DECISION MAKING PROBLEM WITH FUZZY SET OF GOAL SETS

Sergiy Mashchenko, Oleksandr Bovsunivskyi

Abstract: *The problem of alternatives rational choice, in which goal of person who makes decision, is defined with the fuzzy set of fuzzy goal (aim) sets. The concept of intersection of fuzzy set of fuzzy sets is introduces and in it also investigated its properties.*

Keywords: *fuzzy set, fuzzy goal, membership function, fuzzy set of type 2, decision making, rational choice.*

ACM Classification Keywords: *H.4.2 Information Systems Applications: Types of Systems: Decision Support.*

Introduction

One of the most important problems, which arise in decision making practice, is the problem of making multi-goal decisions under fuzzy information.

In this paper we consider the problem of alternatives rational choice in an environment where goal of person who makes decision (PMD) is defined with fuzzy set of fuzzy goal sets. These models generalize well-known decision making problems, in which the PMD aim is characterized by a clear set of fuzzy sets (Bellman-Lotfi Zadeh approach, [Bellman, 1970]). On the one hand, such a generalization allows to analyze the situation in the cases when it is impossible to clearly specify which sets are characterizing the PMD aim, on the other - helps deeply and more accurately understand the decision making process, ways of finding and selecting reasonable alternatives under fuzzy information.

Decision making problem with fuzzy aim

Let be X – universal set of alternatives in which defined fuzzy set of alternatives D by the membership function $\mu_D : X \to [0,1]$. The fussy subset (let be marked G) of the universal set X is named as fussy goal (aim) [Bellman, 1970]. The fussy set G will be defined by the membership function $\mu_G : X \to [0,1]$.

According to Bellman - Lotfi-Zadeh approach the decision making problem with fuzzy defined aim is to reach the goal G within a fuzzy set of alternatives D. And, in this fuzzy formulation it is not simply taking about achieving goal, sand its achievement of one or another degree, taking into account also the membership degree to fuzzy set of alternatives.

Suppose, for example, that some alternative x provides a $\mu_G(x)$ degree of achievement of goals, and it belongs to fuzzy set of alternatives with degree $\mu_D(x)$. Then by [Bellman, 1970] it considered that the degree of membership of this alternative to the fuzzy set of problem solutions is equal to the minimum of these values. In other words, the alternative with the degree of membership, such as 0.3, with the same degree belongs to fuzzy set of solutions, despite the fact that it provides to achieve goal with a degree equal to, for example, 0.8.

So, the fuzzy solution of achievement of fuzzy goal problem on fuzzy set of alternatives by the Bellman – Lotfi-Zadeh approach – is called intersection of fuzzy sets of goal and alternatives. Membership function of fuzzy set of solutions to this problem has the form: $\mu(x) = \min\{\mu_G(x), \mu_D(x)\}$. If there are few goals, then fuzzy set of solutions is described by membership function as follows: $\mu(x) = \min\left\{\mu_{G_1}(x), ..., \mu_{G_n}(x), \mu_D(x)\right\}$.

Let be $G = \bigcap_{i \in N} G_i$, where $N = \{1,...,n\}$ - PMD's set of fuzzy goals. Then the fuzzy set G, that is characterized by the membership function $\mu_G(x) = \min_{i \in N} \mu_{G_i}(x)$, will be the PMD's fuzzy goal set. The fuzzy set of solutions $X^* = G \bigcap D$ will be described by membership function $\mu(x) = \min\{\mu_G(x), \mu_D(x)\}$. If PMD interested in any particular alternative, then the rational choice is the so-called maximizing alternative [Bellman, 1970], which satisfies the condition $x^* = \arg\max_{x \in X} \mu(x)$.

Definition of the decision making problem with fuzzy set of goals

Sometimes PMD can't clearly specify which fuzzy sets G_i, $i \in N$, characterizing its goal, but may ask some fuzzy subset $\tilde{N} \subseteq N$ of these sets. Note, that the set N should be called, down in this article, like universal set of goals indexes.

