

## OPTIMIZATION OF CONNECTION'S STRUCTURE OF REMOTELY-OPERATED PORTABLES TO INFORMATION NETWORKS' BASIC EQUIPMENT

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**Abstract:** *The abstract model of information network's evolution's process is considered. Three classes of equipment's types on the network are specified. Classification of information network's structure's optimization problems by a time sign is offered and proved. Possibilities of the further research of modernized network's development's general problem according to the offered classes of problems are defined.*

**Keywords:** *information network, basic equipment, portable, station, structure of connection.*

**ACM classification keywords:** *H. Information Systems - H.1 MODELS AND PRINCIPLES.*

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### Introduction

Information networks' (IN) designing includes various problems from mathematical and technical to economic and political. Recently so-called portable or remotely-operated equipment which is connected to the equipment named basic and functionally interconnected with it is more widely used on information networks. Information networks' basic equipment can work as stand-alone, and together with the considered portable equipment. There is great number of systems working in such way: digital switching systems with portable remote modules connected to them - digital concentrators or subscriber's multiplexers, computer systems using the «Client - server» connection principle and control systems co-operating on the «control device - operated object» principle. Base stations' controllers connected to mobile communication's cellular networks' switching centers work by such principle.

Earlier network development's process was planned for each station independently. Occurrence of portable equipment which is usually named portable modules (PM) creates additional logic connection between stations and brings about necessity of integrated approach for planning. Taking into account entered equipment's features at definition of network's modernization strategy it is important to consider a way of connection between PM and basic equipment. This work is devoted to one of possible approaches for the decision of this problem.

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### Problem statement

Let's consider IN's development model at introduction of the new equipment differing by a number of governing modernized networks' structure parameters. The general formulation of a research's problem is given in [1,2], some private problems' decision's aspects within the formulated problem's limits in works [3-5]. For a considered problem network development's strategy is defined by type, site and placing time of anew entered equipment.

Superposition method at which existing equipment keeping in network's stations and development is carried out by the new one and a replacement method at which existing equipment is replaced with the new are considered. At the same time network's development model considers network's interrelations in space and in time.

According to the problem put by let's allocate three classes of equipment's types (fig. 1) used on the network:

- existing technology's equipment;
- new technology's basic equipment;
- new technology's portable equipment.

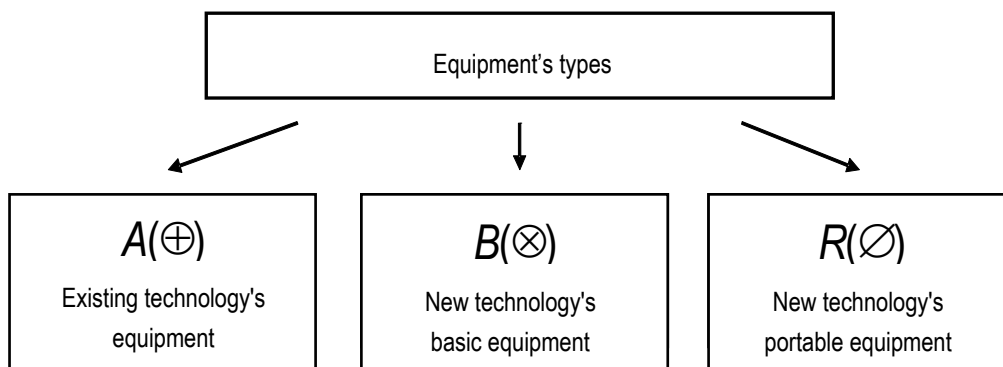


Fig. 1 Classification of equipment's types

Let's designate these equipment's classes in the form of the sets  $A$ ,  $B$  and  $R$ . It is natural to consider these sets final which each element  $a_i$ ,  $b_i$  or  $r_i$  represents equipment's type of corresponding class.

