

PREDICTIVE MODELING OF SPATIAL REDISTRIBUTION IN DYNAMICAL MODELS OF GLOBAL VEGETATION PATTERNS UNDER CLIMATE CHANGE

Nelli Ajabyan

Abstract: *The paper discusses stability concepts and methods of stability investigation in models describing the climate-biosphere interaction for a hypothetical zero-dimensional model. The climate-biosphere model proposed by [Svirezhev-& von Bloh, 1997], exhibited interesting features concerning the stability and sensitivity to perturbations of the global system. The work is focused on the determination of transition times between the equilibriums due to random perturbations. In investigate bifurcations in the model to describe complex behavior in the model with multiple equilibriums. Global vegetation pattern dynamics strongly depends on production and hence evolutionary models must explicitly use this concept. We have shown analytically that GVP dynamics could demonstrate complex behavior with multiple equilibria, chaotic dynamics associated with them as well as transition zones between consequent states.*

Keywords: *global vegetation pattern, modeling*

ACM Classification Keywords J.2 PHYSICAL SCIENCES AND ENGINEERING, G.1.7 Ordinary Differential Equations: *Chaotic systems*

Introduction

Global vegetation pattern dynamics strongly depends on production and hence evolutionary models must explicitly use this concept. The description of global vegetation dynamics explicitly including the production as parameter was suggested in [Svirezhev Y. 2000]. In this model the influence of different factors on vegetation dynamics on concentrated in the function that describes the dependence of production on them. The evolution of GVP under climate change is described by the change of model parameters. It was shown that using the formalism of Lotka-Volterra model for competing species it revealed possible to construct some discrete structures remaining meanwhile in the framework of a continuous model description. In this work we will investigate bifurcations in the same model to describe complex behavior in the model with multiple equilibria.

Competition models, including diffusive Lotka-Volterra models, proved to be subject of intensive exploration recently. The neutral stability theory has been widely applied to modeling of species richness in tropical forests. One model developed by Tilman [Tilman, 1994] was used to give alternative explanation to that of neutral theory.

Bampfyld [Bampfyld, 2005] adapted Tilman's competition model to gain deeper understanding of mechanisms for coexistence of many species exhibited in rain forests.

Vandermeer [Vandermeer, 2004] provided examples demonstrating complicated patterns in tree species distribution of tropical rain forests. He proposed that transient dynamics represents the continual shuffling among various basins of attraction under perturbation caused by disturbance events repeatedly affecting the forest. The most interesting point here, as it is noted in [Bampfyld, 2005], is finding empirical evidence since the recruitment limitation (defined as the failure of species to colonize a suitable vacant site) is acute in forests when generation times are many decades.

An attempt is made for analytical investigation of special class of Lotka-Volterra competition models directed to description of global vegetation pattern proposed by Svirezhev [Svirezhev, 2000]. The state variables are interpreted as different types of vegetation which correspond to the density of living biomass of a selected type at some geographical point (x, y) . We study the change of dynamics of model with respect to discrete values of parameters with further analysis of stability under continuous in some vicinity of a selected value. Under this condition the model can be described by ordinary differential equations. A movement in the transition zone between types of vegetation then is determined in parametric space rather than considering solutions of partial differential equation system. The most important is GVP map, which represents a geographical distribution of different types of vegetation. As it is indicated in [Svirezhev Y. 2000] the map is a space of discrete structure. To interpret the results we will use the approach developed for the original model, based on definition of a mapping from parameter space to geographical points.

The complex behavior is formally determined in the general dynamic model with three species when unperturbed system can itself possess chaotic orbits. The third component is included in the model as a pseudo-type designed to integrate different disturbances in the system, examples of which present seasonal growth of parasite insects or insect invasion, consequences of fires that cause a relatively long term impact on the change of dynamics. The resulting system hence is a dynamic system of one predator with two competing species as a resource type. In this model free spaces which occur after some events can be latter occupied by the species with faster response to occurrence of fruiting conditions. In a long term scale later it can be replaced by the second competitor, thus moving the border between species. The latter is not necessarily a straight line on a specific spot but could be a line of wavelike form.

The method of investigation is based on identifying invariant sets of the dynamical system. The smallest of them are the equilibria. The stability of an equilibrium can change in result of bifurcations as parameters of the system change. New invariant sets defined in result through a prescribed tuning of multiple parameters in the system. These sets are idealizations but are associated with a range of definite behavior in case of slightly weaker assumptions on parameters than the tuned ones. In more than two-dimensional case the invariant sets are destroyed by local or global bifurcations. Local bifurcations may generate periodic solutions or new homoclinic or heteroclinic cycles, while global destroy links between two invariant sets. It has been recognized recently (see, for

example, J. Porter, 2004, Magnitski 2004) that heteroclinic cycles are associated with bursting and intermittent behavior, as well as give rise to chaotic dynamics. In general the trajectories escape from the invariant sets when small symmetry-breaking terms are considered or under stochastic perturbations. A weakly broken symmetry gives rise to intermittent dynamics in some nearby invariant region, while noise may allow the trajectories to "jump across" the invariant space. Due this process of potential random switching the partition of phase space and definition of transitions through it is very important for predicting the behavior of the system

Routes to chaos and parameter combinations in which there exist alternative limit cycles need detailed examination to reveal whether the jumps in dynamics will be attained in a real system.

