DISCRETE ARTIFICIAL INTELLIGENCE PROBLEMS AND NUMBER OF STEPS OF THEIR SOLUTION

Tatiana Kosovskaya

Abstract: *Aggregate characteristics of discrete models appearing in different artificial intelligence problems are considered. It is shown that if an investigated object is a collection of its elements and its description contains properties of these elements and relations between them then a predicate calculus language is convinient for its simulation. In such a case a lot of problems are NP-hard. Upper bounds of steps for two essentially different decision algorithms are presented. A problem of transformation of an investigated object and the number of its decision steps is regarded. A many-level approach (consisting in the extraction of subformulas of goal conditions) to the decision of these problems is described. It allows to decrease the used time.*

Keywords: *artificial intelligence, pattern recognition, analysis of situation, transformation, predicate calculus, complexity of algorithm.*

ACM Classification Keywords: *I.2.4 ARTIFICIAL INTELLIGENCE Knowledge Representation Formalisms and Methods – Predicate logic, F.2.2 ANALYSIS OF ALGORITHMS AND PROBLEM COMPLEXITY Nonnumerical Algorithms and Problems – Complexity of proof procedures.*

Introduction

Algorithmical complexity of a lot of artificial intelligence problems permitting its simulation by means of predicate formulas is considered. Examples of such problems are: pattern recognition, chess and draught playing, market situation analysis, intelligent robot movement, medical diagnostics and treatment choice.

It is shown that for the most of the problems under consideration we can construct a model described by simple type predicate formulas. In such a case the problem decision is equivalent to the proof of a logical sequent of the form "If elementary conditions for an object are fulfilled then there exist a list of different in pairs values for variables such that the goal condition is valid for this list of values". Such a problem is NP-complete.

The upper bounds of step number of such a sequent proof are done for two different approaches. These bounds have different parameters in the exponent of the power. The number of a solution steps may be rather different in dependence of the chosen elementary features and the goal condition structure.

Problems in which an object may be transformed by means of an action from the done set of transformations are regarded. Examples of such problems are: recognition of a distorted image, the choice of a strategy in chess playing, the choice of an action upon the market objects to receive a favorable situation, the search of the intelligent robot movement sequence which carry it into the done position, medical treatment choice. To solve such a problem one can add descriptions of possible transformations to the premise of the main sequent.

Location of important parts of an object permits to decrease the used time because of not great complexity of such parts. A many-level approach to the solution of the described problems (consisting in the location of important parts with not great complexity of an object) is described.

Attributes in Discrete Simulation

The choice of initial attributes for description of an object for solving an artificial intelligence problem is the first stage of a discrete simulation of an informational process (representation of information for its further use). Examples of such attributes for rather different problems are as following.

1. Pattern recognition problems. Characteristics of the recognized objects or their parts are attributes in the terms of which a recognizable object and the classes of them are described.

2. Chess or draught games simulation. The state of a cell (what figure is situated in the cell) may be regarded as an attribute.

3. Market situation analysis. Qualitative and quantitative characteristics of a market participant are attributes.

4. Simulation of an intelligent robot movement. A graph of all possible pairwise connected situations of a robot may be a model for such a problem. A relation of two verteces to be adjoining and a property of a vertex to have a special mark may be regarded as an attribute.

5. Medical diagnostics. Symptoms of a patient are attributes.

Different researchers use different types of initial attributes representation to describe a model. But all of them have one common property – elementary character, i.e. the value of an attribute may be easily measured for every object of the model. Denote these attributes by

$$p_1, \dots , p_n .$$

Some examples of such types of attributes are the following.

– Propositional (boolean) variables (for problems 1, 5).

– Predicates describing properties of an object part or relations between them (for problems 1, 2, 3, 4, 5).

– Many-valued attributes having values from the done set D (for problems 1, 2, 3, 4, 5). Fuzzy and probabilistic characteristics may be regarded as many-valued attributes.