### Definition of information network's evolution's strategy

Let's consider IN's development model at new equipment's introduction differing in parameters influencing modernized network's structure. Network's evolution strategy is defined by type, site and placing time of the new equipment.

With use of works' [1-5] results decision of considered problem can be examined in a kind of pair  $(X, Y)$ , where  $X : I \times T \rightarrow 2^{A \cup B \cup R}$  defines used equipment's types on stations. Such task  $X$  means that there can be some types of equipment  $X(i, t) \subseteq A \cup B \cup R$  simultaneously established on some station  $i$  during each moment of time  $t$ .

Let's put additional designations of this characteristic for what we will fix some station  $i$  and define  $x_i : T \rightarrow 2^{A \cup B \cup R}$ ,  $x_i(t) = X(i, t)$ .

Let's name set of equipment's types used at present on station and accordingly functions which are carried out in station - station's condition. Then  $\Psi = 2^{A \cup B \cup R}$  - set of station's conditions.

For stations with established portable modules let's define basic station to which concrete PM is connected at present time. For this purpose let's set representation

$$Y : E_R \rightarrow I_B,$$

where  $E_R = \{ \langle i, t \rangle \mid i \in I, t \in T : X(i, t) \cap R \neq \emptyset \}$  - set of spatio-temporal system's points demanding the basic equipment that is ordered pairs from time's moment and station on which PM is located.

$I_B = \{ i \mid i \in I : \exists t \in T X(i, t) \cap B \neq \emptyset \}$  - set of potentially basic spatial points that is stations on which at replacement basic equipment is entered during the investigated period.

This representation in its range of definition sets basic station  $Y(i, t) \in I$  to which station  $i$  is connected at the moment of  $t$ .

Then the decision of a problem of a choice of IN's evolution strategy can be considered in a kind of pair  $(X, Y)$  where  $Y : I \rightarrow I_0$  defines connection's structure of the portable equipment. Let's formulate this statement in the form of a lemma.

**Lemma 1.** *It is always possible to define network's evolution strategy on the basis of pair  $(X, Y)$  and thus only one.*

**The proof.** Admissible network evolution strategies are only that one which allow either full replacement of previous type's equipment with new type's equipment, or preservation of previous type's equipment and usage of new type's equipment only for expansion of station's capacity, during the introduction of new type of the equipment. During the concrete moment of time introduction only of one equipment's type on one station is supposed.

From the foregoing follows that if there is a change of a set of used equipment's types at the moment of time  $t$  at station  $i$

$$X(i, t) \neq X(i, t-1),$$

only two variants are possible

$$X(i, t) = X(i, t-1) \cup \{\chi_2\}$$

or

$$X(i, t) = (X(i, t-1) \setminus \{\chi_1\}) \cup \{\chi_2\},$$

where  $\chi_2 \in B \cup R$  - the equipment's type entered during moment  $t$ ;

$\chi_1 \in B \cup R$  - the equipment's type entered directly ahead of  $\chi_2$ .

The first variant corresponds to imposing of the type  $\chi_2$  equipment on the existing equipment and the second - full replacement of the type  $\chi_1$  equipment by the type  $\chi_2$  equipment. These variants for each station  $i$  and time moment  $t$  are defined unequivocally and in their turn unequivocally define development's strategy as was to be proved. ■

Let's carry out classification of IN's evolution problem by complexity level. For this purpose let's enter several definitions [3].

Let's name station's  $j$  on which PM is established work remote control by station  $i$  supplied with the basic equipment as **portable's  $j$  connection** to basic station  $i$  during some moment  $t$ .

$$y_j(t) = i$$

$$x_j(t) \cap R \neq \emptyset$$

$$x_i(t) \cap B \neq \emptyset$$

Let's name basic station's change for PM on station  $j$  as **portable's  $j$  connection change**

$$\exists t \in T, i, k \in I, i \neq k, t > 0: y_j(t-1) = i \wedge y_j(t) = k.$$

Let's name set of each network station's connections as network's **portables' connection structure**.