For example, if the buyer chooses a product that should belong to certain goal sets, which define: price, calorie content, quality, freshness, prestige - that all these sets are not necessarily characterize his goal with the degree of membership equal to one. Therefore, the buyer may ask some fuzzy subset of universal set of goals that will adequately characterize the true goal.

Denote $\eta : N \to [0,1]$ like the membership function of fuzzy set $\tilde{N}$ of fuzzy sets G_i that characterize the PMD goal. Then the whole goal may be defined by the intersection $\tilde{G} = \bigcap_{i \in \tilde{N}} G_i$ of fuzzy set $\tilde{N}$ of fuzzy sets G_i, $i \in N$.

Define the concept according to the approach that was proposed in [Mashchenko, 2010].

Let be $\mu_{G_j} : X \to [0,1]$ - membership function of fuzzy set G_j, $j \in N$. For the first let's consider the set $G = \bigcap_{i \in N} G_i$, which is the intersection of a clear set N of fuzzy sets G_i, $i \in N$. According to the classical theory [Zadeh, 1973] $G = \bigcap_{i \in N} G_i$ - is a fuzzy set, which is given by the membership function $\mu_G(x) = \min_{j \in N} \mu_{G_j}(x)$, $x \in X$. It is easy to see, that the value of membership function $\mu_G(x)$ for each fixed alternative $x \in X$ is actually defined as the value of objective function of trivial problem "clear" mathematical programming $\mu_G = \min_{j \in N} \mu_{G_j}$ (in this record for the visual perception is not specified a fixed value $x \in X$).

Let consider the intersection $\tilde{G} = \bigcap_{i \in \tilde{N}} G_i$ of fuzzy set $\tilde{N}$ of fuzzy sets G_i, $i \in N$. Generalization of classical operation of intersection clear set N of fuzzy sets naturally leads to the fact that the set $\tilde{G}$ is defined by the membership function:

$$\mu_{\tilde{G}}(x) = \min_{j \in \tilde{N}} \mu_{G_j}(x), \ x \in X. \quad (1)$$

It is clear that the value of this membership function $\mu_{\tilde{G}}$ for each alternative $x \in X$ will be defined as the value of the objective function for the fuzzy mathematical programming problem:

$$\mu_{\tilde{G}} = \min_{j \in \tilde{N}} \mu_{G_j} \tag{2}$$

(in this record, as in the previous case, also not specified a fixed $x \in X$).

Fuzzy mathematical programming problems are sufficiently well studied. According [Orlovsky, 1981], the solution of problem (2) is fuzzy set $\tilde{N}^{\cap}$, a carrier which is the set of Pareto optimal solutions (denote it by $N^{\cap}$) next two-criteria problem:

$$\mu_{G_i} \to \min,\ \eta(i) \to \max,\ i \in N. \tag{3}$$

Membership function $\eta^{\cap}$ of fuzzy set $\tilde{N}^{\cap}$ is a narrowing of the membership function $\eta(i)$, $i \in N$ from universal set of criteria indexes on the set $N^{\cap}$.

In other words, this membership function is as follows: $\eta^{\cap}(i) = \begin{cases} \eta(i), & i \in N^{\cap}, \\ 0, & i \notin N^{\cap}. \end{cases}$

According to the solution of problem (2), which is a fuzzy set $\tilde{N}^{\cap}$, by [Orlovsky, 1981] defined fuzzy set $\Re$ of optimal values of objective function of this problem. It is defined by the membership function $\rho : [0,1] \to [0,1]$, $\rho(z) = \max\limits_{\mu_{G_j} = z} \eta^{\cap}(j)$, $z \in [0,1]$. It should be noted that the membership function $\rho(z)$, $z \in [0,1]$, of fuzzy set $\Re$ of optimal values of objective function of problem (2) defined on the interval $[0,1]$. This explained by the fact that this segment is a strongly possible set of values of membership function $\mu_{\tilde{G}}(x) = \min\limits_{j \in \tilde{N}} \mu_{G_j}(x)$ of fuzzy set $\tilde{G} = \bigcap\limits_{i \in \tilde{N}} G_i$ in any fixed alternative $x \in X$.

