Jayasena S., Madnick S. & Bressan, S. (2004) "Financial Information Mediation: A case study of standards integration for Electronic Bill Presentment and Payment using the COIN mediation technology (TES/VLDB)", Working Paper CISL# 2004-12, Sloan School of Management, MIT

Madnick S. (1996) "Are we moving toward an information superhighway or a Tower of Babel? The challenge of largescale semantic heterogeneity," Proc. IEEE International Conference on Data Engineering, 1996, pp. 2-8.

Siegel M.& Madnick S.(1991) "A metadata approach to resolving semantic conflicts," Proc. 17th International Conference on Very Large Data Bases, 1991, pp. 133-145.

Thompson, J. (1967) Organisations in Action, New York: McGraw-Hill

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RECENT RESULTS ON STABILITY ANALYSIS OF AN OPTIMAL ASSEMBLY LINE BALANCE

Yuri Sotskov

***Abstract**: Two assembly line balancing problems are addressed. The first problem (called SALBP-1) is to minimize number of linearly ordered stations for processing n partially ordered operations $V = \{1, 2, ..., n\}$ within the fixed cycle time c. The second problem (called SALBP-2) is to minimize cycle time for processing partially ordered operations V on the fixed set of m linearly ordered stations. The processing time t_i of each operation $i \in V$ is known before solving problems SALBP-1 and SALBP-2. However, during the life cycle of the assembly line the values t_i are definitely fixed only for the subset of automated operations $V \backslash \widetilde{V}$. Another subset $\widetilde{V} \subseteq V$ includes manual operations, for which it is impossible to fix exact processing times during the whole life cycle of the assembly line. If $j \in \widetilde{V}$, then operation times t_j can differ for different cycles of the production process. For the optimal line balance **b** of the assembly line with operation times $t_1, t_2, \ldots, t_n$, we investigate stability of its optimality with respect to possible variations of the processing times t_j of the manual operations $j \in \widetilde{V}$.*

***Keywords**: Scheduling, robustness and sensitivity analysis, assembly line.*

***ACM Classification Keywords**: F.2.2 Nonnumerical algorithms and problems: Sequencing and scheduling.*

Introduction

A single-model paced assembly line, which manufactures homogeneous product in large quantities, is addressed (we use terminology given in monograph [Scholl, 1999]). The assembly line is a sequence of *m* linearly ordered stations, which are linked by a conveyor belt or other material handling equipment. Each station of the assembly line has to perform the same set of operations repeatedly during the life cycle of the assembly line. Set of operations *V,* which has to be processed on the assembly line within one cycle time *c,* is fixed. Each operation $i \in V$ is considered indivisible: An operation has to be completely processed on one station within one cycle time. All the *m* stations start simultaneously the sequences of their operations and buffers between stations are absent. *Simple Assembly Line Balancing Problem* is to find an optimal balance of the assembly line for the fixed cycle time c, i.e., to find a feasible assignment of operations V into a minimal possible number m of stations. In [Scholl, 1999], abbreviation SALBP-1 is used for such a problem.

In this paper, it is assumed that set V includes operations of two types. Subset $\widetilde{V} \subseteq V$ includes all the operations for which it is impossible to fix exact processing times for the whole life cycle of the assembly line (manual operations). An operation $i \in V \setminus \widetilde{V}$ is one with operation time t_i being fixed during the life cycle of the assembly line (automated operations). The technological factors define a partial order on the set of operations V. Digraph $G = (V, A)$ with vertices V and arcs A defines partially ordered set of operations $V = \{1, 2, ..., n\}$, which have to be processed on the assembly line within cycle time c. Without loss of generality, it is assumed that $\widetilde{V} = \{1, 2, ..., \widetilde{n}\}$, $V \backslash \widetilde{V} = \{\widetilde{n} + 1, \widetilde{n} + 2, ..., n\}$, and $0 \le \widetilde{n} \le n$. The vectors of the operation times are denoted as follows: $\widetilde{t} = (t_1, t_2, ..., t_{\widetilde{n}})$, $\bar{t} = (t_{\widetilde{n}+1}, t_{\widetilde{n}+2}, ..., t_n)$, $t = (\widetilde{t}, \bar{t}) = (t_1, t_2, ..., t_n)$. The set of n operations is presented as follows: $V = \{1, 2, ..., \widetilde{n}, \widetilde{n}+1, ..., n\}$. If $\widetilde{n} = 0$, then $\widetilde{V} = \varnothing$. If $\widetilde{n} = n$, then $\widetilde{V} = V$. The assignment $V = V_1 \cup V_2 \cup ... \cup V_m$ of the set of n operations to m linearly ordered stations $S = (S_1, S_2, ..., S_m)$ (i.e., partition of set V into m mutually disjoint non-empty subsets V_k, $k \in \{1, 2, ..., m\}$) is feasible operation assignment (called also *line balance*) if the following two conditions hold.