– Multi-sets of objects different parts of which have the same property and, consequently, this property must be presented in the object description several times (for problems 1, 4, 5).

– Graphs and marked graphs (for problems 1, 2, 3, 4).

All these types of attributes may be simulated by means of predicates.

– A boolean variable is a 0-ary predicate.

– To simulate a many-valued attribute $p(x)$ it is sufficient to have a predicate p' with an additional argument d: $p'(x,d) \Leftrightarrow p(x) = d$ (where $d \in D$).

– For a multi-set it is sufficient to have an additional integer argument in its characteristic function $\chi_A(x)$ which points out the number of appearance of the element x in the multi-set A: $p_A(x, n) \Leftrightarrow \chi_A(x) = n$.

– Graph $G = (V,E)$ may be represented by a set of atomic formulas with a binary predicate p defined by the equality $p(x,y) \Leftrightarrow (\{x,y\} \in E)$. To set a marked graph it is sufficient to have additional arguments for the marks.

Predicate formulas as a model for goal conditions

The definition of a goal condition providing the solution of a problem under consideration is the second stage of a discrete simulation of an informational process for an artificial intelligence problem.

Such a goal condition may be formulated in the terms of the chosen initial attributes and be written as such a formula $A(x)$ of a formalized language that if the formula $A(\omega)$ is valid for an investigated object ω then the problem has a positive solution. Moreover the goal condition may be represented by a quatifier-free formula in the form of disjunction of elementary conjunction of atjmic formulas.

For a lot of artificial intelligence problems it is important if there exists a part of the investigated object ω which satisfies the formula $A(x)$. Such a situation appears, for example, in the problem of a compound scene (it is denoted as ω) analysis which has several similar (from the same class) images situated in the different places of the scene: $\omega^1, \ldots, \omega^r$ such that $\omega^j \subset \omega$ and $A(\omega^j)$ is valid for all $j = 1, \ldots, r$.

While analysis of a market situation (ω is the whole market) there may appear several market participants or their collections $\omega^1, \ldots, \omega^r$ ($\omega^j \subset \omega$ for all $j = 1, \ldots, r$) such that every of them satisfies the same goal condition $A(\omega^j)$ for all $j = 1, \ldots, r$.

In the medical diagnostics problems (ω is a patient) there may be several parts

$$\omega^1, \ldots, \omega^r \ (\omega^j \subset \omega \text{ for all } j = 1, \ldots, r)$$

such that every of them satisfies the same or different goal conditions $A_1(\omega^1), \ldots, A_r(\omega^r)$.

This is the reason to represent the investigated object as a set of elementary objects $\omega = \{\omega_1, \ldots, \omega_\tau\}$. In such a case the attributes will be measured for the elements of the object and the goal condition will be represented by a formula with variables for elementary objects $A(x_1, \ldots, x_m)$ (or briefly $A(\mathbf{x})$ where $\mathbf{x}$ is a notation for the list of variables $x_1, \ldots, x_m$).

A description of an investigated object ω in the chosen model is a set of all properties of its elements and relations between them:

$$S(\omega) = \{p_1(\omega_1), \ldots, p_1(\omega_t), p_2(\omega_1), \ldots, p_n(\omega_i, \ldots, \omega_j)\}.$$

So the solution of the above mentioned problems may be reduced to the checking of a logical sequent of the form

$$S(\omega) \Rightarrow x_{\neq} \ A(x), \qquad (1)$$

where $\exists \mathcal{J}_{\neq}$ denotes "there exists a string of different in pairs values for the list of variables $\mathcal{J}$". For a lot of problems it is important not only to check out whether there exists a string of different in pairs values for variables $\mathcal{J}$ satisfying the formula $A(\mathcal{J})$ but to find such a string.

The proof of the logical sequent (1) is an NP-complete problem [Kosovskaya, 2007] and hence the determination of the string of different in pairs elementary objects satisfying the formula $A(\mathcal{J})$ is an NP-hard problem.

