ON HYPERSIMPLE wtt-MITOTIC SETS, WHICH ARE NOT tt-MITOTIC

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Abstract: *A T-complete wtt-mitotic set is composed, which is not tt-mitotic. A relation is found out between structure of computably enumerable sets and the density of their unsolvability degrees.*

Let us adduce some definitions:

A computably enumerable (c.e.) set is tt - mitotic (wtt - mitotic) set if it is the disjoint union of two c.e. sets both of the same tt -degree (wtt -degree) of unsolvability.

Let A be an infinite set. f majorize A if $(\forall n[f(n) \geq z_n])$, where $z_0, z_1, \ldots$ are the members of A in strictly increasing order.

A is hyperimmune (abbreviated h-immune) if A is infinite and ($\forall$ recursive f) [f does not majorize A].

A is hypersimple if A is c.e. and $\overline{A}$ is hyperimmune.

A is hyperhyperimmune if A is infinite and $\neg(\exists$ recursive $f)$ so that

[$(\forall u)[W_{f(u)}$ is finite $\& W_{f(u)} \cap A \neq \varnothing] \& (\forall u)(\forall v)[u \neq v \Rightarrow W_{f(u)} \cap W_{f(v)} = \varnothing]$].

A is hyperhypersimple if A is computably enumerable and $\overline{A}$ is hyperhyperimmune.

We shall denote T-degrees by small bold Latin letters.

A degree $\mathbf{a} \leq \mathbf{0}'$ is low if $\mathbf{a}' = \mathbf{0}'$ (i.e. if the jump $\mathbf{a}'$ has the lowest degree possible).

*Theorem (Martin [6]). **a** is the degree of a maximal set $\Leftrightarrow$ **a** is the degree of a hypersimple set $\Leftrightarrow$ [**a** is c.e. and $\mathbf{a}' = \mathbf{0}''$].*

*Theorem (R. Robinson [8]). Let **b** and **c** be c.e. degrees such that $\mathbf{c} < \mathbf{b}$ and **c** is low. Then there exist incomparable low c.e. degrees $\mathbf{a_0}$ and $\mathbf{a}_1$, such that $\mathbf{b} = \mathbf{a_0} \cup \mathbf{a_1}$ and $\mathbf{a}_i > \mathbf{c}$, for $i < 2$.*

*Griffiths ([3]) proved that there is a low c.e. T-degree **u** such that if **v** is a c.e. T-degree and **u** $\leq$ **v** then **v** is not completely mitotic.*

In this article it is proved the following theorem:

Theorem. *There exists a low c.e. T-degree **u** such that if **v** is a c.e. T-degree and **u** $\leq$ **v** then **v** contains hypersimple wtt -mitotic set, which is not tt -mitotic.*

From the abovementioned theorems of Martin and R. Robinson follows that it is impossible to replace hypersimple by hyperhypersimple.

Keywords: *computably enumerable (c.e.) set, mitotic, wtt -reducibility, tt -reducibility, hypersimple set, low degree.*

ACM Classification Keywords: *F. Theory of Computation, F.1.3 Complexity Measures and Classes.*

Introduction

We shall use notions and terminology introduced in [9], [10].

The definitions of tt - and wtt - reducibilities are from [9].

$\varphi(x)\downarrow$ denotes, that $\varphi(x)$ is defined, and $\varphi(x)\uparrow$ denotes, that $\varphi(x)$ is undefined.

Definition. The order pair **<<** $x_1,\cdots,x_k$ **>,** α **>**, where **<** $x_1,\cdots,x_k$ **>** is a k -tuple of integers and α is a k -ary Boolean function ($k>0$) is called a *truth-table condition* (or tt *-condition*) *of norm* k . The set $\{x_1,\cdots,x_k\}$ is called the *associated set of the* tt *-condition*.

Definition. The tt -condition **<<** $x_1,\cdots,x_k$ **>,** α **>**, is *satisfied* by A if $\alpha(c_A(x_1),\cdots,c_A(x_k))=1$, where c_A is characteristic function for A .