Thus, for each fixed alternative $x \in X$ value of membership function (1) of fuzzy set $\tilde{G} = \bigcap\limits_{i \in \tilde{N}} G_i$ also forms fuzzy set. Hence we can conclude that the fuzzy set $\tilde{G}$ is the so-called [Zadeh, 1973], fuzzy set of type 2.

So, our research can now formalize the notion of intersection $\tilde{G} = \bigcap\limits_{i \in \tilde{N}} G_i$ of fuzzy set $\tilde{N}$ of fuzzy sets G_i, $i \in N$.

For any alternative $x \in X$ lets have a look at the ratio of dominance, which is generated by the goal sets of the problem (3) in a universal set of goals N.

We say that the goal with index $i \in N$ dominated by the goal with index $j \in N$ for alternative $x \in X$ and mark it $i \overset{(x,y)}{\succ} j$, if there are such inequalities taking true: $\mu_{G_i}(x) \le \mu_{G_j}(x)$, $\eta(i) \ge \eta(j)$, and at least one of them is strict.

This concept allows to define a set of Pareto optimal solutions for two-criteria problem (3), which will be the carrier of fuzzy set of solutions of problem (2). For any $x \in X$ let denote this carrier like this:

$$N^{\cap}(x) = \left\{ i \in N \mid j \overset{(x)}{\not\succ} i,\ \forall j \in N \right\} \tag{4}$$

For all $x \in X$, $i \in N$, let make a definition of the membership function of fuzzy set of solutions of problem (2):

$$\eta^{\bigcap}(x,i)=\begin{cases}\eta(i), & i\in N^{\bigcap}(x),\\ 0, & i\notin N^{\bigcap}(x),\end{cases} \tag{5}$$

Then the intersection of fuzzy set $\tilde{N}$ of fuzzy sets G_i, $i\in N$, will be called $\tilde{G}=\bigcap_{i\in\tilde{N}}G_i$ - fuzzy set of type 2, which is given by pairs $(x,\mu_{\tilde{G}}(x,z))$, where x - the element of the set of alternatives X, and $\mu_{\tilde{G}}(x,z)$ - fuzzy image $\mu_{\tilde{G}}:X\times[0,1]\to[0,1]$, which serves as its fuzzy membership function and defined as follows:

$$\mu_{\tilde{G}}(x,z)=\max_{i\in N}\{\eta^{\bigcap}(x,i)\mid\mu_{G_i}(x)=z\}\text{, if }\exists i\in N\text{, }\mu_{G_i}(x)=z\text{, }z\in[0,1]; \tag{6}$$

$$\mu_{\tilde{G}}(x,z)=0\text{, if }\mu_{G_i}(x)\neq z,\forall i\in N\text{, }z\in[0,1] \tag{7}$$

Calculation of the membership function $\mu_{\tilde{G}}(x,z)$ by (4) - (7) can be simplified if you use the following theorem.

Theorem 1. Let G_i, $i\in N$, - some fuzzy sets defined on the set of alternatives X, which are set by membership functions $\mu_{G_i}(x)$, $x\in X$, $i\in N$; $\tilde{N}$ - fuzzy subset of N with the membership function $\eta(i)$, $i\in N$. Then the membership function of fuzzy set $\tilde{G}=\bigcap_{i\in\tilde{N}}G_i$ of type 2 is given by the following formula:

$$\mu(x,z)=\begin{cases}\max\limits_{i\in N^{\bigcap}(x,z)}\eta(i), & N^{\bigcap}(x,z)\neq\varnothing,\\ 0, & N^{\bigcap}(x,z)=\varnothing,\end{cases} \tag{8}$$

and $N^{\bigcap}(x,y,z)=\left\{i\in N\mid\mu_{G_i}(x)=z,\ z=\min_{j\in N}\{\mu_{G_j}(x)\big|\eta(j)\geq\eta(i)\},\eta(i)=\max_{j\in N}\{\eta(j)\big|\mu_{G_j}(x)\leq z\}\right\}$, (9)

$\forall x,y\in X$, $z\in[0,1]$.

