VECTORS AND MATRIXES IN GROUPING INFORMATION PROBLEM

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Abstract: *Grouping information problem appears in application in two main forms. These are the problem of recovering function, represented by its observations and the problem of classification (clusterization).It is very important for both them which are the "representatives" of the objects under investigations: scalars, vectors or objects of other kinds. This choice is determined by the math technique can be used for handling with the "representatives". Using the real valued vectors and Euclidean spaces correspondingly is therefore usual. Development of the technique, including SVD and Moore-Penrose inversion on the base of special "cortege operators" for Euclidean space of $R^{m\times n}$ type, is proposed in the article*

Keywords: *Feature vectors, information aggregating, generalized artificial neuronets, vector corteges, matrix corteges, linear operator between cortege spaces, Single Valued Decomposition for cortege linear operators.*

ACM Classification Keywords: *G.2.m. Discrete mathematics: miscellaneous,G.2.1 Combinatorics*. *G.3 Probability and statistics, G.1.6. Numerical analysis I.5.1.Pattern Recognition H.1.m. Models and Principles: miscellaneous:*

Introduction

The problem of grouping the information (grouping problem) is the fundamental problem of applied investigations. It appears in various forms and manifestations. All of them eventually are reduced to two forms. Namely, these are: the problem of recovering the function represented by their observations and the problem of clustering, classification and pattern recognition. State of art in the field is represented perfectly in [Kohonen, 2001; Vapnik, 1998; Haykin, 2001; Friedman, Kandel, 2000; Berry, 2004].

It’s opportune to mark what the information regarding the object or a collection of similar object is exposed to aggregating is. It is of principal importance that an object is considered as a set of its main components and fundamental for the object ties between them. Such consideration and only this one enable application of the math in object description, namely, for math modeling. It is due the fact that after Georg Cantor the objects of investigation in math (math structures) are the sets plus “ties” between its elements. There are only four (may be, five) fundamental mathematical means to describe these “ties”. Namely, these are: relations, operations, functions and collections of subsets (or combinations of mentioned above). Thus, the mathematical description of the object (mathematical modeling) can not be anything other than representing the object structure by the means of mathematical structuring. It is applicable to the full extent to that objects which indicated by the term “complex system”. A “complex system” should be understanding and, correspondingly, determined, as an objects with complex structure (complex “ties”). Namely, when reading attentively manuals by the theme (see, for example, [Yeates, Wakefield, 2004; Forster, Hölzl, 2004]) one could find correspondent allusions. It is reasonable understanding of “complex systems” instead of the its understanding as the “objects, consisting of numerous parts, functioning as an organic whole”.

So, math modeling is designing in math “parts plus ties”, which reproduce “part plus ties” in reality.

So it is principal question in math modeling which math objects represents “part” of the object and which the “ties” ones. The math object - representative should be chosen in such a way that variety of math structuring means were sufficient to convey the object structure.

It is commonly used approach for designing objects - representative to construct them as an finite ordered collection of characteristics: quantitative (numerical) or qualitative (non numerical). Such ordered collection of characteristics is determined by term cortege in math. Cortege is called vector when its components are numerical. In the function recovering problem objects - representatives are vectors and functions are used as a rule to design correspond mathematical “ties”. In clustering and classification problem the collection may be both qualitative and quantitative. In last case correspond collection is called feature vector. It is reasonable to note that term “vector” means more, than simply ordered numerical collection. It means that curtain standard math “ties” are applicable to them. These “ties” are adjectives of the math structure called Euclidean space denoted be R^n. Namely these are: linear operations (addition and scalar multiplying), scalar product and correspond norm.