Condition 1: Feasible operation assignment does not violate the precedence constraints given by digraph $G = (V, A)$, i.e., inclusion $(i, j) \in A$ implies that operation i is assigned to station S_k: $i \in V_k$, and operation j is assigned to station S_r: $j \in V_r$, such that $1 \le k \le r \le m$.

Condition 2: *Cycle time c is not violated for each station S_k, $k \in \{1, 2, ..., m\}$, i.e., sum of the processing times of all the operations assigned to station S_k (called station time), has to be not greater than cycle time c:*

$$\sum_{i \in V_k} t_i \le c . \tag{1}$$

For problem SALBP-1, line balance b is optimal when it uses the minimal number of m stations and when both Condition 1 and Condition 2 are satisfied.

Lemma 1: *Constructing an optimal line balance for problem SALBP-1 is binary NP-hard problem even for the case of two stations used in the optimal line balance ($S = (S_1, S_2)$), empty precedence constraints ($A = \varnothing$), and fixed processing times of all the operations V processed on the assembly line ($\widetilde{V} = \varnothing$).*

Lemma 1 may be easily proven by polynomial reduction of NP-complete partition problem [Garey, Johnson, 1979] to problem SALBP-1 with two stations and $A = \varnothing$ (see, e.g., [Scholl, 1999]).

For the sake of simplicity, notation

$$t(V_k) = \sum_{i \in V_k} t_i \tag{2}$$

is used for the original vector $t = (t_1, t_2, ..., t_n)$ of the operation times. We assume that, if $j \in \widetilde{V}$, then operation time t_j is a given non-negative real number: $t_j \ge 0$. The value of this operation time can vary during life cycle of the assembly line and can even be equal to zero. Zero operation time t'_j will mean that operation $j \in V_k \cap \widetilde{V}$ will be processed (e.g., by an additional worker) in such a way that processing operation j will do not increase station time for S_k for the new vector $t = (\widetilde{t}\,', \bar{t}) = (t'_1, t'_2, ..., t'_{\widetilde{n}}, t_{\widetilde{n}+1}, ..., t_n)$ of the operation times: $\sum_{i \in V_k} t_i' = \sum_{i \in V_k \setminus \{j\}} t_i'$. The latter equality is only possible if $t'_j = 0$.

If $i \in V \setminus \widetilde{V}$, then operation time t_i is given real number fixed during the whole life cycle of the assembly line. We assume that $t_i > 0$ for each operation $i \in V \setminus \widetilde{V}$. As far as the processing time of an automated operation is fixed, one can consider only automated operations, which have strictly positive processing times. Indeed, an operation with fixed zero processing time has no influence on the solution to problem SALBP-1. In contrast to usual stochastic problems (see surveys [Erel, Sarin, 1998; Sarin, Erel, Dar-El, 1999]), we do not assume any probability distribution known in advance for the random processing times of the manual operations. Moreover, it is assumed that optimal line balance ***b*** is already constructed for the given vector $t = (t_1, t_2, ..., t_n)$ of the operation times and the aim is to investigate the stability of optimality of line balance ***b*** with respect to independent variations of the processing times of the manual operations $\widetilde{V} = \{1, 2, \ldots, \widetilde{n}\}$. More precisely, we investigate *stability radius* of an optimal line balance ***b***, which may be interpreted as the maximum of simultaneous independent variations of the manual operation times with definitely keeping optimality of line balance ***b***.

Motivation and Definitions

Problem SALBP-1 arises when a new assembly line must be installed and the internal demands and properties of the assembly line have to be estimated. Cycle time *c* is defined on the basis of customer demands in the finished products. The value of *c* may be calculated as the ratio of available operating time of the assembly line and production volume for the same calendar interval. This problem may also arise when cycle time *c* of acting assembly line has to be changed because of changing customer demands in the finished product.

In the real-world assemble lines, processing times of some operations may be known exactly and fixed for a long time (if an operation has to be done by fully-automated or semi-automated machine). Indeed, modern machines and robots are able to work permanently at a constant speed for a long time. However, in some cases it is not realistic to assume constant processing time for an operation, if it has to be done by a human operator with rather simple tools. In the case of a human work, operation time is subject to physical, psychological, and other factors. Due to the learning of operators, the operation times during the first days (weeks, months) of a life cycle of the assembly line may differ from the processing times of the same operations during the later days (weeks, months). Moreover, some workers can leave the plant and new workers with lower or higher skills have to replace them.