Proof. To prove the theorem we should show that $\mu(x,z)=\mu_{\tilde{G}}(x,z)$ for $\forall x\in X$, $z\in[0,1]$.

Suppose that for some $x\in X$, $z\in[0,1]$ $N^{\bigcap}(x,z)=\varnothing$, than from (8) $\mu(x,z)=0$, and by (9) for $\forall i\in N$ should be implemented at least one of the following conditions:

$$\mu_{G_i}(x)\neq z, \tag{10}$$

$$\exists j\in N:\mu_{G_i}(x)>\mu_{G_j}(x),\ \eta(j)\geq\eta(i) \tag{11}$$

$$\exists j\in N:\eta(i)<\eta(j),\ \mu_{G_j}(x)\leq\mu_{G_i}(x) \tag{12}$$

If condition (10) became true, then by (7) we obtain $\mu_{\tilde{G}}(x,z)=0$. If condition (11) or (12) became true, then $j\overset{(x)}{\succ}i$. Then according (4) $i\notin N^{\bigcap}(x)$. Hence by (5) $\eta^{\bigcap}(x,i)=0$. Therefore by (6) we get also $\mu_{\tilde{G}}(x,z)=0$. Thus, $\mu(x,z)=\mu_{\tilde{G}}(x,z)=0$.

Suppose that for some $x\in X$, $z\in[0,1]$ $N^{\bigcap}(x,z)\neq\varnothing$. Hence by (8) we get $\mu(x,z)=\max\limits_{i\in N^{\bigcap}(x,z)}\eta(i)$. Denote $i^*=\arg\max\limits_{i\in N^{\bigcap}(x,z)}\eta(i)$. Then from (9) it follows next:

$$r_{i^*}(x) = z\,,\ j \overset{(x,y)}{\not\succ} i^*,\ \forall j \in N\,. \tag{13}$$

So according to (4) $i^* \in N^{\cap}(x) \neq \varnothing$, $\eta^{\cap}(x,i^*) = \eta(i^*)$. Hence $\mu_{\tilde{G}}(x,z) \geq \eta(i^*)$.

Let show that $\mu_{\tilde{G}}(x,z) = \eta(i^*)$. Assume contrary that $\mu_{\tilde{G}}(x,z) > \eta(i^*)$. Denote $j^* = \arg\max_{i \in N}\{\eta^{\cap}(x,i) \mid r_i(x) = z\}$.

Then $\eta(j^*) > \eta(i^*)$, $\mu_{G_{j^*}}(x) = z$ and by (5) $j^* \in N^{\cap}(x)$. As for (13) $\mu_{G_{i^*}}(x) = z$, then $\mu_{G_{i^*}}(x) = \mu_{G_{j^*}}(x)$. So $j^* \overset{(x,y)}{\not\succ} i^*$. Hence $j^* \notin N^{\cap}(x)$. We obtained contradiction. So, $\mu_{\tilde{G}}(x,z) = \eta(i^*) = \mu(x,z)$. Theorem is proved.

To illustrate the intersection of fuzzy set of fuzzy sets let take a look on an example.

Example 1. Let be the set of alternatives X which consists four alternatives: *a*, *b*, *c*, *d*. The set of fuzzy alternatives D is defined by the membership function $\mu_D(x)$ (Tab.1). On the set X also defined two fuzzy sets G_1, G_2 with membership functions respectively $\mu_{G_1}(x)$ and $\mu_{G_2}(x)$, and whose values are listed in Tab.1. Let also define fuzzy subset $\tilde{N}$ from indexes set of these relations $N = \{1,2\}$ with membership function $\eta(i)$, $i \in N$, which takes the values: $\eta(1) = 0.5$, $\eta(2) = 0.8$. So, lets find the intersection $\tilde{G} = \bigcap_{i \in \tilde{N}} G_i$ of fuzzy set $\tilde{N}$ of fuzzy relations G_1, G_2.

