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CONSTRUCTING OF A CONSENSUS OF SEVERAL EXPERTS STATEMENTS*

Gennadiy Lbov, Maxim Gerasimov

***Abstract**: Let Γ be a population of elements or objects concerned by the problem of recognition. By assumption, some experts give probabilistic predictions of unknown belonging classes γ of objects $a \in \Gamma$, being already aware of their description $X(a)$. In this paper, we present a method of aggregating sets of individual statements into a collective one using distances / similarities between multidimensional sets in heterogeneous feature space.*

***Keywords**: pattern recognition, distance between experts statements, consensus.*

***ACM Classification Keywords**: I.2.6. Artificial Intelligence - knowledge acquisition.*

Introduction

We assume that $X(a) = (X_1(a),..., X_j(a),..., X_n(a))$, where the set X may simultaneously contain qualitative and quantitative features X_j, $j = \overline{1,n}$. Let D_j be the domain of the feature X_j, $j = \overline{1,n}$. The feature space is given by the product set $D = \prod_{j=1}^{n} D_j$. In this paper, we consider statements S^i, $i = \overline{1,M}$; represented as sentences of type "if $X(a) \in E^i$, then the object a belongs to the γ-th pattern with probability p^i", where $\gamma \in \{1,...,k\}$, $E^i = \prod_{j=1}^{n} E_j^i$, $E_j^i \subseteq D_j$, $E_j^i = [\alpha_j^i, \beta_j^i]$ if X_j is a quantitative feature, E_j^i is a finite subset of feature values if X_j is a nominal feature. By assumption, each statement S^i has its own weight w^i. Such a value is like a measure of "assurance".

Without loss of generality, we can limit our discussion to the case of two classes, $k = 2$.

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Distances between Multidimensional Sets

In the works [1, 2] we proposed a method to measure the distances between sets (e.g., E^1 and E^2) in heterogeneous feature space. Consider some modification of this method. By definition, put

$\rho(E^1,E^2)=\sum_{j=1}^{n}k_j\rho_j(E_j^1,E_j^2)$ or $\rho(E^1,E^2)=\sqrt{\sum_{j=1}^{n}k_j(\rho_j(E_j^1,E_j^2))^2}$,

where $0\le k_j\le 1$, $\sum_{j=1}^{n}k_j=1$.

Values $\rho_j(E_j^1,E_j^2)$ are given by: $\rho_j(E_j^1,E_j^2)=\frac{|E_j^1\Delta E_j^2|}{|D_j|}$ if X_j is a nominal feature, $\rho_j(E_j^1,E_j^2)=\frac{r_j^{12}+\theta|E_j^1\Delta E_j^2|}{|D_j|}$ if X_j is a quantitative feature, where $r_j^{12}=\left|\frac{\alpha_j^1+\beta_j^1}{2}-\frac{\alpha_j^2+\beta_j^2}{2}\right|$.

It can be proved that the triangle inequality is fulfilled if and only if $0\le\theta\le 1/2$.

The proposed measure ρ satisfies the requirements of distance there may be.

Consider the set $\Omega_{(1)}=\{S_{(1)}^1,...,S_{(1)}^{m_1}\}$, where $S_{(1)}^u$ is a statement concerned to the first pattern class, $u=\overline{1,m_1}$. Let E^u be the relative sets to statements $S_{(1)}^u$, $E^u\subseteq D$, $u=\overline{1,m_1}$. By analogy, determine $\Omega_{(2)}=\{S_{(2)}^1,...,S_{(2)}^{m_2}\}$, $S_{(2)}^v$, $\tilde{E}^v$ as before, but for the second class.

By definition, put $k_j=\frac{\tau_j}{\sum_{i=1}^{n}\tau_i}$, where $\tau_j=\sum_{u=1}^{m_1}\sum_{v=1}^{m_2}\rho_j(E_j^u,\tilde{E}_j^v)$, $j=\overline{1,n}$.

Consensus

We first treat single expert's statements concerned to a certain pattern class: let Ω be a set of such statements, $\Omega=\{S^1,...,S^m\}$, E^i be the relative set to a statement S^i, $i=\overline{1,m}$.

Denote by $E^{i_1i_2}:=E^{i_1}\oplus E^{i_2}=\prod_{j=1}^{n}(E_j^{i_1}\oplus E_j^{i_2})$, where $E_j^{i_1}\oplus E_j^{i_2}$ is the Cartesian join of feature values $E_j^{i_1}$ and $E_j^{i_2}$ for feature X_j and is defined as follows.

