ON STRUCTURAL RECOGNITION WITH LOGIC AND DISCRETE ANALYSIS

Levon Aslanyan, Hasmik Sahakyan

Abstract*: The paper addresses issues of special style structuring of learning set in pattern recognition area. Above the regular means of ranking of objects and properties, which also use the structure of learning set, the logic separation hypotheses is treated over the multi value features area, which structures the learning set and tries to recover more valuable relations for better recognition. Algorithmically the model is equivalent to constructing the reduced disjunctive normal form of Boolean functions. The multi valued case which is considered is as harder as the binary case but it uses approximately the same structures.*

Keywords*: Learning, Boolean function, logic separation.*

ACM Classification Keywords*: F.2.2 Nonnumerical Algorithms and Problems:* Computations on discrete structures

Introduction

Pattern recognition undergoes an important developing for many years. As a research area this is not uni-modular like classic mathematical sciences, it has a long history of establishment. Inside the theory there are a number of sub disciplines such as – feature selection, object and feature ranking, analogy measures, supervised and unsupervised classification, etc. The same time pattern recognition is indeed an integrated theory studying object descriptions and their classification models. This is a collection of mathematical, statistical, heuristic and inductive techniques of fundamental role in executing the intellectual tasks, typical for a human being, – but on computers.

In many applied problems with multidimensional experimental data the object description is often non-classical, that is, - not exclusively in terms of only numerical or only categorical features, but simultaneously by both kinds of values. Sometimes, the missing value is introduced so that finally we deal with *mixed and incomplete* descriptions of objects as elements of Cartesian product of feature values, without any algebraic, logical or topological properties assumed in applied area. How then, do we select in these cases the most informative features, classify a new object given a (*training*) sample or find the relationships between all objects based on a certain measure of similarity? Logic Combinatorial Algebraic Pattern Recognition is a research area formed since 70's, that uses a mix of discrete descriptors – and similarities, separation, frequencies, integration and corrections, optimization, and solves the whole spectrum of pattern recognition tasks.

This approach is originated by the work [Dmitriev et al, 1966] that transfers the engineering domain technique of tests for electrical schemes [Chegis, Yablonskii, 1958] to the feature selection and object classification area. The applied task of [Dmitriev et al, 1966] is prognosis for mineral resources. In a basic model authors consider that all features are Boolean. Later formal extensions with different kinds of features appeared, as we do it below for the logic separation analysis. Consider the general form of learning set data, L :

FEATURES					
OBJECTS	x_1	x_2	$\cdots$	x_n	CLASSES
$\tilde{a}_1^1$	a_{11}^1	a_{12}^1	$\cdots$	a_{1n}^1	C_1
$\tilde{a}_2^1$	a_{21}^1	a_{22}^1	$\cdots$	a_{2n}^1	
$\cdots$					
$\tilde{a}_{l_1}^1$	$a_{l_1 1}^1$	$a_{l_1 2}^1$	$\cdots$	$a_{l_1 n}^1$	
$\cdots$					
$\tilde{a}_1^m$	a_{11}^m	a_{12}^m	$\cdots$	a_{1n}^m	C_m
$\tilde{a}_2^m$	a_{21}^m	a_{22}^m	$\cdots$	a_{2n}^m	
$\cdots$					
$\tilde{a}_{l_m}^m$	$a_{l_m 1}^m$	$a_{l_m 2}^m$	$\cdots$	$a_{l_m n}^m$	

Features $x_1, x_2 \cdots, x_n$ are categorical or numerical properties represented by their domains of vales $M_1, M_2 \cdots, M_n$. Categorical feature takes values from a set $H_t^s = \{s_1, s_2 \cdots, s_t\}$. Numerical features are from metric spaces assuming the following two types:

a) $H_{k,r}^z \equiv \{k, k+1, \cdots, r\}$, where k, r are nonnegative integers, and $k < r$,

b) $H_{k,r}^r \equiv \{\alpha : \alpha \in (k, r)\}$, where (k, r) is an interval on the real number line, and $k < r$.

Distances between the space elements are usual (numerical). This choice of primary/elementary attribute spaces reflects the common/usual situation that exists in application areas of pattern recognition and classification tasks. A space M in which metric spaces $M_i, i = 1, \cdots, n$, are of H_t^s, $H_{k,r}^z$ and/or $H_{k,r}^r$ types, we call n dimensional attribute space.

