GENERALIZATION OF DISTRIBUTING METHODS FOR FUZZY PROBLEMS

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Abstract: *We consider rationing problem as a group decision making problem. Fuzzy and crisp generalizations of the classical rationing methods are proposed. The proposed generalizations are illustrated by numerical examples.*

Keywords: *rationing problem, fuzzy models and methods, decision making.*

ACM Classification Keywords*: I. Computing Methodologies – I.6. Simulation and modeling – I.6.5. Model Development –Modeling Methodologies.*

Introduction

Resource distributing problems (natural, financial resources, labor etc.) as a group decision making problem [Voloshyn, 2010] on different levels of hierarchy (individual, interpersonal, social, regional, state and interstate) became one of the most important problems nowadays. In a certain sense, all other problems are subtasks that provide input data to the different allocation problems. The importance of the issues, considered in this paper, can be illustrated by the number of Nobel Prize winners in the last few years, who got this prestigious award for researches in the fields related to the resource allocation problems: Robert Aumann, Tomas Schelling (2005), Leonid Hurwicz, Eric Maskin, Roger Myerson (2007), Alvin E. Roth, Lloyd S. Shapley (2012) [nobelprize.org, 2012]. Also, the importance of these issues can be shown by the fact that at KDS-2010 the [Voloshyn, Mashchenko] paper was acknowledged the best.

Fair allocation of joint cost (or joint surplus) is one of the central themes in cooperative games theory. In particular, the distribution of some homogeneous good between agents according to a certain profile of claims was popular problem for axiomatic analysis. This distribution model is often referenced as "the bankruptcy problem" [Moulin, 2001].

The most popular rationing methods are: uniform gains method, uniform losses method, proportional and Talmudic method [Herrero, Villar, 2001]. In this paper we consider uniform gains, uniform losses and Talmudic methods. Most of the researchers dedicated their work to the axiomatic characterization of the above-mentioned methods. For axiomatic characterization of uniform gains rule see, for example, [Moulin, 2001], [Dagan,1996], [Herrero, Villar, 2001]. The Talmudic method was introduced in [O'Neill, 1982], [Aumann, Maschler, 1985] (who argue convincingly that its intuition was present already in the ancient Talmudic literature).

Some authors proposed a dynamic approach to the allocation problem with the generalizations of the uniform gains\losses methods being considered [Marchant, 2004]. But all of the above-mentioned papers deal with crisp data. In the series of papers [Voloshyn, Laver, 2009 - 2012] the classical rationing problem [Voloshyn, 2010] and its generalizations are considered, in particular in case of fuzzy conditions [Voloshyn, Laver, 2010]. Also the applications of the proposed models and methods to the real problems are given [Laver, 2010, 2011]. Unlike the "axiomatic" approach, which is used in the cooperative games theory, in this papers the "algorithmic" approach is applied [Moulin, 1991]. According to this approach the principles of finding the solution are given and the analysis of the final solution is provided to the decision maker (DM). In this paper some generalizations of the rationing methods (algorithms) that were considered in [Voloshyn, Laver, 2010] are proposed, in particular, for the rationing models in "cooperative" form [Moulin, 1991].

Rationing problem

Rationing problem is a triple (*N*,*c*,*b*), where *N* is a finite set of agents, the nonnegative real number *c* represents the amount of resources to be divided, the vector $b=(b_i)_{i\in N}$ specifies for each agent *i* a claim b_i , and these numbers are such that

$$0\le b_i,\ \forall i\in N\text{: } 0\le c\le \sum_{i\in N} b_i. \tag{1}$$

A solution to the rationing problem is a vector $x=(x_i)_{i\in N}$, specifying a share x_i for each agent *i*, such that

$$0\le x_i\le b_i,\ \forall i\in N:\ \sum_{i\in N} x_i=c. \tag{2}$$

There are different ways of interpreting the rationing problem. One of the oldest interpretations is the inheritance problem (here *c* is the liquidation value of the bankrupt firm; b_i is the debt owed to creditor *i* [Aumann, Maschler, 1985]). Other important examples include taxation and cost sharing of the indivisible public good.

Rationing occurs in markets where the price of a commodity is fixed (for instance, at zero): *c* is the available supply and b_i is agent *i*'s demand. Medical triage is an example: *c* measures the available medical resources and b_i is the quantity needed by agent *i* for full treatment.