Let's name invariant throughout all period of research connection as **station's  $j$  strictly stationary connection**.

It means that basic station for this PM  $j$  stays invariable within all considered time area  $y_j(t) \equiv \text{const}$ .

Let's name invariant on segment  $[\theta_j, h]$ , where  $\theta_j$  - the portable equipment's introduction moment on station  $j$ , connection as **station's  $j$  generalized stationary connection**.

$$\forall t \in T: \theta_j < t \leq h \quad y_j(t) = y_j(t-1).$$

Let's name connections' structure on all network as **network's strictly (generalized) stationary connections' structure** if each of station's connections are strictly (generalized) permanently.

Directly from the entered definitions follow statements.

**Lemma 2.** Any strictly stationary connection is generalized stationary.

**Lemma 3.** Any strictly stationary connections' structure is generalized stationary.

Let's allocate problems' classes (fig. 2) corresponding to assumptions:

- class  $K_1$ : the connections' structure of network is strictly stationary;

$$\forall j \in I \quad y_j(t) \equiv \text{const} \tag{1}$$

- class  $K_2$ : the connections' structure of network is generalized stationary;

$$\forall j \in I, t_1, t_2 \in T: \theta_j < t_1, t_2 \leq h \quad y_j(t_1) = y_j(t_2) \tag{2}$$

- class  $K_3$ : any connections' structure in which there are no connections' changes is possible;

$$\forall j \in I, t_1, t_2 \in T: t > 0 \quad \begin{cases} y_j(t_1) \in I \\ y_j(t_2) \in I \end{cases} \Rightarrow y_j(t_1) = y_j(t_2) \tag{3}$$

- class  $K_4$ : arbitrary connections' structure is admissible.

Construction of problems' classes indicates that they make a chain of inclusions, namely  $K_1 \subset K_2 \subset K_3 \subset K_4$  where the first class problems actually exclude possibility of the portable equipment's introduction.

**Lemma 4.** Let  $(X, Y)$  the decision of the first class problem. Then

$$f(t) = \text{sgn card } x_i(t) \cap R = \text{const} \quad \forall i \in I \tag{4}$$

**The proof.** Let's carry out the proof by contradiction method. Let's assume that the condition (4) is not executed, that is

$$\exists i^* \in I, t^* \in T, t^* > 0: \text{sgn card } x_{i^*}(t^*) \cap R \neq \text{sgn card } x_{i^*}(t^* - 1) \cap R \tag{5}$$

Considering the set's cardinal number's properties it is possible only in the event that  $\text{sgn card } x_{i^*}(t^* - 1) \cap R = 0 \wedge \text{sgn card } x_{i^*}(t^*) \cap R = 1$ , that is  $x_{i^*}(t^* - 1) \cap R = \emptyset \wedge x_{i^*}(t^*) \cap R \neq \emptyset$ . But then by definition  $y_i(t) \ y_{i^*}(t^* - 1) = 0 \wedge y_{i^*}(t^*) \neq 0$ . That is  $\exists i^* \in I, t^* \in T : y_{i^*}(t^* - 1) \neq y_{i^*}(t^*)$ .

However it means that portable's connection for some station is not strictly stationary, that is the network's portables' connection's structure as a whole cannot be strictly stationary that contradicts a condition.