In the case of changeable operation times, it is important to know the credibility of the optimal line balance at hand with respect to possible variations of all or a portion of the operation times. Line balance ***b***, which is optimal for the original vector $t = (\widetilde{t}, \bar{t})$ of the operation times, may lose its optimality or even feasibility for a new vector $t' = (\widetilde{t}', \bar{t})$ of the operation times. For example, due to increasing of operation times, line balance ***b*** may become infeasible for cycle time *c* since inequality (1) may be violated. In such a case, it is necessary to look for another line balance and to use it for a suitable modification of production process on the assembly line. Also, line balance ***b*** may lose its optimality with saving feasibility. It may occur if another operation assignment $\boldsymbol{b_s}$ becomes feasible for the new vector $t' = (\widetilde{t}', \bar{t})$ of the operation times and $\boldsymbol{b_s}$ uses less stations than line balance ***b*** uses.

Of course, each re-engineering of the assembly line being in process takes an additional time and other expenditure. So, assembly line modification has to be started, if it is really necessary: When the income from the re-engineering will be larger than the total expenditure caused by this re-engineering. Therefore, an evaluation of expenditures and benefits should be conducted before deciding whether re-engineering of the assembly line is necessary. However, these expenditures and benefits are difficult to evaluate before the end of the re-engineering process. In this paper, we survey some sufficient conditions for keeping the optimality of the line balance being in process.

To test whether line balance ***b*** remains feasible for the new vector $t' = (\widetilde{t}', \bar{t})$ of the operation times takes $O(\widetilde{n})$ time (if station times are included in the input data) or $O(n)$ time (otherwise). Indeed, for the new operation times we have to verify inequality (1) for each subset $V_k, k = 1,2,...,m,$ that includes at least one manual operation with changed processing time in vector t'. In the case of feasibility of the line balance ***b*** for the new vector t', in order to test its optimality for t' one has to solve NP-hard problem SALBP-1. Intuitively, it is clear that sufficiently small changes of the operation times $t_1, t_2, ..., t_{\widetilde{n}}$ may keep line balance ***b*** optimal for the new vector $t' = (\widetilde{t}', \bar{t})$ of the operation times. Our aim is to estimate or (what is better) to calculate the largest simultaneous and independent variations of the operation times $t_i, i \in \widetilde{V},$ that do not violate the feasibility and optimality of the line balance ***b***. At the stage of the design of the assembly line, there exists a lot of optimal line balances. Using stability analysis, one can select such an optimal line balance, which optimality is more stable with respect to possible variations of the operation times $t_i, i \in \widetilde{V}$.

Let B denote the set of all assignments of operations V to stations S_1, S_2, … , S_m (for possible numbers m of the stations: $1 \le m \le n$), which satisfy Condition 1. Subset of set B of all operation assignments which also satisfy Condition 2 for the given vector $t = (t_1, t_2, \dots, t_n)$ of the operation times is denoted by $B(t) = \{b_0\ b_1, \dots, b_h\}$ where b_k, $k \in \{0, 1, ..., h\}$, means line balance $V = V_1^{\boldsymbol{b}_k} \cup V_2^{\boldsymbol{b}_k} \cup \ldots \cup V_{m_{\boldsymbol{b}_k}}^{\boldsymbol{b}_k}$. Let subset of set $B(t)$ of all the optimal line balances be denoted by $B_{opt}(t)$. Inclusion $\boldsymbol{b} \in B_{opt}(t)$ implies that line balance $\boldsymbol{b}$ with partition $V = V_1^{\boldsymbol{b}} \cup V_2^{\boldsymbol{b}} \cup \ldots \cup V_{m_{\boldsymbol{b}}}^{\boldsymbol{b}}$ satisfies Condition 1, Condition 2, and the following optimality condition for the vector $t = (t_1, t_2, \dots, t_n)$ of the operation times.

Condition 3: $m_b = \min\{ m_{\boldsymbol{b}_k} : b_k \in B(t)\}$.

Since line balance $\boldsymbol{b}$ is contained in the set $B(t)$, we obtain $\boldsymbol{b} = \boldsymbol{b}_r \in B(t)$ for some index $r \in \{0, 1, \dots, h\}$. As a matter of convenience, index r will be omitted for the optimal line balance $\boldsymbol{b}$, which stability will be investigated. In both definitions of set B and set $B(t)$, number m of stations is not fixed: For line balance $\boldsymbol{b}_k$ from the set $B(t)$, inequalities $m_{\boldsymbol{b}} \le m_{\boldsymbol{b}_k} \le n$ must hold and number of stations in an operation assignment from set B has to belong to set $\{1, 2, \dots, n\}$. The question under consideration may be formulated as follows. How much can be modified the components of the vector $\tilde{t}$ simultaneously and independently from each other that the given line balance $\boldsymbol{b}$ remains optimal?