Each tt -condition is a finite object; clearly an effective coding can be chosen which maps all tt -conditions (of varying norm) onto ω , on condition that

$$(\forall x)\left(\max\{z \mid z \text{ is the member of associated set of the } tt\text{ - condition } x\}\le x\right).$$

Assume henceforth that a particular such coding has been chosen. Where we speak of “ tt -condition x ”, we shall mean the tt -condition with the code number x .

Definition. A is *truth-table reducible to* B (notation: $A\le_{tt} B$) if there is a computable function f such that for all x , [$x\in A \Leftrightarrow tt$ -condition $f(x)$ is satisfied by B]. We also abbreviate “truth-table reducibility” as “ tt -reducibility”.

Definition. A is weak truth-table reducible to B (notation: $A\le_{wtt} B$) if

($\exists z$)[$c_A=\varphi_z^B$ ($\exists$ computable f)

($\forall x$)[$D_{f(x)}$ contains all integers whose membership in B is used in the computation of $\varphi_z^B(x)$]].

Definition. A c.e. set is tt - mitotic (wtt - mitotic) set if it is the disjoint union of two c.e. sets both of the same tt -degree (wtt -degree) of unsolvability.

Let $A \leq_{tt} B$ and $(\forall x)$ [$x \in A \Leftrightarrow tt$ -condition $f(x)$ is satisfied by B] and $\varphi_n = f$. Then we say that $A \leq_{tt} B$ *by* φ_n.

Let us modify denotations defined in [4] with the purpose to adapt them to our theorem.

We say that $(A_o, A_1, \varphi_o, \varphi_1)$ is *tt -mitotic splitting of* A if A_o and A_1 are c.e., $A_o \bigcup A_1 = A$, $A_o \bigcap A_1 = \varnothing$, $A \leq_{tt} A_0$ by ψ_0 and $A \leq_{tt} A_0$ by ψ_0 $A \leq_{tt} A_1$ by ψ_1.

Let h be a recursive function from ω onto ω^4.

Define $(Y_i, Z_i, \vartheta_i, \psi_i)$ to be a quadruple $(W_{i_o}, W_{i_1}, \varphi_{i_2}, \varphi_{i_3})$, where $h(i) = (i_0, i_1, i_2, i_3)$. If A is c.e. then we say that the *non-* tt *-mitotic condition of* i *order is satisfied for* A , if it is not the case that $(Y_i, Z_i, \vartheta_i, \psi_i)$ is a tt *-mitotic splitting of* A .

Denotation. $u^{tt}(i,n,s) = \begin{cases} x^i_{k_n}, & if\ \varphi_{i,s},(n)\downarrow, \\ 0, & otherwise \end{cases}$,

where *tt*-condition $\varphi_i(n)$ = << $x^i_1, \cdots, x^i_{k_n}$ >, α^i_n > .

We define two computable functions that will be of use later.

1. $k(i,n,s) = \max\{n, \{u^{tt}(i_2, Y^s, m, s) : m \leq n\} \bigcup \{u^{tt}(i_3, Z^s, m, s) : m \leq n\}\}$,
2. $L(A,i,s) = \mu n[\neg(c_A(n) = 1 \Leftrightarrow tt\text{ - condition } \vartheta_i(n) \text{ satisfied by } Y_i) \vee$
$\neg(c_A(n) = 1 \Leftrightarrow tt\text{ - condition } \psi_i(n) \text{ satisfied by } Z_i)]$,

where $h(i) = (i_0, i_1, i_2, i_3)$.

Adduce some information, concerning hypersimple sets.

Definitions.

Let *A* be an infinite set. f *majorizes A* if $(\forall n[f(n) \geq z_n])$, where $z_0, z_1, \ldots$ are the members of *A* in strictly increasing order.

A is *hyperimmune* (abbreviated *h-immune*) if *A* is infinite and ($\forall$ recursive f) [*f* does not majorize *A*].

A is *hypersimple* if A is computably enumerable and $\overline{A}$ is hyperimmune.

A useful characterization of hyperimmune sets is given in the following theorem.