Often the resources to be divided come in indivisible units: organs for transplants, seats in crowded airplanes or in popular sports events, visas to potential immigrants as well as cars allocated by General Motors to its car dealers. In this case b_i and *c* are integers (e.g. in the case of visas or organs b_i can only be 0 or 1) [Moulin, 2001].

Without reducing the generality, let us consider the rationing problem as a cost sharing problem. Thus, *c* is interpreted as the production cost of an indivisible public good, b_i are interpreted as initial amount of money of agent *i.* Both *c* and b_i are non-negative real numbers.

Uniform gains and uniform losses

There are three main principles of cost allocation [Волошин, Мащенко, 2010]:

- Equalizing the gains: $x_i=b_i-\left(\sum_{i\in N} b_i-c\right)/n,\quad \forall i\in N$;
- Equalizing the losses : $x_i=\dfrac{c}{n},\ \forall i\in N$;
- Proportional principle.

In case if the constraint (2) is violated some agent can be forced to pay more than its initial money amount. In this case the agent can refuse to cooperate. When equalizing the losses, it is possible that one agent (or more agents) will be subsidized by others. In this case, all the other agents may refuse to cooperate and the coalition falls apart. If the constraints (2) are satisfied, the principle equalization of gains principle is generalized as uniform gains method and, respectively, equalization of losses is generalized as uniform losses method [Voloshin, Mashchenko, 2010].

Fuzzy generalization of uniform gains and uniform losses methods

Let us consider a rationing problem (*N*,*c*,*b*), where *N* is a set of agents, |*N*|=*n*, *c* is the value of the costs to be distributed, vector $b=(b_1,b_2,...,b_n)$ is the vector of initial money amounts.

Uniform losses method is defined as $x_i = ul_i(N,c,b) = \min\{\lambda, b_i\}$, where λ is the solution of $\sum_{i\in N}\min\{\lambda, b_i\} = c$.

Suppose that there are agents whose shares are equal to their money amounts. Let us denote this set of agents as N_1. Also, let us assume that there are agents, who agree to pay more than their shares, so the "poor" agents can pay less than their money amount.

Let denote $\hat{x}_i, i \in N$ the share, which uniform losses method assigns to the agent i. We can set the threshold values: α_i – how many percents of his share agent *i* could pay without complaints$(i \in N_1)$; β_j – how many percents of his share agent *j* ($j \in N_2 = N \setminus N_1$) could overpay to cover the "deficit". In general, these values are determined by the initial money amounts of the agents ("progressive taxation").

Denote $\underline{x_i}$ - maximal amount of money, that agent *i* $(i \in N_1)$ can pay without complaints, $\underline{x_i} = b_i(1-\alpha_i)$. The final share of agent *i* will belong to the interval $[\hat{x}_i, \underline{x_i}]$. Denote $\Delta_i = \hat{x}_i - \underline{x_i}$ the occurring "deficit". Total deficit we denote as $\Delta = \sum_{i\in N_1}\Delta_i$. This deficit is covered by agents from the subset N_2 .

Consider the set N_2 . Denote $\hat{x}_j$ the share of agent *j*, where $j \in N_2$; $\overline{x_j}$ - maximal amount of money, which agent *j* can pay to cover the deficit, $\overline{x_j} = \hat{x}_j(1+\beta_j)$. The inequality $\sum_{j\in N_2}\left(\overline{x_j} - \hat{x}_j\right) \geq \Delta$ has to be satisfied. If it's not so, we have to change α_i, β_j.

Similarly for the uniform gains method. It is defined as $x_i = ug_i(N,c,b) = (x_i - \mu)_+$, where μ is the solution of $\sum_{i\in N}(x_i - \mu)_+ = c$ (where $(z)_+ = \max\{z, 0\}$).

Denote N_1 the set of agent, for which the share equals zero. Maybe for the other agents it is acceptable to subsidize the agents from the subset N_1 (for example they may want to prevent the breaking up of the grand coalition). In this case α_i denotes how many percents of his share agent *i* is subsidized ($i \in N_1$), β_j – how many percents of his share agent *j* ($j \in N_2 = N \setminus N_1$) can overpay to subsidize the "poor" agents.

Denote $\hat{x}_i$ the value of subsidy for the agent *i*, where $i \in N_1$, $\hat{x}_i = \alpha_i b_i$. Then $\Delta = \sum_{i\in N_1}\hat{x}_i$ is the total amount of the subsidies, which has to be covered by agents from the subset $N_2 = N \setminus N_1$.