If a researcher proves that the logical sequent (1) may be checked out by an offered by him method in a polynomial (under notation lengths of a goal condition and an object description) number of steps then he will prove that **P ≠ NP** what is one of seven problems claimed to be the most complicated mathematical problems of the XXI century.

Methods of proof and upper bounds of their number of steps

Below for a step of computation we take a substitution of variable values into a formula $A(\mathcal{J})$ or a comparison of a conjunct of a formula $A(\mathcal{J})$ with a formula of the set $S(\omega)$ for their graphical coincidence.

The exhaustive search method has the upper bound of steps

$$O(t^m ||A||\ ||S||),$$

where $||A||$ is the number of atomic formulas in the formula $A(\mathcal{J})$, $||S||$ is the number of atomic formulas in the description $S(\omega)$ [Kosovskaya, 2007]. Note that this estimate coincides with the one for simulation of predicate approach to the artificial intelligence problems by boolean variables [Russel, 2003].

Logical methods (namely logical derivation in a sequent calculus [Kosovsky, 1981] or by resolution method [Russel, 2003]) has the upper bound of steps

$$O(s^a),$$

where s and a are the maximal number of occurrences of the same predicate in the description $S(\omega)$ and in the formula $A(\mathcal{J})$ respectively.

One can see that these estimates have different parameters in the exponent of the power. So a researcher may choose the method in applications in dependance of the structure of the attributes and the goal condition.

Actions upon an object involving transformation of its parts properties and relations

The solution of many problems assumes the existence of some actions upon an object which transform the initial properties of its elements and their relations.

Among the pattern recognition problems there is a problem of recognition of an object distorted by a transformation from a known set of transformations.

While simulation of chess game it is important not only to estimate a situation but to find a sequence of moves leading to a “successful” situation.

While projecting a model of intelligent robot movement it is required to construct a sequence of permutations providing a necessary position of it.

In the problem of the market situation analysis it is useful to find an action upon the market members leading to a required state of the whole market.

In the frameworks of a medical diagnostics problem a problem of treatment choice may be set up. It consists in the finding of such a sequence of medical actions upon a patient which transfer him to a state with the done condition (for example, to the class of practically healthy people).

To set an artificial intellegence problem dealing with a set of transformations acting an object it is important to know properties of such a set of transformations.

Let a collection of transformations be a group with a finite number of generatrices. The set of all generatrices will be denoted by $G = \{g_1, ..., g_T\}$ and the group itself by G^*.

Let the change of a single predicate or their couple value may be pointed out for every transformation g_j ($j = 1, ... ,T$) acting upon an investigated object. There may be several changes for every transformation g_j. Denote the number of such changes by l_j. These changes will be written down as an equivalences between attributes of objects ω and $g_j(\omega)$

$$B_j^l(\omega) \Leftrightarrow C_j^l(g_j(\omega)), \quad (2)$$

where $B_j^l(\omega)$ and $C_j^l(g_j(\omega))$ are elementary conjunctions of atomic formulas, $l = l_1, \dots, l_j$. An equivalence of the form (2) will be called a description of the transformation g_j and denoted by $\Gamma_j^l(\omega)$. The set of all descriptions of all transformations will be denoted by $\Gamma(\omega) = \{\Gamma_j^l(\omega_j^l) : j = 1, \dots, T, l = 1, \dots, l_j\}$.

The group properties of a transformation set (i.e. the existence of an inverse transformation for every one) are important if it is necessary not only to find such a transformation which transfers an object to a state satisfying the goal condition but to have an opportunity of its reverse transformation to the initial state.

For a lot of problems the group properties of a transformation set are not fulfilled. Chess game and choice of a medical treatment are examples of such problems. One deals only with a semigroup of transformations, i.e. a composition of allowed transformations is an allowed transformation. In such a case instead of equivalences in the form (2) we have only logical sequents

$$B_j^l(\omega) \Rightarrow C_j^l(g_j(\omega)), \quad (2')$$

every of which will be also called a description of the transformation g_j and the set of all descriptions of all transformations will be denoted by $\Gamma(\omega) = \{\Gamma_j^l(\omega_j^l) : j = 1, \dots, T, l = 1, \dots, l_j\}$.