1. Model description

To consider dynamics of two-species model we assume, following approach in [Svirezhev Y. 2000], that types of vegetation are ordered along a special coordinate x , for instance along a meridian. Then the model is described by the following system:

$$\begin{aligned}\frac{\partial B_1}{\partial t} &= B_1(\varepsilon_1(x) - \gamma_{11}B_1 - \gamma_{12}B_2) \\ \frac{\partial B_2}{\partial t} &= B_2(\varepsilon_2(x) - \gamma_{21}B_1 - \gamma_{22}B_2)\end{aligned}\quad (1.1)$$

where $\varepsilon_i(x) = \varepsilon_i(T(x), H(x)) = g_i(T(x), H(x)) - m_i = g_i(x) - m_i$

We will further adapt the model by assuming ε_i being a parameter that can vary in some limits, but we exclude direct indication of dependence from spatial coordinate x . Then the species compete for a site the characteristics of which are specified by ε_i . In this system one type of vegetation could replace the other or they could coexist, the dynamics of whole area is determined by the dynamics of spots, the system is similar to that of the metapopulation model. The next change consists in introduction of scaled variables defined by the following:

$$\begin{aligned}\gamma_{11}B_1 &\rightarrow \tilde{B}_1 & a_2 &= \frac{\gamma_{12}}{\gamma_{11}} & \gamma_{11} &= \gamma_{22} = 1 \\ \gamma_{22}B_2 &\rightarrow \tilde{B}_2 & a_1 &= \frac{\gamma_{21}}{\gamma_{11}} & \gamma_{12} &= a_2, \gamma_{21} = a_1\end{aligned}$$

However we will use the same notation B_i for the sake of simplicity, then the model is described by the following autonomous system of ordinary differential equations

$$\begin{aligned}\frac{dB_1}{dt} &= B_1(\varepsilon_1 - B_1 - a_2B_2) \\ \frac{dB_2}{dt} &= B_2(\varepsilon_2 - a_1B_1 - B_2)\end{aligned}\quad (1.2)$$

The state (0, 0) is a steady state for (1.2). i.e. dead spaces without any vegetation or free gaps are assumed in this model. The non-trivial equilibria of the system are:

1. $\{B_1^{(1)}, 0\}$ $B_1^{(1)} = \varepsilon_1$
2. $\{B_2^{(1)}, 0\}$ $B_2^{(1)} = \varepsilon_2$
3. $\{B_1^{(2)}, B_2^{(2)}\}$ $B_1^{(2)} = \frac{\varepsilon_2 - a_2\varepsilon_1}{1 - a_1a_2}$ $B_2^{(2)} = \frac{\varepsilon_1 - a_1\varepsilon_2}{1 - a_1a_2}$

From the linear stability conditions the first equilibrium is a stable node if both ε_i are negative since the eigenvalues of the system at this equilibrium are $\lambda_i = \varepsilon_i$ and it is unstable if ε_i are positive. In fact positive values provide growth for both competitors, which is limited by carrying capacity of a spot or growth thresholds for the competing species.

The origin is dead space if ε_i are zero; when they are negative the system to asymptotically stable but dead biomass state. The eigenvalues calculated for the second fixed point $\{B_2^{(1)}, 0\}$ are $\lambda_1 = \varepsilon_1 - a_2\varepsilon_2$, $\lambda_2 = -\varepsilon_2$

The table summarizes the description of stability regions for the state $\{B_2^{(1)}, 0\}$

$\varepsilon_2 > 0$ $\varepsilon_1 > 0$	$\varepsilon_1 < a_2\varepsilon_2$	asymptotically stable coexistence	
	$\varepsilon_1 > a_2\varepsilon_2$	saddle point unstable B_1 is dominant	
	$\varepsilon_1 = a_2\varepsilon_2$	the stability is determined by nonlinear part	
$\varepsilon_2 < 0$ $\varepsilon_1 > 0$	$\lambda_2 > 0$ $\lambda_1 > 0$	unstable	species coexistence
$\varepsilon_2 < 0$ $\varepsilon_1 < 0$	$\lambda_2 > 0$ $\lambda_1 = \varepsilon_1 + a_2\varepsilon_2$ $\lambda_1 < 0, a_2 \varepsilon_2 < \varepsilon_1$	saddle point unstable B_1 dominant	
	$a_2 \varepsilon_2 > \varepsilon_1$	$\lambda_1 > 0, \lambda_2 > 0$ both species develop	
$\varepsilon_2 > 0$ $\varepsilon_1 < 0$	$\lambda_2 < 0, \lambda_1 > 0$	asymptotically stable $ \varepsilon_1 < a_2\varepsilon_2$	
	$a_2 \varepsilon_2 < \varepsilon_1$	unstable	

By symmetry we can simply describe the state $\{B_1^{(1)}, 0\}$ eigenvalues for which are $\lambda_1 = -\varepsilon_1$, $\lambda_2 = \varepsilon_2 - a_1\varepsilon_1$ asymptotically stable

1. $\varepsilon_i > 0, \varepsilon_2 < a_1\varepsilon_1$
2. $\varepsilon_i < 0, \varepsilon_2 < 0, \lambda_i > 0$ Unstable
3. $\varepsilon_i < 0$ saddle point if $\varepsilon_2 > a_1|\varepsilon_1|$ otherwise $\lambda_i > 0$