Denote $\hat{x}_j = b_j$ the share of agent *j* where $j \in N_2$; $\overline{x_j}$ - the maximal amount of money the agent *j* can pay, $\overline{x_j} = b_j(1+\beta_j)$. The inequality $\sum_{j\in N_2}\overline{x_j} \geq \Delta + c$ has to be satisfied. If it is not so, we have to change the threshold values.

For the case of the uniform losses method the cost sharing algorithm is the following:

1. We set the shares of the agents i $(i \in N_1)$ as $x_i = \underline{x_i}$.
2. Set the shares of the agents j $(j \in N_2)$ as $x_j = \hat{x}_j + x_j^{\Delta}$, where x_j^{Δ} is the share of the deficit Δ, and it can be computed using any method (for example uniform gains or uniform losses).
3. If the result is acceptable, the process stops. If not – we correct the thresholds and return to the step 1.

Algorithm for the uniform gains method is similar:

1. Set the shares for the agents i $(i \in N_1)$ as $x_i = -\hat{x}_i$.

2. Set the shares for the agents $j\ (j \in N_2)$ as $x_j = \hat{x}_j + x_j^{\Delta}$, where x_j^{Δ} share of the deficit Δ, and it can be computed using any method (for example uniform gains or uniform losses).
3. If the result is acceptable, the process stops. If not – we correct the thresholds and return to the step 1.

So we can consider four ways of sharing: UG+UG (we compute both – the initial shares and the shares of the deficit by the uniform gains method), UG+UL (the initial shares are computed using uniform gains method, the shares of the deficit are computed using the uniform losses method), UL+UG (the initial shares are computed using uniform losses method, the shares of the deficit are computed using the uniform gains method), UL+UL (the initial shares and the shares of the deficit are computed by the uniform losses method). Selection of the specific cost allocation mechanism is left for the decision maker.

Fuzzy generalizations of the uniform gains and the uniform losses methods

Let us consider the rationing problem using some results of fuzzy sets theory [Zgurovskyj, Zajchenko , 2011].

Assume that we consider the case of uniform losses and the share of agent $i\ (i \in N_1)$ is a fuzzy number with the following membership function:

$$\mu_i(x_i) = \begin{cases} 1,\ 0 \le x_i \le \underline{x}_i; \\ \dfrac{x_i - \hat{x}_i}{\underline{x}_i - \hat{x}_i},\ \underline{x}_i \le x_i \le \hat{x}_i; \\ 0,\ in\, other\, case. \end{cases}$$

So $\mu_i(x_i)$ are right sided trapezoidal fuzzy numbers.

Similarly for agents $j\left(j \in N_j\right)$ we have:

$$\mu_j(x_j) = \begin{cases} 1,\ 0 \le x_j \le \hat{x}_j; \\ \dfrac{x_j - \overline{x}_j}{\hat{x}_j - \overline{x}_j},\ \hat{x}_j \le x_j \le \overline{x}_j; \\ 0,\ in\, other\, case. \end{cases}$$

Denote

$$\tau(x,c) = \begin{cases} 1,\ \sum\limits_{i \in N} x_i = c; \\ 0,\ \sum\limits_{i \in N} x_i \ne c. \end{cases}$$

We can consider $\tau(x,c)$ as membership function of the fuzzy goal and $\mu_k(x_k)(k \in N)$ as the membership function of the fuzzy constraints. Then, according to the Bellman-Zadeh approach, the decision's membership function will be

$$\mu_D(x_1, x_2, \ldots, x_n) = \min_i \{\mu_i(x_i); \tau(x,c)\}$$

We consider $x = (x_1, x_2, \ldots, x_n)$, on which the function $\mu_D(x)$ reaches its maximum, as the crisp solution of the fuzzy rationing problem.

Thus, the process of finding the optimal share is reduced to the following linear programming problem:

$$\begin{aligned}
&\lambda \to \max, \\
&\mu_k(x_k) \geq \lambda, \forall k \in N, \\
&\sum_{i \in N} x_i = c, \\
&0 \leq \lambda \leq 1, x_k \geq 0, k \in N.
\end{aligned}$$

If the obtained solution does not satisfy the decision maker, we need to change the threshold values.