Test (testor) theory [Dmitriev et al, 1966] is based on a restriction of feature sets and learning elements - with preservation of the basic learning set property – preservation of membership to different classes. A features subset $T = \{x_{i_1}, x_{i_2} \cdots, x_{i_k}\}$ is called a testor of L, if prjection of L on T keeps the classes nonintersecting (learning set property). The testor is irreducible when no one x_{i_j} feature may be eliminated with conservation of the learning set property. Further a feature ranking is made taking into consideration the frequency of belonging x_{i_j} to the testors (irreducible testors). Other measure is introduced taking into account frequency of testors on classes. Testor based supervised classification algorithms are constructed by use of frequency similarity measures. Which is the structural property used from the learning set? This is the pair wise difference of learning set elements from different classes (learning set property). We may suppose that this is part or consequence of the well accepted compactness hypothesis.

On formal basis testor technology is well visible on binary tables. Now it is known that constructing all irreducible testors is an NP hard problem. Approximations are studied as well. There is a high similarity with association rule mining models, especially in part of learning of monotone set structures – be it with frequent itemsets in associative rule mining or irreducible tests and testers in this theory.

Consider n-vector $\tilde{\tau} = (\tau_1, \tau_2 \cdots, \tau_n)$, that indicates the types of elementary attribute spaces of space M. In further consideration without loss of generality we may suppose that $\tau_j \in \{z, r\}$ (this may be coded by binary 0,1 values). Hence $\mathrm{M} = H^{\tau_1}_{c_1,d_1} \times H^{\tau_2}_{c_2,d_2} \times \cdots \times H^{\tau_n}_{c_n,d_n}$. Then the space M can be adjusted by a pair of n-vectors $\tilde{c} = (c_1, c_2 \cdots, c_n)$ and $\tilde{d} = (d_1, d_2 \cdots, d_n)$ above the $\tilde{\tau} = (\tau_1, \tau_2 \cdots, \tau_n)$. n-dimensional attribute space $\breve{\mathrm{M}}$ is called sub-space of M, if its each metric sub-space $\breve{M}_i, i = 1, \cdots, n$ is a sub-space of corresponding to it in M, metric space $M_i, i = 1, \cdots, n$. Obviously such space $\breve{\mathrm{M}}$ can be presented by a pair of vectors $\tilde{c}' = (c'_1, \cdots, c'_n)$, $\tilde{d}' = (d'_1, \cdots, d'_n)$, where $c'_i \geq c_i$ and $d'_i \leq d_i$, $i = 1, \cdots, n$. We call the number of positions with $c'_i \neq d'_i$ the dimension of $\breve{\mathrm{M}}$.

[Vaintsvaig, 1973] formulated one of the basic concept in pattern recognition – the KORA algorithm. KORA is constructing elementary conjunctions that intersect with only one class C_i of learning set L. Contrary, [Aslanyan, 1975] considers all irreducible conjunction forms that intersects with only one class C_i of learning set L. This is not the generating idea of this work but is the consequence of the Logic Separation Principle. It is to think, that the Logic Separation Principle is some kind of completion of the well known Compactness hypotheses. The work [Aslanyan, 1975], factually for the first time, is considering learning set elements in a non separated/isolated manner. The concept is used that an element spreads its "similarity", reduced with distance measure of course, which interrupts facing the different class object. An extension may consider not only pairs of learning set elements – one spreading the similarity measure and the second interrupting that - but also arbitrary subsets/fragments of learning set. Several comments: logically, it is evident that the best learning algorithm must be suited best to the learning set itself (at least the learning set is reconstructable by the information on learning set used by algorithm). This also may use the recognition hypothesis when available. The same learning set fragments play a crucial role in estimating the choice of algorithm by the given learning data. The technical solution of Logical Separation (LS) is by Reduced Disjunctive Normal Forms (RDNF), which answers to all issues, - implementation, complexity, interpretation. Further is important to mention that advanced data mining technique IREP (Incremental Reduced Error Pruning) finds its theoretical interpretation in terms of LS framework mentioned above.

Sub-spaces of M we will also call intervals of M. Similarly to the testor theory the irreducible subspace cover technology is well visible on binary tables. The mentioned two visible formalisms, in addition to the basic principles by Yu. Zhuravlev [Dmitriev et al, 1966], that appeared in early 70's, present the basics of Logic Combinatorial Algebraic Pattern Recognition research area.