The characteristic equation for $(B_1^{(2)}, B_2^{(2)})$ is

$$\lambda^2 + \lambda \frac{\varepsilon_2(a_2 - 1) - \varepsilon_1(a_1 - 1)}{a_1a_2 - 1} + M = 0$$

$$\text{Where } M = \varepsilon_1\varepsilon_2 + \frac{a_1\varepsilon_1^2}{a_2a_1 - 1} + \frac{a_2\varepsilon_2^2}{a_2a_1 - 1} + \frac{\varepsilon_1(a_2a_1 + 1)}{a_2a_1 - 1}$$

Then the bifurcation parameter is: $M_c = 0$ if $a_2 = a_1 \neq 1$

The parameter can become zero under

$$\varepsilon_2 = -\varepsilon_1 \ \& \ a_2 = a_1, M_c = \varepsilon_1 + \varepsilon_2 - (a_1\varepsilon_1 + a_2\varepsilon_2), \lambda_i = -\frac{1}{2}M_c \pm \sqrt{\frac{M_c^2}{4} - M}$$

The system will have purely imaginary eigenvalues $\pm i\sqrt{M}$ at $\mu=0$, since the values $a_1 = a_2 = 1$

are restricted and interspecific competition parameters are always positive M will be always positive, but it can become zero only at $\varepsilon_i = 0$ for both i .

Thus when ε_i take values on the straight line l the limit cycle arises from Hopf bifurcation (Figure 1.).

We can check the condition $\frac{\partial \text{Re } \lambda(\mu)}{\partial \mu} > 0$, where $\frac{\partial \text{Re } \lambda(\mu)}{\partial \mu} = \varepsilon_1 + \varepsilon_2 - (a_1\varepsilon_1 + a_2\varepsilon_2)$.

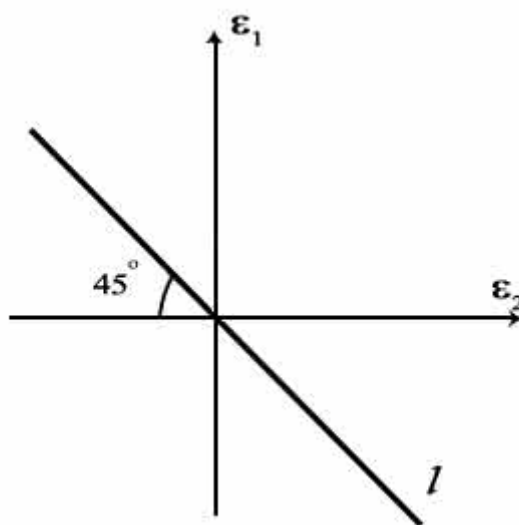


Figure 1.

For the points in part I the expression will be positive, then the system has a stable focus, while in the upper part II it will become negative, hence $\lambda_i > 0$. This means that at $\mu=0$ oscillatory solutions occur in the system with an amplitude proportional to $\sqrt{\rho}$ and the period $T \approx \frac{2\pi}{g}$, where $\lambda_i = \rho \pm i g$. At $\varepsilon_i = 0$ this point becomes a point of center type and it is attracting from one side and repelling from the other. In the vicinity of the limit cycle the system can be brought to the normal form defined as

$$\dot{B}_1 = -gB_2 + [\rho \pm (B_1^2 + B_2^2)]B_1$$

$$\dot{B}_2 = -gB_1 + [\rho \pm (B_1^2 + B_2^2)]B_2$$

where the signs +, - stand for subcritical and supercritical bifurcations accordingly. By estimating the nonlinear component for B_i it is possible to prove that the bifurcation is supercritical in large interval of parameter values. The corresponding transition is soft since a continuous change occurs.

Consider the transition in the system for the particular case with parameter values determined by

$$\gamma_{12} = a_2 = 2, \gamma_{21} = a_1 = 2$$

The system in nontrivial equilibrium $(B_1^{(2)}, B_2^{(2)})$, $B_1^{(2)} = \frac{2\varepsilon_2 - \varepsilon_1}{3}$, $B_2^{(2)} = 2\varepsilon_1 - \varepsilon_2$ has Jacobi matrix

$$\begin{vmatrix} \frac{-7\varepsilon_1 + 4\varepsilon_2}{3} & \frac{\varepsilon_1 - 2\varepsilon_2}{3} \\ -2(2\varepsilon_1 - \varepsilon_2) & \frac{8\varepsilon_2 - 10\varepsilon_1}{3} \end{vmatrix}$$

$$\frac{dB_1}{dt} = B_1(\varepsilon_1 - B_1 - 2B_2)$$

$$\frac{dB_2}{dt} = B_2(\varepsilon_2 - 2B_1 - B_2)$$

$$\lambda^2 - \lambda \frac{12\varepsilon_2 - 17\varepsilon_1}{3} + \frac{82\varepsilon_1^2 + 44\varepsilon_2^2 - 106\varepsilon_1\varepsilon_2}{9} = 0$$

The eigenvalues are

$$\lambda_{1,2} = -\frac{17\varepsilon_1 - 12\varepsilon_2}{6} \pm \sqrt{\left(\frac{17\varepsilon_1 - 12\varepsilon_2}{6}\right)^2 - \frac{82\varepsilon_1^2 + 44\varepsilon_2^2 - 106\varepsilon_1\varepsilon_2}{9}} \quad (1.3)$$

From (3) it is clear that bifurcation of the limit cycle is possible if $17\varepsilon_1 = 12\varepsilon_2$

At this point the has a pair of imaginary eigenvalues given by

$$\lambda_i \approx \pm \varepsilon_1 i \sqrt{21,6}, \text{ where } \varepsilon_2 = \frac{17}{12} \varepsilon_1 \approx 1,4\varepsilon_1$$

The necessary conditions for Hopf bifurcation hold, the imaginary part will not turn to zero unless $\varepsilon_2 \neq 0$.