Thus the assumption (4) is incorrect. The lemma is proved. ■

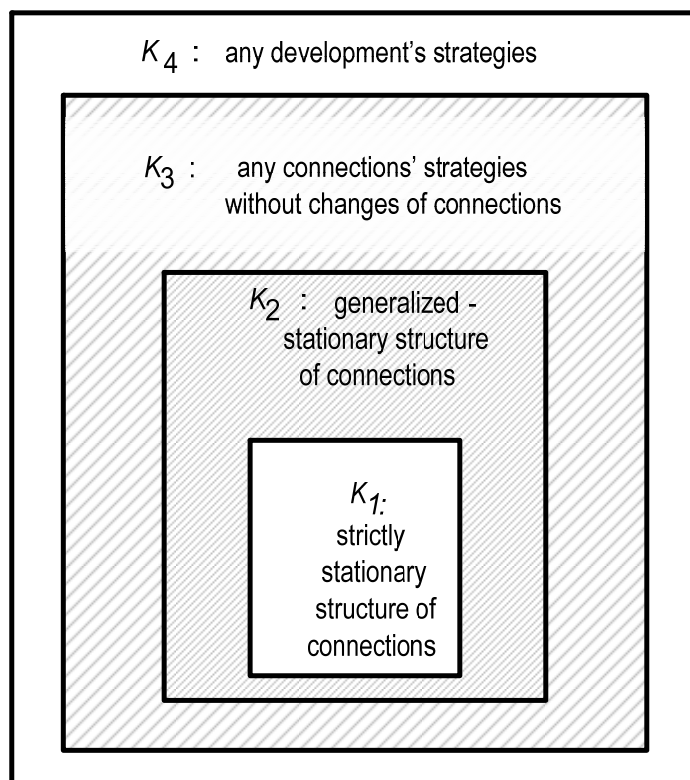


Fig. 2 Classes of problems on complexity

If thus during the initial time moment the portable equipment has not been established it is possible to formulate more essential conclusion.

**Consequence.** If in the lemma's 4 conditions moreover the condition of portable equipment's absence on all network's stations during the initial time moment  $\forall i \in I \ x_i(0) \cap R = \emptyset$  is satisfied, the statement that this equipment will not be established during the investigated period and not one station will not be served by another as basic is fair

$$\begin{aligned} \forall i \in I, t \in T \ X(i, t) \cup R = \emptyset \\ Y(i, t) = 0 \end{aligned} \tag{6}$$

**The proof.** Let's prove the statement by time  $t$  induction.

Induction's base: at  $t = 0$  (6) is true on the condition.

Let's assume that the lemma's statement is fair  $\forall t \leq k, k \geq 0$ . Let's show that it is true for  $t = k + 1$  too.

According to the lemma 4  $\text{sgn card } x_i(k+1) \cap R = \text{sgn card } x_i(k) \cap R$ .

But under the induction's assumption  $x_i(k) \cap R = \emptyset$ ,

that is

$$\text{sgn card } x_i(k) \cap R = 0.$$

And it attracts equality

$$\text{sgn card } x_i(k+1) \cap R = 0,$$

in other words

$$x_i(k+1) \cap R = \emptyset.$$

And from this follows

$$y_i(k+1) = 0.$$

That is the lemma's statement is true for  $t = k + 1$  too.

The step is executed. The lemma is proved. ■

The proved one indicates that the class 1 problems are too narrow and uninteresting.

Class  $K_4$  though is more general and does not produce any restrictions to problem's condition insignificantly differs from class  $K_3$ . The difference consists only in admissibility or inadmissibility of serving station's change for some portable.

**Lemma 5.**  $(X, Y)$  is the decision of class  $K_4 \setminus K_3$  problem in only case when

$$\exists j^* \in I, t_1^*, t_2^* \in T : t_1^*, t_2^* > 0 \quad y_{j^*}(t_1^*), y_{j^*}(t_2^*) \in I, \text{ but } y_{j^*}(t_1^*) \neq y_{j^*}(t_2^*).$$

The proof evidently follows from classes  $K_3$  and  $K_4$  definition.

Let's consider the general aspects of the classes  $K_2$  and  $K_3$  problems' decision. The basic distinction between these classes consists that  $K_3$  supposes replacement of the portable equipment on basic, and class  $K_2$  - no.