For the uniform gains method we have to find the subsidy values first. Then we have to share *Δ+c* among the agents from the subset N_2 (taking in count the following inequality: $\sum_{j \in N_2} \overline{x_j} \geq \Delta + c$). As a result we obtain the following problem:

$$\begin{aligned}
&\lambda \to \max, \\
&\mu_j(x_j) \geq \lambda, \ \forall j \in N_2, \\
&\sum_{j \in N_2} x_i = c + \Delta, \ 0 \leq \lambda \leq 1, x_j \geq 0, \ j \in N_2.
\end{aligned}$$

Solution of this problem is vector $(x_1, x_2, ..., x_n)$ an agents' "level of satisfaction" *λ*. If this value does not satisfy the decision maker, we have to set new threshold values and find new shares.

The case of fuzzy costs

Let the costs is a triangular fuzzy number: $c = (\underline{c}, \hat{c}, \overline{c})$. In this case we need to consider two problems – the problem in optimistic case ($c \in [\underline{c}, \hat{c}]$) and the problem in pessimistic case ($c = [\hat{c}, \overline{c}]$) [Zgurovskyj, Zajchenko, 2011].

In the optimistic case the fuzzy uniform losses method will be reduced to a following linear programming problem:

$$\begin{aligned}
&\lambda \to \max, \\
&\mu_k(x_k) \geq \lambda, \forall k \in N, \\
&\mu_c(c) \geq \lambda, \ \sum_{i \in N} x_i = c, \\
&0 \leq \lambda \leq 1, x_k \geq 0, k \in N, c \in [\underline{c}, \hat{c}].
\end{aligned}$$

$\mu_c(c)$ **–** the fuzzy costs membership function, the agents' membership functions are computed considering that the uniform losses method is applied for $\hat{c}$.

For the pessimistic case we must recomputed the agents' shares for $\overline{c}$. The problem in this case will have similar look:

$$\begin{gathered}
\lambda \to \max, \\
\overline{\mu}_k(x_k) \geq \lambda, \forall k \in N, \\
\mu_c(c) \geq \lambda, \sum_{i \in N} x_i = c, \\
0 \leq \lambda \leq 1, x_k \geq 0, k \in N, c \in [\hat{c}, \overline{c}]
\end{gathered}$$

$\overline{\mu}_k(x_k)$ - new membership functions of the agents.

When the both problems are solved, we compare λ in both cases. The solution of the problem will be the share with maximal λ. In the case when λ is equal in both cases, the choice of the solution is left to the decision maker.

Numerical example

Consider n=5 agents. We have to share among them c=30 units of cost. Initial money amounts are, respectively, 4, 12, 20, 24, 30 [Волошин, Мащенко, 2010]. Let α=25%, β=20%.

For the case of crisp generalizations of uniform losses and uniform gains method we have the following shares:

Number of the agent	1	2	3	4	5	c
Money amount	4	12	20	24	30	
UL	4	6,5	6,5	6,5	6,5	30
UL+UL	3	6,75	6,75	6,75	6,75	30
UL+UG	3	6,5	6,5	6,5	7,5	30
UG	0	0	16/3	28/3	46/3	30
UG+UL	-1	-3	20/3	32/3	50/3	30
UG+UG	-1	-3	16/3	28/3	58/3	30

For fuzzy generalizations (in particular, for the fuzzy c=(29,30,31)) we obtain:

Number of the agent	1	2	3	4	5	c	λ
Money amount	4	12	20	24	30		
UL	4	6,5	6,5	6,5	6,5	30	
Fuzzy UL	98/31	208/31	208/31	208/31	208/31	30	26/31
FUL+fuzzy c	113/36	481/72	481/72	481/72	481/72	1075/36	31/36
UL	0	0	16/3	28/3	46/3	30	
Fuzzy UL	-1	-3	272/45	476/45	782/45	30	1/3
FUL+fuzzy c	-1	-3	578/93	986/93	1598/93	30	16/31

For the both methods the higher levels of λ are obtained when the cost, that has to be allocated, is a fuzzy number. The intuitive explanation is that the higher level of fuzziness allows agent to deviate more from his share, so he can choose the share which is the most comfortable for him.

Fuzzy generalization of the Talmudic method

When reducing the rationing problem to a cooperative game and using the egalitarian principle (which coincides with nucleolus of the cooperative game [Voloshyn, 2010]) we have to use uniform gains or uniform losses method, according to the following theorem [Aumann, Maschler, 1985]:

Theorem. Nucleolus corresponds to the following shares, depending on c:

1. $$c \le \frac{1}{2}\sum_{i=1}^{n} b_i : \sum_{i=1}^{n} \min\left\{\alpha, \frac{b_i}{2}\right\} = c \Rightarrow x_i = \min\left\{\alpha, \frac{b_i}{2}\right\}, i = \overline{1,n}.$$
2. $$c \ge \frac{1}{2}\sum_{i=1}^{n} b_i : \sum_{i=1}^{n} \min\left\{\alpha, \frac{b_i}{2}\right\} = \sum_{i=1}^{n} b_i - c \Rightarrow x_i = b_i - \min\left\{\alpha, \frac{b_i}{2}\right\}, i = \overline{1,n}.$$

The shares, chosen by Talmudic method coincide with the nucleolus of the cooperative game.