2. Routes to chaos in two-species dynamic model with disturbance factors.

To consider dynamics of two-species model we assume, following the approach in [Svirezhev, 2000], that types of vegetation are ordered along a special coordinate x , for instance along a meridian. We will further adapt the model by assuming ε_i being a parameter that can vary in some limits, but we will exclude direct indication of dependence from spatial coordinate x . Then the species compete for a site the characteristics of which are specified by ε_i . In this system one type of vegetation could replace the other or they could coexist, the dynamics

of whole area is determined by the dynamics of spots, the system is similar to that of the metapopulation model. The next change consists in introduction of scaled variables defined by the following:

$$\begin{aligned} \gamma_{11}B_1 \rightarrow \tilde{B}_1 & \quad a_1 = \frac{\gamma_{21}}{\gamma_{11}} & \quad \gamma_{11} = \gamma_{22} = 1 \\ \gamma_{22}B_2 \rightarrow \tilde{B}_2 & \quad a_1 = \frac{\gamma_{12}}{\gamma_{11}} & \quad a_2 = \gamma_{12}, \gamma_{21} = a_1 \end{aligned}$$

However we will use the same notation B_i for the sake of simplicity, then the model is described by the following autonomous system of ordinary differential equations:

$$\begin{aligned} \dot{B}_1 &= B_1(\varepsilon_1 - B_1 - a_2B_2) \\ \dot{B}_2 &= B_2(\varepsilon_2 - B_2 - a_1B_1) \end{aligned} \quad (2.1).$$

To define shifts in dynamics of the two-species model generated by factors that can make severe impact such as fires we introduce a new variable B_3 . We assume B_3 time dependent, dynamics of which is given by the equation

$$\dot{B}_3(t) \cong e^t \int f_3(B_1, B_2) dt$$

We assume B_3 of the order ε but are interested in impact on the system dynamics provided on relatively short time intervals. Since f_3 is a vanishing function of order ε we will apply averaging and write

$$\dot{B}_3(t) \cong e^{\beta t} \left\{ const + \frac{1}{T} \int_0^t f_3(B_1, B_2) dt \right\}$$

The three variable systems now can be written in the form

$$\begin{aligned} \dot{B}_1 &= B_1(\varepsilon_1 - B_1 - a_2B_2 - \gamma_{13}B_3) \\ \dot{B}_2 &= B_2(\varepsilon_2 - B_2 - a_1B_1 - \gamma_{23}B_3) \\ \dot{B}_3 &= B_3(\beta - \gamma_{31}B_1 - \gamma_{32}B_2) \end{aligned} \quad (2.2).$$

The parameters γ_{i3} characterize the strength of B_3 impact on species B_i , γ_{3i} the species survival potential.

The state $(0,0,0)$ is a steady state for (2.2). i. e. dead spaces without any vegetation or free gaps are assumed in this model. The parameter β can be positive and then has interpretation in terms of intensity of the impact, while

negative values correspond to the damping rate of the disturbance. We will show that one of the equilibria of the system is a saddle-focus, while two others can be stable, unstable nodes or saddle-nodes.

The three equilibria of the system along $B_3 = 0$ are:

$$1. \{B_1^{(1)}, 0, 0\}, \quad 2. \{0, B_2^{(1)}, 0\}, \quad 3. \{B_1^{(2)}, B_2^{(2)}, 0\}$$

$$\text{Where } B_1^{(1)} = \varepsilon_1, \quad B_2^{(1)} = \varepsilon_1, \quad B_2^{(1)} = \frac{\varepsilon_2 - a_1\varepsilon_1}{1 - a_1a_2}, \quad B_1^{(2)} = \frac{\varepsilon_1 - a_2\varepsilon_2}{1 - a_1a_2}$$

The eigenvalues for the first equilibrium are $\lambda_3 = \beta - \gamma_3\varepsilon_1$, $\lambda_1 = \varepsilon_1$, $\lambda_2 = \varepsilon_1 - a_2\varepsilon_2$.

From these expressions we can see that the eigenvalues are always real, but the signs are determined from relations among B_1 , ε_1 , ε_2 .

The characteristic equation of the system for a non-trivial equilibrium can be easily defined from the linear stability matrix evaluated at $\{B_1, B_2, 0\}$.

$$\begin{vmatrix} \frac{a_2\varepsilon_2 - \varepsilon_1}{1 - a_1a_2} & a_2 \frac{a_2\varepsilon_2 - \varepsilon_1}{1 - a_1a_2} & -\gamma_{13} \frac{\varepsilon_1 - a_2\varepsilon_2}{1 - a_1a_2} \\ a_1 \frac{a_1\varepsilon_1 - \varepsilon_2}{1 - a_1a_2} & \frac{a_1\varepsilon_1 - \varepsilon_2}{1 - a_1a_2} & -\gamma_{23} \frac{\varepsilon_2 - a_1\varepsilon_1}{1 - a_1a_2} \\ 0 & 0 & \beta - \gamma_{31} \frac{\varepsilon_1 - a_2\varepsilon_2}{1 - a_1a_2} - \gamma_{32} \frac{\varepsilon_2 - a_1\varepsilon_1}{1 - a_1a_2} \end{vmatrix} \quad (2.3)$$