Theorem (Kuznecov, Medvedev, Uspenskii [7]). A is hyperimmune $\Leftrightarrow$ A is infinite and

$\neg(\exists \text{ recursive } f)[(\forall u)[D_{f(u)} \cap A \neq \varnothing] \ \& (\forall u)(\forall v)[u \neq v \Rightarrow D_{f(u)} \cap D_{f(v)} = \varnothing]]$.

Definitions.

A is *hyperhyperrimmune* if A is infinite and $\neg(\exists \text{ recursive } f)$

$[(\forall u)[W_{f(u)}$ is finite & $W_{f(u)} \cap A \neq \varnothing] \ \& (\forall u)(\forall v)[u \neq v \Rightarrow W_{f(u)} \cap W_{f(v)} = \varnothing]]$.

A is *hyperhypersimple* if A is computably enumarable and $\overline{A}$ is hyperhyperimmune.

A degree $\mathbf{a} \leq \mathbf{0}'$ is low if $\mathbf{a}' = \mathbf{0}'$ (i.e. if the jump $\mathbf{a}'$ has the lowest degree possible).

Theorem (Martin [6]). **a** is the degree of a maximal set $\Leftrightarrow$ **a** is the degree of a hypersimple set $\Leftrightarrow$ [**a** is c.e. and $\mathbf{a}' = \mathbf{0}''$].

Theorem (R. Robinson [8]). Let **b** and **c** be c.e. degrees such that $\mathbf{c} < \mathbf{b}$ and **c** is low. Then there exist incomparable low c.e. degrees $\mathbf{a_0}$ and $\mathbf{a}_1$, such that $\mathbf{b} = \mathbf{a_0} \cup \mathbf{a_1}$ and $\mathbf{a}_i > \mathbf{c}$, for $i < 2$.

Griffiths ([3]) proved that there is a low c.e. T-degree **u** such that if **v** is a c.e. T-degree and **u** $\leq$ **v** then **v** is not completely mitotic.

Let us prove the following theorem.

<u>Theorem.</u> There exists a low c.e. T-degree **u** such that if v is a c.e. T-degree and **u** $\leq$ **v** then **v** contains hypersimple wtt-mitotic set, which is not tt-mitotic.

From the abovementioned theorems of Martin and R. Robinson follows that it is impossible to replace hypersimple by hyperhypersimple.

Proof. This statement is proved using a finite injury priority argument. We construct a member U of $\mathbf{u}$ in stages s, $U=\bigcup_{s\in\omega}U_s$. We also construct sets $\{V_e\}_{e\in\omega}$ to witness that each c.e. T-degree in upper cone of $\mathbf{u}$ contains a wtt-mitotic but non-tt-mitotic set.

Denote $\omega^0=\{x:(\exists y)\,(2y=x)\},\ \omega^1=\omega\setminus\omega^0$.

Construct U, $\{V_e\}_{e\in\omega}$ to satisfy, for all $e\in\omega$, the requirements:

N_e: $\{e\}^U(e)\downarrow$ has a limit in s, the stage.

$R_{\langle e,i\rangle}$: The non-*tt*-mitotic condition of order i is satisfied for V_e.

$P_{<e,i>}$: $(\varphi_i$ is total computable & $(\forall u)[D_{f(u)}\cap A\neq\varnothing]$ &

$(\forall u)(\forall v)[u\neq v\Rightarrow D_{f(u)}\cap D_{f(v)}=\varnothing])\Rightarrow(\exists z)(D_{\varphi_i(z)}\subset V_e)$.

$\widetilde{P}_e$: $W_e=\Lambda^{V_e}$ for some computable functional Λ.

We also ensure by permitting that $V_e\equiv_T U\oplus W_e$ and else $V_e^0\equiv_{wtt}V_e^1$ (where $V_e^0=V_e\cap\omega^0$ & $V_e^1=V_e\cap\omega^1$).

If $U\leq_T W_e$ then the above ensure that $V_e\equiv_T U\oplus W_e\equiv_T W_e$ and V_e is not *tt*-mitotic. Hence, $deg(W_e)$ is not *tt*-mitotic but is wtt-mitotic, and $\mathbf{u}=deg(U)$ is the required degree.