In Tab.1 also indicated the sets $N^{\cap}(x)$, $x \in X$, and membership function $\eta^{\cap}(x,i)$, $x \in X$, $i \in N$. The values of membership function $\mu_{\tilde{G}}(x,z)$ of fuzzy set $\tilde{G}$ of type 2 are listed in Tab.2.

Table 1 Function and sets

x	a	b	c	d
$\mu_D(x)$	0.1	0.2	0.3	0.5
$\mu_{G_1}(x)$	0	0.5	0.3	0
$\mu_{G_2}(x)$	0	0.3	1	0
$N^{\cap}(x)$	{2}	{2}	{1,2}	{2}
$\eta^{\cap}(x,1)$	0	0	0.5	0
$\eta^{\cap}(x,2)$	0.8	0.8	0.8	0.8

Table 2 Membership function $\mu_{\tilde{G}}(x,z)$

z	x			
	a	b	c	d
0	0.8	0	0	0.8
0.3	0	0.8	0.5	0
0.5	0	0	0	0
1	0	0	0.8	0

Note that in Tab.2 recorded only those values of variable $z \in [0,1]$ that meet a priori non-zero value of membership function $\mu_{\tilde{G}}(x,z)$.

The solution of decision making problem with fuzzy set goals

Proceed to the construction of solution of the problem of rational choice of alternatives for the fuzzy goal, which is defined by the fuzzy set $\tilde{G} = \bigcap_{i \in \tilde{N}} G_i$ of type 2.

To do this it firstly need to construct the general solution, which is fuzzy set $X^* = \tilde{G} \bigcap D$ of type 2, and then must determine the approach to the selection of specific alternatives in this set.

According to [Zadeh, 1973] the intersection of two fuzzy sets A and B of type 2, which are defined by fuzzy reflection (they act as their fuzzy membership functions), respectively, $\mu_A(x,z)$ and $\mu_B(x,z)$, $x \in X$, $z \in [0,1]$ - is fuzzy set of type 2, which is given by membership function

$$\mu_{A \cap B}(x,z) = \max_{\substack{z_A, z_B \in [0,1], \\ z = \min\{z_A, z_B\}}} \min\{\mu_A(x,z_A), \mu_B(x,z_B)\} \tag{14}$$

Table 3 Membership function $\mu_D(x,z)$

z	x			
	a	b	c	d
0	0	0	0	0
0.1	1	0	0	0
0.2	0	0	1	0
0.3	0	0	0	1
0.5	0	1	0	0
1	0	0	0	0

Table 4 Membership function $\mu(x,z)$

z	x			
	a	b	c	d
0	0.8	0	0	0.8
0.1	0	0	0	0
0.2	0	0	0.8	0
0.3	0	0.8	0	0
0.5	0	0	0	0
1	0	0	0	0

Therefore, fuzzy reflection, which define the fuzzy membership function of fuzzy set of solutions $X^* = \tilde{G} \bigcap D$ of type 2 will look like $\mu(x,z) = \max_{\substack{z_D, z_G \in [0,1], \\ z = \min\{z_D, z_G\}}} \min\{\mu_D(x,z_D), \mu_{\tilde{G}}(x,z_G)\}$, where $\mu_D(x,z_D)$ - reflection, which define the fuzzy set of alternatives D and determined to $\forall x \in X$, $\forall z \in [0,1]$, as follows: $\mu_D(x,z) = \begin{cases} 1, & z = \mu_D(x), \\ 0, & z \neq \mu_D(x). \end{cases}$

For example 1, a membership function of fuzzy sets of type 2 $\mu_D(x,z)$ and $\mu(x,z)$ are respectively defined in the Tab. 3 and 4.

Since PMD can interest a specific alternative, then there is the problem of rational choice from fuzzy set X^* of type 2. It is clear that for PMD is important to choose the alternative that from one side will maximize the fuzzy value $z \in [0,1]$ of dominates degree, and, on the other hand, maximize the fuzzy value μ, which is characterizing the degree of membership the z value to the fuzzy set of its values.