It is evident that the system has one real eigenvalue λ_3 and two complex-conjugate $\lambda_{1,2}$

$$\lambda_3 = \beta - \frac{\gamma_{31}(\varepsilon_1 - a_2\varepsilon_2) + \gamma_{32}(\varepsilon_2 - a_1\varepsilon_1)}{1 - a_1a_2} \quad \lambda_{1,2} = \rho \pm i\vartheta$$

$$\text{Where } \vartheta = \frac{1}{2(1 - a_1a_2)}\sqrt{M}, \quad \rho = -\frac{1}{2} \left[\frac{\varepsilon_1(1 - a_1) + \varepsilon_2(1 - a_2)}{1 - a_1a_2} \right] \quad (2.4)$$

$$M = 4(1 - a_1a_2) [\varepsilon_1\varepsilon_2(1 + a_1a_2) - a_1\varepsilon_1^2 - a_2\varepsilon_2^2] - (\varepsilon_1 - \varepsilon_2 + a_1\varepsilon_1 - a_2\varepsilon_2)^2,$$

We assume $a_1 = 1$ & $a_2 = 1$, $M > 0$.

The emergence of a two-dimensional stable manifold is a consequence of the stable limit cycle existence in the two-dimensional system with parameter values $\varepsilon_1 = \frac{1-a_2}{a_2-1} \varepsilon_2$. This relation means that ε_1 and ε_2 must have different signs providing a_i are positive. Let's remind that $\varepsilon_i(x)$ is $\varepsilon_i(x) = g_i(x) - m_i$ present the difference between the annual productivity and mortality. The shift of border means that destructive process prevails over reproductive. However, by translating the origin into point $(B_1^{(2)}, B_2^{(2)}, 0)$ more transparent conditions for variables in the transition zone. Let B_1^*, B_2^* be

$$B_1^* = B_1 - B_1^{(2)}$$

$$B_2^* = B_2 - B_2^{(2)}$$

Clearly, new B_i^* could be either positive or negative, but we can include as the direction of shift. Suppose the shift from the forest to steppe or to bare soil temporarily, it is possible to write explicitly

$$B_1^* = B_1 - B_1^{(2)}$$

$$B_2^* = B_2 + B_2^{(2)}$$

Then the Jacobian matrix (2) will take the form

$$\begin{vmatrix} \varepsilon_1 & -a_1 B_1^{(2)} & -\gamma_{13} \frac{\varepsilon_1 - a_2 \varepsilon_2}{1 - a_1 a_2} \\ a_1 B_2^* & \varepsilon_2 & -\gamma_{23} \frac{\varepsilon_2 - a_1 \varepsilon_1}{1 - a_1 a_2} \\ 0 & 0 & \beta - \gamma_{31} \frac{\varepsilon_1 - a_2 \varepsilon_2}{1 - a_1 a_2} - \gamma_{32} \frac{\varepsilon_2 - a_1 \varepsilon_1}{1 - a_1 a_2} \end{vmatrix}$$

Shil'nikov's theorem states the existence of Smale's horseshoe and related chaotic irregular dynamics in the vicinity of the robust saddle- focus, $\lambda_3 > 0$ & $\rho < 0$. As it is emphasized in [[Magnitski, 2004, p. 92] this theorem does not prove the existence of an invariant set specific the horseshoe mapping. Following the elaboration in [Magnitski 2004] we bring the model equations to the form

$$\begin{aligned} \dot{B}_1 &= \rho B_1 - \mathcal{G} B_2 + P(B_1, B_2, B_3) \\ \dot{B}_2 &= \mathcal{G} B_1 + \rho B_2 + Q(B_1, B_2, B_3) \\ \dot{B}_3 &= \lambda_3 B_3 + R(B_1, B_2) \end{aligned} \quad (2.5)$$

where P, Q, R some functions containing the components from the second power. The system has two manifolds: a stable and unstable ones. The chaotic dynamics emerges in case $|\rho| < \lambda_3$ unless the speed of movement away from the fixed point on unstable manifold is greater than the speed of attraction to the point.

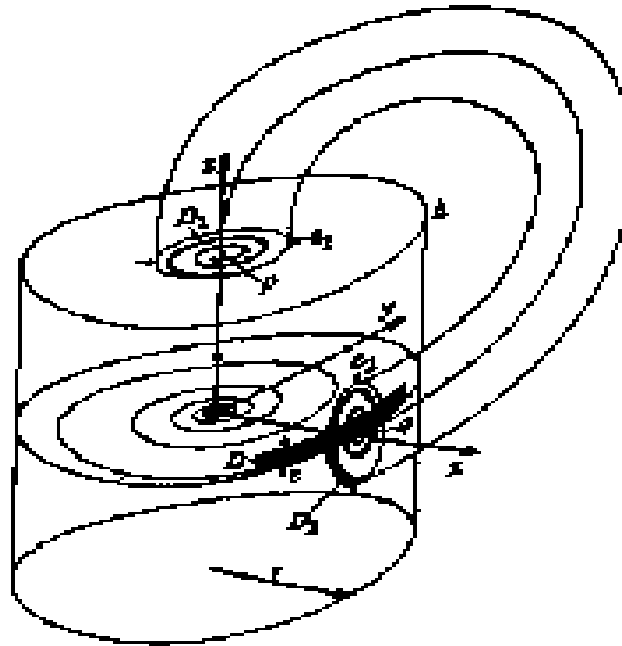


Fig. 2 illustrates the change of trajectories in the phase space (adopted from [Magnitski, 2004]).