Just the belonging to the base math structure (Euclidean space) determines advantages of the “vectors” against “corteges”. It is noteworthy to say, that this variant of Euclidean space is not unique: the space $R^{m\times n}$ of all matrixes of a fixed dimension $m\times n$ may represent alternative example. The choice of the R^n space as “environmental” structure is determined by perfect technique developed for manipulation with vectors. These include classical matrix methods and classical linear algebra methods. SVD-technique and methods of Generalized or Pseudo Inverse according Moore – Penrose are comparatively new elements of linear matrix algebra technique [Nashed, 1978] (see, also, [Albert, 1972; Ben-Israel, Greville, 2002]). Outstanding impacts and achievements in this area are due to N.F Kirichenko (especially, [Кириченко, 1997; Kirichenko, 1997], see also [Кириченко, Лепеха, 2002]). Greville’s formulas: forward and inverse - for pseudo inverse matrixes, formulas of analytical representation for disturbances of pseudo inverse - are among them. Additional results in the theme as to further development of the technique and correspondent applications one can find in [Кириченко, Лепеха, 2001; Donchenko, Kirichenko, Serbaev, 2004; Кириченко, Крак, Полищук, 2004; Kirichenko, Donchenko, Serbaev, 2005; Кириченко, Донченко, 2005; Donchenko, Kirichenko, Krivonos, 2007; Кириченко, Донченко, 2007; Кириченко, Кривонос, Лепеха, 2007; Кириченко, Донченко, Кривонос, Крак, Куляс, 2009].

As to technique designing for the Euclidean space $R^{m\times n}$ as “environmental” one see, for example [Донченко, 2011]. Speech recognition with the spectrograms as the representative and the images in the problem of image recognition are the natural application area for the correspond technique.

As to the choice of the collection (design of cortege or vector) it is necessary to note, that good “feature” selection (components for feature vector or cortege or an arguments for correspond functions) determines largely the efficiency of the problem solution.

As noted above, the efficiency of problem solving group, the choice of representatives of right: space arguments or values of functions and suitable families past or range of convenient features vectors. This phase in solving the grouping information problem must be a special step of the correspondent algorithm. Experience showed the effectiveness of recurrent procedures in passing through selection features step. For correspond examples see, [Ivachnenko, 1969] with Ivachnenko’s GMDH (Group Method Data Handling), [Vapnik, 1998] with Vapnik’s Support Vector Machine. Further development of the recurrent technique one may find in [Donchenko, Kirichenko, Serbaev, 2004; Кириченко, Крак, Полищук, 2004; Kirichenko, Donchenko, Serbaev, 2005; Кириченко, Донченко, 2005; Donchenko, Kirichenko, Krivonos, 2007; Кириченко, Донченко, 2007; Кириченко, Кривонос, Лепеха, 2007]. The idea of nonlinear recursive regressive transformations (generalized neuron nets or neurofunctional transformations) due to Professor N.F Kirichenko is represented in the works referred earlier in its development. Correspondent technique has been designed in this works separately for each of two its basic form f the grouping information problem. The united form of the grouping problem solution is represented here in further consideration. The fundamental basis of the recursive neurofunctional technique include the development of pseudo inverse theory in the publications mentioned earlier first of all due to Professor N.F. Kirichenko and his disciples.

The essence of the idea mentioned above is thorough choice of the primary collection and changing it if necessary by standard recursive procedure. Each step of the procedure include detecting of insignificant components, excluding or purposeful its changing, control of efficiency of changes has been made. Correspondingly, the means for implementing the correspondent operations of the step must be designed. Methods of neurofunctional transformation (NfT) (generalized neural nets, nonlinear recursive regressive transformation: [Donchenko, Kirichenko, Serbaev, 2004; Кириченко, Крак, Полищук, 2004; Кириченко, Донченко, Сербаєв, 2005]).