Transformation descriptions must be taken in account if an object may be changed by transformations from G^*. That is why the set of formulas $\Gamma(\omega)$ must be included to the formula (1). Let $\forall\sim\Gamma(\omega)$ be a closure of all formulas in $\Gamma(\omega)$ by an universal quantifier. Than we have a logical sequent

$$S(\omega)\ \ \sim\Gamma(x) \Rightarrow\ \ x_{\neq}\ \ A(x). \quad (3)$$

In the case of an infinite G^* the problem of checking the logical sequent (3) is algorithmically undecidable. If G^* is finite and has R elements or the number of transformations acting an object is not more than R then the number of steps of checking the logical sequent (3) differs from the one for the logical sequent (1) by a multiplicative factor T^R, where T is the number of generatrices of G^* [Kosovskaya, 2009].

Multi-level approach to the decision of the formulated problems

Let $A_1(\mathbf{x_1}), \dots, A_K(\mathbf{x_K})$ be a set of goal conditions. Such a situation appears, for example, in pattern recognition problems every goal condition of which is a description of a class.

Find all subformulas $P_i^j(\mathbf{y_i^j})$ ($\mathbf{y_i^j} \subseteq \mathbf{x^1} \cup \dots \cup \mathbf{x^K}$) with the “small complexity” which “frequently” appear in goal formulas $A_1(\mathbf{x_1}), \dots, A_K(\mathbf{x_K})$ and denote them by atomic formulas with new predicates p_i^j with new first-level arguments y_i^j for a list $\mathbf{y_i^j}$ of initial variables. Write down a system of equivalences

$$p_i^j(y_i^j) \Leftrightarrow P_i^j(\mathbf{y_i^j}).$$

Let $A_k^1(\mathbf{x_k^1})$ be a formula received from $A_k(\mathbf{x_k})$ by substitution of $p_i^j(y_i^j)$ instead of $P_i^j(\mathbf{y_i^j})$. Here $\mathbf{x_k^1}$ is a list of all variables in $A_k^1(\mathbf{x_k^1})$ including both some (may be all) initial variables of $A_k(\mathbf{x_k})$ and first-level variables appeared in the formula $A_k^1(\mathbf{x_k^1})$.

A set of all atomic formulas of the type $p_i^j(\omega_i^j)$ for which a formula $P(\tau_{ij}^1)$ (for some $\tau_{ij}^1 \subset \omega$) is valid is called a first-level object description and denoted by $S^1(\omega)$. Such a way extracted subsets τ_{ij}^1 are called first-level objects.

Repeat the above described procedure with formulas $A_k^1(\mathbf{x}_k^1)$. After L repetitions L-level goal conditions in the following form will be received [Kosovskaya, 2008].

$$A_k^L(\mathbf{x}_k^L)$$

$$p_1^1(y_1^1) \Leftrightarrow P_1^1(\mathbf{y}_1^1)$$

$$\ldots$$

$$p_{n1}^1(y_{n1}^1) \Leftrightarrow P_{n1}^1(\mathbf{y}_{n1}^1)$$

$$\ldots$$

$$p_i^l(y_i^l) \Leftrightarrow P_i^l(\mathbf{y}_i^l)$$

$$\ldots$$

$$p_{nL}^L(y_{nL}^L) \Leftrightarrow P_{nL}^L(\mathbf{y}_{nL}^L).$$

Such L-level goal conditions may be used for efficiency of an algorithm solving a problem formalized in the form of logical sequent (1).

To decrease the number of steps of an exhaustive algorithm (for every t greater than some t_0) with the use of 2-level goal description it is sufficient

$$n_1 t^r + t^{s1+n1} < t^m,$$

where r is a maximal number of arguments in the formulas $p_i^1(y_i^1) \Leftrightarrow P_i^1(\mathbf{y}_i^1)$, n_1 is the number of first-level predicates, s^1 is the number of atomic formulas in the first-level description, m is the number of variables in the initial goal condition [Kosovskaya, 2008].