Formal extensions

This point considers the LS model with similarity spread and interruption by learning set elements. We try to extend the basic constructions to the case of complicated attribute spaces. Let us introduce partially defined characteristic functions $f_i(x_1, x_2, \cdots, x_n), \quad i = 1, \cdots, m$, which separate some class L_i from the rest of classes, $L_j, j \neq i, j = 1, \cdots, n$. The separation is based on the information about the classification given by subsets L_i, $i = 1, \cdots, m$ in learning set.

$$\tilde{f}_i = f(\alpha_1, \alpha_2, \cdots, \alpha_n) =$$

$$\begin{cases} 1, if \quad \tilde{\alpha} \in L_i \\ 0, if \quad \tilde{\alpha} \in L \setminus L_i \\ undefined, if \ \tilde{\alpha} \in \mathrm{M} \setminus L \end{cases}$$

This system of m partly defined functions will be considered in an analogous way to the class of the algebra of logic (Boolean) functions. For system $\tilde{f}_i$, $i=1,\cdots,m$ we will use analogous constructions [Yablinskii, 1958], to the concepts that are well known for functions of the algebra of logic.

The subspace $\breve{\mathrm{M}}$ of M we call an interval of the function $\tilde{f}_i(x_1,\cdots x_n)$ if

1) $\breve{\mathrm{M}} \subseteq \mathrm{M} \setminus \bigcup_{j\neq i} L_j$

2) $\breve{\mathrm{M}} \cap L_i \neq 0$

Let $\breve{U}$ be an interval of $\tilde{f}_i(x_1,\cdots x_n)$. Then $\breve{U}$ is called the maximal interval if there is no other interval $\breve{V}$ of $\tilde{f}_i(x_1,\cdots x_n)$, such that $\breve{V} \supset \breve{U}$.

We denote by U_i, $i=1,\cdots,p_i$ the set of all maximal intervals of $\tilde{f}_i(x_1,\cdots x_n)$. By analogy to the similar constructions of the algebra of logic, we call the set of U_i reduced interval normal form /or interval cover form/ of $\tilde{f}_i(x_1,\cdots x_n)$.

Further, for each function $\tilde{f}_i(x_1,\cdots x_n)$, $i=1,\cdots,m$ we need to construct U_i, - the set of its all maximal intervals. This question is addressed by [Aslanyan, 1975] in the particular case when $\mathrm{M}_i \equiv H_{0,1}^z$, $i=1,\cdots,m$.

Other methods designed for the mentioned constructions in the class of the algebra of logic are given in [Yablinskii, 1958] /for k -valued logic/. But the learning set L in our tasks is very specific – it consists of a finite and very low number of points, and therefore growing type methods analogous to those given in [Yablinskii, 1958] for constructing reduced interval forms, are less efficient than the step by step constructions given by sets L_i, $i=1,\cdots,m$. Notice also that if among the metric spaces M_i there exists a space of the type $H_{k,r}^r$, then the direct use of methods from [Yablonskii, 1958] is simply impossible. Below we describe the geometric constructions that appear in step by step procedures for reduced interval covers of systems of partial separation functions. Our goal is to demonstrate that there is a high similarity with the binary case with additions, that constructed intervals are accompanied with small size descriptive data.

Reduced interval covers of systems if partial separation functions

Before starting to construct reduced interval forms U_i of $\tilde{f}_i(x_1,x_2,\cdots x_n)$ let us assume that intervals $\{k,r\}$ of individual directions defined above /specifically the real value case/ include their endpoints in a following way: point is inclusive iff it is a boarder point of corresponding M_i and doesn’t match to one of the considered vertices of the learning set. Different coding is possible. For $H_{k,r}^z$ we may use the instant intervals, - all end points included. For $H_{k,r}^r$ we use the scheme that the end point is an exclusive real value. We need this for only internal point indication, and end points of M_i are always inclusive when doesn’t match to one of the considered vertices of learning set. Algorithmically, exclusive end point is very easy to code – the use of negative value of corresponding coordinate may code this case.

Thus we consider functions $\tilde{f}_i(x_1,\cdots x_n)$ which separate the sets L_i from the complementary sets L_j, $j\neq i$, $j=1,\cdots,m$. Construct sets U_i - the sets that consist of corresponding maximal intervals. Let $\tilde{\beta}_1,\tilde{\beta}_2,\cdots,\tilde{\beta}_{k_i}$ be points of M, that belong to L_j, $j\neq i$, $j=1,\cdots,m$, where $k_i = m-(m_i - m_{i-1})$. Let $\tilde{\gamma}_1,\tilde{\gamma}_2,\cdots,\tilde{\gamma}_{r_i}$ be points of L_i, where $r_i = m_i - m_{i-1}$ (m_i are defined by table of learning set given above as the last enumerated index in the class L_i).