Neurofunctional transformation in recovering function problem

The fundament of the Math truth is the conception of deducibility. It means that the status of truth (proved statement) has the statement which is terminal in the specially constructed sequence of statements, which called its proof. The peculiarity in sequence constructing means, that a next one in it produced by previous by special admissible rules (deduction rules) from initial admissible statements (axioms and premises of a theorem). As a rule, corresponded admissible statements have the form of equations with the formulas in both its sides. So, each next statement in the sequence-proof of the terminal statement is produced by previous member of sequence (equation) by changing some part of formulas in left or right it side on another: from another side of equations-axioms or equations premises. The specification of the restrictions on admissible statements and the deduction rules are the object of math logic.

As it was already marked, the idea of neurofunctional transformation (NfT-) or neurofunctional transformation in recovering function problem in the variant of inverse recursion was offered in [Кириченко, Крак, Полищук, 2004], and in variant of forward recursion - in [Donchenko, Kirichenko, Serbaev, 2004; Кириченко, Донченко, Сербаєв, 2005]. References on neuronets is determined by the fact that NfT generalizes artificial neuronets: in possibilities of the standard functional elements (ERRT (elementary recursive regression transformation) in NfT): in topology of its connection; in adaptive design of NfT structure in the whole; in adequate math for its description. Just this forward variant will considered below. Namely, NfT- is the transformation built by recursive application of the certain standard element, which will be designated by abbreviation ERRT (Elementary Recursive Regression Transformer). Process of construction of the NfT- transformation consists in connection of the next ERRT (or certain number of it) to already constructed during previous steps transformer according to one of three possible types of connection (connection topology). Types of connection which will be designated as "parinput", "paroutput" and "seq", realize natural variants of use of an input signal: parallel or sequential over input, - and parallel over output. An input of the Output of current step of recursion is input of the next step.

The basic structural element of the NfT- -transformer is ERRT - an element [Кириченко, Донченко, Сербаєв, 2005], which is determined as mapping from R^{n-1} in R^m of a kind:

$$y = A_+\Psi_u\left(C\begin{pmatrix} x \\ 1 \end{pmatrix}\right),$$

which approximates the dependence represented by training sample $(x_1^{(0)}, y_1^{0}), ..., (x_M^{(0)}, y_M^{(0)})$, $x_i^{(0)} \in R^{n-1}$,

$$y_i^{(0)} \in R^m \ , \ i = \overline{1,M} \ ,$$

where:

- C–(n×n) - matrix, which performs affine transformation of the vector $x \in R^{n-1}$ - an input of the system; it is considered to be given at the stage of synthesis of ERRT;
- Ψ_u – nonlinear mapping from R^n in R^n, which consists in component-wise application of scalar functions of scalar argument $u_i \in \Im$, $i = \overline{1,n}$ from the given final set $\Im$ of allowable transformations, including identical transformation: must be selected to minimize residual between input and output on training sample during synthesis of ERRT;
- A+– solution A with minimal trace norm of the matrix equation

$$AX_{\Psi_u C} = Y ,$$

in which matrix $X_{\Psi_u C}$ formed from vector-columns $\Psi_u(C\begin{pmatrix} x_i^{(0)} \\ 1 \end{pmatrix}) = \Psi_u(z_i^{(0)})$, and Y – from columns, $y_i^{(0)}, i = \overline{1,M}$.

In effect, ERRT represents empirical regression for linear regression y on $\Psi_u\left(C\begin{pmatrix} x \\ 1 \end{pmatrix}\right)$, constructed with method of the least squares, with previous affine transformation of system of coordinates for vector regressor x and following nonlinear transformation of each received coordinate separately.

Remark 1. Further we shall assume that functions of component-wise transformations from $\Im$ would have a necessary degree of smoothness where it is necessary.

Task of synthesis of ERRT by an optimal selection of nonlinear transformations of coordinates on the given training sample was introduced and solved in already quoted above work [Кириченко, Донченко, Сербаев, 2005]. The solution of a task of synthesis is based on methods of the analysis and synthesis of the pseudoinverse matrices, developed in [Кириченко, 1997]. Particularly, reversion of Grevil's formula [10] was prooved in these works, that recurrently allows to recalculate pseudoinverse matrices when a column or a row of the matrix changed by another one.