Let D be some region on the surface of the cylinder, the points in which are specified with two variables ξ, θ , where $0 < \xi < \theta$, $|\theta| < \theta_{\max}$ is the angle between the old position of some point q and the transferred one. Since the linear approximation is valid inside the cylinder the solutions of the three-dimensional system are defined by

$$\dot{B}_1 = r \exp(\rho t) \sin(\mathcal{G}t + \theta)$$

$$\dot{B}_2 = r \exp(\rho t) \cos(\mathcal{G}t + \theta)$$

$$\dot{B}_3 = \xi \exp(\rho t) \sin(\lambda_3 t)$$

On the starting point the movement along the separatrix will bring some point q from D to a new area D_1 situated on the upper surface of the cylinder. The size of the region is restricted by a cycle of a diameter

$$\varepsilon_1 = 2r \left(\frac{\varepsilon}{h} \right)^{\lambda_3}. \text{ On the following transition the central point p of } D_1 \text{ makes transition to the original position q.}$$

The mapping of the whole D_1 to new D_2 can be approximated by a spiral lying inside the cycle with the diameter of

$$k \text{ fold the one of } D_1, \text{ i.e. } \varepsilon_2 = 2\pi r \left(\frac{\varepsilon}{h} \right)^{\frac{|\rho|}{\lambda_3}}$$

When $\varepsilon_1 < 2\varepsilon$ and small ε the spiraling curve will remain in D while at $\frac{|\rho|}{\lambda_3} < 1$ change $\varepsilon_1 > 2\varepsilon$ specific for the horseshoe mapping occurs.

In the limit $\varepsilon \rightarrow 0$ a countable number of horseshoes appear, but introduction of small perturbations will leave a finite number of horseshoes persistent.

Let us select two strips V_1, V_2 in the population area each containing the points that belonged to the state at initial moment, this could be biomasses with values in some attracting zone of the limit cycle then the second picture we can present the set of points survived after two steps of the iteration process. There can be a sequence of iterations from the formal point of view. In the model the iteration correspond to some shifts that then could occur in reverse direction. Under decay or some slow recovery from the disturbance can the species can return in time run to some state close to the initial or with a distorted shape of the border.

Another version of Shil'nikov's theorem is valid for the case $\rho > 0, \lambda_3 < 0$, where the condition for the emergence of chaotic dynamics is $\rho < |\lambda_3|$

The point now is to consider the behavior of the system with $\rho = \lambda_3$. The case with both ρ, λ_3 positive corresponds to the emergence of a unstable focus, in fact this means that the third component is not disturbing, it could be even fruiting. It is not feasible for GVP model in this approach and we will not take it into account. Thus we shall consider the conditions under which the trajectories of the system will “jump” across the invariant subspace of saddle focus stuck to another state, thus bringing the original model to destruction of the existing pattern and replacing it by new one. The boundary which has just been the intersection of the stable and unstable manifolds in the consequence of perturbations now will become a connection between two fixed points. To define the properties of the heteroclinic cycle separatrix that emerges in result of this new non local bifurcation. Let us consider the change in parameter values that can push the system to a new bifurcation.

The transition along the connecting manifold can correspond either to substantial growth for one of the species or the number and size of free spots growth. The opportunities for further “recolonization” of the spot depend however from the selected species potential to react on new conditions and not only the absence of the competitor, as well as disturbing factor.

From the expressions for $B_1^{(2)}, B_2^{(2)}$ it is easy to conclude that the system can potentially come to $(0, B_2^{(1)}, 0)$ when $\varepsilon_1 \rightarrow a_2 \varepsilon_2$ or to the state $(B_1^{(1)}, 0, 0)$ in case $\varepsilon_2 \rightarrow a_1 \varepsilon_1$.

Another point is made by a change in the sign of M , after which the system will have real eigenvalues. Consider first the type of the equilibrium $\{0, B_2^{(1)}, 0\}$.

Let's put $\delta = \varepsilon_1 - a_2 \varepsilon_2$ then the eigenvalues are:

$$\lambda_1 = \delta, \lambda_2 = -\varepsilon_2, \lambda_3 = \beta - \gamma_{32} \varepsilon_2$$

Assuming $\varepsilon_2 > 0, \beta > 0, \beta > \gamma_{32} \varepsilon_2$ the state will be a saddle node with two-dimensional stable node and one-dimensional unstable.

There are two points in the system: a saddle-focus with one-dimensional unstable and two-dimensional stable and a saddle-node with the contrary. Based on numerical investigations it can be concluded [Magnitski 2004, p. 93] that the contour connecting these two points is not robust. An irregular dynamics is observed in the neighborhood of the bifurcation point with infinitely many stable and unstable limit cycles. A special closed contour connecting the saddle focus with saddle -node emerges at $\varepsilon_i \rightarrow 0$. From the model point of view this means that the presence of a disturbing factor will continue to provide impact bringing to oscillations in the system.