Thus, we can formulate the following 2-criteria problem:

$$z \to \max,\ \mu(x,z) \to \max,\ x \in X,\ z \in [0,1].$$

Let be $x^* \in X$, $z^* \in [0,1]$ - a solution to this problem, then the value z^* characterizes the dominates degree of alternative x^*, and the value $\mu(x^*,z^*)$ - the membership degree of z^* to fuzzy set with membership function $\mu(x^*,z)$, $z \in [0,1]$. To distinguish between these concepts, then the value $\mu(x^*,z^*)$ will be called the degree of certainty dominates z^* of alternative x^*.

Depending to the comparing methods of alternatives [Podinovsky, 1982] by the criteria of 2-criteria problem (14) consider two definitions.

Alternative $x^* \in X$, which is strongly no-dominates with degree z^* of credibility $\mu(x^*,z^*)$ will be called fuzzy weakly-effective alternative (it set we denote $S(X)$), if $\nexists x \in X\ \nexists z \in [0,1]$, for which: $z > z^*$, $\mu_{ND}(x,z) > \mu_{ND}(x^*,z^*)$.

Alternative $x^* \in X$, which is no-dominates with degree z^* of credibility $\mu(x^*,z^*)$ will be called fuzzy effective alternative (it set we denote $P(X)$), if $\nexists x \in X\ \nexists z \in [0,1]$, for which the condition or $\mu_{ND}(x,z) \geq \mu_{ND}(x^*,z^*)$, $z > z^*$, or $\mu_{ND}(x,z) > \mu_{ND}(x^*,z^*)$, $z \geq z^*$.

It is clear, that $S(X) \supseteq P(X)$.

For example 1 according to the Tab.4 it is easy to verify that alternative *b* is only one fuzzy effective alternative with the membership degree 0.3 and with reliability 0.8.

In general, those alternatives which have a maximum no-dominates degree may have a low degree of reliability and vice versa.

From the famous theorem [Podinovsky, 1982] as a consequence follows the criterion of efficiency alternatives.

Corollary. The alternative $x^* \in X$, which is no-dominates with degree z^* of credibility $\mu(x^*,z^*)$, is effective if and only if it is the best solution of pair of optimizations problems:

$$\begin{aligned} &\mu(x,z) \to \max, \\ &z \in [z^*,1],\ x \in X; \end{aligned} \quad (15)$$

$$\begin{aligned} & z \to \max, \\ & \mu(x,z) \ge \mu(x^*,z^*), \\ & z \in [0,1],\ \ x \in X. \end{aligned} \tag{16}$$

Construction of all fuzzy sets of effective alternatives is a difficult task, but as often PMD interesting choice of specific alternative, then this is not necessary.

One of the possible variants of rational choice alternatives can be considered using one of problems (15), (16).

First consider the searching problem of fuzzy alternative x^* that maximizes the degree of credibility μ of no-dominates degree *z*, not less than a value $\tilde{z} \in [0,1]$, ie:

$$\begin{aligned} & \mu(x,z) \to \max, \\ & z \in [\tilde{z},1],\ x \in X; \end{aligned} \tag{17}$$

Fair such a statement.

Proposition 1. Let the pair (x^*,z^*) is a solution of problem (17) for some value $\tilde{z} \in [0,1]$. Then there is x^* which is weakly-efficient alternative that is no-dominates with degree z^* and with degree of credibility $\mu(x^*,z^*)$.

Proof. Denote μ^* the maximum value of the objective function of problem (17). Suppose contrary to that $x^* \notin S(X)$. Then, by definition, $\exists \hat{x} \in X\ \exists \hat{z} \in [0,1]$ for which are performed next inequalities: $\mu(\hat{x},\hat{z}) > \mu(x^*,z^*)$, $\hat{z} > z^*$. According to (17) $\tilde{z} \le z^* \le 1$, then $\tilde{z} \le z^* < \hat{z} \le 1$. Therefore a pair $(\hat{x},\hat{z})$ satisfies to the conditions of problem (17), and $\mu(\hat{x},\hat{z}) > \mu^*$. Obtained contradiction. The claim is proved.