Details of designing the NfT one can find in [Donchenko at al, 2012]. Details regarding NfT classification and clastrerization problems one can find in [Кириченко, Кривонос, Лепеха, 2007; Donchenko, Krak, Krivonos, 2012].

Development of Pseudo Inverse Technique for matrixes Euclidean spaces

The following are results that transfer basic features of describing the basic structures of Euclidean spaces [Донченко, 2011] matrix Euclidean spaces. These are, first of all General Single Valued Decomposition (SVD) theorems and then determination of Pseudo Inverse (PdI) and designing the constructive methods for manipulating with basic structures within matrixes spaces on the base of the Pseudo Inverse.

Matrixes spaces and cortege operators

Theorem 3. For an arbitrary linear operator between a pair of Euclidean spaces $(E_i,(,)_i), i = 1,2 : \wp_E : E_1 \to E_2$, the collection of singularities (v_i, λ_i^2), $(u_i, \lambda_i^2) i = \overline{1,r}$, $r = rank\wp_E$ exists for the operators

$\wp_E^*\wp : E_1 \to E_1$, $\wp\,\wp_E^* : E_2 \to E_2$ correspondingly, with a common for both operators $\wp_E^*\wp, \wp\,\wp_E^*$ set of Eigen values $\lambda_i^2, i = \overline{1,r}$: $\lambda_{i-1} \ge \lambda_i > 0$, $\overline{i = 2,r}$ such that

$$\wp_E x = \sum_{i=1}^{r} \lambda_i u_i (v_i, x)_1, \qquad \wp_E^* y = \sum_{i=1}^{r} \lambda_i v_i (u_i, y)_2$$

Besides, the following relations take place: $u_i = \lambda_i^{-1} \wp v_i, i = \overline{1,r}$, $v_i = \lambda_i^{-1} \wp_E^* u_i, i = \overline{1,r}$.

SVD – technique for matrixes spaces

We denote by $R^{(m\times n),K}$ - Euclidean space of all matrixes K-corteges from $m\times n$ matrixes: $\alpha = (A_1 \vdots \ldots \vdots A_K) \in R^{(m\times n),K}$ with a "natural" component wise trace inner product:

$$(\alpha, \beta)_{cort} = \sum_{k=1}^{K} (A_k, B_k)_{tr} = \sum_{k=1}^{K} trA_k^T B_k ,$$

$$\alpha = (A_1 \vdots \ldots \vdots A_K), \beta = (B_1 \vdots \ldots \vdots B_K) \in R^{(m\times n),K} .$$

1. We also denote by $\wp_\alpha : R^K \to R^{m\times n}$ a linear operator between the Euclidean space, determined by the relation:

$$\wp_\alpha y = \sum_{k=1}^{K} y_k A_k, \alpha = (A_1 \vdots \ldots \vdots A_K) \in R^{(m\times n),K}, y = \begin{pmatrix} y_1 \\ \cdots \\ y_K \end{pmatrix} \in R^K \tag{1}$$

2. **Theorem 4**. Range $\Re(\wp_\alpha) = L_{\wp_\alpha}$, which is linear subspace of R^{m*n} , is the subspace spanned on the components of cortege $\alpha = (A_1 \vdots \ldots \vdots A_K) \in R^{(m\times n),K}$, that determines $\wp_\alpha$:

$$\Re(\wp_\alpha) = L_{\wp_\alpha} = L(A_1, \ldots, A_K) .$$

3. Theorem 5. Conjugate for the operator, determined by (1) is a linear operator, which, obviously, acts in the opposite direction: $\wp_\alpha^* : R^{m\times n} \to R^K$, and defined as:

$$\wp_\alpha^* X = \begin{pmatrix} trA_1^T X \\ \cdots \\ trA_K^T X \end{pmatrix} = \begin{pmatrix} trX^T A_1 \\ \cdots \\ trX^T A_K \end{pmatrix}.$$