Let $\boldsymbol{R}^n$ ($\boldsymbol{R}_+^{\boldsymbol{n}}$ respectively) denote the space of n-dimensional real vectors $t = (t_1, t_2, \dots, t_n)$ (the space of n-dimensional non-negative real vectors) with the maximum metric, i.e., distance $d(t, t^*)$ between vector $t \in \boldsymbol{R}^n$ and vector $t^* = (t_1^*, t_2^*, \dots, t_n^*) \in \boldsymbol{R}^n$ is calculated as follows: $d(t, t^*) = \max\{ \left| t_i - t_i^* \right| : i \in V\}$, where $\left| t_i - t_i^* \right|$ denotes the absolute value of the difference $t_i - t_i^*$. Let line balance $\boldsymbol{b}$ be optimal for the given non-negative real vector $t = (\tilde{t}, \bar{t}) = (t_1, t_2, \dots, t_n) \in \boldsymbol{R}_+^{\boldsymbol{n}}$ of the operation times, i.e., $\boldsymbol{b} \in B_{opt}(t)$. For problem SALBP-1, the definition of stability radius of an optimal line balance is introduced as follows.

Definition 1: *The open ball* $O_\rho(\tilde{t})$ *with radius* $\rho \in \boldsymbol{R}_+^{\boldsymbol{1}}$ *and center* $\tilde{t} \in \boldsymbol{R}_+^{\tilde{n}}$ *in the space* $\boldsymbol{R}^{\tilde{n}}$ *is called a stability ball of the line balance* $\boldsymbol{b} \in B_{opt}(t)$, *if for each vector* $t^* = (\tilde{t}^*, \bar{t})$ *of the operation times with* $\tilde{t}^* \in O_\rho(\tilde{t}) \bigcap \boldsymbol{R}_+^{\tilde{\boldsymbol{n}}}$ *line balance* $\boldsymbol{b}$ *remains optimal. The maximal value of the radius* ρ *of stability ball* $O_\rho(\tilde{t})$ *of the line balance* $\boldsymbol{b}$ *is called stability radius denoted by* $\rho_b(t)$.

In Definition 1, vector $\bar{t} = (t_{\tilde{n}+1}, t_{\tilde{n}+2}, \dots, t_n)$ of the processing times of the automated operations and the complete vector $t = (\tilde{t}, \bar{t}) = (t_1, t_2, \dots, t_n)$ of the operation times are fixed, while vector $\tilde{t}^* = (t_1^*, t_2^*, \dots, t_{\tilde{n}}^*)$ may vary within the intersection of the open ball $O_\rho(\tilde{t}) \subset \boldsymbol{R}^n$ with the space $\boldsymbol{R}_+^{\tilde{n}}$ of the non-negative real vectors. Stability radius $\rho_b(t)$ is equal to the minimal upper bound of independent variations ε_i of the processing times t_i of all the manual operations $i \in \tilde{V}$ which definitely keep the optimality of the line balance $\boldsymbol{b}$, i.e., inclusion $\boldsymbol{b} \in B_{opt}(t^*)$ holds with $t_i^* = \max\{0, t_i - \varepsilon_i\}$ or $t_i^* = t_i + \varepsilon_i$.

In the rest of this paper, we survey recent results proven in [Sotskov, Dolgui, 2001; Sotskov, Dolgui, Portmann, 2006; Sotskov *et al*, 2005] on stability analysis of an optimal line balance for problems SALBP-1 and SALBP-2.

Stability Radius of an Optimal Line Balance for Problem SALBP-1

Let $\tilde{V}_k^{\boldsymbol{b}}$ denote the subset of manual operations of set $V_k^{\boldsymbol{b}}$ and let $\bar{V}_k^{\boldsymbol{b}}$ denote the subset of automated operations of set $V_k^{\boldsymbol{b}}$. For each index $k \in \{1, 2, \dots, m_b\}$, equality $V_k^{\boldsymbol{b}} = \tilde{V}_k^{\boldsymbol{b}} \cup \bar{V}_k^{\boldsymbol{b}}$ holds. The following remark is used in stability analysis of an optimal line balance for problem SALBP-1.

Remark 1: *Let us consider the line balance* **b** $\in B_{opt}(t)$ *being in process and the modified vector* $t' = (\tilde{t}\,', \bar{t})$ *of the operation times. If there exists subset* $V_k^{\boldsymbol{b}}$, $k \in \{1, 2, ..., m_{\boldsymbol{b}}\}$, *in the line balance* **b** *such that*

$$\sum_{i \in V_k^{\boldsymbol{b}}} t_i' = 0, \tag{3}$$

we continue to affirm that line balance **b** *uses* m_b *stations for the modified vector* $t' = (\tilde{t}\,', \bar{t})$ as well.