Analogous condition for decreasing the number of steps of a logical algorithm solving the problem (1) is

$$\Sigma_{k=1\ldots K} s^{ak} - \Sigma_{j=1\ldots n1} s^{\rho j} \geq \Sigma_{\kappa=1\ldots K} (s^1)^{ak1},$$

where a_k and a_k^1 are maximal numbers of atomic formulas in $A_k(\mathbf{x}_k)$ and $A_k^1(\mathbf{x}_k^1)$ respectively, s and s^1 are numbers of atomic formulas in $S(\omega)$ and $S(\omega) \cup S^1(\omega)$ respectively, ρ_j is the number of atomic formula in $P_i^1(\mathbf{y}_i^1)$ [Kosovskaya, 2008].

Conclusion

The offered approach to the solution of artificial intelligence problems reduces them to the checking of a logical sequent (1). The problem (1) is NP-complete but different algorithms of its solution give different exponents in the upper bounds of their steps. An exhaustive algorithm is preferable if the number of variables in the goal condition is not great. If the number of atomic formulas in the goal condition is less then the number of its variables then the search of logical inference of (1) is preferable. These characteristics of the goal condition depend on the way of formalization of a problem.

In the framework of the offered approach it is possible to include descriptions of transformations acting upon an object into the main formula (1) and to receive the formula (3). Independently of the method used for (1) the number of steps of an algorithm solving the problem (3) increases in the same times.

Many-level approach to the description of goal conditions allows decreasing the number of steps of both an algorithm solving the problem (1) and an algorithm solving the problem (3). In such a case the term "small complexity" of an extracted formula means small number of variables in it if we use an exhaustive algorithm. The term "small complexity" of an extracted formula for an algorithm based on construction of a logical inference means small number of atomic formulas and decreasing the goal condition notation length after replacement the extracted formulas by atomic formulas with new first-level predicates.

Bibliography

[Kosovskaya, 2007] T.M. Kosovskaya. Proofs of the number of steps bounds for solving of some pattern recognition problems with logical description. In: Vestnik of St.Petersburg University. Ser. 1. 2007. No. 4. P. 82 – 90. (In Russian)

[Kosovskaya, 2008] T.M. Kosovskaya. Level descriptions of classes for decreasing step number of pattern recognition problem solving described by predicate calculus formulas. In: Vestnik of St.Petersburg University. Ser. 10. 2008. No. 1. P. 64 – 72. (In Russian)

[Kosovskaya, 2009] T.M. Kosovskaya. Recognition of objects from classes closed under a group of transformations. In: Vestnik of St.Petersburg University. Ser. 10. 2009. No. 3. P. 45 – 55. (In Russian)

[Kosovsky, 1981] N.K.Kosovsky. Elements of Mathematical Logic and its applications to subrecursive algorithms theory. Leningrad University, 1981. 192 p. (In Russian)

[Russel, 2003] S.J.Russel and P.Norvig. Artificial Intelligence. A Modern Approach. Pearson Education, Inc. 2003. ISBN 0-13-790396-2.

Authors' Information

Tatiana Kosovskaya *– Senior reasercher, St.Petersburg Institute of Informatics and Automatization of Russian Academy of Science, 14 line, 39, St.Petersburg, 199178, Russia; Professor of St.Petersburg State Marine Technical University, Lotsmanskaya ul., 3, St.Petersburg, 190008, Russia; e-mail: kosov@NK1022.spb.edu*

Major Fields of Scientific Research: Logical approach to artificial intelligence problems, theory of complexity of algorithms.

The paper is published with financial support of the project ITHEA XXI of the Institute of Information Theories and Applications FOI ITHEA (www.ithea.org) and the Association of Developers and Users of Intelligent Systems ADUIS Ukraine (www.aduis.com.ua).