4. Theorem 6. A product of two operators $\wp_\alpha^*\wp_\alpha : R^K \to R^K$ is a linear operator, defined by the matrix from the next equation:

$$\wp_\alpha^*\wp = \begin{pmatrix} trA_1^T A_1, \ldots, trA_1^T A_K \\ \cdots \\ trA_K^T A_1, \ldots, trA_K^T A_K \end{pmatrix} \tag{2}$$

Remark. Matrix defined by (2) is the Gram' matrix for the elements of the cortege $\alpha = (A_1 \vdots \ldots \vdots A_K) \in R^{(m\times n),K}$, which determines the operator.

5. Singular value decomposition for a matrix (2) is obvious, as it is the classical matrix: symmetric and positive semi-definite, on vector Euclidean R^K. It is defined by a collection of singularities $(v_i, \lambda_i^2), i, j = \overline{1,r}$:

$$\| v_i \| = 1, v_i \perp v_j, i \neq j; i, j = \overline{1,r}; \lambda_1 > \lambda_2 > \ldots > \lambda_r > 0,$$

$$\wp_\alpha^* \wp_\alpha v_i = \lambda_i^2 v_i, i = \overline{1,r}.$$

The operator $\wp_\alpha^* \wp_\alpha$ by itself and is determined by the relation

$$\wp_\alpha^* \wp_\alpha = \sum_{i=1}^{r} \lambda_i^2 v_i v_i^T = \sum_{i=1}^{r} \lambda_i^2 v_i (v_i, \ \cdot \ ).$$

Each of the row - vectors $v_i^T, i = \overline{1,r}$ will be written by their components:

$$v_i^T = (v_{i1}, \ldots, v_{iK}), i = \overline{1,r},$$

i.e. $v_{ik}, i = \overline{1,r}, k = \overline{1,K}$ is the component with the number k of a vector v with a number I.

6. **Theorem 7**. Matrices $U_i \in R^{m \times n} : U_i = \frac{1}{\lambda_i} \wp_\alpha v_i = \frac{1}{\lambda_i} \sum_{k=1}^{K} A_k v_{ik}, i = \overline{1,r}$, defined by the singularities $(v_i, \lambda_i^2), i = \overline{1,r}$ of the operator $\wp_\alpha^* \wp_\alpha$ are elements of a complete collection of singularities $(U_i, \lambda_i^2), i = \overline{1,r}$ of the operator. $\wp_\alpha^* : R^K \to R^{m \times n}$

Proof. This follows from Theorem 1, and the standard relations between singularities of the $\wp_\alpha^* \wp_\alpha$, $\wp_\alpha \wp_\alpha^*$ operators.

7. Theorem 4 (Singular Value Decomposition (SVD) for cortege operator). Singularity of two operators $\wp_\alpha^* \wp_\alpha, \wp_\alpha \wp_\alpha^*$, obviously determine the singular value decomposition of operators $\wp_\alpha$, $\wp_\alpha^*$:

$$\wp_\alpha y = \sum_{i=1}^{r} \lambda_i U_i v_i^T y, y \in R^K,$$

$$\wp_\alpha^* X = \sum_{i=1}^{r} \lambda_i v_i (U_i, X)_{tr}, X \in R^{m \times n}.$$

8. Corollary. A variant is a SVD for the operator $\wp_\alpha$ is represented by the next relation:

$$\wp_\alpha = \sum_{k=1}^{r} \lambda_k U_k v_k^T = \sum_{k=1}^{r} \left(\wp_\alpha v_k \right) v_k^T.$$

Pseudo Inverse Technique for matrixes Euclidean spaces

Basic operators PdI theory for a cortege operators: pseudo inverse by SVD-representation.

