CONSTRUCTING OF A CONSENSUS OF SEVERAL EXPERTS STATEMENTS∗

Gennadiy Lbov, Maxim Gerasimov

Abstract: Let \( \Gamma \) be a population of elements or objects concerned by the problem of recognition. By assumption, some experts give probabilistic predictions of unknown belonging classes \( \gamma \) of objects \( a \in \Gamma \), being already aware of their description \( X(a) \). In this paper, we present a method of aggregating sets of individual statements into a collective one using distances / similarities between multidimensional sets in heterogeneous feature space.

Keywords: pattern recognition, distance between experts statements, consensus.

ACM Classification Keywords: I.2.6. Artificial Intelligence - knowledge acquisition.

Introduction

We assume that \( X(a) = (X_{j}(a),...,X_{j}(a),...,X_{n}(a)) \), where the set \( X \) may simultaneously contain qualitative and quantitative features \( X_{j}, \ j = 1,n \). Let \( D_{j} \) be the domain of the feature \( X_{j}, \ j = 1,n \). The feature space is given by the product set \( D = \prod_{j=1}^{n} D_{j} \). In this paper, we consider statements \( S_{i}, \ i = 1,M \); represented as sentences of type “if \( X_{j} \in E_{j}^{i} \), then the object \( a \) belongs to the \( \gamma \)-th pattern with probability \( p_{i}^{\gamma} \), where \( \gamma \in \{1,...,k\}, \ E_{j}^{i} = \prod_{j=1}^{n} E_{j}^{i}, \ E_{j}^{i} \subseteq D_{j}, \ E_{j}^{i} = [\alpha_{j},\beta_{j}] \) if \( X_{j} \) is a quantitative feature, \( E_{j}^{i} \) is a finite subset of feature values if \( X_{j} \) is a nominal feature. By assumption, each statement \( S_{i} \) has its own weight \( w_{i} \). Such a value is like a measure of “assurance”.

Without loss of generality, we can limit our discussion to the case of two classes, \( k = 2 \).

∗ The work was supported by the RFBR under Grant N04-01-00858.
Distances between Multidimensional Sets

In the works [1, 2] we proposed a method to measure the distances between sets (e.g., \( E^1 \) and \( E^2 \)) in heterogeneous feature space. Consider some modification of this method. By definition, put

\[
\sum_{j=1}^{n} k_j \rho_j(E^1_j, E^2_j) = \rho(E^1, E^2) = \left( \sum_{j=1}^{n} k_j (\rho_j(E^1_j, E^2_j))^2 \right)^{1/2},
\]

where \( 0 \leq k_j \leq 1 \), \( \sum_{j=1}^{n} k_j = 1 \).

Values \( \rho_j(E^1_j, E^2_j) \) are given by:

\[
\rho_j(E^1_j, E^2_j) = \frac{|E^1_j \Delta E^2_j|}{|D_j|} \text{ if } X_j \text{ is a nominal feature},
\]

\[
\rho_j(E^1_j, E^2_j) = \frac{\theta |E^1_j \Delta E^2_j|}{|D_j|} \text{ if } X_j \text{ is a quantitative feature, where } r_j^{12} = \frac{\alpha_j^2 + \beta_j^2}{2}.
\]

It can be proved that the triangle inequality is fulfilled if and only if \( 0 \leq \theta \leq 1/2 \).

The proposed measure \( \rho \) satisfies the requirements of distance there may be.

Consider the set \( \Omega_{(1)} = \{S^1_1, ..., S^m_1\} \), where \( S^u_1 \) is a statement concerned to the first pattern class, \( u = 1, m \). Let \( E^u \) be the relative sets to statements \( S^u_1 \), \( E^u \subseteq D \), \( u = 1, m \). By analogy, determine \( \Omega_{(2)} = \{S^1_2, ..., S^m_2\} \), \( S^v_2 \), \( \tilde{E}^v \) as before, but for the second class.

By definition, put \( k_j = \frac{\tau_j}{\sum_{j=1}^{n} \tau_j} \), where \( \tau_j = \sum_{u=1}^{m} \sum_{v=1}^{m} \rho_j(E^u_j, \tilde{E}^v_j) \), \( j = 1, n \).

Consensus

We first treat single expert's statements concerned to a certain pattern class: let \( \Omega \) be a set of such statements, \( \Omega = \{S^1, ..., S^m\} \), \( E^i \) be the relative set to a statement \( S^i \), \( i = 1, m \).

Denote by \( E^i_0 := E^i \oplus E^s = \prod_{j=1}^{n} (E^i_j \oplus E^s_j) \), where \( E^i_j \oplus E^s_j \) is the Cartesian join of feature values \( E^i_j \) for feature \( X_j \) and is defined as follows.

When \( X_j \) is a nominal feature, \( E^i_j \oplus E^s_j \) is the union: \( E^i_j \oplus E^s_j = E^i_j \cup E^s_j \).

When \( X_j \) is a quantitative feature, \( E^i_j \oplus E^s_j \) is a minimal closed interval such that \( E^i_j \cup E^s_j \subseteq E^i_j \oplus E^s_j \).