Let $\langle\ ,\ \rangle$ be computable bijective pairing function increasing in both coordinates. At each stage s place markets $\lambda(e,x,s)$ on elements of $\overline{V}_{e,s}$. Values of λ will be used both as witnesses to prevent the *tt*-mitoticity of V_e sets (by corresponding $Y_i,Z_i,\vartheta_i,\psi_i$) and to ensure that W_e is T-reducible to V_e. Initially

$\lambda(e,x,0)=4(\langle e,x\rangle+1)-2)$ for all $e,x\in\omega$.

Also define a function $\xi(e,i,s)$ for all $e,i\in\omega$ (at each stage s), $\xi(e,i,0)=i$ for all $e,i\in\omega$. We use ξ to ensure that only members of sufficiently large magnitude enter U at stage s, so we can satisfy the lowness requirements N_e.

According to the theorem (Kuznecov, Medvedev, Uspenskii) the satisfaction of $P_{<e,i>}$ (for all i) ensure the hypersimplicity of V_e.

Order the requirements in the following priority ranking:

$N_0,R_0,P_0,N_1,R_1,P_1,N_2,R_2,P_2,\ldots$

The $\{\widetilde{P}_e\}_{e\in\omega}$ do not appear in this ranking.

N_e *requires attention* if it is not *satisfied* and $\{e\}^U(e)[s]\downarrow$.

$R_{\langle e,i\rangle}$ *requires attention* if it is not *satisfied* and

$(\forall x_{\leq y})\left(\vartheta_i^s(x)\downarrow \ \& \ \psi_i^s(x)\downarrow\right)$, where $y=\lambda(e,\xi(e,i,s)s)$.

$(Y_i,Z_i,\vartheta_i,\psi_i)$ is *threatening A through x at stage s* if it is partially satisfied and all the following hold:

i) $i\leq s$,

ii) $x<L(A,i,s)$,

iii) $Y_i^s\bigcap Z_i^s=\varnothing$,

iv) $c_A^s(m)=(Y_i^s\bigcup Z_i^s)(m)$ for all $m\leq k(i,x,s)$.

(Note, that actually $R_{\langle e,i\rangle}$ is partially satisfied, if $R_{\langle e,i\rangle}$ requires attention (via some $y=\lambda(\ ,\ ,s)$) and corresponding $y-1,\ y-2$ belong to V_e, $y-1$ belongs to U_{s+1}. See Construction, Part A , a)).

We will build $U=\bigcup_s U_s$ and $V_s=\bigcup_s V_{e,s}$ for all $e\in\omega$. Initially all requirements N_e, $R_{\langle e,i\rangle}$ are declared *unsatisfied*.

Construction

Stage $s=0$. Let $U_0=\varnothing$, $V_{e,0}=\varnothing$ for all $e\in\omega$.

Stage $s+1$.

Part A. Act on the highest priority requirement which requires attention, if such a requirement exists.

a) If N_e *requires attention* then set $\xi\left(\hat{e},\hat{i},s+1\right)=\xi\left(\hat{e},\hat{i}+s,s\right)$ for each $\langle\hat{e},\hat{i}\rangle\geq e$. This action prevents injury to N_e by lower priority requirements as we assume that s bounds the use of the halting computation.

Define $\xi(\ ,\ ,s+1)$ not specified in Part A to be the same as $\xi(\ ,\ ,s)$.

Declare N_e satisfied; declare all lower priority R, N unsatisfied.

If $R_{\langle e,i\rangle}$ require attention via $y=\lambda(e,\xi(e,i,s+1),s)$ then set $\widetilde{V}_{e,s+1}=V_{e,s}\bigcup\{y-1,y-2\}$ and $\widetilde{U}_{s+1}=U_s\bigcup\{y-1\}$. (Note that such $\langle e,i\rangle$ cannot be $\geq e$, so $\xi(e,i,s+1)=\xi(e,i,s)$). Declare $R_{\langle e,i\rangle}$ partially satisfied.

