, which reflects the total declination of j-object $j \in J$ from the optimum meanings, will be presented as

$$D(x^j, x^{opt}) = \sum_{i\in I}\rho_i \omega(x^j{}_i) = \sum_{i\in I}\rho_i \frac{x^{opt}{}_i - x^j{}_i}{x^{opt}{}_i - x^0{}_i},\ j \in J\,.$$

The last formula is the proximity metrics of vector $x^j \in X$, $j \in J$, which presents the distribution of goods units j-excerpts to some ideal (optimum) vector of distributions $x^{opt} = (x^{opt}{}_1, x^{opt}{}_2 \ldots x^{opt}{}_n)$ weighted in the dimension of goods. Formula (1) will be presented as:

$$x^1Rx^2 \ \Leftrightarrow\ \sum_{i\in I}\rho_i\omega_i(x^1) \le \sum_{i\in I}\rho_i\omega_i(x^2) \ \Leftrightarrow\ D(x^1, x^{opt}) \le D(x^1, x^{opt}),\ x^1, x^2 \in X\,.$$

Last inequality can be interpreted in the following way: the statement "excerpt X.. is preferable than the excerpt X.." means that in the dimension of goods Ω^n the point which corresponds to excerpt x^1 is located within less distance according to the ideal point than the point which corresponds to excerpt x^2. In case of ratio of equality of excerpts points in Ω^n which correspond to them are located within the same distance from the point corresponding to the ideal object.

The procedures of localization of vector of weighting coefficients (2) represent factually two procedures: the procedure of intervals of weighting coefficients specifying (3) and the siftings procedure of "not perspective" excerpts from the original set of teaching excerpts under consideration.

The procedures are based correspondingly on the statements 1 and 2 which are given further. The evidences of these statements which are generalized for the case of defining preferences by the expert in metrical form are given in the work [Волошин, Гнатиенко, Дробот, 2003].

Statement 1. Vector of preferences, which corresponds to equal excerpts in the dimension of preferences E^1, defines the intervals limits of the change of goods' weighting coefficients. That is expressed numerically

$$\sum_{\substack{i\in I_1\\ \rho_i^s\in\Pi^s}}\rho_i^{(s)B}\omega_i(x^1) + \sum_{\substack{i\in I_2\\ \rho_j^s\in\Pi^s}}\rho_i^{(s)H}\omega_i(x^2) = \sum_{\substack{i\in I_1\\ \rho_i^s\in\Pi^s}}\rho_i^{(s)B}\omega_i(x^1) + \sum_{\substack{i\in I_2\\ \rho_j^s\in\Pi^s}}\rho_i^{(s)H}\omega_i(x^2),\ x^1, x^2 \in X\,,$$

where $\rho_i^{(s)B}$, $\rho_i^{(s)H}$, $i\in I$ - correspondingly the upper and the lower borders of I-interval of weighting coefficients on s-repetition of comparisons; $I_1 = (i : \omega_i(x^1) > \omega_i(x^2)) \neq 0$, $I_2 = (i : \omega_i(x^1) \le \omega_i(x^2)) \neq 0$, $i \in I = I_1 \cup I_2$.

So hyper-parallelepiped of weighting coefficients (HWC) on s+1 step will be equal

$$\Pi^{s+1} = \prod_{i\in I_1}[\rho_i^{(s)H}, \rho_i^{(s+1)B}] \times \prod_{i\in I_2}\left[\rho^{(s+1)H}, \rho^{(s)B}\right]\ . \tag{5}$$

The discovering of preferences vector, which corresponds to equal objects [Волошин, Гнатиенко, Дробот, 2003] is suggested to accomplish by solving of n equations of the type:

$$\rho\,{}_{\substack{i\\ i\in I_1}}(\omega_i(x^1) - \omega_i(x^2)) - \rho_{\substack{i\\ i\in I_2}}(\omega_i(x^1) - (\omega_i(x^2)) = 0\,,\ x^1, x^2 \in X\,, \tag{6}$$

$$\sum_{i\in I}\rho_i = 1,\ \ \rho_i > 0,\ \ i \in I\,.$$

Statement 2. The condition of object selection ω^j, $j \in J$, from the set X^s is unbelongingness of HWC vector which passes through the coordinates beginning and point $\omega(x^j)$, $x \in X^s$, $j \in J$, namely $\rho(\omega(x^j)) \notin \Pi^{(s+1)}$. Vector of weighting coefficients is defined according to the formula given in [Волкович, Волошин, Заславский, 1993]:

$$\rho = \rho(\omega(a^j)) = \{\rho_i : \rho_i = \prod_{\substack{t\in I \\ t\neq i}} \omega(a^j{}_t) / \sum_{\substack{q\in I \\ l\neq q}} \prod_{l\in I} \omega(a^j{}_l)\} .$$

The procedures of localization of vector of weighting coefficients are used in the following person-computer procedure.

Step 1. Pointing out the ultimate set of teaching excerpts X on the unlimited consumer goods' set. The very first HWC is set as equal to single hypercube.

Step 2. Expert's choice of two excerpts x^1 and x^2 from the set X^s in HWC $П^s$, s =1,2,... (step of limiting HWC) with stating of the preference and equivalence fact.

Step 3. Constructing equation system of type (6). Finding solution of the equation system.

Step 4. Specifying HWC limits according to formula (5). If hypercube $П^{(s+1)}$ satisfies the expert it means the end of the procedure. Otherwise we pass by to the next step.

Step 5. Pointing out the set of "perspective" excerpts $X^{(s+1)}$ ($X^{(s+1)} \subseteq X^{(s)}$) in HWC $П^{(s+1)}$ and presentation of them to the expert for the choice of next two objects with stating for them the preference attitude.

Step 6. Uniting the excerpts, chosen by the expert on the previous step to the set of discussed excerpts and analysis on the given set of transitiveness. If the transitiveness is not destroyed then increase of iteration number s=s+1 and passing by to step 2. If the transitiveness is destroyed then the exclusion of these excerpts from the set of considered objects and passing by to step 6.

Repeated process is finished when the expert is satisfied with the discovered intervals of changing of goals weighting coefficients.

To find team solutions on the basis of interval team evaluations which are formalized in such a way, we can use, for example, the methods, suggested in [Гнатієнко, Дробот, Санько-Новік, 2002].

Conclusion

The suggested procedures do not demand the complete metrics of binary comparisons of teaching excerpts and allow restoring the function of consumer's value on the binary relations' set.

Besides the task of vector of weighting coefficients in the kind of intervals can be interpreted as the reflection of indistinctness in social-economical systems. That is why the suggested procedures allow reducing the uncertainty level in indistinct models of making decisions.

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Author's Information

Elena V. Drobot – Kirovograd Pedagogical University, Kirovograd, Ukraine, e-mail: elena_drobot@ukr.net