To consider global bifurcations let's introduce two parameters: δ for the difference between ρ and λ_3, d_{12} and d_{21} for the coefficients at in the nonlinear part of the normal form and write the equations in the form:

$$\dot{B}_1 = -\vartheta B_2 + \rho B_1 + d_{12} (B_1^2 + B_2^2) B_1$$

$$\dot{B}_2 = -\vartheta B_1 + \rho B_2 + d_{21} (B_1^2 + B_2^2) B_2, \quad \delta = \lambda_3 - \rho, \quad B_1^2 + B_2^2 = r^2$$

$$\dot{B}_3 = (\delta + \rho) B_3$$

When $\delta = 0$ Hopf bifurcation is supercritical unless $d_{1,2}$ and $d_{2,1}$ are negative, when the signs of d_{ij} change (under the impact of disturbance) the bifurcation will become subcritical. The bifurcation diagram (Fig. 3) shows the scheme for the upper part of parameter space partition, with $\lambda_3 > 0$.

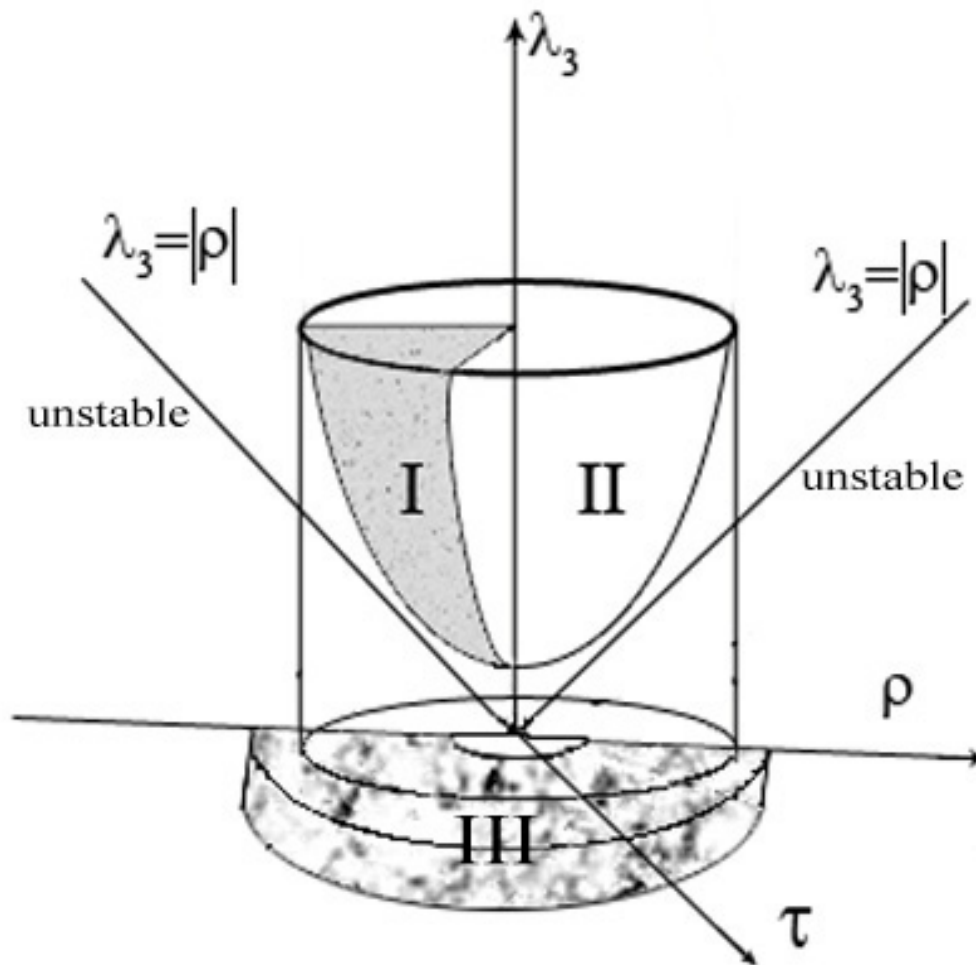


Fig.3. The part of the conic surface shows the region where Silnikov's type chaotic dynamics emerges. The bisectors divide the quadrants to the parts based on restrictions $\lambda_3 = -\rho$, $\lambda_3 = \rho$. Region III is marked with Hopf bifurcation.

The chaotic dynamics is associated with some type of local or global bifurcation. When δ is small and λ_3 is negative the limit cycle created by Hopf bifurcation surrounding the nontrivial equilibrium on the phase portrait of the system persists. With a change in λ_3 and, hence the increasing disturbance, a saddle-focus bifurcations take place. At the border (line l_1) in some small part defined. While the region enlarges a new closed loop encircling both saddle focus and saddle node equilibrium emerges. At the saddle node one of the eigenvalues of the system is negative and the second is positive. After the saddle-node gets destroyed, the new emergent dynamics with $\{B_1^{(1)}, 0\}$ makes the equilibrium relatively stable, but with only one of the species survived. When the destabilizing impact of B_3 decreases or even disappears, the new distribution will depend on several factors.

Thus, if a small collection of the removed species seedlings remains and the survivors can use new beneficial conditions (such as free gaps, for example) the system has an opportunity to restore the primary border.