We can argue Remark 1 as follows. In spite of equality (3) valid for the new vector $t' = (\tilde{t}\,', \bar{t})$, station S_k is still exists in the assembly line with line balance ***b***. At very least to delete station S_k causes additional cost and additional time for re-engineering the assembly line. Moreover, after deleting station S_k we obtain another line balance, say, $b^* \in B$: $V = \bigcup_{i \in \{1,2,...,m_b\}, i \neq k} V_i^{\boldsymbol{b}} = V_1^{\boldsymbol{b}^*} \cup V_2^{\boldsymbol{b}^*} \cup ... \cup V_{m_{\boldsymbol{b}^*}}^{\boldsymbol{b}^*}$, where $m_{\boldsymbol{b}^*} = m_{\boldsymbol{b}} - 1$. Due to the validity of inequality $t_i > 0$ for each automated operation $i \in V \backslash \tilde{V}$, equality (3) is only possible if $V_k^{\boldsymbol{b}} = \tilde{V}_k^{\boldsymbol{b}}$.

In [Sotskov, Dolgui, Portmann, 2006], it was proven the following necessary and sufficient condition for the case when optimality of the line balance $b \in B_{opt}(t)$ is unstable.

Theorem 1: *For line balance* $b \in B_{opt}(t)$ *equality* $\rho_b(t) = 0$ *holds if and only if there exists a subset* $V_k^{\boldsymbol{b}}$, $k \in \{1, 2, \ldots, m_{\boldsymbol{b}}\}$, *such that* $\tilde{V}_k^{\boldsymbol{b}} \neq \emptyset$ *and* $t(V_k^{\boldsymbol{b}}) = c$.

To present a formula for exact value of stability radius $\rho_b(t)$ of optimal line balance $b \in B_{opt}(t)$ we need the following notation:

$$\delta^{\boldsymbol{b}} = \min\{\delta_k^{\boldsymbol{b}} : \tilde{V}_k^{\boldsymbol{b}} \neq \emptyset,\ k \in \{1, 2, ..., m_{\boldsymbol{b}}\}\}, \tag{4}$$

where $\delta_k^{\boldsymbol{b}} = \dfrac{c - t(V_k^{\boldsymbol{b}})}{|\tilde{V}_k^{\boldsymbol{b}}|}$ and value $t(V_k)$ is defined in (2). It is easy to see that testing criterion given in Theorem 1 takes $O(n)$ time. This asymptotic bound is defined due to calculating station times $t(V_p^{\boldsymbol{b}}), p = 1, 2, ..., m_{\boldsymbol{b}}$. If station times are included in the input data, then testing criterion given in Theorem 1 takes $O(m_b)$ time.

The following lower bound of stability radius has been obtained within the proof of Theorem 1 given in [Sotskov, Dolgui, Portmann, 2006].

Corollary 1*: If optimality of line balance* $b \in B_{opt}(t)$ *is stable, then* $\rho_{\boldsymbol{b}}(t) \geq \min\{\delta^{\boldsymbol{b}}, \Delta^{\boldsymbol{b}}\}$, *where*

$$\Delta^{\boldsymbol{b}} = \min\{\Delta(V_p^{\boldsymbol{b}^*}) : \boldsymbol{b}^* \in B\} \text{ and } \Delta(V_p^{\boldsymbol{b}^*}) = \frac{t(V_p^{\boldsymbol{b}^*}) - c}{|\tilde{V}_p^{\boldsymbol{b}^*}|}.$$

Let $\tilde{V}_p^{\boldsymbol{b}(d)} = \{i_1, i_2, ..., i_u\}$, where $u = |\tilde{V}_p^{\boldsymbol{b}(d)}|$, and indices v of operations i_v are assigned in such a way that the following inequalities hold: $t_{i_1} \leq t_{i_2} \leq \ldots \leq t_{i_u}$. It is assumed that $t_{i_0} = 0$. Vector $t'' = (\tilde{t}\,'', \bar{t}) \in \boldsymbol{R}_+^n$ closest to t, for which subset $V_p^{\boldsymbol{b}(d)}$ is feasible (inequality (1) holds for subset $V_p^{\boldsymbol{b}(d)}$ with vector $t'' = (\tilde{t}\,'', \bar{t})$), can be obtained if for each operation $i_q \in \tilde{V}_p^{\boldsymbol{b}(d)}$ we set $t''_{i_q} = \max\{0, t_j - \hat{\Delta}(V_p^{\boldsymbol{b}(d)})\}$ where j and i_q denotes the same manual operation ($j = i_q$) and value $\hat{\Delta}(V_p^{\boldsymbol{b}(d)})$ is calculated as follows:

$$\hat{\Delta}(V_p^{\boldsymbol{b}(d)}) = \max\left\{ \frac{\sum_{i \in V_p^{\boldsymbol{b}(d)}} t_i - c - \sum_{\alpha=0}^{\beta} t_{i_\alpha}}{|\tilde{V}_p^{\boldsymbol{b}(d)}| - \beta} : \beta = 0, 1, ..., |\tilde{V}_p^{\boldsymbol{b}(d)}| - 1 \right\}$$