In general we are given also the point $\tilde{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_n) \in \mathrm{M}$ that is to be recognized/arranged into one of the classes $C_i,\ i = 1, \cdots, m$.

Consider the first point $\tilde{\beta}_1 = (\beta_{11}, \beta_{12}, \cdots, \beta_{1n})$. Then an interval $\breve{U}$ will be maximal out of $\{\tilde{\beta}_1\}$ if and only if at least one of its elementary intervals $\{c'_i, d'_i\}$, $i = 1, \cdots, n$ has no intersection with the corresponding β_{1i}, and all these intervals are maximal in $\{c_i, d_i\}$ out of the corresponding β_{1i}. Besides, it is clear that all but one elementary intervals from $\{c'_i, d'_i\}$, $i = 1, \cdots, n$ should be coincide with the corresponding $\{c_i, d_i\}$, and the presence of exactly one interval $\{c'_i, d'_i\}$ which does not contain β_{1i}, - is sufficient. Let $s_1, \cdots, s_k$ be the coordinates of $\tilde{\beta}_1$ such that the following relations take place simultaneously: $\beta_{1s_j} \neq c_{s_j}$ and $\beta_{1s_j} \neq d_{s_j}$ for $j = 1, \cdots, k$. Let $s_{k+1}, \cdots, s_n$ - are the rest of coordinates of $\tilde{\beta}_1$. In accordance with the reasoning above, we make the following table for maximal intervals after $\tilde{\beta}_1$:

$\{c_{11}, d_{11}\}, \{c_{12}, d_{12}\}, \cdots, \{c_{1n}, d_{1n}\}$

$\cdots$

$\{c_{\nu 1}, d_{\nu 1}\}, \{c_{\nu 2}, d_{\nu 2}\}, \cdots, \{c_{\nu n}, d_{\nu n}\}$

Here several cases are possible. $\nu = 1$ and $c_{1i} = c_i, d_{1i} = d_i, i = 1, \cdots, n$ if $\tilde{\beta}_1$ doesn't prick interval $\breve{U}$. This is when at least in one direction, β_{1i} is out of $\{c_i, d_i\}$ or when β_{1i} is the repeated (or boarder) point. This case is not possible in step 1 but can appear later in iterations. Other special cases appear when $\breve{U}$ became one dimensional. The general construction is as follows in supposition that $\breve{U}$ is of dimension n:

$$
c_{tr} = \begin{cases}
c_{s_j} & for \quad t = 2j-1 \quad and \quad r = s_j \quad if \quad \tau_{s_j} = 0 \\
\beta_{1s_j} + 1 & for \quad t = 2j \quad and \quad r = s_j \quad if \quad \tau_{s_j} = 0 \\
c_{s_j} & for \quad t = 2j-1 \quad and \quad r = s_j \quad if \quad \tau_{s_j} = 1 \\
\beta_{1s_j} & for \quad t = 2j \quad and \quad r = s_j \quad if \quad \tau_{s_j} = 1 \\
 & j = 1, \cdots, k \\
c_{s_j} & for \quad t = k+j \quad and \quad r = s_j \quad if \quad d_{s_j} = \beta_{1s_j} \\
c_{s_j} + 1 & for \quad t = k+j \quad and \quad r = s_j \quad if \quad c_{s_j} = \beta_{1s_j} \\
 & j = k+1, \cdots n
\end{cases}
$$

$$
d_{tr} = \begin{cases}
\beta_{1s_j} - 1 & for \quad t = 2j-1 \quad and \quad r = s_j \quad if \quad \tau_{s_j} = 0 \\
d_{s_j} & for \quad t = 2j \quad and \quad r = s_j \quad if \quad \tau_{s_j} = 0 \\
\beta_{1s_j} & for \quad t = 2j-1 \quad and \quad r = s_j \quad if \quad \tau_{s_j} = 1 \\
d_{s_j} & for \quad t = 2j \quad and \quad r = s_j \quad if \quad \tau_{s_j} = 1 \\
 & j = 1, \cdots, k \\
d_{is_j} - 1 & for \quad t = k+j \quad and \quad r = s_j \quad if \quad d_{s_j} = \beta_{1s_j} \\
d_{s_j} & for \quad t = k+j \quad and \quad r = s_j \quad if \quad c_{s_j} = \beta_{1s_j} \\
 & j = k+1, \cdots n
\end{cases}
$$