1. **Theorem 8**. The PdI operators for $\wp_\alpha$, $\wp_\alpha^*$ are determined, correspondingly, by the relations

$$\wp_\alpha^+ X = \sum_{k=1}^{r} \lambda^{-1} v_k \left(U_k, X\right)_{tr} = \sum_{k=1}^{r} \lambda^{-2} v_k \left(\wp_\alpha v_k, X\right)_{tr}, \forall X \in R^{m\times n},$$

$$\left(\wp_\alpha^*\right)^+ y = \sum_{i=1}^{r} \lambda^{-1} U_i v_i^T y, \forall y \in R^K.$$

2. Basic operators PdI theory for a cortege operators:: basic orthogonal projectors. The basic orthogonal projectors PdI-theory are two pairs of orthogonal projectors. The first one is the pair of orthogonal projectors on the pair principal subspaces of $\wp_\alpha, \wp_\alpha^* : \Re(\wp_\alpha) = L_{\wp_\alpha}, \Re(\wp_\alpha^*) = L_{\wp_\alpha^*}$ -their ranges. These orthogonal projections will be designated in one of two equivalent ways:

$$P(\wp_\alpha^*) \equiv P_{L_{\wp_\alpha}} = P_{(A_1,\dots,A_K)}, L_{L_{\wp_\alpha}} \subseteq R^{m\times n}, P(\wp_\alpha) \equiv P_{L_{\wp_\alpha^*}}, L_{\wp_\alpha^*} \subseteq R^K.$$

The second pair is a pair of orthogonal projectors onto the orthogonal complement $L_{\wp_\alpha}^\perp \subseteq R^{m\times n}, L_{\wp_\alpha^*}^\perp \subseteq R^K$ of the first pair of the subspaces. The complements, namely, are the Kernels of the correspondent operators. Each of these projectors will be denoted in one of two equivalent ways:

$$Z(\wp_\alpha) \equiv P_{L_{\wp_\alpha^*}^\perp}, Z(\wp_\alpha^*) \equiv P_{L_{\wp_\alpha}^\perp},$$

Obviously:

$$Z(\wp_\alpha) \equiv E_K - P(\wp_\alpha), Z(\wp_\alpha^*) \equiv E_{m\times n} - P(\wp_\alpha^*) \tag{3}$$

In accordance with the general properties of PdI, the next properties are valid:

$$P(\wp_\alpha) = \wp_\alpha^+ \cdot \wp_\alpha, P(\wp_\alpha^*) = \left(\wp_\alpha^*\right)^+ \cdot \wp_\alpha^* = \wp_\alpha \cdot \wp_\alpha^+.$$

Correspondingly:

$$Z(\wp_\alpha) \equiv E_K - \wp_\alpha^+ \cdot \wp_\alpha, \quad Z(\wp_\alpha^*) \equiv E_{m\times n} - \wp_\alpha \cdot \wp_\alpha^+.$$

3. Basic operators PdI theory for a cortege operators: basic orthogonal projectors. Grouping operators, denoted below as $R(\wp_\alpha)$, $R(\wp_\alpha^*)$, are also "paired" operators, and are determined by the relations:

$$R(\wp_\alpha) = \wp_\alpha^+ \left(\wp_\alpha^+\right)^* = \wp_\alpha^+ \left(\wp_\alpha^*\right)^+, \; R(\wp_\alpha^*) = \left(\wp_\alpha^*\right)^+ \left(\left(\wp_\alpha^*\right)^+\right)^* = \left(\wp_\alpha^+\right)^* \wp_\alpha^+.$$