Here maximum is taken among right-hand fractions calculated for $\beta = 0, 1, ..., \left| \widetilde{V}_p^{\boldsymbol{b}^{(d)}} \right| - 1$. We define

$$\Delta(\boldsymbol{b}^{(d)}) = \max\left\{ \hat{\Delta}(V_p^{\boldsymbol{b}^{(d)}}) : t(V_p^{\boldsymbol{b}^{(d)}}) > c \right\},$$
$$\hat{\Delta}^{\boldsymbol{b}} = \min\{ \Delta(\boldsymbol{b}^{(d)}) : \boldsymbol{b}^{(d)} \in B^{(m_b - 1)} \}. \tag{5}$$

In [Sotskov, Dolgui, Portmann, 2006], the following formula for calculating the exact value of stability radius $\rho_b(t)$ has been derived.

Theorem 2: *If optimality of line balance* $\boldsymbol{b} \in B_{opt}(t)$ *is stable, then* $\rho_b(t) = \min\{\delta^{\boldsymbol{b}}, \hat{\Delta}^{\boldsymbol{b}}\}$ *with* $\delta^{\boldsymbol{b}}$ *being defined in (4) and* $\hat{\Delta}^{\boldsymbol{b}}$ *in (5).*

Let $\lceil a \rceil$ denote the smallest integer greater than or equal to real number a. Theorem 2 implies the following five corollaries.

Corollary 2: *If* $m_b = \left\lceil \frac{t(V)}{c} \right\rceil$, *then* $\rho_{\boldsymbol{b}}(t) \geq \min\left\{ \delta^{\boldsymbol{b}}, \frac{t(V) - c(m_{\boldsymbol{b}} - 1)}{\widetilde{n}} \right\}$.

Corollary 3: *If* $m_b = \left\lceil \frac{t(V)}{c} \right\rceil$ *and* $\delta^{\boldsymbol{b}} \leq \frac{t(V) - c(m_{\boldsymbol{b}} - 1)}{\widetilde{n}}$, *then* $\rho_{\boldsymbol{b}}(t) = \delta^{\boldsymbol{b}}$.

Corollary 4: *If* $\boldsymbol{b} \in B_{opt}(t)$, *then* $\rho_{\boldsymbol{b}}(t) \leq \min\{\delta^{\boldsymbol{b}}, \max_{i \in \widetilde{V}} t_i\}$.

Corollary 5: *If* $\boldsymbol{b} \in B_{opt}(t)$, *then* $\rho_{\boldsymbol{b}}(t) \leq \min\{c - \max_{i \in \widetilde{V}} t_j, \max_{i \in \widetilde{V}} t_j\}$.

Corollary 6: *If* $\boldsymbol{b} \in B_{opt}(t)$ and $\boldsymbol{b}^{(d)} \in B^{(m_{\boldsymbol{b}} - 1)}$, *then* $\rho_{\boldsymbol{b}}(t) \leq \min\{\delta^{\boldsymbol{b}}, \Delta(\boldsymbol{b}^{(d)})\}$.

Stability of an Optimal Line Balance for Problem SALBP-2

In this section, we consider problem SALBP-2: To find an optimal balance of the assembly line for a fixed number of stations, i.e., to find a feasible assignment of all operations *V* to exactly *m* stations in such a way that the cycle-time *c* is minimal. For problem SALBP-2, line balance $\boldsymbol{b}_r$ is *optimal* if along with Condition 1 and Condition 2, it has the minimal cycle time. We denote the cycle time for line balance $\boldsymbol{b}_r$ with the vector *t* of the operation times as $c(\boldsymbol{b}_r, t)$:

$$c(\boldsymbol{b}_r, t) = \max_{k=1}^{m} \sum_{i \in V_k^{b_r}} t_i .$$

For problem SALBP-2, optimality of line balance $\boldsymbol{b} = \boldsymbol{b}_s$ with vector *t* of the operation times may be defined via the following condition.

Condition 4: $c(\boldsymbol{b}_s, t) = \min\{c(\boldsymbol{b}_r, t) : \boldsymbol{b}_r \in B(t)\}$, *where* $B(t) = \{\boldsymbol{b}_0, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_h\}$ *is the set of all line balances.*

For problem SALBP-2, definition of the stability radius of an optimal line balance is introduced as follows.