**Lemma 6.** If  $(X, Y)$  is the decision of class  $K_3 \setminus K_2$  problem following

$$\exists j^* \in I, t_1^*, t_2^* \in T : t_1^*, t_2^* > 0 \quad x_{j^*}(t_1^*) \cap R \neq \emptyset \wedge x_{j^*}(t_2^*) \cap R = \emptyset \text{ is true.}$$

**The proof.** If  $(X, Y)$  simultaneously is the class  $K_3$  problem's decision and is not the class  $K_2$  problem's decision by definition for  $(X, Y)$  it is carried out (2) and it is not carried out (3). That is

$$\exists j^* \in I, t_1^{**}, t_2^{**} \in T : \theta_{j^*} < t_1^{**}, t_2^{**} \leq h \quad y_j(t_1^{**}) \neq y_j(t_2^{**})$$

besides, that

$$\begin{cases} y_{j^*}(t_1^{**}) \in I \\ y_{j^*}(t_2^{**}) \in I \end{cases} \Rightarrow y_{j^*}(t_1^{**}) = y_{j^*}(t_2^{**}).$$

It means one of condition's fulfillment

$$\begin{cases} y_{j^*}(t_1^{**}) \notin I, \\ y_{j^*}(t_2^{**}) \notin I. \end{cases}$$

Really if it was not so owing to (3)  $y_{j^*}(t_1^*)$  it would be equal  $y_{j^*}(t_2^*)$ . From definition of function  $y_j(t)$  expansion,

$$\begin{cases} x_{j^*}(t_1^{**}) \cap R = \emptyset, \\ x_{j^*}(t_2^{**}) \cap R = \emptyset. \end{cases} \quad (7)$$

But also  $t_1^{**}$  and  $t_2^{**}$  are strictly more then the first imposing  $\theta_{j^*}$  time for which by definition it is carried out  $x_{j^*}(\theta_{j^*}) \cap R \neq \emptyset$ .

Let's designate  $t_2^*$  that one from variables  $t_1^{**}$  and  $t_2^{**}$  which provides fulfillment of set (7),  $x_{j^*}(t_2^*) \cap R = \emptyset$ .

Let also  $t_1^* = \theta_{j^*}$ . We have received

$$\exists j^* \in I, t_1^*, t_2^* \in T : t_1^*, t_2^* > 0 \quad x_{j^*}(t_1^*) \cap R \neq \emptyset \wedge x_{j^*}(t_2^*) \cap R = \emptyset.$$

Which was to be proved.

This classification is convenient because it allows concretizing station's evolution's strategies. In particular for a case with one type of the existing equipment, one type of new portable and one type of new basic it is possible to assert that there are strategies only on one type of station's evolution in each of classes  $K_1$ ,  $K_2 \setminus K_1$  and  $K_3 \setminus (K_2 \cup K_1)$ .

These station evolution's types are on fig. 3 and defined as follows:

- imposing of the new basic equipment and the subsequent replacement of the existing equipment by it;
- imposing of the new portable module and the subsequent replacement of the existing equipment by it;
- imposing of the new portable module and the subsequent replacement of all equipment by the new basic.

The listed types of station's evolution assume only potential possibility of imposition's and replacement's events, that is, this or that event can not occur at all. For example, strategies of the second and third type include preservation's possibility of the imposed portable equipment on the station till the end of the investigated period. Also it is necessary to notice that all three strategy's types include strategy of proceeding growth. Similar classification's ambiguity can be eliminated easily enough if we agree to carry the strategy belonging simultaneously to several station's evolution's types only to type with minimum number. Then for each station it will be possible to define unequivocally one of three strategies of its evolution.

It is necessary to notice that information which gives that the station develops by the second or third type strategy is insufficient it demands specifications. Namely it is necessary to specify also what station is basic for the given portable.