It is easy to check that the rows of the constructed above table are all maximal intervals in M out of the point $\tilde{\beta}_1$. For $s_{k+1},\cdots,s_n$ just change of the code sign on positive coordinates is applied. Assume that we have constructed a table whose rows are all maximal intervals in M out of $\{\tilde{\beta}_1,\tilde{\beta}_2,\cdots,\tilde{\beta}_{i-1}\}$. Then for constructing an analogous table specifying the set $S_{\{\tilde{\beta}_1,\tilde{\beta}_2,\cdots,\tilde{\beta}_i\}}$, it is sufficient to do the following: for each row of constructed in the previous stage table that specifies the maximal intervals of M out of $\{\tilde{\beta}_1,\tilde{\beta}_2,\cdots,\tilde{\beta}_{i-1}\}$, we construct the set of its maximal subintervals out of $\tilde{\beta}_i$, - and list them as strokes of corresponding tables. It is easy to check that among the maximal intervals out of $\{\tilde{\beta}_1,\tilde{\beta}_2,\cdots,\tilde{\beta}_{i-1}\}$, those that have no intersection with $\tilde{\beta}_i$, will not be changed, and thus the construction of subintervals out of $\tilde{\beta}_i$ is similar to the construction of maximal intervals of M out of $\tilde{\beta}_1$ given in the first step, and because instead of M we can deal with an interval which is maximal out of $\{\tilde{\beta}_1,\tilde{\beta}_2,\cdots,\tilde{\beta}_{i-1}\}$, and instead of $\tilde{\beta}_1$ we take $\tilde{\beta}_i$. We take into consideration the dimension and fixed coordinates of current interval to prick. It is worth to mention that all new intervals that we get in i th step, are pair wise different and are not contained in each other. Therefore for the final construction of $S_{\{\tilde{\beta}_1,\tilde{\beta}_2,\cdots,\tilde{\beta}_i\}}$ all we have to do is to remove from the set of all constructed intervals, those intervals constructed in i th step, which are included in others. The algorithm is complete.

Further, for recognition, we will consider the sets $S_i(\tilde{\gamma}_i)$ and $S_i(\tilde{\alpha},\tilde{\gamma}_i)$ of all maximal intervals of M out of $\{\tilde{\beta}_1,\tilde{\beta}_2,\cdots,\tilde{\beta}_{s_i}\}$ passing trough the points $\tilde{\gamma}_i$ and $\tilde{\alpha}$ and $\tilde{\gamma}_i$ respectively. The construction of these sets $S_i(\tilde{\gamma}_i)$ and $S_i(\tilde{\alpha},\tilde{\gamma}_i)$ can be done either in an analogue way as the construction of S_i, or they might be derived from S_i by choosing those maximal intervals that pass trough the points $\tilde{\gamma}_i$ and $\tilde{\alpha}$ and $\tilde{\gamma}_i$ respectively.

Let $\tilde{u}=(u_1,u_2,\cdots,u_n)$ is a point from M, where each coordinate u_i is equal either to c_i or to d_i. There are 2^n such points assuming that $c_i \neq d_i, i=1,\cdots,n$. Let $\tilde{u}_1,\tilde{u}_2,\cdots\tilde{u}_{2_n}$ denote these points. We call them corner/angular points of M. Let $\tilde{\alpha}\in\mathrm{M}$. We consider intervals of the form $\breve{U}(\tilde{\alpha},\tilde{u}_i)$, $i=1,\cdots,2^n$ in M. We call them intervals of directions $\tilde{u}_i$, outgoing from the point $\tilde{\alpha}$. Such way we get in M a system of independent directions, - outgoing from $\tilde{\alpha}$, which simplifies the study of such point $\tilde{\alpha}$. This study is fallen into iterations, studying the corresponding pictures/situations in spaces $\breve{U}(\tilde{\alpha},\tilde{u}_i)$, $i=1,\cdots,2^n$, where $\tilde{\alpha}$ itself is a corner point of these spaces.

Clearly the above constructions are valid for any subspace (intervals) of M. These are useful also for studying S_i sets of maximal intervals of the function $\tilde{f}_i$, $i=1,\cdots,m$, as well as for discovering interrelations of points of M and constructions given by the sets S_i.