Definition 2: *The closed ball* $\underline{O}_\rho(\widetilde{t})$ *in the space* $\boldsymbol{R}^{\widetilde{n}}$ *with the radius* $\rho \in \boldsymbol{R}_+^1$ *and center* $\widetilde{t} \in \boldsymbol{R}_+^{\widetilde{n}}$ *is called a stability ball of the line balance* $\boldsymbol{b} \in B(t)$, *if for each vector* $t^* = (\widetilde{t}^*, \bar{t})$ *of the operation times with* $\widetilde{t}^* \in \underline{O}_\rho(\widetilde{t}) \cap \boldsymbol{R}_+^{\widetilde{n}}$ *line balance* $\boldsymbol{b}$ *remains optimal. The maximal value of the radius* ρ *of stability ball* $\underline{O}_\rho(\widetilde{t})$ *of the line balance* $\boldsymbol{b}$ *is called stability radius denoted by* $\underline{\rho}_b(t)$.

In Definition 2, vector $\bar{t} = (t_{\widetilde{n}+1}, t_{\widetilde{n}+2}, \ldots, t_n)$ of the automated operation times and vector $t = (\widetilde{t}, \bar{t}) = (t_1, t_2, \ldots, t_n)$ of all the operation times are fixed, while vector $\widetilde{t}^* = (t_1^*, t_2^*, ..., t_{\widetilde{n}}^*)$ of the manual operation times may vary within the intersection of the closed ball $\underline{O}_\rho(\widetilde{t})$ with the space $\boldsymbol{R}_+^{\widetilde{n}}$. For each optimal line balance $\boldsymbol{b}_r \in B_{opt}(t)$, we define a set $W(\boldsymbol{b}_r)$ of all subsets $\widetilde{V}_k^{b_r}$, $k \in \{1, 2, \ldots, m\}$, such that

$\sum_{i \in V_k^{b_r}} t_i$ = *c(b, t)*. It should be noted that set *W(b$_r$)* may include the empty set as its element. In [Sotskov *et al*, 2005], the following claims have been proven.

Theorem 3: *Let inequality $t_i > 0$ hold for each manual operation $i \in \widetilde{V}$. Then for line balance* **b**$_s \in$ *B(t), equality $\underline{\rho}_{b_s}(t) = 0$ holds if and only if there exists a line balance* **b**$_r \in B_{opt}(t)$ *such that condition W(***b**$_s$*)* $\subseteq$ *W(***b**$_r$*) does not hold.*

Corollary 7: *If $B_{opt}(t) = \{$***b**$_s$*$\}$, then $\underline{\rho}_{b_s}(t) > 0$.*

If there exists an index $k \in \{1, 2, \ldots, m\}$ such that

$$\sum_{i \in V_k^{b_0}} t_i < c(\mathbf{b}_0,t), \tag{6}$$

then we set $\lambda(\mathbf{b}_0) = \{c(\mathbf{b}_0,t) - \max\{\sum_{i \in V_k^{b_0}} t_i : \widetilde{V}_k^{b_0} \notin W(\mathbf{b}_0)\}\} \big/ \widetilde{n}$. Due to (6), the strict inequality $\lambda(\mathbf{b}_0) > 0$ must hold. If $\sum_{i \in V_k^{b_0}} t_i = c(\mathbf{b}_0,t)$ for each index $k \in \{1, 2, \ldots, m\}$, then we set $\lambda(\mathbf{b}_0) = \min\{t_i : i \in \widetilde{V}\}$. We denote

$\Delta = \min\{\Delta(\mathbf{b}_s) : \mathbf{b}_s \in B \setminus B(t)\}$, where $\Delta(\mathbf{b}_s) = \dfrac{c(\mathbf{b}_s,t) - c(\mathbf{b}_0,t)}{\widetilde{n}}$. Theorem 3 implies the following claim.

Corollary 8: *If $\underline{\rho}_{b_s}(t) > 0$, then $\underline{\rho}_{b_s}(t) \geq \min\{\Delta, \lambda(\mathbf{b}_0)\}$.*

Calculating exact value of stability radius $\underline{\rho}_b(t)$ is close to calculating stability radius of the optimal schedule for the makespan criterion (see [Sotskov, 1991; Sotskov, *et al.* 1998]).

Conclusion

In the above sections, the recent results on stability analysis of an optimal line balance are presented. We used the notion of stability radius, which is similar to the stability radius of an optimal schedule introduced in [Sotskov, 1991] for shop-scheduling problems. If stability radius of an optimal line balance is strictly positive, then any independent changes of the operation times t_j, $j \in \widetilde{V}$, within the ball with this radius definitely keep the optimality of this line balance. On the other hand, if stability radius of optimal line balance ***b*** is equal to zero (Theorems 1 and 3), then even small changes of the processing times of all or a portion of the manual operations may deprive the optimality of line balance ***b***. It is worth noting that conditions presented in this paper (except Theorem 2, Theorem 3, Corollary 1, and Corollary 8) may be tested in polynomial time, which is important for real-world assembly lines with large numbers of operations. Moreover, for exact value of stability radius, feasibility of line balance ***b***, which is defined by the value $\delta^{\mathbf{b}}$, may be tested in polynomial time due to Theorem 2 as well.