So there are two extreme cases – when the shares are computed using uniform losses (when $c \le \frac{1}{2}\sum_{i=1}^{n} b_i$) or uniform gains method (when $c \le \frac{1}{2}\sum_{i=1}^{n} b_i$) for the following initial money amounts vector: $\frac{1}{2}b = \left(\frac{1}{2}b_1, \frac{1}{2}b_2, \ldots, \frac{1}{2}b_n\right)$.

Between these extreme cases there are many compromise allocations.

Let us consider fuzzy generalizations of the Talmudic method. We can break the set of agents in two subsets - N_1 and N_2. In the first subset we include the agents, who want to pay less than their share; in the second group we include the agents, who can pay more than their shares, to cover the deficit.

Consider the agents of the first group. Denote $\underline{x}_i$ the desired share, $\hat{x}_i$ - the share obtained using the Talmudic method. The shares will be right sided trapezoidal fuzzy numbers $(\underline{x}_i, \hat{x}_i)$. For the second group we denote as $\hat{x}_i$ the corresponding nucleolus value, $\overline{x}_i$ – the maximal value that agent agrees to pay. Then the shares are trapezoidal fuzzy numbers $(\hat{x}_i, \overline{x}_i)$. Thus, to find the shares we have to solve the linear programming problem:

$$
\begin{aligned}
&\lambda \to \max, \\
&\mu_k(x_k) \ge \lambda,\ \forall k \in N_1, \mu_j(x_j) \ge \lambda,\ \forall j \in N_2, \\
&\sum_{i \in N} x_i = c, \\
&0 \le \lambda \le 1, x_i \ge 0, i \in N.
\end{aligned}
$$

The task of finding the optimal solutions can be solved in several stages: if the λ is not satisfying the decision maker, we can change the fuzzy numbers that correspond to the agents' shares making them narrower. The process continues till we obtain the optimal λ. Note that if $\lambda=1$ the shares will coincide with the shares given by the Talmudic method. Let us consider a numerical example. There are three agents with initial money amounts 100, 200, 300. We have to distribute 100 units of cost. Talmudic method gives each agent a share that is equal $33\frac{1}{3}$. Consider the extreme cases (uniform losses and uniform gains for $\frac{1}{2}b$):

Money amounts	100	200	300
UL($\frac{1}{2}$)	$33\frac{1}{3}$	$33\frac{1}{3}$	$33\frac{1}{3}$
UG($\frac{1}{2}$)	0	25	75
TM	$33\frac{1}{3}$	$33\frac{1}{3}$	$33\frac{1}{3}$

As $c \le \frac{1}{2}\sum_{i=1}^{n} b_i$, the shares given by Talmudic method TM coincide with UL($\frac{1}{2}$).

For the first and the second agent UG($\frac{1}{2}$) is less than TM. Thus, they would rather pay UG($\frac{1}{2}$). We include them in the first subset - N_1. In the second subset (N_2) we include only one agent – agent 3, because for him UG($\frac{1}{2}$) is more than TM. Assume that this is the maximal share, he is willing to pay.

The shares will be right sided trapezoidal fuzzy numbers (0, $33\frac{1}{3}$), (25, $33\frac{1}{3}$), ($33\frac{1}{3}$, 75).

Solving the above-mentioned linear programming problem with this data, we obtain a solution - (14.79; 26.93; 58.28) and λ=0.528. Assume that the result does not satisfy the decision maker. We make the next iteration with fuzzy shares (14.79; $33\frac{1}{3}$), (26.93; $33\frac{1}{3}$), ($33\frac{1}{3}$; 58.28). The solution is (24.06; 30.14; 45.8) and λ=0.5. We continue the process, till the obtained result satisfies the decision maker.

Conclusion

Fuzzy models and methods of rationing allow us to take in count the fuzziness of input data, typical for real processes (see in particular [Laver, 2010; Laver, Malyar, 2011]). These methods give us results different from their crisp analogs, although very close to them. Ultimately, the choice of the rationing method is left to the decision maker.

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