Define $\widetilde{V}_{e,s+1},\widetilde{U}_{e,s+1}$ not specified in Part A, a) to be the same as $V_{e,s+1},U_{e,s+1}$ respectively.

b) If ($Y_i,Z_i,\vartheta_i,\psi_i$) is threatening $\widetilde{V}_{e,s+1}$ through y at stage $s+1$ (so $R_{\langle e,i\rangle}$ is partially satisfied via $y=\lambda(e,\xi(e,i,s),s)$), then set $\widetilde{\widetilde{V}}_{e,s+1}=\widetilde{V}_{e,s+1}\bigcup\{y\}$ and $\widetilde{\widetilde{U}}_{s+1}=\widetilde{U}_{s+1}\bigcup\{y\}$.

If $R_{\langle e,i\rangle}$ is partially satisfied, via $y=\lambda(e,\xi(e,i,s),s)$, whether ($Y_i,Z_i,\vartheta_i,\psi_i$) is threatening $\widetilde{V}_{e,s+1}$ through y at stage $s+1$ or not define $\lambda^1(e,\xi(e,i,s),s+1)=$ $\lambda(e,\xi(e,i+s,s),s)$.

Define $\widetilde{\widetilde{V}}_{e,s+1},\widetilde{\widetilde{U}}_{e,s+1},\lambda^1(\ ,\ ,s+1)$ not specified in Part A, b) to be the same as $\widetilde{V}_{e,s+1},\widetilde{U}_{e,s+1},\lambda(\ ,\ ,s)$ respectively.

Such definition of λ^1 allow us to satisfy $R_{\langle e,i\rangle}$ requirement (after Part A) whether ($Y_i,Z_i,\vartheta_i,\psi_i$) is threatening $\widetilde{V}_{e,s+1}$ through y or not (if don't take into consideration higher priority requirements).

Declare $R_{\langle e,i\rangle}$ satisfied; declare all lower priority $R,\ N$ unsatisfied.

Part B. If $x\in W_{e,s+1}\setminus W_{e,s}$ then set

$$V^*_{e,s+1}=\widetilde{\widetilde{V}}_{e,s+1}\bigcup\left\{\lambda^1(e,x,s+1)\right\} \quad \text{and}$$

$$\lambda^2(e,x+j,s+1)=\lambda^1(e,\xi(e,x+j+1,s+1),s+1) \text{ for all } j\in\omega.$$

Find all $\hat{i}$ such that $\lambda(e,\xi(e,\hat{i},s+1),s)\geq\lambda^1(e,x,s+1)$ and declare $R_{\langle e,\hat{i}\rangle}$ *unsatisfied* for each such $\hat{i}$.

Define $V^*_{e,s+1},\lambda^2(\ ,\ ,s+1)$ not specified in Part B to be the same as $\widetilde{\widetilde{V}}_{e,s+1},\lambda^1(\ ,\ ,s+1)$ respectively.

Note that for all $s,\ \xi(e,i,s)$ is increasing in both e and i.

Part C. Let $m_{s+1}=\max\left\{\{\xi(\hat{e},\hat{i},s+1)\right.\mid$ for all $\hat{e},\hat{i}$ with $\langle\hat{e},\hat{i}\rangle\leq\langle e,i\rangle\}$,

$$\{\lambda^2(\hat{e},\hat{\imath},s+1) \mid \text{for all } \hat{e},\hat{\imath} \text{ with } \langle \hat{e},i\rangle \le \langle e,i\rangle\}\} .$$

If $(\exists\, z)(\varphi_{i,s+1}(z)\downarrow)$, denote $z_0 = \mu y\,(y = D_{\varphi_i}(z))$. Then, if $z_0 > m_0$ and $P_{\langle e,i\rangle}$ is not satisfied,

set $(\forall y)\,(y = D_{\varphi_i}(z)) \Rightarrow V_{e,s+1} = V^*_{e,s+1} \cup \{y, y+1\}\ \&U_{s+1} = \tilde{\tilde{U}}_{s+1} \cup \{z_0\}$.

Set $\lambda\,(e,\xi(e,\hat{\imath},s+1),s+1) = \lambda^2(e,\xi(e,\hat{\imath}+s,s+1),s+1)$, for all $\hat{\imath} \ge i$.

Define $V_{e,s+1}, U_{e,s+1}, \lambda(\,,\,,s+1)$ not specified in Part C, to be the same as $V^*_{e,s+1}, \tilde{\tilde{U}}_{e,s+1}, \lambda^2(\,,\,,s+1)$ respectively.