In practice, the tendency at the design stage must be to find optimal line balance for which stability is as much as possible. The common objective is to assign to each station a set of operations with roughly the same total operation time [Bukchin, Tzur, 2000; Erel, Sarin, 1998; Lee, Johnson, 1991; Sarin, Erel, Dar-El, 1999]. Due to the above results, we have to defer stations with manual operations and stations without manual operations. Theorem 1 shows that for the station with manual operations it is desirable to have some slack between cycle time and station time. The larger this slack is the larger stability radius of the line balance may be. On the other hand, for the stations loaded by only automated operations, such a slack may be as small as possible, which gives the possibility to increase slacks for stations with manual operations.

Since stability radius $\rho_b(t)$ cannot be larger than $c/2$, one has to pay special attention to the manual operations with possible variations of the processing times more than $c/2$ (such operations may cause instability of optimality of the line balance being in process). At the design stage, such an operation has to be divided into shorter operations. If line balance will be used for a long time for assembling the same finished product, it is desirable at the design stage, to construct several optimal line balances, and select among them the one with the best stability characteristics. To this end, it is useful to develop algorithms, which construct a set of optimal line balances (instead of only one optimal line balance), in order to carry out a stability analysis for them. Or, better yet, it is useful to include in the branch-and-bound algorithm or other algorithms used for solving problem SALBP-1 and problem SALBP-2 specific rules in order to construct optimal line balance with larger stability radius. In a concrete study, the set $\widetilde{V}$ of manual operations can be reduced (e.g., only critical manual operations may be considered) or, on the contrary, set $\widetilde{V}$ may be completed by some unstable automated operations. By changing the set of operations with variable times, the designer of the assembly line can study the influence of different operations on stability of optimality and feasibility of line balances.

In [Sotskov, Dolgui, 2001], slightly different definition of stability radius of an optimal line balance has been used for problem SALBP-1. Namely, it was assumed that $O_{\rho_b(t)} \subset \boldsymbol{R}_+^{\widetilde{n}}$ (as a result, such a stability radius cannot be greater than $\min\{t_i : i \in \widetilde{V}\}$). The above Definition 1 is more appropriate for practical assembly lines. As a subject for future research, it is important to consider stability characteristics of an assembly line balance provided that for each operation time an interval of possible variation is known in advance. Such an assumption seems to be realistic for many real-word assembly lines.

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Bibliography

[Bukchin, Tzur, 2000] J. Bukchin, M. Tzur, Design of flexible assembly line to minimize equipment cost, *IIE Transactions* 32 (2000) 585-598.

[Erel, Sarin, 1998] E. Erel, S.C. Sarin, A survey of the assembly line balancing procedures, *Production Planning & Control* 9 (5) (1998) 414-434.

[Garey, Johnson, 1979] M.R. Garey, D.S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness.* Freeman, San Francisco, USA, 1979.

[Lee, Johnson, 1991] H.F. Lee, R.V. Johnson, A line-balancing strategy for designing flexible assembly systems, *The International Journal of Flexible Manufacturing Systems* 3 (1991) 91-120.

[Sarin, Erel, Dar-El, 1999] S.C. Sarin, E. Erel, E.M. Dar-El, A methodology for solving single-model, stochastic assembly line balancing problem, *OMEGA – International Journal of Management Science* 27 (1999) 525-535.

[Scholl, 1999] A. Scholl, *Balancing and Sequencing of Assembly Lines*, Heidelberg: Physica-Verlag, Germany,1999.

[Sotskov, 1991] Yu.N. Sotskov, Stability of an optimal schedule, *European Journal of Operational Research* 55 (1991) 91-102.

[Sotskov, Dolgui, 2001] Yu.N. Sotskov, A. Dolgui, Stability radius of the optimal assembly line balance with fixed cycle time. In: *Proceedings of the IEEE Conference ETFA'2001* (2002) 623-628.

[Sotskov, Dolgui, Portmann, 2006] Yu.N. Sotskov, A. Dolgui, M-C. Portmann, Stability of an optimal balance for an assembly line with fixed cycle time, *European Journal of Operational Research* 168 (2006) 783-797.

[Sotskov, *et al.* 2005] Yu.N. Sotskov, A. Dolgui, N. Sotskova, F. Werner, Stability of optimal line balance with given station set. In: *Supply Chain Optimisation* (2005) 135-149.

[Sotskov, *et al.* 1998] Yu.N. Sotskov, V.S. Tanaev, F. Werner, Stability radius of an optimal schedule: A survey and recent developments, Chapter in: G. Yu (Ed.) *"Industrial Applications of Combinatorial Optimization"* 16, Kluwer Academic Publishers, Boston, USA, 1998, 72-108.

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