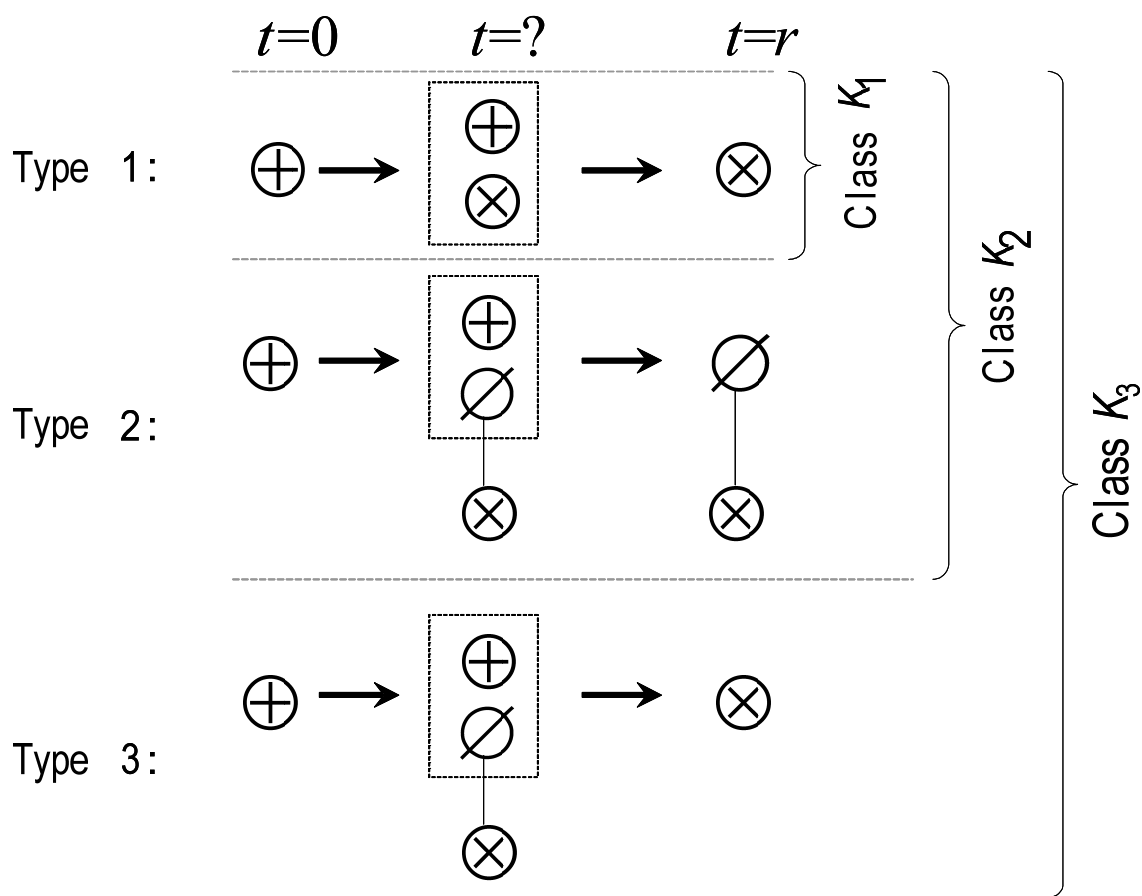


Fig . 3 Three types of station evolution's strategy

According to this let's define binary variables  $\chi_{ij}$  and divide them into three groups corresponding to types of station evolution's strategies  $\forall i, j \in I$

$$\chi_j^1 = \begin{cases} 1, & \text{if station } j \text{ develops by type 1 strategy;} \\ 0, & \text{otherwise.} \end{cases}$$

$$\chi_{ij}^2 = \begin{cases} 1, & \text{if station } j \text{ develops by type 2 strategy and connects to station } i; \\ 0, & \text{otherwise.} \end{cases}$$

$$\chi_{ij}^3 = \begin{cases} 1, & \text{if station } j \text{ develops by type 3 strategy and connects to station } i; \\ 0, & \text{otherwise.} \end{cases}$$