Denote by \( v^{i_0} := d(E^i_0, E^s \cup E^i) \).

The value \( d(E, F) \) is defined as follows: \( d(E, F) = \max_{E \subseteq E^i} \min_{E^j \subseteq E^s} \frac{k_j | E^j_j |}{diam(E)} \), where \( E^* \) is any subset such that its projection on subspace of quantitative features is a convex set.

By definition, put \( I_i = \{1, ..., m\} \), \( I_q = \{1, ..., q\} \) \( I_q \subseteq I_{q+1} \) \( \forall q, \forall \in \mathbb{Q} \), \( \mathbb{Q} \leq m \).

Take any set \( J_q \subseteq \{1, ..., q\} \) of indices such that \( J_q \in I_q \) and \( J_{q+1} \nsubseteq J_{q+1} \) \( \forall J_{q+1} \in I_{q+1} \).

Now, we can aggregate the statements \( S^{i_1}, ..., S^{i_q} \) into the statement \( S^{i_q} \):
\[ S_{\gamma} = \text{if } X(a) \in E_{\gamma}^{s}, \text{ then the object } a \text{ belongs to the } \gamma \text{-th pattern with probability } p_{\gamma}^{s}, \text{ where} \]
\[ E_{\gamma}^{s} = E_{1}^{s} \oplus ... \oplus E_{s}^{s}, \quad p_{\gamma}^{s} = \frac{\sum_{i \in J_{\gamma}} c_{ij}^{s} w^{i} p^{i}}{\sum_{i \in J_{\gamma}} c_{ij}^{s} w^{i}}, \quad c_{ij}^{s} = 1 - \rho(E_{i}^{s}, E_{\gamma}^{s}). \]

By definition, put to the statement \( S_{\gamma} \) the weight \( w_{\gamma} = \left(1 - d(E_{\gamma}^{s}, \bigcup_{i \in J_{\gamma}} E_{i}^{s})\right) \frac{\sum_{i \in J_{\gamma}} c_{ij} w^{i}}{\sum_{i \in J_{\gamma}} c_{ij} w^{i}}. \)

The procedure of forming a consensus of single expert’s statements consists in aggregating into statements \( S_{\gamma}^{s} \) for all \( J_{\gamma} \) under previous conditions, \( q = 1, Q \).

After coordinating each expert’s statements separately, we can construct an agreement of several independent experts for each pattern class. The procedure is as above, except the weights: \( w_{\gamma} = \sum_{i \in J_{\gamma}} c_{ij} w^{i} \).

**Solution of Disagreements**

After constructing of a consensus for each pattern, we must make decision rule in the case of contradictory statements. Take any sets \( E_{(1)}^{w} \) and \( E_{(2)}^{w} \) such that \( E_{(1)}^{w} \cap E_{(2)}^{w} = E_{\gamma}^{w} \neq \emptyset \), where the set \( E_{\gamma}^{w} \) corresponds to a statement \( S_{\gamma}^{w} \) from the experts agreement concerned to the \( \gamma \)-th pattern class, \( \gamma = 1,2 \).

Consider the sets \( I_{(\gamma)}^{w} = \left\{ \{S^{i} \in \Omega_{(\gamma)}\} \text{ and } (\rho(E_{\gamma}^{i}, E_{\gamma}^{w}) < \varepsilon^{*}\} \right\} \), where \( \varepsilon^{*} \) is a threshold, \( 0 < \varepsilon^{*} < 1 \).

By definition, put \( p_{(\gamma)}^{w} = \frac{\sum_{i \in I_{(\gamma)}^{w}} (1 - \rho(E_{(\gamma)}^{i}, E_{\gamma}^{w})) w^{i} p^{i}}{\sum_{i \in I_{(\gamma)}^{w}} (1 - \rho(E_{(\gamma)}^{i}, E_{\gamma}^{w})) w^{i}} \). Denote by \( \gamma^{*} := \arg \max_{\gamma} (p_{(\gamma)}^{w}) \).

Thus, we can make decision statement:
\[ S_{\gamma}^{w} = \text{if } X(a) \in E_{\gamma}^{w}, \text{ then the object } a \text{ belongs to the } \gamma^{*} \text{-th pattern with probability } p_{(\gamma^{*})}^{w}. \]

with the weight \( w_{\gamma}^{w} = \frac{\sum_{i \in I_{(\gamma^{*})}^{w}} (1 - \rho(E_{(1)}^{i}, E_{\gamma}^{w})) w^{i} - \sum_{i \in I_{(\gamma^{*})}^{w}} (1 - \rho(E_{(2)}^{i}, E_{\gamma}^{w})) w^{i}}{\sum_{i \in I_{(\gamma^{*})}^{w}} (1 - \rho(E_{(1)}^{i}, E_{\gamma}^{w}))}. \)

**Bibliography**


**Authors' Information**

Gennadiy Lbov – Institute of Mathematics, SB RAS, Koptyug St., bl.4, Novosibirsk, Russia; e-mail: lbov@math.nsc.ru

Maxim Gerasimov – Institute of Mathematics, SB RAS, Koptyug St., bl.4, Novosibirsk, Russia; e-mail: max_post@bk.ru