Lets consider the searching problem of fuzzy alternative x^* with maximum no-dominates degree *z* that has degree of credibility μ_{ND}, not less than a value $\tilde{\mu} \in [0,1]$, ie:

$$\begin{aligned} & z \to \max, \\ & \mu(x,z) \ge \tilde{\mu}, \\ & z \in [0,1],\ \ x \in X. \end{aligned} \tag{18}$$

Fair such a statement.

Proposition 2. Let the pair (x^*,z^*) is a solution of problem (18) for some value $\tilde{\mu} \in [0,1]$. Then there is x^* which is weakly-efficient alternative that is no-dominates with degree z^* and with degree of credibility $\mu(x^*,z^*)$.

Proof. Note that z^* is the maximum value of the objective function of problem (18). Suppose contrary to that $x^* \notin S(X)$. Then, by definition, $\exists \hat{x} \in X\ \exists \hat{z} \in [0,1]$ for which are performed next inequalities: $\mu(\hat{x},\hat{z}) > \mu(x^*,z^*)$, $\hat{z} > z^*$. According to (18) $\mu(x^*,z^*) \ge \tilde{\mu}$ then $\mu(\hat{x},\hat{z}) > \mu(x^*,z^*) \ge \tilde{\mu}$. Therefore a pair $(\hat{x},\hat{z})$ satisfies to the conditions of problem (18), and $\hat{z} > z^*$. Obtained contradiction. The claim is proved.

Conclusion

In the end it should be noted that considered in this paper an approach to "rational" choice of alternatives in decision-making problem with the goal set, which is defined by the fuzzy set of fuzzy goal sets is another view to this problem than the method that was developed in the works of Zadeh in particular in [Bellman, 1970], using weights coefficients that characterize PMD advantage on the set of goal sets.

It should also be noted that the new operation of intersection of fuzzy set of fuzzy relations, which is formalized in this paper presents an independent interest and can be used in various productions of new decision making problems. The concept of fuzzy effective alternatives will be correct and has some interest for the decision making problem with goal of which will be defined by fuzzy set of clear goal sets.

Acknowledgments

The paper is partially financed by the project ITHEA XXIII of the Institute of Information Theories and Applications FOI ITHEA and the Consortium FOI Bulgaria (www.itea.org, www.foibg.com).

Literature

[Bellman, 1970] Bellman R. E., Zadeh L. A. Decision-Making in a Fuzzy Environment // Management Science.- 1970.- N17.-p. 141-164.

[Mashchenko, 2010] Mashchenko S. O. Fuzzy individual-optimum equilibriums // Cybernetics and computing engineering. – 2010. – N 159. – p. 19 – 29 (In Russian).

[Zadeh, 1973] Zadeh L.A. Outline of a new approach to the analysis of complex systems and decision processes.- IEEE Transactions System, Man Cybernetics, SMC-3. – 1973.- N1.- p. 28-44.

[Orlovsky, 1981] Orlovsky S. A. Decisions making problems at conditions of fuzzy initial information. - M.: Science, 1981. – 208 p. (In Russian).

[Podinovsky, 1982] Podinovsky V. V., Nogin V. D. Pareto-optimum decisions of multicriterial problems. – M. : Science, 1982. – 255 p. (In Russian).

Authors' Information

Sergiy Mashchenko *– math Ph.D., associate professor; Taras Shevchenko National University of Kyiv, Ukraine, 01017 Kyiv, Volodymyrs'ka st.64.*

e-mail: msomail@yandex.ru

Research interests: *optimization methods, decision making, decision support systems, game theory*

Oleksandr Bovsunivskyi *– postgraduate student; Taras Shevchenko National University of Kyiv, Ukraine, 01017 Kyiv, Volodymyrs'ka st.64.*

e-mail: bovs@email.ua

Research interests: *decision making, decision support systems, game theory*