Let's define sets  $X^1$ ,  $X^2$  and  $X^3$  as sets of the binary variables corresponding to each type of strategies

$$X^1 = \{x_j^1 \mid j \in I\},$$

$$X^2 = \{x_{ij}^2 \mid i, j \in I, i \neq j\},$$

$$X^3 = \{x_{ij}^3 \mid i, j \in I, i \neq j\}.$$



As the new equipment's imposition's moment is defined by value  $\theta_j \in T$  and the replacement moment -  $r_j \in T$  for any station  $j$  pair  $\theta_j - r_j$  should satisfy to following parity  $0 \leq \theta_j \leq r_j \leq h$ .

Let's define sets  $P_\theta$  and  $P_r$  as set of the imposition's or replacement's moments accordingly

$$P_\theta = \{\theta_j \mid j \in I\},$$

$$P_r = \{r_j \mid j \in I\}.$$

Then it is possible to present network evolution's strategy in the form of system  $P$

$$P = (P_\theta, P_r, X).$$

To finish the description of network evolution's strategy let's define connections' structure  $X$  as follows

$$X = \{\chi_{ij} \mid i, j \in I\},$$

where  $\chi_{ij} = \begin{cases} 1, & \text{if station } j \text{ connects to the basic station } i, \\ 0, & \text{otherwise.} \end{cases}$

Let's notice also that  $\chi_{ij} = 1$  means installation of the new basic equipment on station  $j$ . If  $\chi_{ij} = 1$  station  $j$  is the portable connected to station  $i$  only if  $\theta_j < h$ . Otherwise the existing equipment remains on station  $j$ .

Thus it is proved that the further researches of the modernized network's development's general problem can be carried out only for set of the second and third classes' problems. Naturally rejection of the portable equipment's replacement's possibility on basic essentially constricts decision's generality however consideration of stationary connections' structure considerably simplifies it. Therefore it is expediently to reduce problems where replacement of portable by basic station is seldom to the second class problem and to use methods of the decision for such problems.

## Conclusion

In work it is proved that the abstract model of network evolution's process can be presented in a kind of pair functions  $(X, Y)$  describing types of the equipment established on each station at each time moment and structure of portable modules' connection. Classification of IN's structure optimization's problems by a time sign is offered and proved. It is shown that problems of the first class are too narrow and mismatch difficult real problems of IN's planning and designing. Therefore at the further researches these problems can be considered as special cases of wider classes' problems.

In the subsequent works the generalized model of network's evolution's spatio-temporal structure's optimization by criterion of the minimum size of the future expenses' valid cost for network's evolution with which use the research of IN's evolution's strategies is executed.

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**Bibliography**

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1. Гайворонская Г.С. Проблема синтеза развивающихся информационных сетей, сборник ДУИКТ №3, 2005, Киев с.14-21
2. Гайворонская Г.С. Формализация модели эволюции телекоммуникационных сетей / Гайворонская Г. С. // Applicable Information Models. – Sofia: ITHEA, 2011. – № 22. – P. 155-169.
3. Гайворонская Г.С. Оптимизация выбора сценария эволюции местной телефонной сети // Зв'язок – 2002, № 1, с. 51 - 54.
4. Гайворонская Г.С., Оптимизация стратегии эволюции узла коммутации методом нелинейного программирования, Праці УНДІРТ, № 4(32) 2002, с. 60 – 62
5. Гайворонская Г.С. Экономические аспекты оптимизации эволюционной политики развития местных сетей связи. Сборник международной академии информатизации при ООН, Acta Academia, 2001, Chisinau: Evrica, 2001, с. 93- 98
6. Гайворонская Г. С. Один из подходов к синтезу топологической структуры телекоммуникационной сети / Г.С. Гайворонская // Вісник ДУИКТ. – 2006. – Т. 4. – №2. – С. 119-122.

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