Declare $P_{\langle e,i\rangle}$ *satisfied*, declare all lower priority R, N unsatisfied.

Verification

Lemma 1. *For all* e, i*:*

1. N_e is met, $\lim_s \xi(e,i,s) = \xi(e,i)$ exists.
2. $R_{\langle e,i\rangle}$ is met, $\lim_s \lambda(e,\xi(e,i,s),s)$ exists.

Proof. By induction on $j = \langle e,i\rangle$.

Suppose there exists a stage s_0 such that for all $\hat{e},\hat{\imath}$ with $\langle \hat{e},\hat{\imath}\rangle < j$:

1. $N_{\langle \hat{e},\hat{\imath}\rangle}$ is met and never acts after stage s_0, $\lim_s \xi(\hat{e},\hat{\imath},s) = \xi(\hat{e},\hat{\imath})$ exists and is attained by s_0.

2. $R_{\langle \hat{e},\hat{\imath}\rangle}$ is met and never acts after stage s_0, $\lim_s \lambda(\hat{e},\xi(\hat{e},\hat{\imath},s),s)$ exists and is attained by s_0.

1). The proof of point 1 is similar to Lemma 1 of Theorem 2.2.2 [3].

After stage s_0 the requirements $N_{\langle \hat{e},\hat{\imath}\rangle}$, $R_{\langle \hat{e},\hat{\imath}\rangle}$ (for all $\hat{e},\hat{\imath}$ with $\langle \hat{e},\hat{\imath}\rangle < j$) do not injury N_j. Positive requirements $P_{\langle \hat{e},\hat{\imath}\rangle}$ for all $\hat{e},\hat{\imath}$ with $\langle \hat{e},\hat{\imath}\rangle < j$, can injury N_j only finitely. So there is stage s_1 after which if N_j receives attention, then it is met and never injured, so there is a $s_2 > s_1$ after which N_j does not receive attention. (Else set $s_2 = s_1$). Thus $\xi(e,i,s_2+1) = \xi(e,i)$, because $\xi(e,i,s_2+1)$ is not changed after.

2). Now consider point 2. Note, that positive requirements $P_{\langle e,i\rangle}$, $\tilde{P}_e$ injury each of the requirements with lower priority only finitely.

Let the stage s_1 is such stage, that $s_1 > s_0$ and $N_{\langle \hat{e},\hat{i} \rangle}$, $R_{\langle \hat{e},\hat{i} \rangle}$, $P_{\langle \hat{e},\hat{i} \rangle}$, $\widetilde{P}_{\langle \hat{e},\hat{i} \rangle}$ (for all $\hat{e},\hat{i}$ with $\left\langle \hat{e},\hat{i} \right\rangle < j$) are met and never acts after stage s_1 .

The following Lemma is used (in [4], [5]) for building the non-T -mitotic set :

Lemma. If $\left(Y_i, Z_{i,} \theta_i, \psi_i\right)$ is threating A through x at stage $s, x \in A - A_s$ and for all $m \neq x$ such that $m \leq k(i,x,s)$ we have $A_m - A_s(m)$, then the non-T -mitotic condition of order i is satisfied for A .

Similar lemma is thrue for tt -reducibility.

Let s_0 is such stage that $N_{<\hat{e},\hat{i}>}$, $R_{<\hat{e},\hat{i}>}$ $P_{<\hat{e},\hat{i}>}$ are met and never acts after stage s_1 .

If there isn't such $y = \lambda(e, \xi(e,i,s'+1), s')$ (where $s' > s_1$), that $R_{\langle e,i \rangle}$ is partially satisfied (via y), then $R_{\langle e,i \rangle}$ is met.

If there exists such y and $\left(Y_i, Z_{i,} \vartheta_i, \psi_i\right)$ never threatens V_e through y after stage s' , then certainly the condition is satisfied. On the other hand, if $\left(Y_i, Z_{i,} \vartheta_i, \psi_i\right)$ threatens V_e through y at time $t > s'$, then put y into V_e at time $t+1$, and never put any other number $\leq k(i,y,t)$ into V_e after stage t , so $R_{\langle e,i \rangle}$ is met.