4. **Theorem 9.** Grouping operators for the cortege operators $\wp_\alpha$, $\wp_\alpha^*$ can be represented by the next expression:

$$R(\wp_\alpha^*) X = \sum_{k=1}^{r} \lambda_k^{-2} U_k (U_k, X)_{tr} = \sum_{k=1}^{r} \lambda_k^{-2} U_k tr U_k^T X = \sum_{k=1}^{r} \lambda_k^{-2} U_k tr X^T U_k,$$

and the quadratic form $(X, R(\wp_\alpha^*) X)_{tr}$ is determined by the relation:

$$(X, R(\wp_\alpha^*) X)_{tr} = \sum_{k=1}^{r} \lambda_k^{-2} (U_k, X)_{tr}^2,$$

where

$$\wp_{\alpha}^{+}X=\sum_{k=1}^{r}\lambda^{-1}v_k\left(U_k,X\right)_{tr}=\sum_{k=1}^{r}\lambda^{-2}v_k\left(\wp_{\alpha}v_k,X\right)_{tr},$$

$$\left(\wp_{\alpha}^{*}\right)^{+}y=\sum_{i=1}^{r}\lambda^{-1}U_i v_i^T y.$$

5. **Theorem 10**. Quadratic form $(X,R(\wp_{\alpha}^{*})X)_{tr}$ may be written as:

$$(X,R(\wp_{\alpha}^{*})X)_{tr}=\sum_{i=1}^{r}\lambda_i^{-4}v_i^T\begin{pmatrix} trA_1^TXtrA_1^TX & trA_2^TXtrA_2^TX & \cdots & trA_1^TXtrA_K^TX \\ trA_2^TXtrA_1^TX & trA_2^TXtrA_2^TX & \cdots & trA_2^TXtrA_K^TX \\ \cdots & \cdots & \cdots & \cdots \\ trA_K^TXtrA_1^TX & trA_K^TXtrA_1^TX & \cdots & trA_K^TXtrA_1^TX \end{pmatrix}v_i=$$

$$=\sum_{i=1}^{r}\lambda_i^{-4}\left\{v_i^T\begin{pmatrix} trA_1^TX \\ \cdots \\ trA_K^TX \end{pmatrix}\right\}^2=\sum_{i=1}^{r}\lambda_i^{-4}\left\{v_i^T\wp_{\alpha}^{*}X\right\}^2.$$

Importance of grouping operators is determined by their properties, represented by the next two theorems.

6. **Theorem 11.** For any $A_i, i=\overline{1,K}$ of $\alpha=(A_1\vdots...\vdots A_K)\in R^{(m\times n),K}$ the next inequalities are fulfilled:

$$(A_i,R(\wp_{\alpha}^{*})A_i)_{tr}\le r, i=\overline{1,K}, r=rank\wp_{\alpha}.$$

7. **Theorem 12**. For any $A_i, i=\overline{1,K}$ of $\alpha=(A_1\vdots...\vdots A_K)\in R^{(m\times n),K}$ the next inequalities are fulfilled:

$$(A_i,R(\wp_{\alpha}^{*})A_i)_{tr}\le r_{\min}\le r, i=\overline{1,K}, r=rank\wp_{\alpha},$$

$$r_{\min}=\min_{i=\overline{1,n}}(A_i,R(\wp_{\alpha}^{*})A_i)_{tr}\le r_{\min}\le r, i=\overline{1,K}, r=rank\wp_{\alpha}.$$

Comment to the theorems 11, 12. These theorems give the minimal grouping ellipsoids for the matrixes $A_i, i=\overline{1,K}$. In order to build it one only has to construct cortege operator $\wp_{\alpha}$ by the cortege $\alpha=(A_1\vdots...\vdots A_K)\in R^{(m\times n),K}$.

Pseudo Inverse Technique for matrixes Euclidean spaces clasterization

The results, represented earlier one can apply to solve the grouping information problem in applied math with matrixes 'representatives': matrixes 'feature vectors' just in the way of the first part of the article.

Conclusion

Development of the technique for manipulating with the basic structures of Euclidean spaces within matrixes spaces is represented. This technique include General SVD theorem and Moore - Penrose pseudo inverse technique for matrixes spaces. Designing the technique demanded introduction matrixes corteges and of special cortege operators associated with them.

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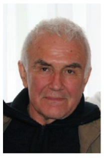

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