When X_j is a nominal feature, $E_j^{i_1}\oplus E_j^{i_2}$ is the union: $E_j^{i_1}\oplus E_j^{i_2}=E_j^{i_1}\bigcup E_j^{i_2}$.

When X_j is a quantitative feature, $E_j^{i_1}\oplus E_j^{i_2}$ is a minimal closed interval such that $E_j^{i_1}\bigcup E_j^{i_2}\subseteq E_j^{i_1}\oplus E_j^{i_2}$.

Denote by $r^{i_1i_2}:=d(E^{i_1i_2},E^{i_1}\bigcup E^{i_2})$.

The value $d(E,F)$ is defined as follows: $d(E,F)=\max\limits_{E'\subseteq E\setminus F}\min\limits_{j\|E'_j|\ne|E_j|}\frac{k_j|E'_j|}{diam(E)}$, where E' is any subset such that its projection on subspace of quantitative features is a convex set.

By definition, put $I_1=\{\{1\},...,\{m\}\}$, ..., $I_q=\{\{i_1,...,i_q\}\,|\,r^{i_ui_v}<\varepsilon\ \ \forall u,v=\overline{1,q}\}$, where ε is a threshold decided by the user, $q=\overline{2,Q}$; $Q\le m$.

Take any set $J_q=\{i_1,...,i_q\}$ of indices such that $J_q\in I_q$ and $J_q\not\subset J_{q+1}\ \ \forall J_{q+1}\in I_{q+1}$.

Now, we can aggregate the statements S^{i_1}, ..., S^{i_q} into the statement S^{J_q}:

S^{J_q} = "if $X(a) \in E^{J_q}$, then the object a belongs to the γ-th pattern with probability p^{J_q}", where

$$E^{J_q} = E^{i_1} \oplus \ldots \oplus E^{i_q}, \; p^{J_q} = \frac{\sum_{i \in J_q} c^{iJ_q} w^i p^i}{\sum_{i \in J_q} c^{iJ_q} w^i}, \; c^{iJ_q} = 1 - \rho(E^i, E^{J_q}).$$

By definition, put to the statement S^{J_q} the weight $w^{J_q} = \left(1 - d(E^{J_q}, \bigcup_{i \in J_q} E^i)\right) \frac{\sum_{i \in J_q} c^{iJ_q} w^i}{\sum_{i \in J_q} c^{iJ_q}}$.

The procedure of forming a consensus of single expert's statements consists in aggregating into statements S^{J_q} for all J_q under previous conditions, $q = \overline{1,Q}$.

After coordinating each expert's statements separately, we can construct an agreement of several independent experts for each pattern class. The procedure is as above, except the weights: $w^{J_q} = \sum_{i \in J_q} c^{iJ_q} w^i$.

Solution of Disagreements

After constructing of a consensus for each pattern, we must make decision rule in the case of contradictory statements. Take any sets $E^u_{(1)}$ and $E^v_{(2)}$ such that $E^u_{(1)} \cap E^v_{(2)} = E^{uv} \neq \varnothing$, where the set $E^u_{(\gamma)}$ corresponds to a statement $S^u_{(\gamma)}$ from the experts agreement concerned to the γ-th pattern class, $\gamma = 1,2$.

Consider the sets $I^{uv}_{\{\gamma\}} = \{ i \,|\, (S^i \in \Omega_{(\gamma)})$ and $(\rho(E^i, E^{uv}) < \varepsilon^*) \}$, where ε^* is a threshold, $0 < \varepsilon^* < 1$.

By definition, put $p^{uv}_{(\gamma)} = \frac{\sum_{i \in I^{uv}_{(\gamma)}} (1 - \rho(E^i_{(\gamma)}, E^{uv})) w^i p^i}{\sum_{i \in I^{uv}_{\{\gamma\}}} (1 - \rho(E^i_{\{\gamma\}}, E^{uv})) w^i}$. Denote by $\gamma^* := \arg\max_{\gamma}(p^{uv}_{(\gamma)})$.

Thus, we can make decision statement:

S^{uv} = " if $X(a) \in E^{uv}$, then the object a belongs to the γ^*-th pattern with probability $p^{uv}_{(\gamma^*)}$ "

with the weight $w^{uv} = \left| \frac{\sum_{i \in I^{uv}_{(1)}} (1 - \rho(E^i_{(1)}, E^{uv})) w^i - \sum_{i \in I^{uv}_{(2)}} (1 - \rho(E^i_{(2)}, E^{uv})) w^i}{\sum_{i \in I^{uv}_{(\gamma^*)}} (1 - \rho(E^i_{(\gamma^*)}, E^{uv}))} \right|$.

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