Conclusion

The description above is an attempt to interlink several basic ideas of Logic Combinatorial pattern recognition. In the point of view given it appears that ideas are around the recovering more valid relations in the learning set. Learning set (plus the global hypotheses is there is one) is the only information about the classes and its best use is related to selecting its characteristic fragments and constructing the classification algorithms on this basis. Two examples considered are the testor scheme with pair of elements from different classes that are different, and logic separation with similarity spread – interruption fragment. These basic ideas were further initiated as the associated rule generation and incremental reduced error pruning schemes in Data Mining theory. After this methodological discussion the paper gives the algorithm of constructing the reduced interval covers for systems of discrete functions that just demonstrate the complexity of tasks and the similarity with the binary case.

Bibliography

[ChegisYablonskii, 1958] Chegis I. A. and Yablonskii S. V., Logical Methods for controlling electrical systems, Proceedings of Mathematical Institute after V. A. Steklov, 51: 270-360, Moscow (In Russian), 1958.

[Dmitriev et al, 1966] Dmitriev A. N., Zhuravlev Yu. I. and Krendelev F.P. On mathematical principles of object and phenomena classification, Discrete Analysis, 7: 3-15, Novosibirsk (In Russian), 1966.

[Bongard, 1967] Bongard M.M. The problem of cognition, Moscow, 1967.

[Vaintsvaig, 1973] Vaintsvaig M.N. Pattern Recognition Learning Algorithm KORA, Pattern recognition learning algorithms, Moscow, Sovetskoe Radio, 1973, pp.110-116.

[Yablonskii, 1958] Yablonskii S. V., Functional constructions in k-valued logic, Proceedings of Mathematical Institute after V. A. Steklov, 51: 5-142, Moscow (In Russian), 1958.

[Zhuravlev, 1958] Zhuravlev Yu. I., On separating subsets of vertices of n-dimensional unit cube, Proceedings of Mathematical Institute after V. A. Steklov, 51: 143-157, 1958.

[Baskakova, Zhuravlev, 1981] Baskakova L.V., Zhuravlev Yu.I. Model of recognition algorithms with representative and support sets, Journal of Comp. Math. and Math. Phys., 1981, Vol. 21, No.5, pp. 1264-1275.

[Zhuravlev, 1978] Zhuravlev Yu.I. On an algebraic approach to the problems of pattern recognition and classification, Problemy Kybernetiki, Vol. 33, 1978, pp.5-68.

[Zhuravlev, Nikiforov, 1971] Zhuravlev Yu.I., Nikiforov V.V. Recognition algorithms based on calculation of estimates. Cybernetics, Vol. 3, 1971, pp.1-11. [Aslanyan, 1975]

Aslanyan L. H. On a pattern recognition method based on the separation by the disjunctive normal forms. Kibernetika, 5, (1975), 103 -110.

[Zhuravlev, 1977] Zhuravlev Yu.I. Correct algebras over the sets of incorrect (heuristical) algorithms: I, Cybernetics, No.4, 1977, pp.5-17., II, Cybernetics, No.6, 1977., III, Cybernetics, No.2, 1978.

[Aslanyan, Zhuravlev, 2001] Aslanyan L., Zhuravlev Yu,.Logic Separation Principle, Computer Science & Information Technologies Conference, Yerevan, September 17-20, 2001, 151-156.

[Zhuravlev, 1998] Zhuravlev Yu. I., Selected Scientific Works (in russian), Magistr, Moscow, 1998, 417 p.

[Aslanyan, Castellanos, 2006] Aslanyan L. and Castellanos J., Logic based pattern recognition – ontology content (1), iTECH-06, 20-25 June 2006, Varna, Bulgaria, Proceedings, pp. 61-66.

[Aslanyan, Ryazanov, 2006] Aslanyan L. and Ryazanov V., Logic based pattern recognition – ontology content (1), iTECH-06, 20-25 June 2006, Varna, Bulgaria, Proceedings, pp. 61-66.

[Aslanyan, 1975] Aslanyan L. H. On a pattern recognition method based on the separation by the disjunctive normal forms. Kibernetika, 5: 103-110, 1975.

Aslanyan L. H. The Discrete Isoperimetry Problem and Related Extremal Problems for Discrete Spaces, Problemy Kibernetiki, 36: 85-128, 1979.

Authors' Information

Levon Aslanyan *– Head of Department, Institute for Informatics and Automation Problems, NAS RA, P.Sevak St. 1, Yerevan 14, Armenia, e-mail: lasl@.sci.am*

Hasmik Sahakyan *– Leading Researcher, Institute for Informatics and Automation Problems, NAS RA, P.Sevak St. 1, Yerevan 14, Armenia, e-mail: hasmik@ipia.sci.am*