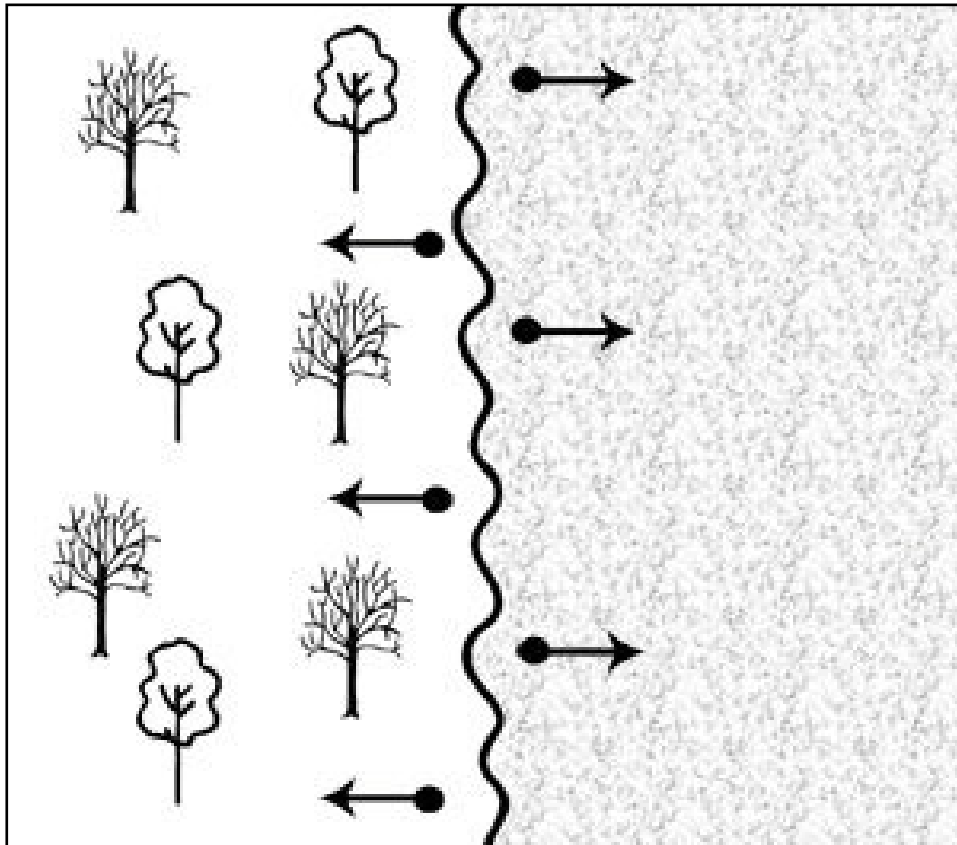


Fig.4. The line shows the border between the part covered with the woods and the steppe.

Conclusion

We have shown analytically that GVP dynamics could demonstrate complex behavior with multiple equilibria, chaotic dynamics associated with them as well as transition zones between two consequent states. The main parameter of the model is defined by distinct values of productivity function. A new variable has been integrated to the system; only general assumptions are made with respect to this function characterization. We model the competition between the destabilizing effects caused by some event with stabilizing potential of the two-species system. We illustrate that in case the strength of destabilization is weak (with respect to existing resource, i.e. biomasses and their potential to renew and survive) and then damping takes place the chaotic dynamics is revealed that can provide a distortion of the borderline or an essential shift, which however will not bring the system to the origin with corresponding dead space in the model.

Acknowledgements

The work was partially supported by DAAD grant Gr 6205-A0538516 and was implemented when the author was in PIC, Germany. The author expresses her gratitude for hospitality and helpful discussions.

Bibliography

[Svirezhev Y., 2000]. Svirezhev Y., 2000, Lotka-Volterra models and the global vegetation pattern. *Ecological Modeling* 135, pp.135-146.

[Svirezhev-von Bloh 1997]. Svirezhev Y. M., W. von Bloh. Climate, vegetation and global carbon cycle: the simplest zero-dimensional model. *Ecological Modeling*, 101, 1997 pp. 75-95.

[Bampfyde C. et al, 2005]. Bampfyde C., Brown N. D., Gavaghan D. J., Maini P. K., 2005, Modeling rain forest diversity: The role of competition. *Ecological modeling*, 188, pp. 253-278.

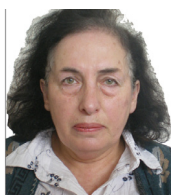
[Vandermeer J., 2004]. Vandermeer J., Granzow I., Perfecto I., Boucher D., Ruiz J., Kaufman A. 2004. Multiple basins of attraction in a tropical forest: evidence for nonequilibrium community structure. *Ecology* 85(2), pp. 575-579.

[Magnitski N.,2004].Magnitski N., Sidorov V., 2004. New methods of chaotic dynamics. Moscow (in Russian).

[Porter J., 2004].Porter J., 2004, Dynamics near robust heteroclinic cycles. In: *Dynamics and bifurcations of patterns in dissipative systems*. pp. 329-356. World Scientific Publishing Co.

[Tilman, 1994]. David Tilman, Competition and Biodiversity in Spatially Structured Habitats.. *Ecology*, Volume 75, Issue 1 (Jan., 1994), 2-16.

Authors' Information



Ajabyan Nelli –Researcher, Institute for Informatics and Automation Problems of NAS RA, e-mail: najabyan@ipia.sci.am.

Major fields of Scientific Research: predictive modeling of ecosystems, problems of assessment of response on disturbances.