Lemma 2. $P_{\langle e,i \rangle}$ is met.

According to Lemma1 $(\exists\, s_0)\left[\xi(\hat{e},\hat{i},s_0) = \xi(\hat{e},\hat{i})\right.$, for all $\hat{e},\hat{i}$ with $(\hat{e},\hat{i}) \leq (e,i)$ &

$\& \lambda\left(\hat{e}, \xi(\hat{e},\hat{i},s_0), s_0\right) = \lambda\left(\hat{e}, \xi(\hat{e},\hat{i})\right)$ for all $\hat{e},\hat{i}$ with $\left.(\hat{e},\hat{i}) \leq (e,i)\right]$.

Denote $m_0 = \max\left\{\{\xi(\hat{e},\hat{i}) \mid \text{for all } (\hat{e},\hat{i}) \leq (e,i)\}, \{\lambda\left(\hat{e}, \xi(\hat{e},\hat{i})\right) \mid \text{for all } \hat{e},\hat{i} \text{ with } (\hat{e},\hat{i}) \leq (e,i)\}\right\}$.

Then if $(\varphi_i$ is total computable & $(\forall u)[D_{f(u)} \cap A \neq \varnothing]$ &

$(\forall u)(\forall v)[u \neq v \Rightarrow D_{f(u)} \cap D_{f(v)} = \varnothing]) \Rightarrow (\exists s_{>s_0})(\exists z)\left(\varphi_{i,s}(z)\downarrow \;\&\; \{0,1,\ldots,m_0\} \cap D_{\varphi_i(z)} = \varnothing\right)$.

Thus, according to constraction $P_{\langle e,i \rangle}$ is met, because $D_{\varphi_i(z)}$ enters V_e .

Lemma 3 . $V_e \leq_T U \oplus W_e$.

Proof. By permitting: in the construction a number k enters V_e only if a number less than or equal to k enters U or enters W_e.

Lemma 4. *For all* e *,* $\widetilde{P}_e$ *is satisfied, that is* $W_e = \Lambda^{V_e}$.

Proof. To determine whether $z \in W_e$ we need to find a stage such that $\lambda(e,z,s)$ has attained its limit. V_e computably determines $\lambda(e,0),\ldots,\lambda(e,z)$ (note that $\lambda(e,y,s)$ changes only if a number $\leq \lambda(e,y,s)$ enters V_e).

Find a stage s_z such that $V_{e,s_z} \upharpoonright \gamma_z + 1 = V_e \upharpoonright \gamma_z + 1,$ where $\gamma_z = \max\{\lambda(e,0),\ldots,\lambda(e,z)\}$. Then $z \in W_e$ iff $z \in W_{e,s_z}$.

Lemma 5. V_e *is* wtt *-mitotic.*

Proof. 1) Prove $V_e^0 \leq_{wtt} V_e^1$ (and hence $V_e \leq_{wtt} V_e^1$).

To determine whether $x \in V_e^0$ find such stage s , that $V_{e,s}^1 \upharpoonright x+2 = V_e^1 \upharpoonright x+2$. Then $x \in V_e^0 \Leftrightarrow x \in V_{e,s}^0$, because

i) if $\neg \exists i, \hat{s}$, such that ($Y_i, Z_i, \vartheta_i, \psi_i$) is threatening $\widetilde{V}_{e,s+1}$ through x at stage $\hat{s}$, then $x \in V_e^0$, only if a number less than or equal to $x+1$ enters,

ii) otherwise, then find a stage s' (this stage s' obligatory is $\leq s$) such that $x-1 \in V_e^1$. If after the stage s' such changes happen in $V_{e,s}^1 \upharpoonright x+2$, which lead to displacement of marker $\lambda(e,x,s')$ and we have $x \notin V_{e,s}^0$, then $x \notin V_e^0$. Thus $x \in V_e^0 \Leftrightarrow x \in V_{e,s}^0$.

2) Prove $V_e^1 \leq_{wtt} V_e^0$.

The proof is similar to abovementioned in item 1), only without point *ii).*

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